Minimum Energy-per-Bit Wireless Multi-Hop Networks with Spatial Reuse

Changhun Bae and Wayne E. Stark

Abstract: **In this paper, a tradeoff between the total energy consumption-per-bit and the end-to-end rate under spatial reuse in wireless multi-hop network is developed and analyzed. The end-to-end rate of the network is the number of information bits transmitted (end-to-end) per channel use by any node in the network that is forwarding the data. In order to increase the bandwidth efficiency, spatial reuse is considered whereby simultaneous relay transmissions are allowed provided there is a minimum separation between such transmitters. The total energy consumption-per-bit includes the energy transmitted and the energy consumed by the receiver to process (demodulate and decoder) the received signal. The total energy consumption-per-bit is normalized by the distance between a source-destination pair in order to be consistent with a direct (single-hop) communication network. Lower bounds on this energy-bandwidth tradeoff are analyzed using convex optimization methods. For a given location of relays, it is shown that the total energy consumption-per-bit is minimized by optimally selecting the end-to-end rate. It is also demonstrated that spatial reuse can improve the bandwidth efficiency for a given total energy consumption-per-bit. However, at the rate that minimizes the total energy consumption-per-bit, spatial reuse does not provide lower energy consumption-per-bit compared to the case without spatial reuse. This is because spatial reuse requires more receiver energy consumption at a given end-to-end rate. Such degraded energy efficiency can be compensated by varying the minimum separation of hops between simultaneous transmitters. In the case of equi-spaced relays, analytical results for the energy-bandwidth tradeoff are provided and it is shown that the minimum energy consumption-per-bit decreases linearly with the end-to-end distance.**

Index Terms: **Cochannel interference, convex optimization, minimum energy-per-bit, relay networks, spatial reuse.**

I. INTRODUCTION

Multi-hop routing for wireless networks has garnered considerable interest recently. Energy optimization is a critical issue in the design of low-power wireless multi-hop networks. Typically, the analysis of such networks considers only the transmitted energy. However, the energy consumption of the receiver may not be negligible compared to the transmitter energy consumption. Thus, reducing the total energy consumption for end-to-end transmission is a critical design objective for such networks. In addition, efficient utilization of the scarce spectrum is important. In these multihop networks, as in point-to-point links, there is a fundamental tradeoff between energy efficiency and bandwidth efficiency.

Recently, research on the performance evaluation of multi-hop networks has focused on the problems of capacity and minimum transmission energy consumption [1]–[7]. Network capacity in random networks was analyzed to find the maximum throughput in both random and arbitrary networks [1]. The maximum achievable rate per bit is analyzed in [2]–[4] while in [5], [6] the minimum transmission energy per bit is analyzed to measure the efficiency of transmission. A potential energy savings of multi-hop transmission results from reducing the distance between communicating nodes [7]. However, emphasis on transmission energy may not be an effective approach when a significant amount of energy relative to the transmission energy is consumed in the receiver. In addition to energy consumption, bandwidth utilization needs to be considered. In [8] and [9], while receiver energy consumption is modeled and incorporated into the total energy consumption, bandwidth utilization is not considered. The minimum energy-bandwidth tradeoff with receiver energy consumption is analyzed in [12] without considering spatial reuse.

Because of the practical constraints in the design of radio devices, multi-hop transmission is usually accomplished with half duplex nodes, which leads to a loss in bandwidth efficiency. However, by reusing the time or frequency, increased utilization of radio resources can be anticipated, since more transmissions are involved. This, however, comes at the price of increased interference. In this paper, we analyze the energy-bandwidth tradeoff with spatial reuse in a multi-hop network where nodes can be placed arbitrarily between a single source-destination pair over an additive white Gaussian noise (AWGN) channel. In this analysis, we formulate the overall performance in terms of how much energy will be consumed and how efficiently the network utilizes the bandwidth to deliver a single information bit using spatial reuse. The main outcome of this paper is characterization of the tradeoff between energy efficiency E_b/N_0 and bandwidth efficiency R_e similar in nature to the tradeoff in a single hop (point-to-point) transmission over an AWGN channel determined by Shannon. Our work differs from previous work in two ways: (1) Unlike the equi-distant assumption, relay nodes can be placed arbitrarily in a 2-dimensional plane with the requirement that the sum of each relaying distance should be equal or greater than the end-to-end distance, which makes it possible to compare the performance of different routing paths; and (2) unlike the typical transmission energy optimization, receiver energy consumption is included to find the minimum total energy consumption-per-bit and corresponding optimal end-to-end rate.

This paper is organized as follows. In Section II, the system model is described. Section III presents the performance measures considered in this work and the problem formulation. Sec-

Manuscript received October 1, 2009.

This research was supported by the Air Force Office of Scientific Research under grant FA9550-07-1-0456.

The authors are with the Electrical Engineering and Computer Science Department at the University of Michigan, Ann Arbor, MI 48109, USA, email: {changhun, stark}@umich.edu.

1229-2370/10/$10.00 © 2010 KICS

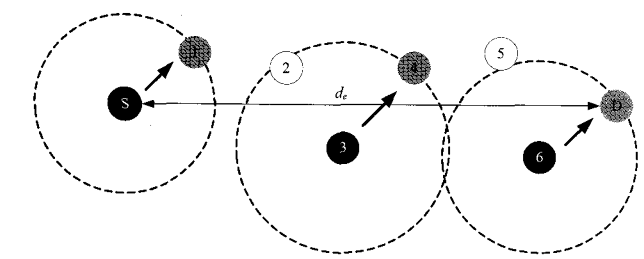


Fig. 1. Illustration of network model with spatial reuse, $\rho = 3$.

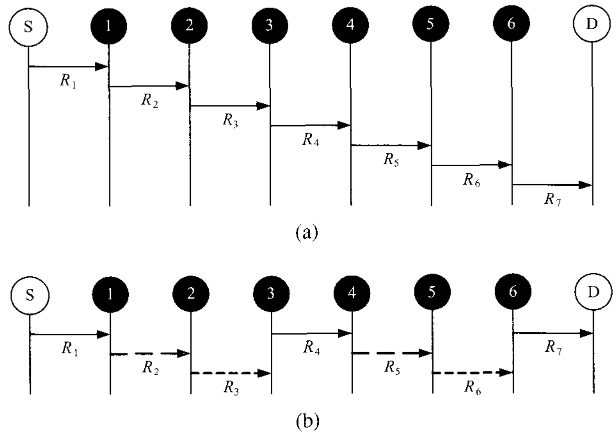


Fig. 2. Illustrative example of multi-hop transmission scenario: (a) Multi-hop communication without spatial reuse and (b) multi-hop communication with spatial reuse.

tion IV investigates lower bounds of the performance in terms of the end-to-end energy-bandwidth tradeoff. In Section V, the special case of equi-spaced relays is considered. Numerical results are presented in Section VI, and finally Section VII concludes the work.

II. SYSTEM MODEL

In this section, we describe the model we use for the network. We consider a stationary network that consists of a single source node and a single destination node, separated by a distance d_e, and $k-1$ relaying nodes located arbitrarily between the source and destination node, as shown in Fig. 1. In particular, we consider a decode-and-forward relaying protocol where relay node $j+1$ decodes the message sent from relay node j, re-encodes it using the same or a different codebook, and then forwards the message to relay node $j+2$. We denote the relay distance of hop i from node $i-1$ to node i as d_i, given by $d_i = \alpha_i d_e$, where $0 < \alpha_i < 1$. Note that $\sum_{i=1}^{k} \alpha_i \geq 1$, which implies that not every relay is necessarily located on the line between the source and destination. Data from the source is forwarded to the destination through $k-1$ relays. The transmission at each hop is implemented using capacity-achieving codes with the same duration for each coded symbol for each link. For reliable communication, the rate of transmission R in information bits per channel use must be less than the capacity C. That is, $R < C(\gamma) \Leftrightarrow \gamma > g(R)$, where γ is the received energy per channel use-to-noise power spectral density ratio and $g(R) = C^{-1}(R)$ is the inverse of the channel capacity $C(\gamma)$. We assume that the channel capacity function $C(\gamma)$ is continuous, twice differentiable, and concave. For the case of an AWGN channel, the capacity and its inverse function are given by $C(\gamma) = \frac{1}{2}\log_2(1+2\gamma)$ and $g(R) = \frac{2^{2R}-1}{2}$, respectively.

We assume that the network operates with spatial reuse to facilitate simultaneous transmissions over the network. The bandwidth is reused by transmitters with a minimum separation of ρ hops between simultaneous transmitters. All interference from simultaneous transmitters is assumed to be modeled as Gaussian noise. Thus, it is important to determine which relays are selected to operate in the same transmission slot. We consider a transmission strategy whereby nodes are separated by at least 3 hops. This strategy avoids simultaneous transmissions by two nodes that are one hop from a neighboring receiving node. Hence, $3 \leq \rho \leq k$ and there are ρ non-overlapping transmission slots. Let S_i be the set of nodes that transmit simultaneously in slot i, $1 \leq S_i \leq \rho$. We assume the propagation of signals follows a ηth path-loss law model with distance. The received power P_r at node $j+1$ is given by $P_r = \delta P_{j,tx}/d_j^{\eta}$, where $P_{j,tx}$ is the transmitter output power of node j. The energy transmitted per channel use (modulation symbol) of node j, $E_{j,tx}$ can then be expressed as $E_{j,tx} = P_{j,tx}T_s$ for a modulation symbol duration T_s. Note that δ is a constant depending on the antenna characteristics. We set $\delta = 1$ throughout this paper. In Fig. 1, as an example with $\rho = 3$, transmission by node 3 creates a certain amount of interference to node 1 and the destination, thus impairing the performance. The link signal-to-interference-and-noise ratio of the jth link in the ith transmission slot is then

$$\Gamma_j = \frac{E_{j,tx}d_j^{-\eta}}{N_0 + \sum_{l \in S_i, l \neq j} E_{l,tx} d_{l,j}^{-\eta}} = \frac{\gamma_j}{1 + \sum_{l \in S_i, l \neq j} \beta_{l,j}\gamma_l} \quad (1)$$

where $\gamma_j = \frac{E_{j,tx}}{N_0} d_j^{-\eta}$ and $\beta_{l,j} = \left(\frac{d_l}{d_{l,j}}\right)^{\eta} \neq \beta_{j,l}$. Note that $\beta_{l,j}$ denotes the relative distance ratio of the relaying distance to the interfering distance of link j in slot i. Therefore the interfering distances from other simultaneous transmitters and the minimum separation of hops affects the possible rate of communication and the performance. The rate of transmission R_j is given by $R_j = C(\Gamma_j)$. We assume each node has the same receiver energy consumption, i.e., E_p Joules are required to process a received coded symbol. In practice, the energy consumption per channel use depends on the power consumption and the processing time in the receiver. For some cases the power consumption of the RF front end can dominate the energy consumption of the digital processing. In this case, the receiver energy consumption can be expressed as a product of the receiver power consumption and the modulation symbol duration. Throughout this paper we assume E_p is constant.

III. PERFORMANCE MEASURES

A. End-to-End Rate

To measure the end-to-end bandwidth efficiency, we consider the number of information bits transmitted (end-to-end) per channel use transmitted by any node in the network that is forwarding the data. Unlike multi-hop transmission without spatial reuse, multi-hop transmission *with spatial reuse* suffers from interference from other active transmissions. As shown in Fig. 2, with spatial reuse there are ρ non-overlapping transmission slots and k hops for the end-to-end transmission. In contrast, for no spatial reuse there are k non-overlapping transmission slots and k hops.

Consider N total uses of the channel by the network. We assume that the same number of channel uses are assigned to the same transmission slot, i.e., N_i channel uses are assigned to the ith slot for $i = 1, \cdots, \rho$. The number of information bits communicated in slot i by node $j \in S_i$ is $N_i R_j$. Let $R_{\min,i}$ denote the minimum rate of all the transmissions in slot i. Then, the minimum number of information bits for the ith slot is $N_i R_{\min,i} = \min_{j \in S_i}(N_i R_j)$. With N_i channel uses, the jth link can deliver $N_i R_j$ information bits while the network can only handle $N_i R_{\min,i}$ information bits. Hence, $N_i R_{\min,i}/R_j$ channel uses are needed to transmit $N_i R_{\min,i}$ information bits. Maximizing the minimum end-to-end number of information bits yields the end-to-end rate as follows

$$\begin{aligned} R_e &= \max_{\sum_{i=1}^{\rho} N_i = N} \min \left\{ \frac{N_i}{N} R_{\min,i} \right\} \\ &= \max_{\sum_{i=1}^{\rho} q_i = 1} \min\{q_i R_{\min,i}\} \end{aligned}$$

where $q_i = N_i/N$. The optimal solution to the above minimax problem can be easily obtained by letting $q_i R_{\min,i} = q_j R_{\min,j}$, which yields $q_i^* = R_{\min,i}^{-1} / \sum_{j=1}^{\rho} R_{\min,j}$. Therefore, the end-to-end rate with spatial reuse is

$$R_e = \left(\sum_{i}^{\rho} R_{\min,i}^{-1} \right)^{-1} \text{ (bits/channel use)} \tag{2}$$

where ρ denotes the minimum separation of hops between simultaneous transmitters. Therefore, with spatial reuse, the end-to-end bandwidth utilization is enhanced by the spatial reuse factor ρ while the rate of each link is degraded by interference from simultaneous transmissions.

B. End-to-End Energy Consumption-per-Bit

Consider the jth link in slot i which is communicating at rate R_j. The energy consumption per information bit of link j, E_j, consists of transmission energy consumption and receiver energy consumption to process $N_i R_{\min,i}/R_j$ received symbols. Thus, we have

$$E_j = \frac{\frac{N_i R_{\min,i}}{R_j} (E_{j,tx} + E_p)}{N_i R_{\min,i}} = \frac{E_{j,tx} + E_p}{R_j}. \tag{3}$$

Note that $E_{j,tx}$ and E_p can be normalized with d_e to be consistent with a single hop case [12] as follows:

$$\frac{E_{j,tx}}{N_0} d_e^{-\eta} = \frac{E_{j,tx}}{N_0} \alpha_j^{\eta} d_j^{-\eta} = \alpha_j^{\eta} \gamma_j \tag{4}$$

$$\gamma_c \triangleq \frac{E_p}{N_0} d_e^{-\eta} \tag{5}$$

where $\alpha_j = d_j/d_e$ is the normalized distance for link j. Our major interest in this paper is to consider the overall energy consumed by all the nodes that forward the data across the network. In order to fairly represent the total energy consumption for different hops, we define the *normalized total energy consumption per information bit-to-noise power spectral density ratio* as

$$\frac{E_{\text{tot}}}{N_0} \triangleq \sum_{j=1}^{k} \frac{E_j}{N_0} d_e^{-\eta} = \sum_{j=1}^{k} \frac{\alpha_j^{\eta}\gamma_j + \gamma_c}{R_j}. \tag{6}$$

Note that, for low rates, the processing energy for the circuitry will dominate the total energy consumption, whereas for high rates the transmitted energy will dominate. In particular, when $\gamma_c = 0$, (6) denotes the sum of received energy per information bit in each link that is involved in the end-to-end transmission. This is consistent with a single link where the received energy is the usual performance measure as opposed to the transmitted energy.

C. Minimum Energy Consumption-per-Bit with Arbitrary Transmission Energy

For a given multi-hop routing path, the problem of minimizing total energy consumption-per-bit with arbitrary transmission energy of each link for a given end-to-end rate can be formulated as the following optimization problem:

$$\begin{aligned} \frac{E_{\text{tot}}(\overline{\alpha}, R_e)}{N_0} = \min_{\gamma_j} \quad & \sum_{j=1}^{k} \frac{\alpha_j^{\eta}\gamma_j + \gamma_c}{R_j} \\ \text{s.t.} \quad & R_j = C(\Gamma_j), j = 1, \cdots, k \\ & \sum_{i=1}^{\rho} R_{\min,i}^{-1} = R_e^{-1} \end{aligned} \tag{7}$$

where the optimization variables are γ_j's for $j = 1, \cdots, k$. Due to the cross coupled terms of variables in the objective function, which can be seen in (1), it is difficult to prove the convexity of the optimization problem for the general case of number of hops and to find the optimal set of transmission energies for each link as well. Instead of finding the optimal solution to the general problem, we propose a common power strategy as a sub-optimal strategy, which was shown to provide a good result for the case without spatial reuse in [12]. We assume that concurrent transmitters in the same transmission slot transmit with the same energy, that is, $E_{j,tx} = E_{S_i,tx}$ for $j \in S_i$. With this assumption we obtain the following relations. For $j, l \in S_i$

$$\alpha_j^{\eta}\gamma_j = \alpha_l^{\eta}\gamma_l \tag{8}$$

$$\Gamma_j = \frac{E_{S_i,tx} d_j^{-\eta}}{N_0 + \sum_{l \in S_i, l \neq j} E_{S_i,tx} d_{q,j}^{-\eta}} = \frac{\gamma_j}{1 + \gamma_j \beta_j} \tag{9}$$

$$\beta_j = \sum_{l \in S_i, l \neq j} \left(\frac{d_j}{d_{l,j}} \right)^{\eta} \tag{10}$$

where β_j denotes the sum of relative distances of link j in slot i. The fundamental goal of this paper is, for a given multi-hop routing path $\overline{\alpha} = (\alpha_1, \cdots, \alpha_k)$ and $\overline{\beta} = (\beta_1, \cdots, \beta_k)$, to find the optimal set of transmission energies $(E_{1,tx}, \cdots, E_{\rho,tx})$ that provides the minimum total energy consumption per information bit for a given end-to-end data rate R_e. We also focus on finding the global minimum total energy consumption and corresponding optimal end-to-end rate.

IV. MINIMUM ENERGY CONSUMPTION-PER-BIT

A. Minimum Energy-per-Bit for a Given End-to-End Rate

We start with the optimization problem for a given end-to-end rate. This general problem of minimizing the total energy consumption-per-bit for a given end-to-end rate is given as

Problem I:

$$\begin{aligned}\frac{E_{\text{tot}}(\overline{\alpha}, R_e)}{N_0} = \min_{E_{S_i,tx}} \quad & \sum_{m=1}^{k} \frac{E_{m,tx} + E_p\, d_e^{-\eta}}{R_m \quad N_0} \\ \text{s.t.} \quad & \textstyle\sum_{i=1}^{\rho} R_{\min,i}^{-1} = R_e^{-1} \\ & E_{m,tx} = E_{S_i,tx} \\ & R_m = C(\Gamma_m) \text{ for } m \in S_i\end{aligned} \tag{11}$$

where the optimization variables are $E_{S_i,tx}$'s for $i = 1, \cdots, \rho$. Because the objective function contains the sum of decreasing convex functions and the constraint contains increasing concave functions, Problem I is not a convex optimization problem. However, by letting $x_m^{-1} = C(\Gamma_m)$, Problem I can be converted into the following problem.

$$\begin{aligned}\frac{E_{\text{tot}}(\overline{\alpha}, R_e)}{N_0} = \min_{x_m} \quad & \sum_{m=1}^{k} x_m \left[\alpha_m^\eta h(x_m^{-1}) + \gamma_c\right] \\ \text{s.t.} \quad & \sum_{i=1}^{\rho} \dot{x}_i = R_e^{-1} \\ & \alpha_j^\eta h(x_j^{-1}) = \dot{\alpha}_i^\eta h(\dot{x}_i^{-1})\end{aligned} \tag{12}$$

where

$$\gamma_m = h(x_m^{-1}) = \frac{g(x_m^{-1})}{1 - \beta_m g(x_m^{-1})} > 0 \tag{13}$$

$$I_i = \operatorname*{argmax}_{j \in S_i} x_j \tag{14}$$

$$\dot{x}_i \equiv x_{I_i} = \max_{j \in S_i} x_j = R_{\min,i}^{-1} \tag{15}$$

$$\dot{\alpha}_i \equiv \alpha_{I_i}, \; \dot{\beta}_i \equiv \beta_{I_i}. \tag{16}$$

The form in (12) is still not a convex optimization problem due to the non-affinity of the second equality constraint. While numerical methods can provide the minimum energy consumption-per-bit for a given end-to-end rate, it is hard to obtain an optimal solution for arbitrary networks.

In summary, the original problem in (7) is simplified to Problem I in (12) by the assumption of equal transmission power for concurrent transmitters. This enables us to simplify the SINR relation in (1) to (9). While optimal solutions for both problems can be obtained by numerical methods, it is not easy to find optimal solutions when the number of nodes becomes large. In particular, Problem I is a nonconvex optimization problem even though it has been converted. Thus, instead of finding the optimal solution of Problem I, a lower bound to Problem I can be obtained by eliminating the equal power constraint in (12), which is applicable to any arbitrary networks with a large number of nodes. This leads to the following convex problem.

Lower bound of Problem I:

$$\begin{aligned}\frac{E_{\text{tot}}(\overline{\alpha}, R_e)}{N_0} \geq \min_{x_m} \quad & \sum_{m=1}^{k} x_m \left[\alpha_m^\eta h(x_m^{-1}) + \gamma_c\right] \\ \text{s.t.} \quad & \sum_{i=1}^{\rho} \dot{x}_i = R_e^{-1}\end{aligned} \tag{17}$$

where the optimization variables are x_m's for $m = 1, \cdots, k$. For (17) to be a convex problem, it is required that $h(x) = \dfrac{g(x)}{1 - \beta g(x)}$ be an increasing convex function for $0 < g(x) < 1/\beta$ and $\beta > 0$ [12], which can be easily shown. Therefore, the optimal parametric solution to the lower bound of Problem I can be obtained from the Karush-Kuhn-Tucker (KKT) conditions.

Parametric solution to lower bound of Problem I: For $m = 1, \cdots, k$, let $x_m^{-1} = C(\Gamma_m)$. For a given Lagrange multiplier λ, and $\overline{\alpha}$ and $\overline{\beta}$, $\dot{x}_i^*$ can be determined by solving the following equation from its Lagrangian function for $\dot{x}_i$, $i = 1, \cdots, \rho$

$$\dot{\alpha}_i^\eta \left[\frac{h'(\dot{x}_i^{-1})}{\dot{x}_i} - h(\dot{x}_i^{-1})\right] = \gamma_c - \lambda \tag{18}$$

and $x_j^* \neq \dot{x}_i$ for $j \in S_i$ can be determined by solving the following equation for x_j

$$\alpha_m^\eta \left[\frac{h'(x_m^{-1})}{x_m} - h(x_m^{-1})\right] = \gamma_c \tag{19}$$

where $0 < x_m^{-1} < C(1/\beta_m)$ for $m = 1, \cdots, k$ and $\gamma_c \geq 0$. Since the function $f(u) = uh'(u) - h(u)$ in the left hand side of (18) and (19) is increasing with $u > 0$ and $f(0) = 0$, it is required that $\lambda \leq \gamma_c$. Hence, the optimal γ_m^* is obtained from x_m^*, which also provides

$$R_e = \left(\sum_{i=1}^{\rho} \dot{x}_i^*\right)^{-1} \leq \left(\sum_{i=1}^{\rho} C^{-1}(1/\dot{\beta}_i)\right)^{-1} \tag{20}$$

where it should be noticed that the end-to-end rate is bounded, since the minimum link rate of each slot i is bounded by the maximum achievable SINR. The resulting minimum total energy consumption-per-bit is lower bounded by

$$\begin{aligned}\frac{E_{\text{tot}}(\overline{\alpha}, R_e)}{N_0} &\geq \sum_{m=1}^{k} x_m^* \left[\alpha_m^\eta h(1/x_m^*) + \gamma_c\right] \\ &= \sum_{i=1}^{\rho} \left[\dot{\alpha}_i^\eta h'\left(\frac{1}{\dot{x}_i^*}\right) + \lambda \dot{x}_i^* + \sum_{\substack{j \in S_i \\ j \neq i}} \alpha_j^\eta h'\left(\frac{1}{x_j^*}\right)\right].\end{aligned} \tag{21}$$

The parametric solution to the lower bound of Problem I is obtained from the following procedure using numerical methods:

Step 1. Fix λ.

Step 2. Solve (18) for $\dot{x}_i$.

Step 3. For $x_j \neq \dot{x}_i$ with $j \in S_i$, solve (19) for x_j.

Step 4. Repeat the above steps for $i = 1, \cdots, \rho$

Step 5. Determine R_e and $\frac{E_{\text{tot}}}{N_0}$ from (20) and (21).

By varying λ we can determine the tradeoff between E_{tot}/N_0 and R_e. Using (13), (18) can be simplified to

$$\begin{aligned}&\dot{\alpha}_i{}^\eta(\dot{x}_i^{-1} g'(\dot{x}_i^{-1}) - g(\dot{x}_i^{-1}) + \dot{\beta}_i g(\dot{x}_i^{-1})^2) \\ &\quad = (\gamma_c - \lambda)(1 - 2\dot{\beta}_i g(\dot{x}_i^{-1}) + (\dot{\beta}_i)^2 g(\dot{x}_i^{-1})^2).\end{aligned} \tag{22}$$

Note that when there is no significant interference, i.e., $\dot{\beta}_i \ll 1$, (22) can be approximated as

$$\dot{\alpha}_i{}^\eta \dot{x}_i^{-1} g(\dot{x}_i^{-1}) + \left[2\dot{\beta}_i(\gamma_c - \lambda) - \dot{\alpha}_i{}^\eta\right] g(\dot{x}_i^{-1}) = \gamma_c - \lambda. \tag{23}$$

For the case of an AWGN channel, we have $g(x) = C^{-1}(x) = 0.5(2^{2x} - 1)$, which leads to $h(x_m^{-1}) = \frac{(2^{2/x_m}-1)}{2-\beta_m(2^{2/x_m}-1)}$. Therefore, it is straightforward to obtain the solution to the lower bound of Problem I for an AWGN channel. In particular, the solution to (22) for an AWGN channel can be approximated as

$$\frac{2\ln 2}{\dot{x}_i^*} \approx \left[\mathcal{W}\left(\frac{\theta_1(1+\dot{\beta}_i)-1}{e}e^{\theta_1\dot{\beta}_i}\right)+1-\theta_1\dot{\beta}_i\right] \quad (24)$$

where $\theta_1 = 2\dot{\alpha}_i{}^{-\eta}(\gamma_c - \lambda)$, and $\mathcal{W}(\cdot)$ is the principal branch of the Lambert W-function, for which $\mathcal{W}(\S) \geq -\infty$ [13]. Note that when there is no interference, i.e., $\beta_i = 0$, the approximated solution (24) is the same as the result of [12].

B. Minimum Energy-per-Bit for an Optimal End-to-End Rate

Now consider the minimum total energy consumption-per-bit by optimally selecting the end-to-end rate for a given multi-hop routing path as follows.

Problem II:

$$\begin{aligned} \frac{E_{\text{tot}}(\overline{\alpha}, R_e^*)}{N_0} = \min_{E_{j,tx}} & \sum_{j=1}^{k} \frac{E_{j,tx}+E_p}{R_j}\frac{d_e^{-\eta}}{N_0} \\ \text{s.t.} \quad & R_j = C(\Gamma_j), j=1,\cdots,k \\ & E_{j,tx} = E_{S_i,tx} \text{ for } j \in S_i \end{aligned} \quad (25)$$

where the optimization variables are $E_{j,tx}$'s. Note that in (11) and (25), both $E_{j,tx}$ and $E_{S_i,tx}$ are the same, but in (25) our goal is to find the global minimum energy-per-bit over the rates by removing the constraint for the given rate related to the first condition related to ρ in (11). Thus, $E_{j,tx}$ is preferred over $E_{S_i,tx}$.

As in Problem I, (25) cannot be converted into an equivalent convex optimization problem. However, a lower bound of Problem II can be obtained from the unconstrained convex optimization problem of (17)

$$\frac{E_{\text{tot}}(\overline{\alpha}, R_e^*)}{N_0} \geq \min_{x_m}\left(\sum_{m=1}^{k} x_m\left[\alpha_m^\eta h(x_m^{-1})+\gamma_c\right]\right). \quad (26)$$

Solution to lower bound of Problem II: Let x_m^* for $m = 1,\cdots,k$ be the solution to

$$\alpha_m^\eta\left[\frac{h'(x_m^{-1})}{x_m} - h(x_m^{-1})\right] = \gamma_c. \quad (27)$$

The solution to the lower bound of Problem II is then as follows

$$\frac{E_{\text{tot}}(\overline{\alpha}, R_e^*)}{N_0} \geq \sum_{m=1}^{k} x_m^*\left[\alpha_m^\eta h(1/x_m^*)+\gamma_c\right] \quad (28)$$

$$= \sum_{m=1}^{k} \alpha_m^\eta h'\left(\frac{1}{x_m^*}\right) \quad (29)$$

where (28) can be obtained from (21) by substituting $\lambda = 0$. For the case of an AWGN channel, when $\beta \ll 1$, the solution to (27) is approximated as

$$\frac{2\ln 2}{x_m^*} \approx \left[\mathcal{W}\left(\frac{\theta_2(1+\beta_m)-1}{e}e^{\theta_2\beta_m}\right)+1-\theta_2\beta_m\right] \quad (30)$$

where $\theta_2 = 2\alpha_m^{-\eta}\gamma_c$. Notice that when there is no interference, i.e., $\beta_m = 0$, the approximated solution (30) is the same as the result of [12]. In particular, when the circuit processing energy is ignored, substituting $\gamma_c = 0$ to (27) yields $w(u) \triangleq ug'(u) - g(u)(1-\beta_m g(u))^2 = 0$, where $u = x_m^{-1}$. It can be easily shown that $w(u)$ is increasing in u for $0 < g(u) < 1/\beta_m$ and $w(0) = 0$. Therefore, we obtain the optimal solution $1/x_m^* = 0$ and, correspondingly, the optimal minimum total energy consumption-per-bit for the case of $\gamma_c = 0$.

Solution to Problem II with $\gamma_c = 0$:

$$\begin{aligned} \frac{E_b(\overline{\alpha}, R_e^*)}{N_0} &= \lim_{1/x_m^* \to 0}\left(\sum_{m=1}^{k} \alpha_m^\eta x_m^* h(1/x_m^*)\right) \\ &= \ln 2 \sum_{m=1}^{k} \alpha_m^\eta. \end{aligned}$$

This implies that the minimum energy consumption-per-bit occurs at $R_e = 0$ when only the transmission energy is considered. Hence, without receiver energy consumption, multi-hop routing with spatial reuse achieves the same minimum total energy consumption per information bit as multi-hop routing without spatial reuse in [12].

V. PERFORMANCE OF EQUI-DISTANT MULTI-HOP ROUTING

In this section, we derive the optimal end-to-end rate and number of hops to minimize the overall energy consumption-per-bit of equi-spaced relays between the source and the destination. We also investigate how the number of hops and the end-to-end distance affect the performance. The optimal solutions are obtained by decoupling the joint optimization problem in rate and number of hops into two sub-problems. We first obtain the optimal number of hops as a function of R_e. Then, the optimal value of R_e is derived [8], [14].

Consider the case of equi-spaced relays using spatial reuse with minimum separation between simultaneous transmitters. In practice, depending on the relay position in the network, each link SINR can vary even if the same transmission energy level is allowed in each link. However, for the purpose of investigating the impact of the end-to-end rate and number of hops, we assume that each link experiences the same amount of interference; this requires $k \geq 2\rho$, since at least one interferer must exist in each transmission slot. Also, the cochannel interference is assumed to be independent of the number of hops by considering the major interference from the immediate neighboring concurrent transmitter, that is, $\beta = (\rho - 1)^{-\eta}$. As such, each link is assumed to communicate at the same rate. Let Γ be the SINR at each link. The end-to-end rate is then

$$R_e = \frac{R}{\rho} = \frac{C(\Gamma)}{\rho} \leq \frac{C(\beta^{-1})}{\rho} \quad (31)$$

and the received SNR of each link is $\gamma = \frac{g(\rho R_e)}{1-\beta g(\rho R_e)} \triangleq h(\rho R_e)$, where $g(\cdot)$ denotes the inverse of the channel capacity function and $0 < g(\rho R_e) < 1/\beta$, since $\gamma > 0$. The total energy consumption per information bit is then

$$\frac{E_{\text{tot}}(k, R_e)}{N_0} = \sum_{i=1}^{k} \frac{k^{-\eta}\gamma + \gamma_c}{\rho R_e} = \frac{k^{1-\eta}h(\rho R_e) + k\gamma_c}{\rho R_e}. \quad (32)$$

With transformation of the variables $k = e^{-x}$ and $\rho R_e = e^{-y}$, the problem of minimizing the total energy consumption-per-bit can be converted into an equivalent optimization problem

$$\frac{E_{\text{tot}}(x^*, y^*)}{N_0} = \min_{x\in\mathbf{R}, y\in\mathbf{R}} e^y \left[e^{(\eta-1)x}h(e^{-y}) + \gamma_c e^{-x}\right] \quad (33)$$

where $h(e^{-y}) = \frac{g(e^{-y})}{1-\beta g(e^{-y})}$.

Proposition 1: The objective function $E_{\text{tot}}(x, y)/N_0$ is convex in x, y for $x, y \in \mathbf{R}$ if $g(u)$ satisfies the following conditions

$$g(u)g'(u) + u\left[g''(u)g(u) - g'(u)^2\right] > 0. \quad (34)$$

Proof: See Appendix 1. □

Therefore, if the sufficient conditions are satisfied, the optimization problem is a convex optimization problem and thus any locally optimal solution is guaranteed to be a globally optimal solution. For the case of an AWGN channel, it can be easily verified that the objective function in (33) is convex (see Appendix 1). Other channels of interest are the binary input AWGN channel and the binary input hard decision AWGN channel, whose channel capacity functions [15] are given by

$$C_{\text{BISO}}(\gamma) = 1 - \int_{-\infty}^{\infty} g(x - \sqrt{2\gamma}) \log_2(1 + e^{-2x\sqrt{2\gamma}})dx$$
$$C_{\text{BIBO}}(\gamma) = 1 - H_2(Q(\sqrt{2\gamma}))$$

where $g(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ and $H_2(x) = -x\log_2(x) - (1 - x)\log_2(1 - x)$ is the binary entropy function. Numerical methods can verify the sufficient condition for convexity of the above two channels. Note that interference from other transmissions is regarded as additional Gaussian noise.

A. Optimization of the Number of Hops

For a given end-to-end rate, treating the number of hops as a continuous variable, we optimize the total energy consumption-per-bit over x:

$$\frac{E_{\text{tot}}(x^*, y)}{N_0} = \min_{x\in\mathbf{R}} e^y \left[e^{(\eta-1)x}h(e^{-y}) + \gamma_c e^{-x}\right]. \quad (35)$$

Setting the derivative with respect to x equal to zero yields the optimal number of hops for a given end-to-end rate as follows

$$k^* = e^{-x^*} = \left[\frac{\eta-1}{\gamma_c}\frac{g(e^{-y})}{1-\beta g(e^{-y})}\right]^{1/\eta}. \quad (36)$$

The minimum total energy consumption-per-bit at the optimal number of hops is then

$$\frac{E_{\text{tot}}(k^*, R_e)}{N_0} = \left[\frac{\eta-1}{\gamma_c}\right]^{1/\eta-1} \frac{\eta}{\rho R_e}\left[\frac{g(\rho R_e)}{1-\beta g(\rho R_e)}\right]^{1/\eta}. \quad (37)$$

B. Optimization of the End-to-End Rate

The problem of determining the optimal end-to-end rate is formulated by substituting (36) into (33) which results in

$$\frac{E_{\text{tot}}(x^*, y^*)}{N_0} = \min_{y}\left(\eta\left[\frac{\eta-1}{\gamma_c}\right]^{\frac{1}{\eta}-1}\left[\frac{e^{\eta y}g(e^{-y})}{1-\beta g(e^{-y})}\right]^{1/\eta}\right).$$

Setting the derivative with respect to y to be equal to zero yields $\eta g(e^{-y})(1-\beta g(e^{-y})) = e^{-y}g'(e^{-y})$. Let u^* be the solution to $\eta g(u)[1-\beta g(u)] = ug'(u)$. The optimal end-to-end rate is then

$$R_e^* = \frac{u^*}{\rho} \quad (38)$$

and the optimal number of hops at the optimal rate is obtained by substituting (38) into (36),

$$k^*(R_e^*) = \left[\frac{\eta-1}{\gamma_c}\frac{g(u^*)}{1-\beta g(u^*)}\right]^{1/\eta}$$
$$= \left[\frac{g(u^*)}{1-\beta g(u^*)}\right]^{1/\eta}\left[\frac{E_p}{N_0}\frac{1}{\eta-1}\right]^{-1/\eta} d_e. \quad (39)$$

The resulting normalized minimum total energy consumption-per-bit at the optimal rate is then

$$\frac{E_{\text{tot}}(k^*, R_e^*)}{N_0} = \eta\left[\frac{\eta-1}{\gamma_c}\right]^{\frac{1}{\eta}-1}\frac{1}{u^*}\left[\frac{g(u^*)}{1-\beta g(u^*)}\right]^{1/\eta}$$
$$= f_c(\eta)\left[\frac{E_p}{N_0}\right]^{1-1/\eta} d_e^{1-\eta}. \quad (40)$$

where

$$f_c(\eta) = \frac{1}{u^*}\left[\frac{g(u^*)}{1-\beta g(u^*)}\right]^{1/\eta}\left[\frac{\eta}{(\eta-1)^{1-1/\eta}}\right] \quad (41)$$

which is a function of the channel and the propagation loss exponent. From (6), the unnormalized minimum total energy consumption-per-bit is given by

$$\frac{E_{\text{tot}}^U(k^*, R_e^*)}{N_0} = \frac{E_{\text{tot}}(k^*, R_e^*)}{N_0}d_e^{\eta} = f_c(\eta)\left[\frac{E_p}{N_0}\right]^{1-1/\eta} d_e. \quad (42)$$

From the above results, it is observed that the optimal rate depends only on the relative distance ratio β if the optimal number of hops satisfies the constraint $k^* \geq 2\rho$, which grows linearly with d_e, as in the case without spatial reuse. This is because the transmitted energy becomes dominant as d_e increases, which implies that more hops yields more energy efficiency for small γ_c. Thus, as γ_c becomes small, networks with more hops can achieve the minimum energy consumption-per-bit, as will be shown in the numerical results. In addition, there are two key observations for the minimum normalized total energy consumption-per-bit: (i) Increases with the $\frac{\eta-1}{\eta}$ power of the processing energy and (ii) decreases with $d_e^{1-\eta}$. Therefore, we can conclude that the actual unnormalized minimum total energy consumption-per-bit increases linearly with d_e from the relation between the normalized total energy consumption-per-bit

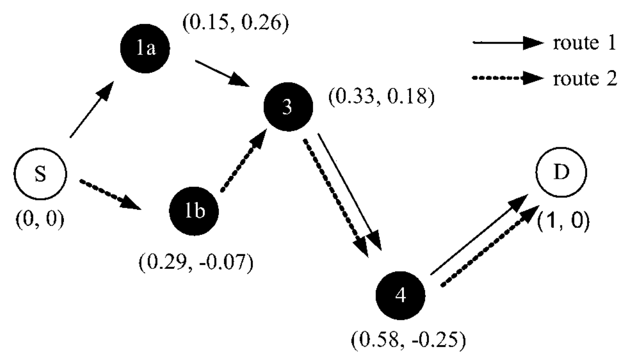


Fig. 3. Example of two possible routes in 2-dimensional network model where distance between any two nodes denotes the normalized relay distance by the end-to-end distance with spatial reuse, $\rho = 3$.

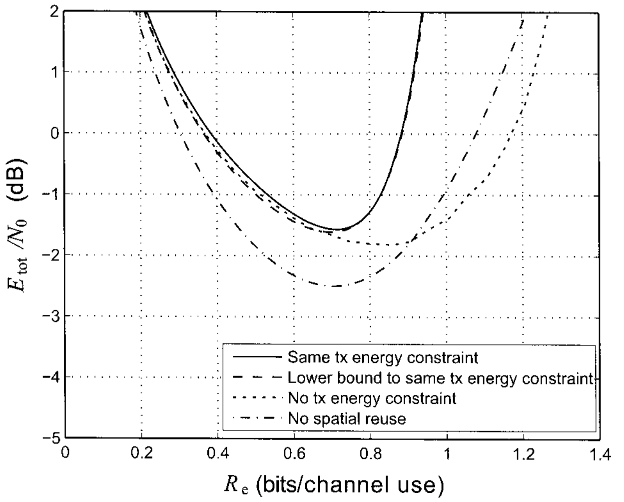


Fig. 4. Comparison of normalized energy-bandwidth tradeoff when $\gamma_c = -5.31$ dB with $k = 4$ hops for selected location of relays at $d_e = 3000$ m and $\rho = 3$.

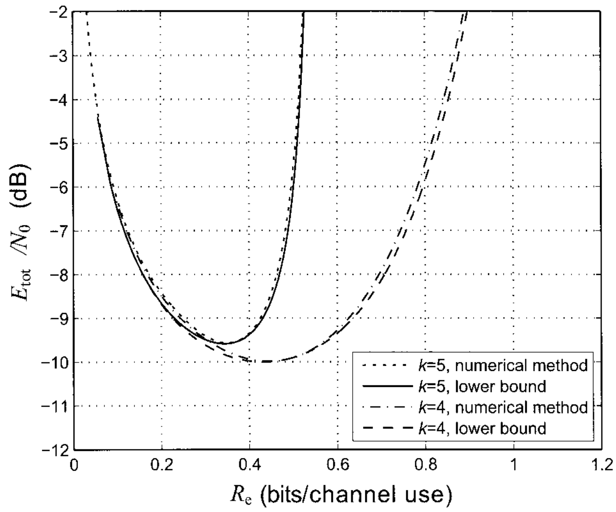


Fig. 5. Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot when $\gamma_c = -17.36$ dB with $k = 4$ and $k = 5$ hops for selected location of relays at $d_e = 3000$ m and $\rho = 3$.

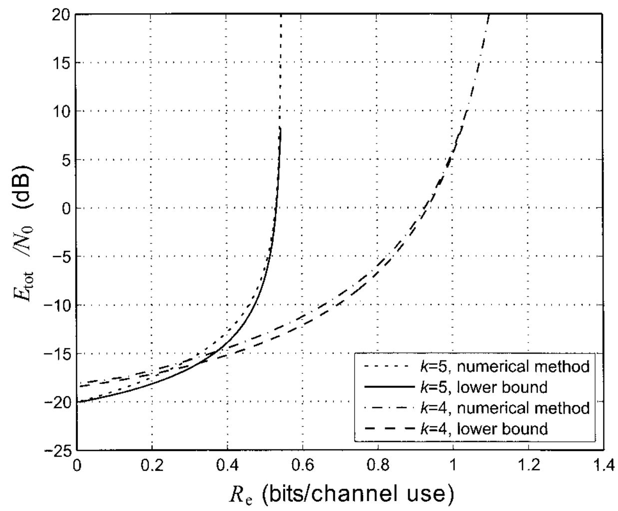


Fig. 6. Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot when $\gamma_c = 0$ with $k = 4$ and $k = 5$ hops for selected location of relays at $d_e = 3000$ m and $\rho = 3$.

and the end-to-end distance in (6). Note that the optimal energy constant in (41) depends on the channel capacity, the path-loss exponent, and the relative distance ratio. For different input/output AWGN channels (e.g., binary input or binary output), the resulting energy consumption constants are given in Table 1 along with the loss incurred relative to the AWGN channel for $\rho = 3$.

VI. NUMERICAL RESULTS

In the numerical results, unless otherwise specified, we assume that the path-loss exponent is $\eta = 4$, the channel is an AWGN, the minimum separation of hops between simultaneous transmissions is $\rho = 3$, the noise power spectral density $N_0 = -174$ dBm/Hz, and the circuit processing energy is $E_p = 0.095$ μJ/symbol, which at an end-to-end distance of $d_e = 3$ km yields $\gamma_c = -5.31$ dB.

In Fig. 4, the normalized energy-bandwidth tradeoffs are plotted, where results are drawn based on numerical optimization with the same transmission energy, the lower bound of Problem I, the numerical result with arbitrary transmission energy, and the optimal result without spatial reuse in [12], respectively. The distance ratio vector is $\overline{\alpha} = (0.3, 0.25, 0.15, 0.3)$ and $\gamma_c = -5.31$ dB. The lower bound on the energy-per-bit with the same transmission energy constraint in the same slot is shown to be very tight to the actual energy-per-bit with the same constraint. The arbitrary transmission energy and the case without spatial reuse are drawn together to observe the effect of transmission energy restriction and the performance loss from spatial reuse. At low rates the energy consumption due to the transmitter alone decreases while the total energy consumption increases due to the energy needed for processing. At high rates, however, the total energy consumption increases as the transmitted energy increases. Thus, such characteristics yield a nonzero optimal rate that minimizes the total energy consumption-per-bit, similar to the result in [12]. It is also seen that the end-to-end bandwidth utilization is limited, because the rate of communication in each slot is bounded by cochannel interference, as indicated in (20). In Figs. 5 and 6,

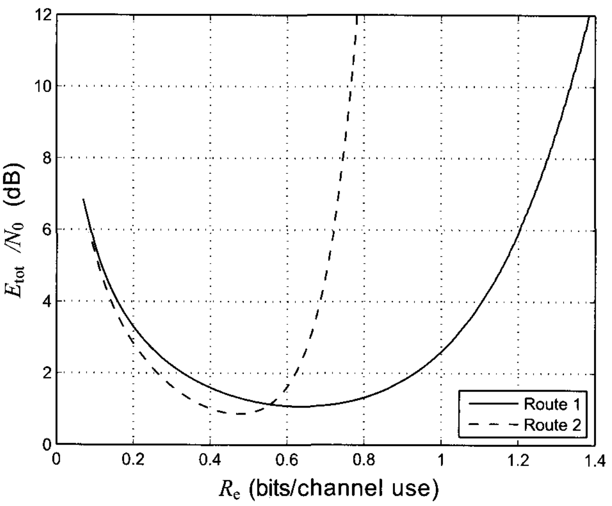


Fig. 7. Comparison of lower bounds of normalized energy-bandwidth tradeoff for two different routes: $k = 4$ hops for selected location of relays when $\gamma_c = -5.31$ dB at $d_e = 3000$ m.

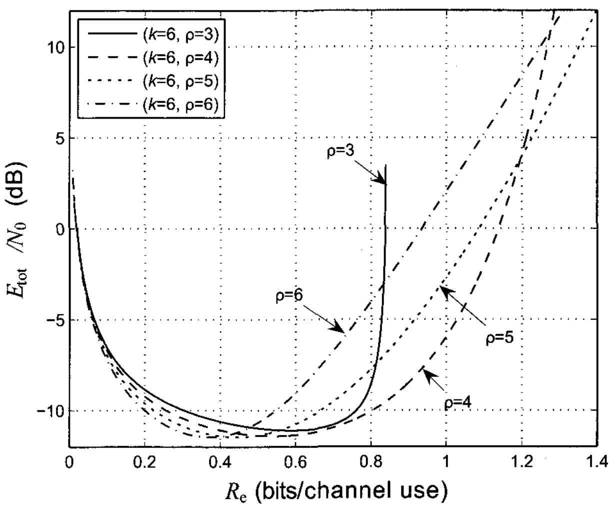


Fig. 9. Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot: $k = 6$ hops for selected minimum separation of simultaneous transmitters when $\gamma_c = -17.36$ dB for equi-distant relays at $d_e = 3000$ m.

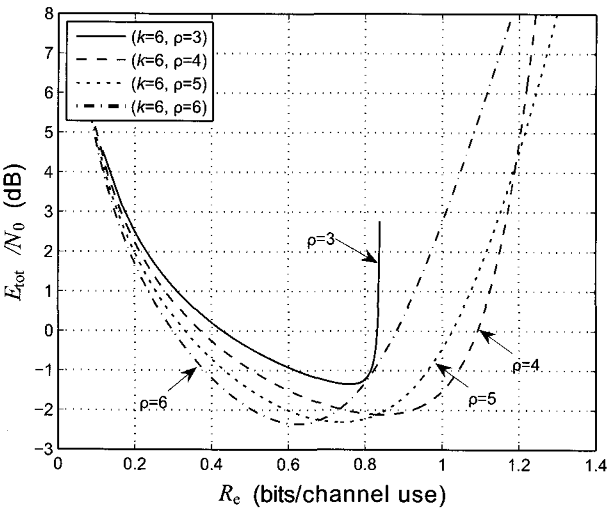


Fig. 8. Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot: $k = 6$ hops for selected minimum separation of simultaneous transmitters when $\gamma_c = -5.31$ dB for equi-distant relays at $d_e = 3000$ m.

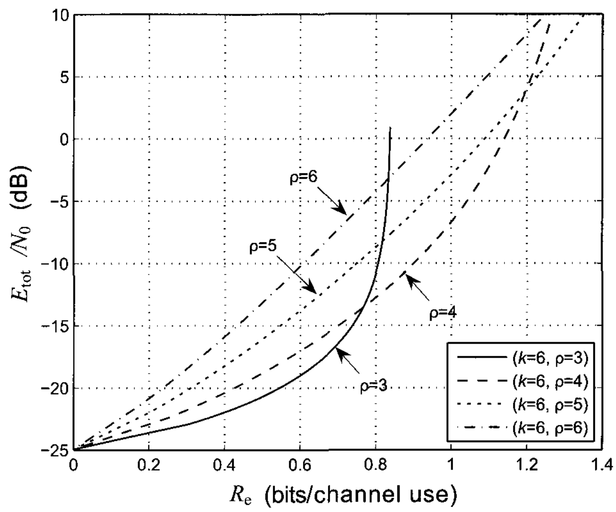


Fig. 10. Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot: $k = 6$ hops for selected minimum separation of simultaneous transmitters when $\gamma_c = 0$ for equi-distant relays $d_e = 3000$ m.

the normalized energy-bandwidth tradeoff with a transmission energy constraint is plotted for different processing energies, $\gamma_c = -17.36$ dB and $\gamma_c = 0$ ($-\infty$ dB) where the distance ratio vectors are given as $\overline{\alpha} = (0.3, 0.25, 0.15, 0.3)$ for $k = 4$ hops and $\overline{\alpha} = (0.3, 0.2, 0.1, 0.25, 0.15)$ for $k = 5$ hops. As the processing energy decreases, the optimal rate minimizing the total energy consumption-per-bit decreases, because the transmitted energy is more dominant as γ_c becomes small. In particular, when $\gamma_c = 0$, the minimum total energy consumption-per-bit occurs at $R_e = 0$, which verifies the result of Problem II with $\gamma_c = 0$. In addition, when $\gamma_c = 0$, it shows that a multi-hop network with more hops is more energy-efficient. This is because more relays reduce the distance between hops, which further contributes to reducing the transmitted energy.

In order to show that different selection of relays causes different amounts of interference because of the different distances from transmitters to the receivers, we consider a wireless network with 3 relays as an example where the locations of the nodes are given in Fig. 3. Using the lower bound to the solution of Problem I, we compare the performance of two possible multihop routes in Fig. 7. It is observed based on the bound that route 2 is better than route 1 at low rates while at high rates route 1 is better, because route 2 suffers more from interference as the transmission energy increases. An efficient algorithm that can find the optimal route for a given rate is currently unknown when the number of nodes is large. However, the insight into which route is better based on bounds is very useful.

Fig. 8 depicts the impact of the minimum separation of simultaneous transmitters when the relays are placed equidistantly for $\gamma_c = -5.31$ dB at $d_e = 3$ km. Instead of the result in Section V where the same interference is assumed in the same transmission slot, lower bounds to the solution of Problem I are used. Note that as ρ increases, fewer simultaneous transmissions are

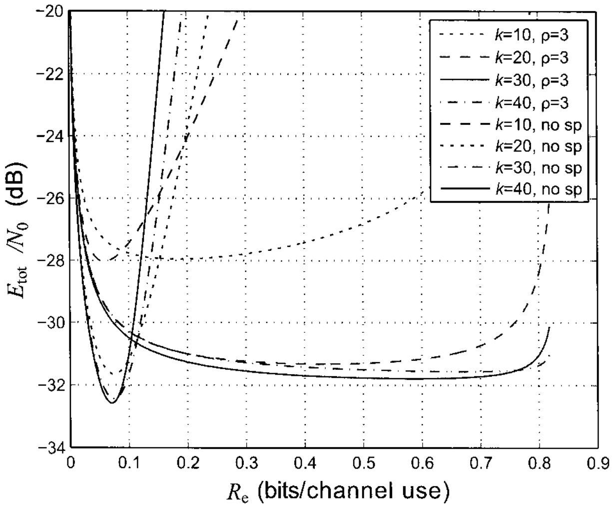


Fig. 11. Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot when $\rho = 3$ and $\gamma_c = -45.31$ dB for equi-distant relays $d_e = 30$ km.

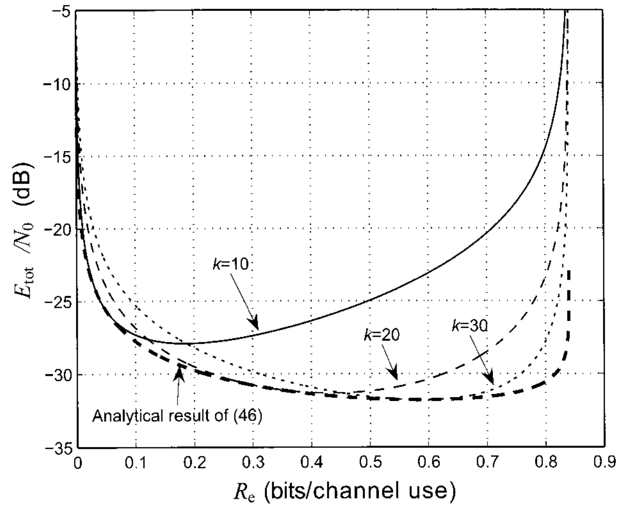


Fig. 13. Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot for selected number of hops, $k = 10, 20, 30$ when $\gamma_c = -45.31$ dB for equi-distant relays $d_e = 3$ km and $\rho = 3$.

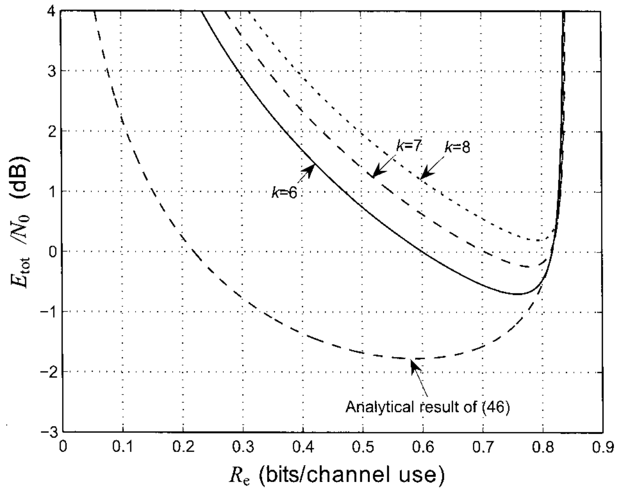


Fig. 12. Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot for selected number of hops, $k = 6, 7, 8$ when $\gamma_c = -5.31$ dB for equi-distant relays $d_e = 3$ km and $\rho = 3$.

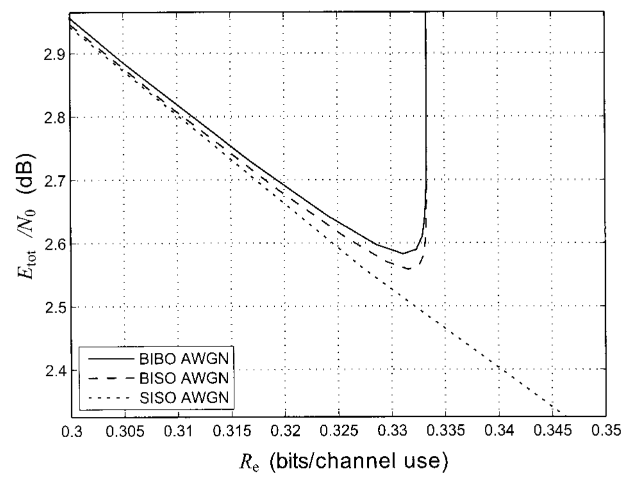


Fig. 14. Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot for various AWGN channel models when $\gamma_c = -5.31$ dB and $k = 6$ hops for equi-distant relays $d_e = 3$ km and $\rho = 3$.

used and $\rho = k$ corresponds to the case without spatial reuse. At low rates, the energy efficiency is improved with fewer simultaneous transmissions. However, at high rates, an optimal selection of ρ is necessary, because more spatial reuse can cause a significant amount of interference whereas less spatial reuse reduces the bandwidth utilization. Hence, it should be noted that whether spatial reuse can improve the bandwidth utilization and contribute to enhancing the energy efficiency depends on the desired end-to-end rate.

In Figs. 9 and 10, lower bounds on the normalized energy-bandwidth tradeoff are plotted for $\gamma_c = -17.36$ dB and $\gamma_c = 0$ to further explore the impact of processing energy. In Fig. 9, where $\gamma_c > 0$, it is seen that a multi-hop transmission without spatial reuse is more energy efficient at low rates. This is because the receiver energy consumption without spatial reuse, $\gamma_c R_e^{-1}$, is less than that with spatial reuse, $\gamma_c(R_e^{-1} + \sum_{R_j \neq R_{\min,i}} R_j^{-1})$, for $i = 1, \cdots, \rho$ at a given end-to-end rate. However, as shown in Fig. 10 for $\gamma_c = 0$, more spatial reuse yields more energy efficiency at low rates, because: (1) Only the transmission energy is considered, and (2) less transmission energy is required for more spatial reuse transmission to produce the same end-to-end rate before it suffers from severe cochannel interference. For short end-to-end distances such as $d_e = 3000$ m in Fig. 9, it is seen that, at high rates, the spatial reuse case cannot outperform the no spatial reuse case even, if the minimum separation is adjusted. However, when d_e becomes large, such as $d_e = 30$ km in Fig. 11, it is observed that the spatial reuse case is better than the no spatial reuse case at high rates. This verifies that, although both cases suffer from bandwidth efficiency loss due to half duplex property, for large end-to-end distance, increased utilization of resources is expected by spatial reuse.

To investigate the relation between the optimal number of hops and the end-to-end distance or equivalently normalized processing energy, we plot the normalized energy-bandwidth

Table 1. Optimal energy consumption constants for various AWGN channels.

Type	Energy constant			Loss (dB)		
	$\eta=2$	$\eta=3$	$\eta=4$	$\eta=2$	$\eta=3$	$\eta=4$
SISO	3.27	2.83	2.25	-	-	-
BISO	3.35	3.03	2.55	-0.10	-0.29	-0.54
BIBO	4.34	3.60	2.86	-1.23	-1.04	-1.04

tradeoff for the equi-distant relays case in Figs. 12 and 13 with the same interference assumption as in Section V. Since we assume that at least one interferer must exist in each transmission slot, $k \geq 6$ hops are considered for $\rho = 3$. In order to assess the performance for very low processing energy, cases with $\gamma_c = -45.31$ dB are compared with cases for $\gamma_c = -5.31$ dB. When $\gamma_c = -5.31$ dB, the minimum energy consumption-per-bit occurs at $k = 6$, which is the smallest number of hops when $\rho = 3$. This implies that when the normalized processing energy consumption is not negligible, a multi-hop network with less hops is more energy efficient. On the contrary, when the normalized receiver energy consumption is very low but not equal to 0, e.g., $\gamma_c = -45.31$ dB, a multi-hop network with more hops is more energy efficient compared to a multi-hop network with less hops. This can be identified from the result in (39) that as γ_c increases, the optimal number of hops decreases. Correspondingly, the minimum energy consumption-per-bit also increases as d_e decreases, which verifies the results in (40). It is also observed that, due to the limited achievable SINR, the end-to-end bandwidth utilization is limited and converges to $C(\beta^{-1})/\rho$, because its immediate neighboring simultaneous transmitter is a major source of interference. Note that when multiple interferers are considered, the end-to-end rate converges to $(\sum_\rho C^{-1}(1/\beta_i))^{-1}$, as described in (20).

Performance comparisons for various AWGN channel models are presented in Fig. 14, where $k = 6$ equally spaced hops are considered. Input constrained AWGN channels, binary input with hard decision (BIBO AWGN) and binary input/unquantized output AWGN (BISO AWGN), show a bounded bandwidth utilization at $R_e = 1/3$ bits/channel, and degraded performance compared to the unquantized input/output AWGN channel (SISO AWGN) due to their inherently limited bandwidth utilization. This can be identified in Table 1, where the optimal normalized energy consumption constants are provided for different channel models. It is seen that the optimal energy consumption per information bit decreases as bandwidth is utilized efficiently through the channel.

VII. CONCLUSION

This paper establishes the minimum energy-bandwidth tradeoff for wireless multi-hop networks where relays can be placed arbitrarily between the source and destination. The end-to-end rate is first introduced to capture the end-to-end bandwidth utilization, in which spatial reuse is used by restricting the minimum number of hops between simultaneous transmitters. Normalized total energy consumption-per-bit for a multi-hop network which includes the transmission energy and required energy for processing at each receiver is considered to be consistent with a single hop network. A parametric solution for lower bounds on the minimum total energy consumption per information bit is derived by solving an equivalent convex optimization problem. An optimal end-to-end rate exists from the relation of the transmission and processing energy at a given end-to-end rate. When the normalized processing energy is small, the optimal end-to-end rate decreases due to the dominance of the transmitted energy consumption. In particular, when the receiver processing energy is not included, multi-hop routing with spatial reuse shows the same performance as multi-hop routing without spatial reuse. It is shown that spatial reuse only contributes to enhancing bandwidth utilization, not reducing energy consumption-per-bit. Instead the energy efficiency can be improved by varying the minimum separation of hops between simultaneous transmitters, which is related to adjusting the resulting cochannel interference.

The paper also establishes a general relationship between the number of hops or equivalently minimum energy consumption and the end-to-end distance by approximating cochannel interference as being equal for each link. It is shown that the normalized minimum energy consumption-per-bit decreases with $1-\eta$ power of the end-to-end distance, which indicates that the unnormalized energy consumption-per-bit increases with the end-to-end distance. In addition, the optimal number of hops is linearly proportional to the end-to-end distance, implying that for large end-to-end distance, more hops achieves the minimum energy consumption.

In conclusion, there exists an optimal transmission strategy that provides the best performance at a given energy-bandwidth tradeoff by considering simultaneous transmissions and the number of hops. Therefore, in order to minimize the energy consumption-per-bit at a given end-to-end rate, the number of hops and the minimum separation of simultaneous transmitters should be considered jointly.

APPENDIX

1. CONDITION FOR CONVEXITY

Proposition 1: The function $f(x,y) = e^{(\eta-1)x+y}h(e^{-y}) + \gamma_c e^{-x+y}$ is convex in x and y for $x \in \mathbf{R}$ and $y \in \mathbf{R}$ if $g(u)$ satisfies the following conditions for $u \geq 0$ and $\beta \geq 0$

$$g(u)g'(u) + u\left[g''(u)g(u) - g'(u)^2\right] > 0 \tag{43}$$

where $h(u) = \frac{g(u)}{1-\beta g(u)} > 0$ for $1 - \beta g(u) > 0$.

Proof: To prove convexity of $f(x,y)$ in x and y, it is sufficient to prove convexity of $f_1(x,y) = e^{(\eta-1)x+y}h(e^{-y})$, since $f_2(x,y) = \gamma_c e^{-x+y}$ is convex in x, y for $\gamma_c \geq 0$ and the non-negative sum of convex functions is convex. The second order partial derivative of $f_1(x,y)$ with respect to x is given by

$$\frac{\partial^2 f_1(x,y)}{\partial x^2} = (\eta-1)^2 e^{(\eta-1)x+y}h \triangleq a > 0$$

and with respect to y

$$\frac{\partial^2 f_1(x,y)}{\partial y^2} = e^{(\eta-1)x+y}\left[h - e^{-y}h' + e^{-2y}h''\right] \triangleq c$$

Finally, with respect to x and y, we have

$$\frac{\partial^2 f_1(x,y)}{\partial x \partial y} = (\eta - 1)e^{(\eta-1)x+y}\left[h - e^{-y}h'\right] \triangleq b$$

where h, h', and h'' denote $h(e^{-y})$,$h'(e^{-y})$, and $h''(e^{-y})$, respectively. Therefore, the Hessian of $f_1(x,y)$ is given by

$$\mathbf{H} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}.$$

A 2×2 matrix of the form $\mathbf{H}$ with $a > 0$ is positive definite if and only if $\Delta = ac - b^2 > 0$ (Schur's complement condition [14]). Let $u = e^{-y} \geq 0$. Then, for $u \geq 0$, its Hessian is given by

$$\Delta = uh(u)h'(u) + u^2\left[h(u)h''(u) - h'(u)^2\right] \quad (44)$$

where

$$h(u) = \frac{g(u)}{1 - \beta g(u)} > 0 \quad (45)$$

$$h'(u) = \frac{g'(u)}{(1 - \beta g(u))^2} > 0 \quad (46)$$

$$h''(u) = \frac{g''(u) - \beta g(u)g''(u) + 2\beta g'(u)^2}{(1 - \beta g(u))^3} \quad (47)$$

since $g(u)$ is increasing convex [12] and $1 - \beta g(u) > 0$. Therefore, $\Delta > 0$ if the following condition is satisfied for $u \geq 0$ and $g(u) < 1/\beta$ with $\beta \geq 0$

$$u\left[1 - \beta g(u)\right]\left[g(u)g'(u) + u(g''(u)g(u) - g'(u)^2)\right] + \beta u^2 g(u)g'(u)^2 > 0.$$

Therefore, for any $\beta \geq 0$, it is sufficient for the Hessian to be positive definite if $g(u)g'(u) + u\left[g''(u)g(u) - g'(u)^2\right] > 0$. □

REFERENCES

[1] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation," *IEEE Trans. Inf. Theory*, vol. 50, no. 5, pp. 748–767, May 2004.

[2] S. Verdu, "On channel capacity per unit cost," *IEEE Trans. Inf. Theory*, vol. 36, no. 5, pp.1019–1030, Sept. 1990.

[3] V. Rodoplu and T. H. Meng, "Bits-per-joule capacity of energy-limited wireless networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 3, pp. 857–865, Mar. 2007.

[4] J. Gomez-Vilardebo and A. I. Perez-Neira, "Bounds on maximum rate-per-energy for orthogonal AWGN multiple relay channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4238–4247, Nov. 2008.

[5] A. El Gamal, M. Mohseni, and S. Zahedi, "Bounds on capacity and minimum energy-per-bit for AWGN relay channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1545–1561, Apr. 2006.

[6] M. Sikora, J. N. Laneman, M. Haenggi, D. J. Costello, and T. E. Fuja, "Bandwidth and power efficient routing in linear wireless networks," *Joint Special Issue of IEEE Trans. Inf. Theory and IEEE Trans. Networking*, vol. 52, pp. 2624–2633, June 2006.

[7] A. J. Goldsmith and S. B. Wicker, "Design challenges for energy-constrained ad hoc wireless networks," *IEEE Wireless Commun. Mag.*, vol. 9, no. 4, pp. 8–27, Aug. 2002.

[8] T. Shu, M. Krunz, and S. Vrudhula, "Joint optimization of transmission power-time and bit energy efficiency in CDMA wireless sensor networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 3109–3118, Nov. 2006.

[9] S. Cui, R. Madan, A. J. Goldsmith, and S. Lall, "Cross-layer energy and delay optimization in small-scale sensor networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 10, pp. 3688–3699, Oct. 2007

[10] A. Agustin and J. Vidal, "Amplify-and-forward cooperation under interference-limited spatial reuse of the relay slot," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1952–1962, May 2008.

[11] O. Gurewitz, A. de Baynast, and E. K. Knightly, "Coopertative strategies and achievable rates for tree networks with optimal spatial reuse," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3596–3614, Oct. 2007.

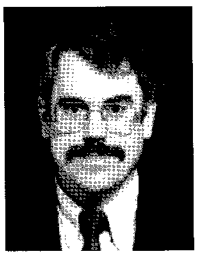
[12] C. Bae and W. E. Stark, "End-to-end energy-bandwidth tradeoff in multihop wireless networks," *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 4051–4066, Sept. 2009.

[13] R. Corless, G. Gonnet, D. Hare, D. Jeffrey, and D. Knuth, "On the Lambert-W function," *Advances in Computational Mathematics*, vol. 5, no. 1, pp. 329–359, 1996.

[14] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge Univ. Press, Cambridge, U.K., 2004

[15] J. G. Proakis, *Digital Communications*, McGraw-Hill Higher Education, 4th ed., 2000

Changhun Bae received B.S. and M.S. degrees in electrical and telecommunication engineering from Hanyang University, Seoul, Korea, in 1998 and 2000, respectively. In 2000, he joined Samsung Electronics Co., Ltd., Wireless Communication Center in Suwon, Korea as an engineer. He is currently pursuing a Ph.D. degree in electrical engineering and computer science at the University of Michigan, Ann Arbor. His current research interests are wireless communication systems and networking, and information theory with an emphasis on multihop cooperative networks.



Wayne E. Stark received B.S. (with highest honors), M.S., and Ph.D. degrees in electrical engineering from the University of Illinois, Urbana-Champaign, in 1978, 1979, and 1982, respectively. Since September 1982, he has been a Faculty Member in the Department of Electrical Engineering and Computer Science at the University of Michigan, Ann Arbor, where he is currently a Professor. His research interests include the areas of coding and communication theory, especially for spread-spectrum and wireless communications networks. He is a Member of Eta Kappa Nu, Phi Kappa Phi, and Tau Beta Phi. He was involved in the planning and organization of the 1986 International Symposium on Information Theory, held in Ann Arbor, MI. From 1984 to 1989, he was Editor for Communication Theory of the IEEE TRANSACTIONS ON COMMUNICATIONS in the area of spread-spectrum communications. He was selected by the National Science Foundation as a 1985 Presidential Young Investigator. He received the IEEE Military Communication Conference Technical Achievement Award in 2002.