Inter J Nav Archit Oc Engng (2010) 2:104~111
DOI 10.3744/JNAOE.2010.2.2.104

A Numerical study for the efficacy of flow injection on the diminution of rudder cavitation

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ABSTRACT: *The complete avoidance of cavitation, as a result of gap flow between the fixed and movable portion of a horn type rudder system, is difficult. To reduce gap flow, it is a common practice to attach a half round prismatic bar that protrudes beyond the concave surface of the horn facing the gap and laid along the centerplane of the rudder. However the employment of such a device does not always yield satisfactory results. Previously, the authors have shown that a pair of blocking bars, attached on the convex surface of the movable portion, better enhance the blocking ability of gap flow to that of a single centre bar installed on the concave surface. This also circumvents difficulties that might occur in practical applications. In the present study, a series of numerical computations show that flow injected into the gap of a rudder may also block the flow within, without employment of any physical devices, such as a half circular bar. This study also shows that the combination of flow injection and blocking bars may result in the synergic augmentation of blocking efficiency of gap flow, as demonstrated in computations for a three dimensional rudder system.*

KEY WORDS: Gap cavitation; Gap flow; Horn-type rudder.

INTRODUCTION

Rudder cavitation is frequently observed in large container ships running at high speed, and can potentially cause serious damage to the rudder device which is consequently detrimental to the safety and cost-effectiveness of the ship. Ships primarily cruise in a straight direction at high speed, which are the conditions under which rudder cavitation occurs more often and with greater severity, than for example when maneuvering slowly with a large rudder angle. Rudder cavitations occur frequently in two regions of the rudder; (1) the leading edge of the rudder, and (2) the gap between the fixed portion (horn and pintle) and close to the movable portion. The former type of cavitation can be mitigated by modifying the rudder section (Kim et al., 2008, Shen et al., 1997, Song et al., 2003). The latter type of cavitation can be countered by reducing gap clearance. However, a substantial reduction in gap clearance may cause serious technical difficulties in the rudder fitting process, and thus other means to circumvent gap cavitation are necessitated. As a result, several studies have developed mechanical devices to block gap flow. For example, Rhee and Kim (2008) suggested a cam device for gap flow blocking, while Rhee et al. (2010) used simulation experiments and computations to demonstrate a device capable of suppressing gap cavitation. However, this device is not easily mounted in the rudder system of a new ship, with the condition worsening if the ship is already in service. Boo et al. (2004) investigated the characteristics of rudder cavitation at various Reynolds numbers, and examined the effectiveness of a half circular shaped blocking bar placed in the middle of a gap. A modest reduction in gap cavitation was achieved with this device. Furthermore, the authors of the current study have previously shown that a pair of blocking bars symmetric to the center plane can enhance blocking performance of gap flow, if attached on the convex surface of the moving portion. The bars can be placed near the outer edges of the gap, which is easily accessible even at the maximum rudder angle, allowing simple installation of the device during routine ship maintenance.

Furthermore, numerical computations have shown that jet flow injection inside the gap can reduce the flow within the rudder system. In a previous study, a flow injection device installed at the horn section was employed to enhance the rudder force by Coanda effects (Seo et al. 2008, 2009). In the present study an identical device has been studied through numerical computations to determine the usefulness of the device through the retardation of the gap flow of a rudder system. The results indicate that considerable retardation of gap flow is achievable with a small amount of flow injection, even in ships traveling at high speeds. The results also found that simultaneous use of flow injection with a pair of

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blocking bars, may further improve the blocking effects. Finally, the findings are applied to a realistic three dimensional horn-type rudder system, and the synergic effect of flow injection combined with the blocking bars is examined through numerical computations and the results are compared to the experimental ones, showing evident improvement.

COMPUTATIONAL ARRANGEMENTS

A scale model of the horn-type rudder of an 8,000 TEU class container ship, with a design speed of 25 *knots* was selected (Fig. 1) for the computational model to study the performance of gap flow blocking devices. The span of the rudder is 12.3 *m* and the mean chord at the horn and pintle portion is 8 *m* and 7.46 *m*, respectively. The scale ratio of the model was selected as 1/10, for comparison with the results of the experimental rudder that will soon be available.

Preliminary numerical studies on typical rudder sections, taken at the middle of the horn and pintle portion of the rudder, were performed to determine the relationship between the amount of flow that was injected and the increase of lift, as well as between the shapes and/or locations of blocking bars and their effectiveness at blocking gap flow. The gap clearance of the sections were fixed at 5 *mm*, and computations were conducted at a rudder angle of 3 ^{o}, based on the fact that the rudder angles of cruising ships are usually within ±3 ^{o}.

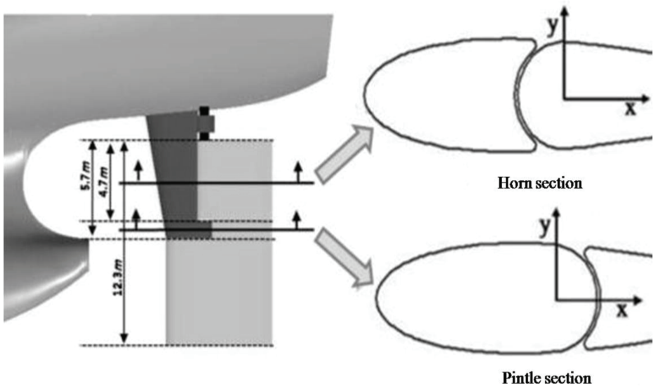
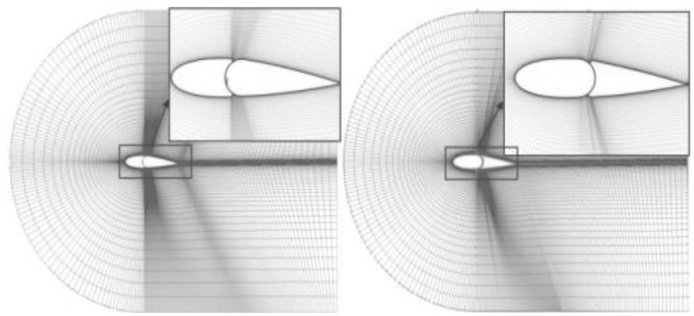


Fig. 1 Horn type rudder of an 8,000*TEU* class container ship and two dimensional sections for computations.

As shown in Fig. 2, numerical grids for two dimensional computations were calculated to have C-type topologies in the Cartesian coordinates (*x, y*), where the positive *x* axis is taken parallel to the incoming flow direction. The computational domains are taken to be $-2.6 \leqq x/c \leqq 3.6$ and $-2.6 \leqq y/c \leqq 2.6$, respectively, based on the chord lengths c of the sections. A total number of 45,000 grid points were used, and Y^{+} was kept below 65 for all two dimensional computations.

Subsequently, the optimum shapes and locations of the flow injection device and blocking bars were selected based on the two dimensional results, and adapted to a three-dimensional rudder model. Numerical computations were conducted to determine the efficacy of the devices on the lift performance of the model through comparison with a rudder without gap flow blocking devices.



(a) Horn section. (b) Pintle section.

Fig. 2 Schematic diagrams of computational domains and grids.

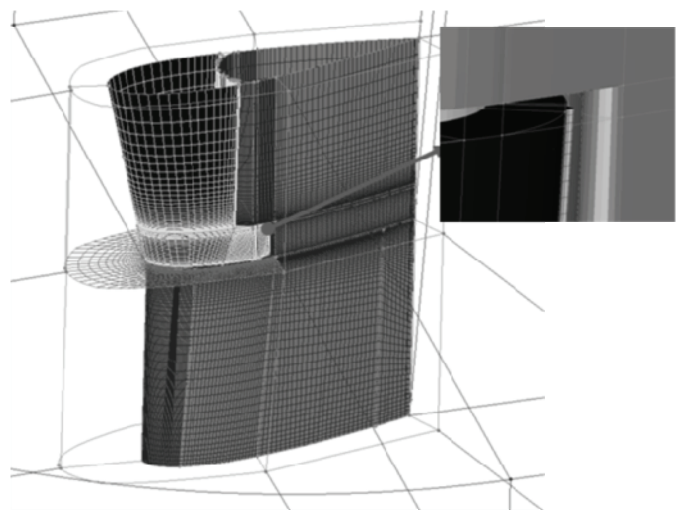


Fig. 3 Three dimensional grids around the model of the horn type rudder. Blocking bars are installed inside the vertical gap of the horn and pintle sections.

The numerical grid for the three-dimensional horn type rudder system is shown in Fig. 3. The grid has a C-H type topology, with computational domain ranges in each coordinates of $-2.6 \leqq x/c \leqq 3.6$, $-2.6 \leqq y/c \leqq 2.6$ and $0 \leqq z/c \leqq 1$ respectively, based on the chord length of 0.746 *m*, identical to the two dimensional computations of the pintle section. A total number of 1.7 *million* grid points was used, with the number of grid points laid on each side of the rudder surface being 75 × 66. Across the gap, between the horn and the rudder, 21 grid lines were laid and the value of Y^{+} was restricted to not exceed 150.

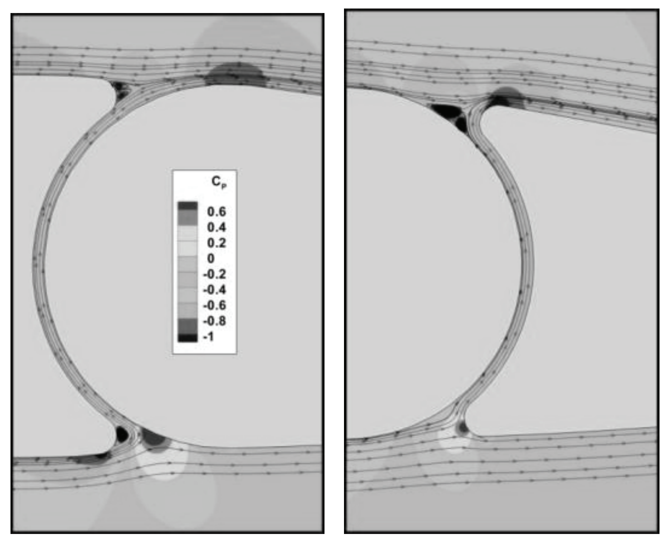
A commercial code, FLEUNT v 6.3, was used for all numerical computations performed in the present study. Note that similar computational methods were successfully used for two-dimensional flow simulations for hydrofoils (Jung et al., 2009). A cell-centered finite-volume method was employed in addition to a a linear reconstruction scheme to allow the use of computational cells of arbitrary polyhedral shapes. The velocity-pressure coupling was based on a SIMPLE type segregated algorithm. Second order accurate discretized schemes were used for pressure and momentum. QUICK and first order upwind schemes were used for turbulent kinetic energy and dissipation rate, respectively. A first order backward implicit scheme was used for time derivatives. Discretized equations were solved using point-wise Gauss-Seidel iterations, while the algebraic multi-grid method accelerated the solution convergence. The relaxation factors were set at 0.3 for pressure, 0.7 for momentum, and 0.8 for turbulence, throughout the present computations. The turbulence of the flow was approximated by the Realizable k-ε method, with a standard wall function. Boundary conditions were defined to have a uniform flow (u = 1, v = w = 0) at the inlet boundary of the domain, and static pressure at the outer boundary of the domain. The speed of the incoming uniform flow to the model was determined as 6 *m/s*, based on the results of Kim et al. (2006), which provided empirical evidence that the flow speed experienced by the rudder is much higher than a ship speed, due to the presence of a propeller upstream. Finally, a cavitation model was adopted to simulate gap cavitations that may occur near the gap of the three-dimensional rudder.

RESULTS AND DISCUSSION

Gaps between the fixed and movable portion of the rudder of a ship are necessary to permit certain allowances for the assembly and operation of the rudder system. Gap clearance may vary with rudder size and/or accuracy in the manufacturing process. The rudders of large commercial ships, considered in the present study, have gap clearances that vary within the range of about 50-100 *mm*. It is well established that gap flow is one of the main sources of rudder cavitation, yet it is impractical to completely block gap flow, even with blocking bars, because gap clearance changes with rudder angle. As a result, oversize bars may result in the rudder jamming during operation. One practical solution is to fillet joint a half-circular cylindrical bar that protrudes over 40% of the gap clearance beyond the concave surface of fixed portion facing the gap in the casting stage of the horns. However, subsequent studies have already indicated that alternative sectional shapes of the blocking bar may provide more optimal results. Additionally, the combination of flow injection and blocking bar calculated to the optimal proportions may provide a more convenient solution for practical installment and operation. Moreover, such a system may result in more optimal performance, the plausibility of which has been investigated in the present study.

Unobstructed two-dimensional gap flow

Numerical computations for the simulation of flow fields around the rudder, in the absence of blocking devices, were conducted for comparative purposes. Fig. 4 shows the computed pressure distributions and streamlining near the gap when free of blocking devices. Here, high and low pressure concentrations are noticeable near the gap on the pressure side, as well as the suction side of the moving portion. This observation indicates that cavitations are likely to occur on both sides near the gap.



(a) Horn section (b) Pintle section

Fig. 4 Pressure distributions and streamlines near the unobstructed gap (Rn =4.5×10^6, rudder angle=3^o)

Table 1 shows that the volume flux through the unit area of the gap for the horn and pintle sections were 4.34 *m/s* and 4.72 *m/s* respectively, when no blocking bar was installed. The obtained values serve as criteria for determining the efficiency of the bars and flow injection devices in blocking gap flow.

Table 1 Flux through unit area of the unobstructed gap.

	Horn section	Pintle section
Flux (*m/s*)	4.34	4.70

Gap flow retardation with blocking bars

In previous research, the authors investigated the effectiveness of various sectional shapes for the blocking bars attached to the horn surface, through numerical computations (Oh et al. 2008). It was found that a rectangle is the most effective sectional shape for the blocking bar attached to the gap of the horn section (Fig. 5 and Table 2).

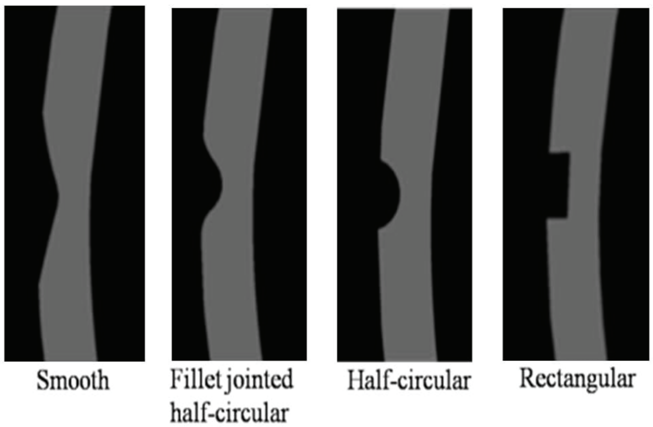


Fig. 5 Sectional shapes of the two-dimensional blocking bars attached on the horn section (Oh et al. 2008).

Table 2 Flux per unit gap area and reduction rate due to two-dimensional blocking bars of various sectional shapes placed in the horn section (Oh et al. 2008)

Sectional shape	Flux (*m/s*)	Flux reduction (%)
Smooth	3.96	8.8
Fillet jointed half-circular bar	3.68	15.2
Half-circular	3.56	18.0
Rectangular	2.54	41.5

In the present study, numerical computations were completed to investigate the effect of the bar sectional shapes attached to the pintle section (as opposed to the horn section in the previous study) on gap flow blocking. The computed results for the bars attached to the pintle section are shown in Fig.6 and Table 3.

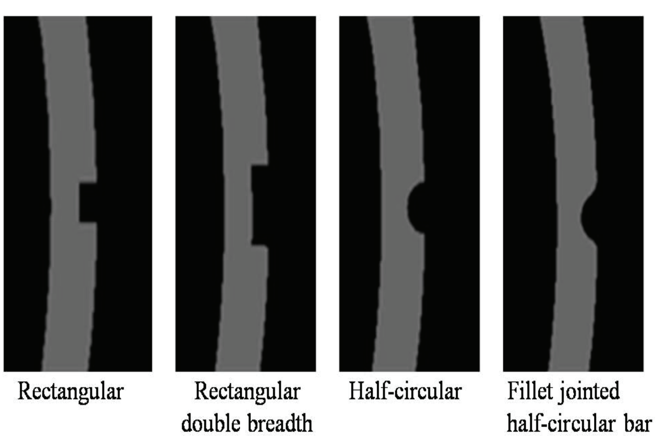
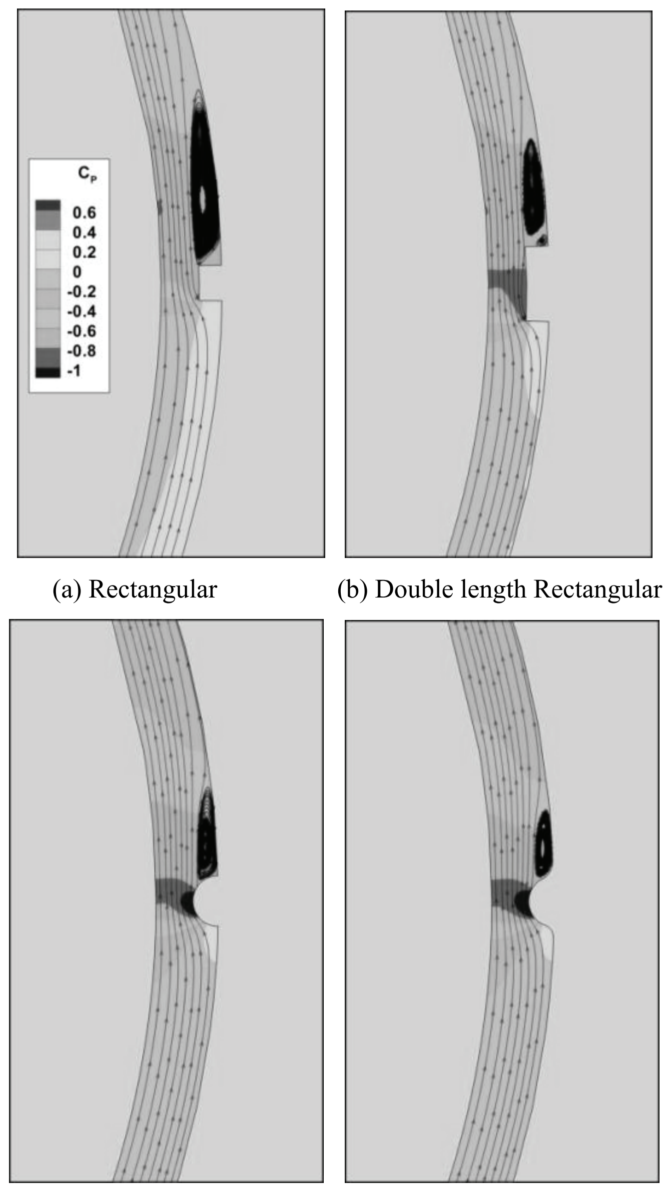


Fig. 6 Sectional shapes of the two-dimensional blocking bars attached on the pintle section

Table 3 Flux per unit gap area and reduction rate due to two-dimensional blocking bars with various sectional shapes and attached on the pintle section

Sectional shape	Flux (*m/s*)	Flux reduction (%)
Rectangular	2.92	38.0
Rectangular double breadth	3.20	32.0
Half-circular	4.00	14.9
Fillet jointed half-circular bar	4.08	13.6



(a) Rectangular (b) Double length Rectangular

(c) Half-circular (d) Fillet jointed half-circular

Fig. 7 Pressure distributions and streamlines about the two-dimensional blocking bars of various sectional shapes attached inside the gap

Tables 2 and 3 indicate that the bar with the rectangular sectional shape is the most effective for blocking gap flow in both the horn and pintle section of the rudder. For the pintle section, the reduction rate of the rectangular bar is about 38%, far exceeding the 13.6% recorded for conventional fillet jointed half-circular cylindrical bars. Fig. 7 compares the pressure distributions and streamlines close to the blocking bars, with various sectional shapes inside the gap. This also clearly indicates that the rectangular sectional bar is the most effective for blocking gap flow. However, in practice the attachment of rectangular bars on the surface of the movable portions in the casting process is not straightforward. Furthermore, the viability of this system is further reduced when using thicker sections, due to an increased possibility of interference with the fixed portion.

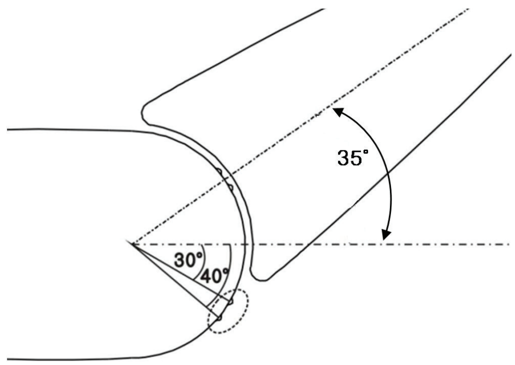


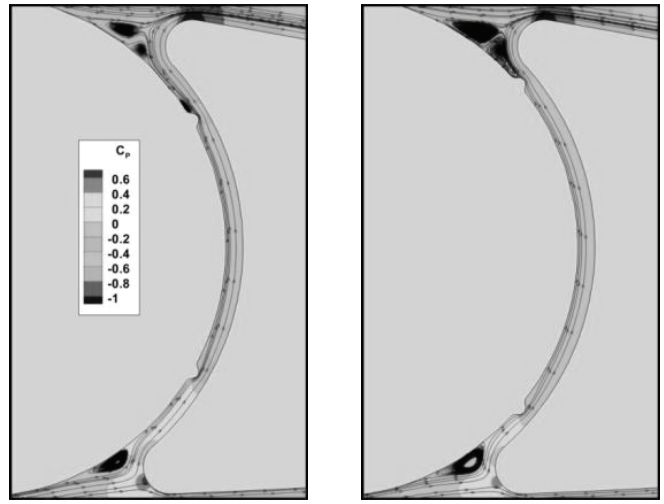
Fig. 8 Locations of symmetric blocking bars installed inside the gap of the pintle section (Oh et al. 2008).

Fig. 8 shows a pair of fillet jointed circular blocking bars that are attached symmetric to the centerplane (termed symmetric bars hereafter), for the purpose of obtaining a degree of efficiency that is equivalent to or better than the rectangular blocking bars in retarding the gap flow. Such a system could potentially reduce the impact of the installment of the bars inside the rudder gap and avoid the previously documented problems.

Table 4. Flux per unit gap area and reduction rate due to the symmetrical placement of a pair of blocking bars in the pintle section at different locations (Oh et al. 2008).

Angle from CL	Flux (*m/s*)	Flux reduction (%)
±30°	2.94	37.5
±40°	2.26	52.0

The numerical computation results for the two different locations of the symmetric blocking bars are summarized in Table 4. The table shows that the symmetric bars placed at ±30 ° produce a blocking efficiency of 37.5%, which is almost equivalent to the single rectangular bar, while the symmetric bars placed at ±40 ° produce a blocking efficiency of 52%, which is much higher than the single bar. Fig. 9 shows the pressure distributions and streamlines around the gap for both examples.



(a) ±30 ° (b) ±40 °

Fig. 9 Pressure distributions and streamlines around the gap due to a pair of symmetric bar attached at the angles of 30° and 40° from the centre line

Gap flow retardation with flow injection

The consequence of flow injection on the retardation of gap flow was investigated in the present section. For this purpose, flow injection devices were installed inside the movable portion of the horn section so that the discharged flow was directed against the incoming gap flow from the pressure side. Numerical simulations on the two-dimensional flow around the rudder and within the gap were conducted for four different flow momentum coefficient values, to estimate the correct amount of the flow injection required. The flow momentum coefficient is defined as in circulation control problems as:

$$C_j = \frac{\dot{m}V_{jet}}{\frac{1}{2}\rho V_\infty^2 CS} \tag{1}$$

V_{jet} : flow velocity through the nozzle

$\dot{m}$: mass flow rate (kg/s)

V_∞ : inflow velocity

ρ : density

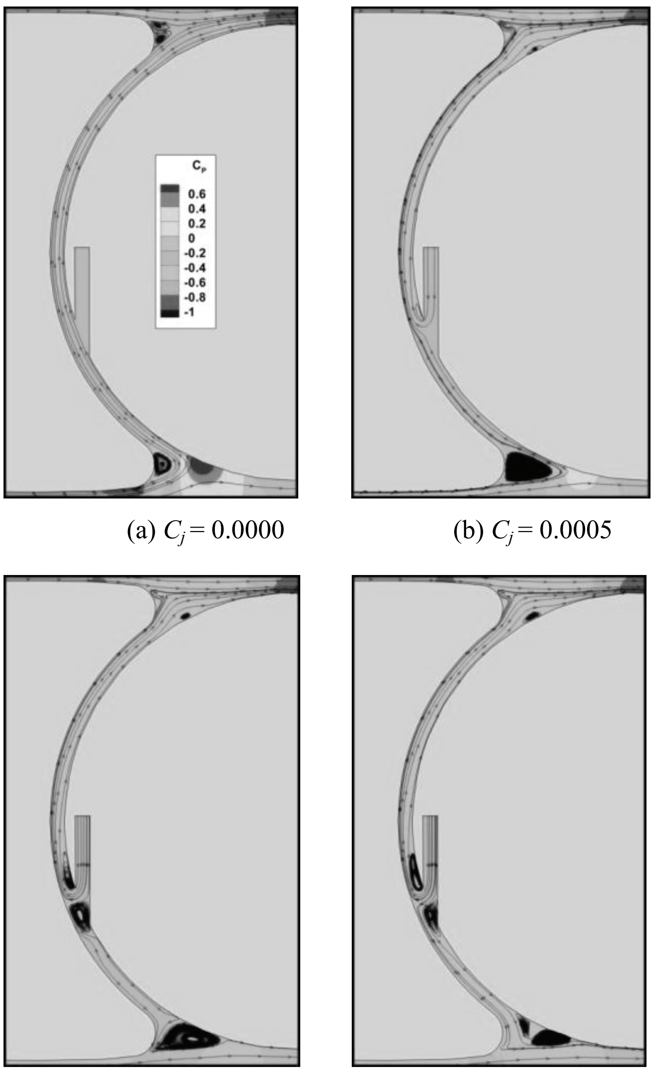
C: chord length

S: effective span length of flow injection

For example, S is taken as a unit length in two-dimensional computations, but 4.7 *m* in three-dimensional simulations for the rudder, as shown in Fig. 1. Also, the flow momentum coefficient C_j values used in the present study are 0.0, 0.0005, 0.001, and 0.0015. The results for the values of C_j are presented in Fig. 10 and Table 5.

Table 5. Comparison of flux per unit gap area and reduction rate through flow injection without a blocking bar.

C_j	Flux (*m/s*)		Flux reduction (%)
	to pressure side	to suction side	
0.0	-4.00	4.00	7.8
0.0005	-0.64	1.78	59.0
0.001	0.08	1.52	65.0
0.0015	0.48	1.50	65.4



(a) $C_j = 0.0000$ (b) $C_j = 0.0005$

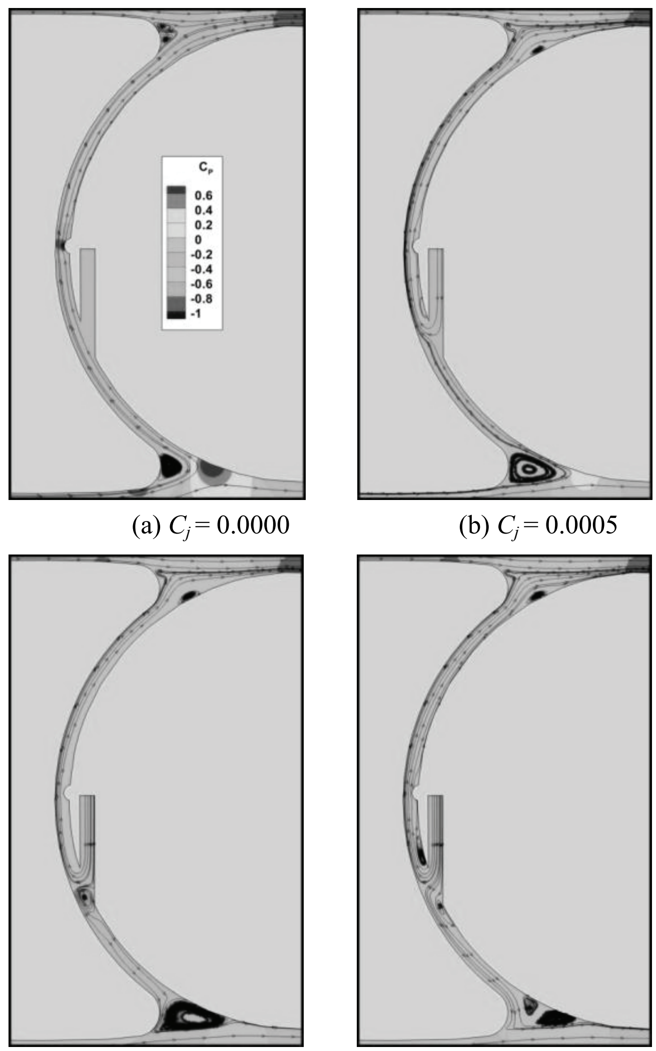
(c) $C_j = 0.001$ (d) $C_j = 0.0015$

Fig. 10 Pressure distributions and streamlines around the gap when flow injected at various flow momentum coefficients C_j ($Rn=4.5\times10^6$, rudder angle=3°).

Fig. 10 shows the influence of flow injection when gap flow is unobstructed. The figure indicates that if flow is injected into the gap towards the pressure side of horn section, the amount of flow entering the gap from the pressure side diminishes and pressure concentrations on the pressure side are weakened as C_j increases. Therefore, it can be concluded that if flow is injected against the flow coming from the pressure side, the cavitation near the gap may be mitigated as much as 65% when $C_j = 0.001$, with gap flow increasing at higher values of C_j. It also shows that the pressure distribution on the suction side near the gap tends to become unfavorable with retardation of the gap flow, which requires further investigation.

Combination of flow injection and blocking bars

In addition to the good performance of the pair of blocking bars, a combination of flow injection and a fillet jointed half circular bar attached on the horn surface was also tested.



(a) $C_j = 0.0000$ (b) $C_j = 0.0005$

(c) $C_j = 0.001$ (d) $C_j = 0.0015$

Fig. 11 Pressure distributions and streamlines around the gap with a blocking bar at various C_j (Rn =4.5×10^6, rudder angle=3°).

The results are shown in Fig. 11 and Table 6. Table 6 compares the flux per unit area of the gap for various values of C_j. The table shows that the gap flow was reduced to almost one third of the gap flux recorded in unobstructed conditions, when a half circular blocking bar was installed and flow was injected at C_j = 0.001. The conditions are sufficient for the efficient retardation of gap flow for the test case, since the additional injection of flux results in an increase in the adverse effects.

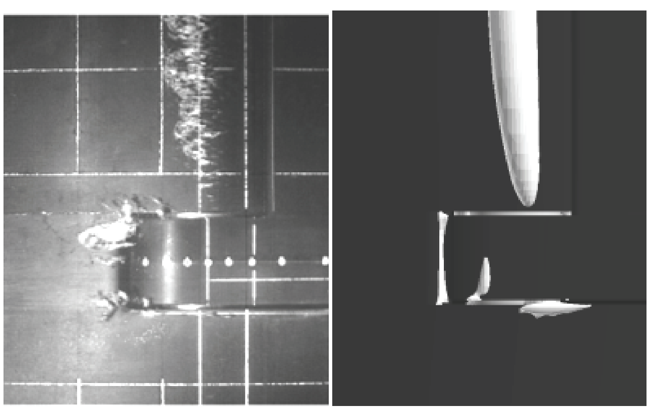
Table 6 Comparison of flux per unit gap area and reduction rate by the combination of flow injection and a blocking bar

C_j	Flux (*m/s*)		Flux reduction (%)
	to pressure side	to suction side	
0.0	-3.62	3.62	16.6
0.0005	-0.42	1.58	63.6
0.001	0.16	1.30	70.0
0.0015	0.56	1.30	70.0

Application to three-dimensional gap flow

To test the realistic efficiency of the gap flow retardation methods, the three-dimensional rudder shown in Fig. 1 was subjected to numerical analyses.

For validation of the numerical scheme, simulations were performed for comparison with existing experiments (Paik et al. 2008). Fig. 12 shows the comparison of the experimental and numerical results for three-dimensional rudder cavitation near the gap.



(a) Experimental (b) Computational

Fig. 12 Comparison between experimental (Paik et al. 2008) and computational results of the cavitation pattern over the suction side (rudder angle of 6^o, σ=1.5, Rn=2.3×10^6)

The photograph in Fig. 12a was taken at a cavitation tunnel at the rudder angle of 6 ^{o}, Reynolds number of 2.3 × 10 ^{6} and cavitation number σ of 1.5. Numerical computations were performed in the same conditions with the experiment, and the results presented in Fig. 12(b) provide qualitative agreement. The void fraction in the computations is taken as 0.1.

Based on the results from the two-dimensional studies, both the flow injection slit and the fillet jointed half circular bar were installed on the rudder surfaces facing the gap. The fillet jointed half circular symmetric bars were also installed at the pintle section at an angle of ±40 ° with respect to the centerline of the rudder. From the slit, flow was injected at C_j = 0.001 towards the pressure side.

Numerical computations were conducted to confirm the synergic effect of the flow injection and the blocking bars at a rudder angle of 3 ^{o}, the results of which are presented in Fig. 13. In Fig. 13(a, b), the value of the void fraction used for the determination of the cavity shape is 0.1, as was used in the earlier computation. Fig. 13(a) shows the pressure distribution and the shape of the gap cavity on the suction side of the rudder. A low pressure region exists along the gap, and a gap cavitation occurs at the upper corner and along the vertical gap of the pintle. Fig. 13(b) shows the results when the rudder is equipped with the combined system to block gap flow. Here, it is apparent that the low pressure region along the gap, and the gap cavitation itself, is much smaller. The retardation of gap flow is also evidenced by the fact that rudder force increases by about 19% with the employment of the blocking device.

(a) Unobstructed

(b) With the gap flow blocking device

Fig. 13 pressure distributions and estimated cavitation areas on suction side of the three-dimensional horn-type rudder (vapor volume fraction=0.1, rudder angle of 3^o, Rn=4.5×10^6, σ=1.5)

In conclusion, the installation of the combined flow injection device and symmetric bars in the three-dimensional horn type rudder are effective for the retardation of gap flow, and as a consequence results in the diminution of gap cavitation and increased rudder force.

CONCLUSIONS

Gap flow between the fixed and movable portion of a rudder system is known to be one of the major sources of rudder cavitation, particularly for large ships operating at high-speeds.

In the present study, numerical computations of two-dimensional rudder sections were performed to show that both the flow injection and the installment of blocking bars inside the gap provide effective measures to reduce the risk of rudder cavitation. The results indicate that up to 38% of gap flow can be retarded with a single blocking bar, 52% by a pair of symmetric blocking bars and 65% by flow injection. The combination of flow injection with blocking bars were speculated to have synergic effect to provide a more effective means of obstructing gap flow, with numerical computations indicating a potential retardation rate in gap flow of more than 70%. For the purposes of validation, numerical computations were conducted for comparison with the results from an existing experiment on the three-dimensional gap cavitation, the results of which indicated a good qualitative agreement with the experiment.

To examine the practical efficacy of the discussed gap flow blocking methods, a rudder system of an 8,000 *TEU* class container ship was selected for the numerical study. A combination of a flow injection device and a half circular bar was installed inside the horn of the three-dimensional rudder, for efficient retardation of gap flow. The momentum coefficient C_j of the flow was taken as 0.001 and a pair of fillet jointed half circular symmetric bars were also installed at the pintle section at angles of ±40 °, with respect to the centerline of the rudder.

The numerical computations that were performed confirm the synergic effect of the flow injection device and the blocking bars at a rudder angle of 3 ^{o}. The results indicate that a low pressure region along the gap with visible cavitation at the upper corner of the pintle was noticeably reduced with the adoption of the combined gap flow blocking device. Hence, it can be concluded that the combination of the flow injection device and symmetric bars installed in the three-dimensional horn type rudder are capable of retarding gap flow effectively.

Additional studies on the subject are required, since the low pressure region upstream of the gap had a tendency to expand when gap flow was blocked. Furthermore, since the numerical codes used in the present study cannot be used for the reliable prediction of gap cavitation, the selection of the void fraction is to a certain extent unpredictable, hence enhancements in the numerical methods and experimental work to validate the present numerical study are required.

ACKOWLEDGEMENTS

This work was supported by the Korean Research Foundation Grant (KRF-2008-005-J01603) and the Korean Science and Engineering Foundation (KOSEF) grant (ROA-2007-000-10028-0).

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