International Journal of Concrete Structures and Materials
Vol.4, No.1, pp. 25~36, June 2010
DOI 10.4334/IJCSM.2010.4.1.025

Interpreting Conservativeness in Design Criteria for Flexural Strengthening of RC Structures Using Externally Bonded FRP

Kunal D. Kansara,[1)] Tim J. Ibell,[2)] Antony P. Darby,[1)] and Mark Evernden[1)]

(Received August 11, 2009, Revised November 19, 2010, Accepted December 30, 2010)

Abstract: This paper presents the influence of various flexural strengthening design criteria specified by three important design guidelines (ACI440, TR55, FIB14) on the resulting strength, ductility and conservativeness of FRP strengthened RC elements. Various generalised mathematical relations in non-dimensional form are presented that can be employed to develop design aids for the FRP-strengthening process. A design methodology is prescribed based on these equations enabling the designer to optimally and intuitively incorporate sufficient ductility while designing for strength. In order to better interpret conservativeness within design codes, four distinct levels of embedded conservativeness are identified, which cover the entire range of sources of conservativeness. Finally, a detailed parametric study is presented, using the proposed design equations and methodology, to determine the influence of each of these four levels of conservativeness on final design solutions. Specific criteria that are useful while calibrating design guidelines are also presented.

Keywords: FRP, strengthening, concrete structure, flexural strengthening, conservativeness.

1. Introduction

Concrete structures constitute a major share of existing civil infrastructure worldwide and each of them is required to serve a specific function at or above a minimum acceptable performance level. Under the influence of environmental and mechanical actions, this infrastructure deteriorates, leading to gradual loss of performance over a period of time.[1] Furthermore, these structures may be under-performing due to changes in the live load requirements, seismic design loads (arising from the revision of design code specifications, especially for older structures) and inherent defects. It is logical to believe that improving or increasing the performance of an under-performing concrete structure, whenever possible, should be the preferred choice over replacing it with a new one. In most situations, this is not only economically more efficient but importantly it also concurs with the global movement towards sustainable development in the construction industry. Structural strengthening through retrofitting with FRP is one such attempt aimed at improving the performance level of under-performing structures.

Since the late 1980's fibre reinforced polymer (FRP) materials have been used prolifically around the world as externally bonded reinforcement for structural strengthening of reinforced concrete (RC) and steel structures. The use of FRP in such applications has proved much more successful in comparison with other traditional methods of strengthening due to a variety of advantages such as superior material properties, good corrosion resistance and adaptability to suit various sectional shapes and corners with considerable ease. Its high strength to weight ratio has made it a much preferred choice over the use of externally bonded steel plates for strengthening purposes.[2]

Many design-specific guidelines[3-7] are presently available for using externally bonded FRP for structural strengthening. However, FRP materials are fairly new to civil engineers and there exists considerable uncertainty in their structural behaviour and long term performance. Hence, the design guidelines for FRP applications for structural strengthening, in general, tend to be more conservative than usual. Furthermore, individual guidelines propose different criteria for ensuring safety, resulting in conflicting levels of conservativeness. In order to arrive at a common, efficient and rational global basis for design, it is important to evaluate the influence of such criteria on strengthening design.

This paper looks at the influence of various parameters in flexural strengthening design specifications, as prescribed by three major design guidelines: ACI440,[3] TR55[4] and FIB14.[5] Particular emphasis is placed upon the strength, ductility and residual conservativeness of the resulting design solutions. Various generalised mathematical relations in non-dimensional form are prescribed. A design methodology is proposed that enables the designer to optimally and intuitively incorporate sufficient ductility while designing for strength. To evaluate the influence of various guideline specifications on final design solutions and to demonstrate the employability of the proposed methodology and design equations, a detailed parametric study is carried out. This study also highlights qualitative and quantitative differences

[1)]Dept. of Architecture and Civil Engineering, University of Bath, Bath, BA2 7AY, UK.
[2)]Dept. of Architecture and Civil Engineering, University of Bath, Bath, BA2 7AY, UK. *Email: T.J.Ibell@bath.ac.uk.*

amongst the specifications prescribed by the three identified design guidelines in terms of resulting strength, ductility and residual conservativeness.

2. Overview of FRP strengthening design guidelines

ACI440 has been in its latest form since 2008, superseding the previous 2002 version with significant changes. It is to be noted that in its present form, ACI440 has the status of a design standard in contrast with TR55 and FIB14, which are still design guidelines. However, for convenience all three design documents are referred to as guidelines hereafter in this paper. TR55 is in its second edition, published in 2004 and superseding the original 2001 version, with significant changes. FIB14 was published in 2001 and there has been no revision since then. As stated earlier, since the use of FRP is reasonably new for civil engineering applications, it typically suffers from data-insufficiency for long-term performance. Also the knowledge on the actual or realistic behaviour that can be confidently incorporated into the design specifications is limited. This lack of confidence is apparently absorbed in all of the three design guidelines by exhibiting additional conservativeness compared with the norm, and this fact can be seen at all levels of design specifications.[8]

A detailed comparative statement representing the complete anatomical features of the three design guidelines covering all the parameters that influence the flexural strengthening design process along with issues related to failure modes, ductility, debonding criteria and philosophical similarity and incompatibility between strengthening design guidelines and conventional RC design codes are presented by Kansara *et al.*[8]. Conservativeness in the design specifications, being of primary focus within this paper, has been dealt with in distinct separate sections.

3. Design equations and methodology

It is to be noted that for the case of externally bonded FRP the possibility exists that the FRP could debond from the concrete substrate. Hence, debonding of FRP, in terms of FRP strain at which it occurs, needs to be considered along with the FRP rupture strain in order to arrive at the effective FRP strain determining the governing failure mechanism of the FRP:

$$\varepsilon_{fd} = Min(\varepsilon_{fd-debond}, \varepsilon_{fd-rupture}) \tag{1}$$

On applying externally-bonded FRP to an existing RC section, three classical failure modes (Fig. 1) are possible,[8] which are important from the design point of view.

3.1 Balanced mode of failure

The balanced mode of failure is the hypothetical mode, in which concrete reaches its ultimate strain value simultaneously with FRP reaching its governing strain limit. The state of strain in the steel at this condition is termed the critical steel strain ($\varepsilon_{st-crit}$) and the corresponding depth of the neutral axis is termed the critical depth of neutral axis (x_{u-crit}). Fig. 1(a) shows the state of strain at this failure mode. Based upon this state of strain, the following mathematical relations can be determined[8]:

$$\varepsilon_c = \varepsilon_{cu} \tag{2}$$

$$\varepsilon_{fe} = \varepsilon_{f0} + \varepsilon_{fd} \quad [\therefore \varepsilon_{fl} = \varepsilon_{fd}] \tag{3}$$

$$\frac{x_u}{d} = \frac{x_{u-crit}}{d} = \frac{1}{K}\left[\frac{\varepsilon_{cu}}{\varepsilon_{cu} + (\varepsilon_{f0} + \varepsilon_{fd})}\right] \tag{4}$$

$$\varepsilon_{st} = \varepsilon_{st-crit} = \left[\frac{1 - \frac{x_{u-crit}}{d}}{\frac{x_{u-crit}}{d}}\right]\varepsilon_{cu} = \left[\frac{1 - \frac{x_{u-crit}}{d}}{\frac{1}{K} - \frac{x_{u-crit}}{d}}\right](\varepsilon_{f0} + \varepsilon_{fd}) \tag{5}$$

$$\varepsilon_{sc} = \varepsilon_{sc-crit} = \left[\frac{x_u}{d}\frac{1}{S} - 1\right]\varepsilon_{cu} \tag{6}$$

$$\rho_f = \frac{\left(k_1 \frac{f_{ck}}{\gamma_c}\frac{x_u}{d}\right) + (\sigma_{sc}\rho_{sc}) - (\sigma_{st}\rho_{st})}{(\varepsilon_{fl}E_{fd})} \tag{7a}$$

$$\frac{M_n}{bd^2} = \overbrace{\left[k_1\frac{f_{ck}}{\gamma_c}(1-k_2)\left(\frac{x_u}{d}\right)^2\right]}^{\textit{Concrete Contribution}} + \overbrace{\left[(\sigma_{sc}\rho_{sc})\left(\frac{x_u}{d} - S\right)\right]}^{\textit{Compression Steel Contribution}} + \overbrace{\left[(\sigma_{st}\rho_{st})\left(1 - \frac{x_u}{d}\right)\right]}^{\textit{Tension Steel Contribution}} + \overbrace{\left[(\varepsilon_{fe}E_{fd}\rho_f)\left(\frac{1}{K} - \frac{x_u}{d}\right)\right]}^{\textit{FRP Contribution}} \tag{7b}$$

Equations (7a) and (7b) are general and with the use of appropriate values of x_u / d, ε_{fl} and ε_{fe} are equally applicable to other failure modes described in subsequent sections. The four square brackets in Eq. (7b) respectively represent contributions of concrete, compression steel, tension steel and FRP components towards the nominal moment of resistance of the section in general. Stresses σ_{sc} and σ_{st} are the actual stresses in the

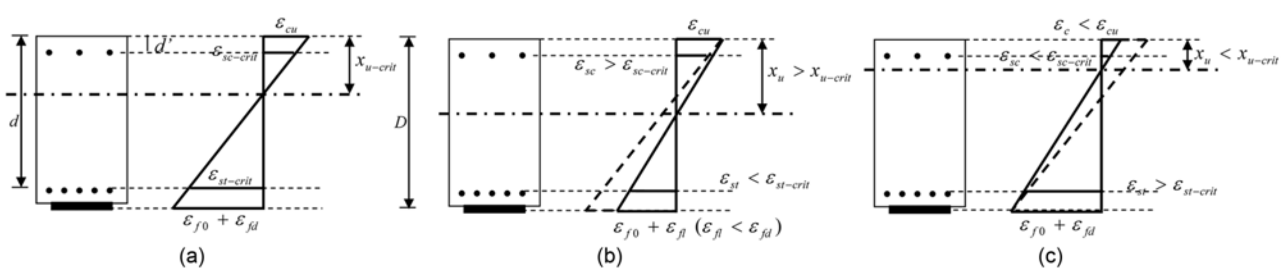

Fig. 1 Classical failure modes for FRP-strengthened RC Elements.

compression steel and tension steel respectively in Eqs. (7a) and (7b). If the strain values corresponding to these stresses are larger than the yield strain of steel (ε_{sy}), these stresses should be replaced by (f_y / γ_s). The stress-block parameters k_1 and k_2 should be as suggested by appropriate conventional RC design codes.

3.2 Failure mode controlled by concrete

For a given combination of an existing RC section with externally bonded FRP, if the resulting strain in the steel is less than $\varepsilon_{st-crit}$, it will lead to concrete crushing prior to the FRP reaching its governing strain limit. Fig. 1(b) shows the state of strain under this condition, based on which the following mathematical relations can be worked out[8]:

$$\varepsilon_{st} < \varepsilon_{st-crit} \tag{8}$$

$$\varepsilon_c < \varepsilon_{cu} \tag{9}$$

$$\frac{x_u}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{st}} \left(> \frac{x_{u-crit}}{d} \right) \tag{10}$$

$$\varepsilon_{fe} = \varepsilon_{f0} + \varepsilon_{fl} = \varepsilon_{f0} + \left[\frac{1 - \left(\frac{x_u}{d}\right) K}{\left(\frac{x_u}{d}\right) K} \varepsilon_{cu} \right] \quad [\varepsilon_{fl} < \varepsilon_{fd}] \tag{11}$$

$$\varepsilon_{sc} = \left[\frac{x_u}{d} \frac{1}{S} - 1 \right] \varepsilon_{cu} \quad (> \varepsilon_{sc-crit}) \tag{12}$$

The required FRP content and normalised moment of resistance can be expressed using Eqs. (7a) and (7b) respectively using the above equations appropriately.

3.3 Failure mode controlled by FRP

If the resulting strain in steel is more than $\varepsilon_{st-crit}$ for a given combination of an existing RC section with externally bonded FRP, it will lead to the FRP failing before the concrete reaches its ultimate strain limit. Fig. 1(c) shows the state of strain under this condition, based on which the following mathematical relations can be worked out[8]:

$$\varepsilon_{st} > \varepsilon_{st-crit} \tag{13}$$

$$\varepsilon_{fe} = \varepsilon_{f0} + \varepsilon_{fd} \quad (\because \ \varepsilon_{fl} = \varepsilon_{fd}) \tag{14}$$

$$\varepsilon_c < \varepsilon_{cu} \tag{15}$$

$$\frac{x_u}{d} = \left[\frac{\left(\frac{\varepsilon_{f0} + \varepsilon_{fd}}{\varepsilon_{st}}\right) - \left(\frac{1}{K}\right)}{\left(\frac{\varepsilon_{f0} + \varepsilon_{fd}}{\varepsilon_{st}}\right) - 1} \right] \left(< \frac{x_{u-crit}}{d} \right) \tag{16}$$

$$\varepsilon_c = \left[\frac{\left(\frac{x_u}{d}\right)}{\left(\frac{1}{K}\right) - \left(\frac{x_u}{d}\right)} \right] (\varepsilon_{f_0} + \varepsilon_{fd}) \quad (< \varepsilon_{cu}) \tag{17}$$

$$\varepsilon_{sc} = \left[\frac{\left(\frac{x_u}{d}\right) - S}{\left(\frac{1}{K}\right) - \left(\frac{x_u}{d}\right)} \right] (\varepsilon_{f_0} + \varepsilon_{fd}) \quad (< \varepsilon_{sc-crit}) \tag{18}$$

The required FRP content and normalised moment of resistance can again be expressed using Eqs. (7a) and (7b) respectively using the above equations appropriately.

Amongst concrete, steel reinforcement and FRP components, it is clear that in the case of an FRP strengthened RC section, reinforcing steel is the only reliably ductile structural component. Thus, it is quite obvious to view the strain in the tension steel reinforcement as the prime design parameter. Therefore Kansara *et al.*[8] has proposed an improved version of the design protocol (Fig. 2) based on tension steel strain (rather than assumed FRP content) which gives better control on section ductility to the designer right from the initial stages of design. Such a methodology is highly suitable for coding the FRP strengthening design process in the form of an expert system (ES) since it gives a clearer and structurally justifiable basis for design iteration.

4. Conservativeness in strengthening design guidelines

In order to provide a better understanding of inherent conservativeness in the three design guidelines, and to arrive at a framework aimed at providing a clear common basis for comparison, four distinct levels at which conservativeness is intended to be inherited in strengthening design process are identified (Fig. 3).[8] These are termed level I, II, III and IV respectively and cover the entire range of sources at which conservativeness is perceived to be inherited by all three design guidelines. Inherited conservativeness ranges from that introduced at the initial stage of defining material

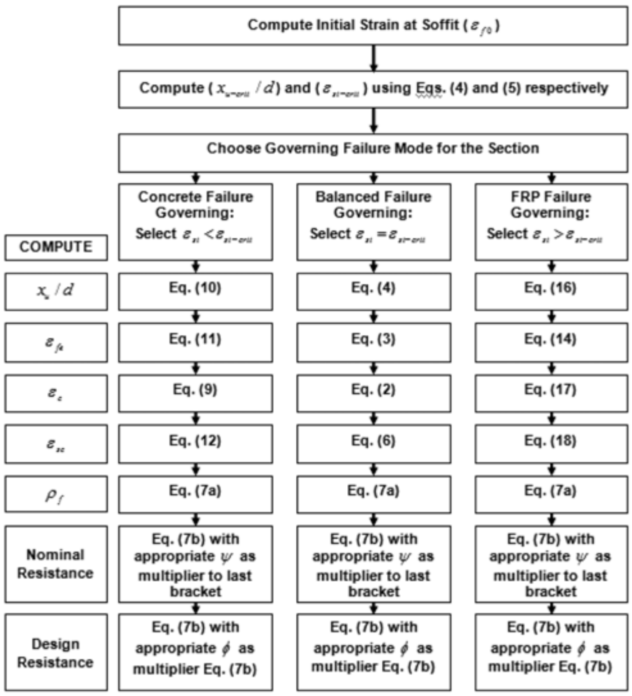

Fig. 2 Proposed design methodology.

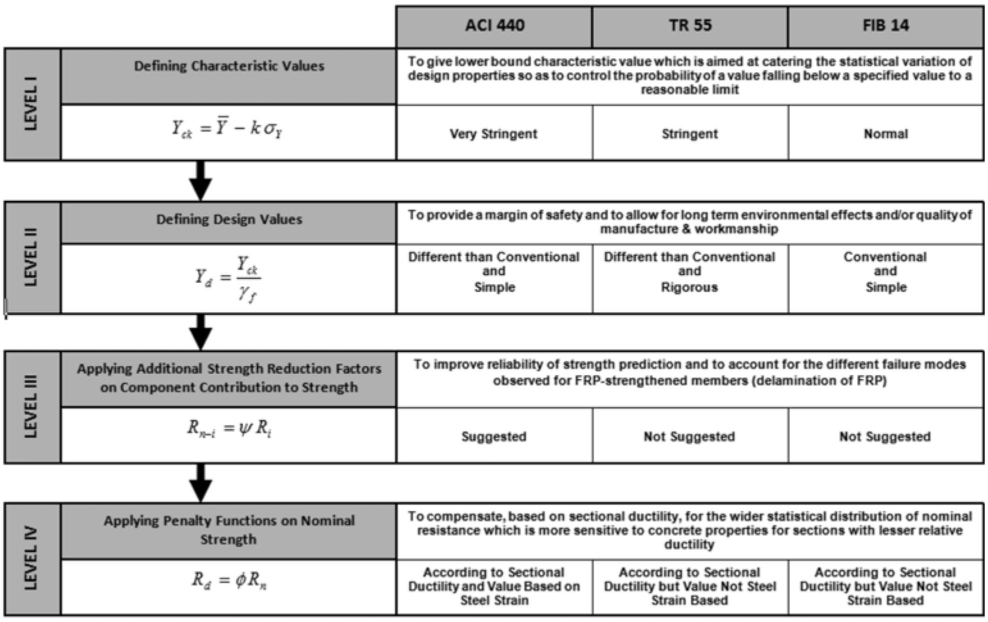

Fig. 3 Four-level conservativeness framework.

properties for design purposes through to conservativeness applied at the final stage on nominal capacity of a section arrived at by the design solution. Conservativeness appears in the order from level I to level IV in all the three design guidelines. Each of the four levels of conservativeness has a specific purpose (see Fig. 3) and is mutually independent and philosophically and mathematically exclusive of one another.[8] Hence, it is possible for a design guideline to drop one or more levels of conservativeness without affecting the design process flow, though the resulting residual conservativeness due to such omissions might vary as an obvious consequence.

4.1 Conservativeness level I: definition of characteristic values of material properties

Level I conservativeness is reflected from the prescription of a higher multiplying factor (k) applied to the statistical standard deviation (σ_Y) to absorb the difference between statistical mean ($\bar{Y}$) and lower or upper bound value of a property (Y) on probabilistic basis for arriving at the characteristic value of that material property (Y_{ck}). The values of factor k suggested by the three design guidelines are presented in Table 1. It can be seen that the ACI440 specification on factor k is most stringent (resulting in 99.87% probability that the actual values of the material properties will exceed their corresponding characteristic values for a standard sample distribution[3,8]) followed by TR55 and FIB14.

4.2 Conservativeness level II: definition of design values of material properties

Level II conservativeness is inherited while applying partial factors of safety on characteristic material properties for arriving at their corresponding design properties.

4.2.1 Partial factors of safety suggested by ACI440

ACI440 suggests use of an environmental reduction factor (C_E), which is based on the type of exposure and the type of fibre used, and is meant to represent the reduction in the material's mechanical properties under long-term environmental exposure. The factor C_E is always less than unity and is applied as a multiplier to the characteristic tensile strength (f_{fm}) and characteristic rupture strain ($\varepsilon_{fm-rupture}$) in order to arrive at the design tensile strength (f_{fd}) and design rupture strain ($\varepsilon_{fd-rupture}$) respectively.[3] The numerical value of the characteristic modulus of elasticity (E_{fm}) and design modulus of elasticity (E_{fd}) remains the same, as indicated below:

$$f_{fd} = C_E f_{fm} \tag{19}$$

$$\varepsilon_{fd-rupture} = C_E \varepsilon_{fm-rupture} \tag{20}$$

$$E_{fd} = \frac{f_{fd}}{\varepsilon_{fd-rupture}} = \frac{C_E f_{fm}}{C_E \varepsilon_{fm-rupture}} = E_{fm} \tag{21}$$

4.2.2 Partial factors of safety suggested by TR55

TR55 suggests a little more rigorous approach of specifying partial factors of safety on materials. Unlike using a single common partial factor of safety as suggested by ACI440, the

Table 1 Characteristic FRP material property definitions.

	Material property	Characteristic value definition	Factor k		
			ACI 440	TR 55	FIB 14
1	Tensile strength	$f_{fm} = \bar{f}_f - k\sigma_{ff}$	3	2	1.64
2	Rupture strain	$\varepsilon_{fm-rupture} = \bar{\varepsilon}_{f-rupture} - k\sigma_{f\varepsilon}$	3	2	1.64
3	Modulus of elasticity	$E_{fm} = \bar{E}_f - k\sigma_{fE}$	3	2	1.64

TR55 suggests two distinct basic partial factors of safety, γ_E and γ_ε, to be applied on E_{fm} and $\varepsilon_{fm-rupture}$ respectively, which are meant to account for the long-term environmental exposure.[4] However, the effect of severity of different environmental exposure classes is not specified in TR 55. Furthermore it is suggested that both of these characteristic properties are also affected by the method of application, manufacturing process and type of FRP system. Hence, an additional partial factor of safety (γ_{mm}) accounting for the variation in these two parameters based upon the application or manufacturing process is suggested[4]. Thus, the net effective partial factors of safety γ_{fE} and $\gamma_{f\varepsilon}$ to be applied to E_{fm} and $\varepsilon_{fm-rupture}$ respectively are the product of their corresponding basic partial factors of safety and additional partial factor of safety as presented below:

$$E_{fd} = \frac{E_{fm}}{\gamma_{fE}} = \frac{E_{fm}}{\gamma_E \gamma_{mm}} \tag{22}$$

$$\varepsilon_{fd-rupture} = \frac{\varepsilon_{f_m-rupture}}{\gamma_{f\varepsilon}} = \frac{\varepsilon_{fm-rupture}}{\gamma_\varepsilon \gamma_{mm}} \tag{23}$$

It is interesting to note that on using these net effective partial factors of safety for arriving at E_{fd} and $\varepsilon_{fd-rupture}$, the resulting net effective partial factor of safety (γ_{ff}) to be applied on f_{fm} carries both the basic partial factors of safety and the square of an additional partial factor of safety as presented below[4]:

$$f_{fd} = \frac{f_{fm}}{\gamma_{ff}} = \frac{E_{fm}\varepsilon_{fm-rupture}}{\gamma_{fE}\,\gamma_{f\varepsilon}} = \frac{f_{fm}}{\gamma_E \gamma_\varepsilon \gamma_{mm}^2} \tag{24}$$

4.2.3 Partial factors of safety suggested by FIB14

FIB14 follows the traditional approach of directly applying a partial factor of safety to the characteristic tensile strength of FRP alone in order to arrive at the design tensile strength, which is intended to represent the reduction in the material's strength under long-term exposure. It is based on the type of fibre and method of application used. In particular, the quality control of the FRP application is addressed.[5]

However, similar to TR55, effects of severity of different environmental exposure classes are not specified by FIB14. In order to incorporate the difference in rupture strain obtained through uniaxial tensile testing (ε_{fum}) and the expected in-situ rupture strain that can be generated within FRP (ε_{fue}), FIB14 suggests to multiply the characteristic tensile strength by a ratio ($\varepsilon_{fue}/\varepsilon_{fum}$) (Eq. (25)).[5] However, the numerical values of this ratio under different application conditions are not suggested by the FIB14.

$$f_{fd} = \frac{f_{fm}}{\gamma_f}\frac{\varepsilon_{fue}}{\varepsilon_{fum}} \tag{25}$$

4.3 Conservativeness level III: penalising elemental flexural capacity contribution

Level III conservativeness gets inherited while defining the contribution of the FRP to the section's nominal flexural capacity. It is meant to account for the fact that some of the design assumptions do not accurately reflect the true fundamental behaviour of externally-bonded FRP flexural reinforcement and to compensate for various secondary discrepancies such as shear deformation in the adhesive layer resulting in relative slip between the FRP and substrate.[3] Therefore, an additional strength reduction factor (ψ) is applied to the FRP contribution to the nominal sectional flexural capacity as a compensation (the last term in Eq. (7b)). The value of factor ψ suggested by ACI440 is 0.85, while TR55 and FIB14 prefer to drop level III conservativeness, and hence factor ψ is to be taken as unity for TR55 and FIB14.[8]

4.4 Conservativeness level IV: adjusting nominal flexural capacity

It is evident that, for given conditions, one can arrive at different possible design solutions each with different relative ductility. Typically, design solutions with a low relative ductility are more sensitive to variations in concrete strength.[3] This makes the statistical distribution of resistance wider for design solutions having lower relative ductility. The consequence of this is an increased probability of failure and reduced reliability for design solutions with low relative ductility.[8] In order to compensate for this reduction in the safety of the section, it is logical to adjust the nominal sectional capacity of the design solutions by either penalising nominal sectional capacity or by imparting over-strength based on relative ductility. Use of externally-bonded FRP on an existing RC section for flexural strengthening either limits or reduces the ductility of the member,[9] hence the concept of adjusting nominal sectional capacity is of particular importance in this case. All three of the strengthening design guidelines suggest this strength adjustment. Such means, in this study, are termed *penalty functions*[8] (ϕ), which are based upon ductility represented by the tension steel strain.

4.4.1 Penalty function suggested by ACI440

ACI440 explicitly proposes use of a strength reduction factor to be used as a multiplier on the nominal strength to arrive at the design strength of the section based on ε_{st} pertaining to a particular design solution at hand. Mathematically, it can be represented as below:

$$\phi = 0.90 \qquad (\text{For} \quad \varepsilon_{st} \geq \varepsilon_{st-adequate}) \tag{26}$$

$$\phi = 0.65 + 0.25\left[\frac{\varepsilon_{st} - \varepsilon_{sy}}{\varepsilon_{st-adequate} - \varepsilon_{sy}}\right] \qquad (\text{For} \quad \varepsilon_{sy} < \varepsilon_{st} < \varepsilon_{st-adequate}) \tag{27}$$

$$\phi = 0.65 \qquad (\text{For} \quad \varepsilon_{st} \leq \varepsilon_{st}) \tag{28}$$

The value of $\varepsilon_{st-adequate}$ is suggested to be 0.005 by ACI440, which is also consistent with ACI318.[10]

4.4.2 Penalty functions suggested by TR55 and FIB14

Penalty functions suggested by TR55 and FIB14 are of similar format and unlike explicitly specifying the use of a strength reduction factor, as suggested by ACI440, both of these guidelines suggest the provision of an over-strength factor instead. The value of $\varepsilon_{st-adequate}$ is suggested to be equal to $\left(\frac{f_y}{\gamma_s E_s} + 0.002\right)$ by TR55. The FIB14 suggests this value to be equal to 0.0043 for concrete grades C35/45 or lower and 0.0065 for concrete grades higher than C35/45. Both the guidelines suggest constant values of over-strength factor (15% and 20% for TR55 and FIB14 respectively) for the design solutions having ε_{st} less than $\varepsilon_{st-adequate}$.[4,5] This is unlike ACI440 which specifies a strength reduction factor in proportion to ε_{st}. No over-strength factor is suggested by either of these two guidelines for design solutions with ductility equal to or more than $\varepsilon_{st-adequate}$.[4,5] Mathematically, the penalty functions for TR55 and FIB14 can be written as follows:

$$\frac{M_{u-strengthened}}{M_{u-reqd}} \geq 1.15 \Rightarrow \phi \leq \frac{1}{1.15} = 0.87$$

(TR55: For $\varepsilon_{st} < \varepsilon_{st-adequate}$) (29)

$$\frac{M_{u-strengthened}}{M_{u-reqd}} \geq 1.20 \Rightarrow \phi \leq \frac{1}{1.20} = 0.83$$

(FIB14: For $\varepsilon_{st} < \varepsilon_{st-adequate}$) (30)

$$\frac{M_{u-strengthened}}{M_{u-reqd}} = 1.00 \Rightarrow \phi = 1.00$$

(TR55 and FIB14: For $\varepsilon_{st} \geq \varepsilon_{st-adequate}$) (31)

5. Influence of conservativeness upon the strengthening design process

The implications of the four identified levels of inherited conservativeness are derived based on a detailed parametric study carried out using the design equations and methodology proposed in this paper. Major variables include $\bar{\varepsilon}_{f-rupture}$ and $\sigma_{f\varepsilon}$ (together these represent rupture strain capacity of FRP statistically), ε_{st} (representing ductility), $\varepsilon_{fd-debond}$, factor k (representing level I conservativeness), general partial factor of safety (γ) on rupture strain (representing level II conservativeness), additional strength reduction factor (ψ) on the FRP component on flexural capacity (representing level III conservativeness) and the penalty function (ϕ) (representing level IV conservativeness). The impact of various parameters are mainly described in the form of normalised flexural capacity (M/bd^2) and an indicative parameter termed as residual conservativeness index (*RCI*).

5.1 Influence of debonding strain limit criterion

For an FRP-strengthened RC element, the FRP component can have two possible failure mechanisms: debonding or rupture of FRP. It is experimentally observed that the former is more likely to occur in most instances.[11] This is evident in all three of the design guidelines in the form of stringent debonding strain limit criteria. ACI440 suggests two empirical criteria for arriving at a debonding strain limit. The minimum of these two values will be the governing debonding strain limit.[3]

$$\varepsilon_{fd-debond1} = 0.41\sqrt{\frac{f_c'}{nE_{fd}\,t_f}} \quad (32)$$

$$\varepsilon_{fd-debond2} = 0.9\,\varepsilon_{fd-rupture} \quad (33)$$

$$\varepsilon_{fd-debond} = Min(\varepsilon_{fd-debond1}, \varepsilon_{fd-debond2}) \quad (34)$$

It can be seen that Eq. (33) is set to always ensure that the debonding strain value is less than the rupture strain value of FRP. This implies that based on ACI440 guideline specifications, the FRP is invariably intended to fail under debonding and not by rupture. Under this condition, level I and II conservativeness associated with rupture of the FRP implicitly finds its way into the final design solution.

TR55, on the other hand, primarily suggests a constant value of 0.008 as a debonding limit strain[4] and hence the design FRP rupture strain may or may not govern, depending upon its relative value compared to the prescribed debonding strain limit of 0.008. FIB14 suggests a strain range of 0.0065 to 0.0085 as the debonding strain limit.[5] However, it also suggests various detailed debonding models and the description of debonding is most rigorously dealt with by FIB14.

By reviewing all such debonding criteria, the resulting strain limit may range approximately from 0.006 to 0.010 and hence this range is utilised for the parametric study in this paper. It is shown that in spite of high potential rupture strain capacity of FRP materials, only a fraction of that is available for design purposes due to the stringent debonding strain limits on the FRP.[8] This observation clearly indicates the need for providing (mechanical or other) means of improving the debonding strain limit in order to make more efficient use of the high strain capacities of FRP materials.

5.2 Interpreting level I and II conservativeness

As mentioned earlier, level I and II conservativeness deal with characteristic material properties and design material properties respectively. The material property of interest here is the FRP failure strain. It is obvious that the inferences derived in this paper for level I and II conservativeness are solely applicable to the failure modes that involve FRP failure. It is shown later in this paper that for the flexural strengthening of RC structures using externally bonded FRP, this failure mode occupies a comparatively small portion of the possible range of practical design solutions and a major portion of this range is occupied by the failure mode that involves concrete crushing without the FRP reaching its governing failure limit. The influence of level I and II conservativeness is derived in terms of a mathematical relation between factor k(representing level I conservativeness), γ (representing level II conservativeness), $\bar{\varepsilon}_{f-rupture}$ and $\varepsilon_{fd-debond}$. The existence of a critical value of factor k (termed as k_{crit}) has been shown here, when level I and II conservativeness are seen together under the influence of a particular debonding strain limit criterion. There is a particular value of factor k for given conditions at which $\varepsilon_{fd-rupture}$ will be numerically equal to $\varepsilon_{fd-debond}$ and can be expressed mathematically as follows[8]:

$$k_{crit} = \frac{\bar{\varepsilon}_{f-rupture} - (\gamma\varepsilon_{fd-debond})}{\sigma_{f\varepsilon}} \quad (35)$$

A little consideration will show that for given other conditions, if a design guideline specifies a value of factor k greater than k_{crit}, the resulting numerical value of design FRP rupture strain will be less than the debonding limit strain value and hence the design solution consequently will involve FRP rupture and not debonding. Thus, better utilisation of FRP rupture strain capacity can be achieved which would otherwise go to waste in cases where debonding strain values govern the FRP failure. These observations, in light of Eq. (1), can be written mathematically as follows[8]:

$$k = k_{crit} \Rightarrow \varepsilon_{fd-rupture} = \varepsilon_{fd-debond} = \varepsilon_{fd} \quad (36)$$

$$k > k_{crit} \Rightarrow \varepsilon_{fd-rupture} < \varepsilon_{fd-debond} \Rightarrow \varepsilon_{fd} = \varepsilon_{fd-rupture} \quad (37)$$

$$k < k_{crit} \Rightarrow \varepsilon_{fd-rupture} > \varepsilon_{fd-debond} \Rightarrow \varepsilon_{fd} = \varepsilon_{fd-debond} \quad (38)$$

It is to be noted that the design solutions involving FRP failure, and belonging to a condition represented by Eq. (38), will be either implicitly sensitive or totally insensitive to the level I & II conservativeness.[8] This helps to identify a lower bound value for factor k for given conditions (i.e. $k \geq k_{crit}$) and is useful when calibrating design guidelines.[8] Fig. 4 shows the sensitivity of factor k to statistical standard deviation of the sample ($\sigma_{f\varepsilon}$) and it can be clearly seen that higher values of k are more sensitive to $\sigma_{f\varepsilon}$. From Fig. 4 it can further be seen that for given conditions there exists a particular value of $\bar{\varepsilon}_{f-rupture}$ corresponding to $k = 0$. From Eq. (35) it can be inferred that this condition is only achieved for non-deterministic representation of $\bar{\varepsilon}_{f-rupture}$ (for defining its characteristic value only) when the numerical value of $\bar{\varepsilon}_{f-rupture}$ becomes equal to the product $\gamma \times \bar{\varepsilon}_{fd-debond}$.[8] It should be noted that for given conditions, this particular value of $\bar{\varepsilon}_{f-rupture}$ is insensitive to $\sigma_{f\varepsilon}$ and hence this value is termed the critical FRP mean rupture strain ($\bar{\varepsilon}_{f-crit}$).[8] It can also be seen from Fig. 4 that with an increase in the numerical value of the debonding limit strain, the (k-$\varepsilon_{f-rupture}$) curves shift towards higher values of $\bar{\varepsilon}_{f-rupture}$. This finding signifies that if by any means (mechanical or otherwise) it is possible to raise the debonding strain limit value, for the same prescribed value of factor k the corresponding value of $\bar{\varepsilon}_{f-crit}$ will be higher, which indicates the potential for more efficient use of higher rupture strain capacity FRP materials. Furthermore, the sensitivity of factor k, as seen from Fig. 4, remains unchanged with varying debonding strain limit values. It is to be noted that very low values of $\bar{\varepsilon}_{f-rupture}$ result in negative corresponding values of factor k, which is practically not a possibility for factor k. However, for the sake of completeness, these values are included in Fig. 4. Figure 5 provides an example of a typical situation represented by $\varepsilon_{fd-debond}$ equal to 0.008 and γ equal to 1.50. This is a better representation of the (k-$\bar{\varepsilon}_{f-rupture}$) relation for design purposes in order to find out a corresponding value of $\bar{\varepsilon}_{f-crit}$ to be used for a prescribed value of factor k under given conditions. The significance of the (k-$\bar{\varepsilon}_{f-rupture}$) relation can be mathematically described as follows[8]:

$$\bar{\varepsilon}_{f-rupture} = \bar{\varepsilon}_{f-crit} \Rightarrow \varepsilon_{fd-rupture} = \varepsilon_{fd-debond} = \varepsilon_{fd} \quad (39)$$

$$\bar{\varepsilon}_{f-rupture} > \bar{\varepsilon}_{f-crit} \Rightarrow \varepsilon_{fd-rupture} > \varepsilon_{fd-debond} \Rightarrow \varepsilon_{fd} = \varepsilon_{fd-debond} \quad (40)$$

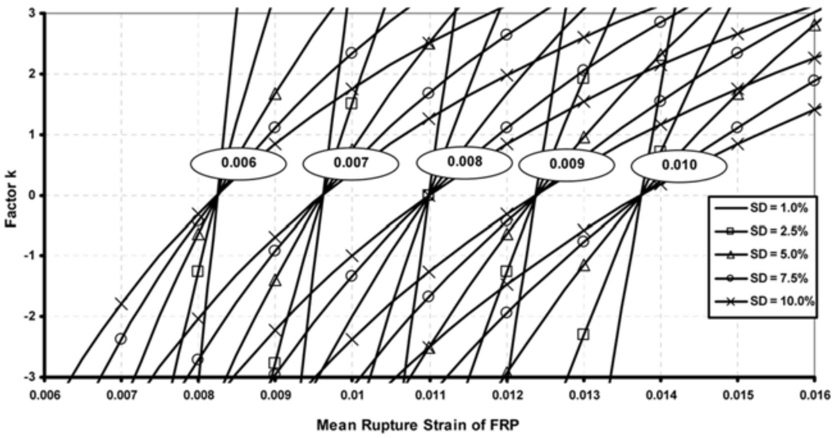

Fig. 4 Typical (k-$\bar{\varepsilon}_{f-rupture}$) plot indicating sensitivity of factor k to statistical standard deviation $\sigma_{f\varepsilon}$.

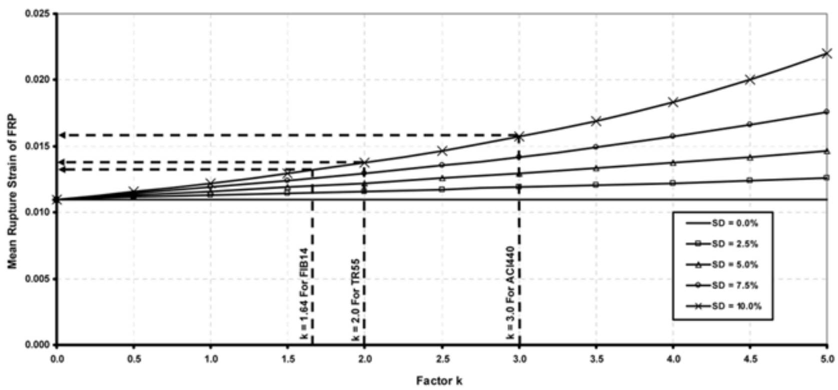

Fig. 5 Typical (k-$\bar{\varepsilon}_{f-rupture}$) plot for design purpose.

$$\bar{\varepsilon}_{f-rupture} < \bar{\varepsilon}_{f-crit} \Rightarrow \varepsilon_{fd-rupture} < \varepsilon_{fd-debond} \Rightarrow \varepsilon_{fd} = \varepsilon_{fd-rupture} \quad (41)$$

Thus, for a prescribed value of factor k under given conditions, it is possible to arrive at an upper bound value of $\bar{\varepsilon}_{f-rupture}$ (i.e. $\bar{\varepsilon}_{f-rupture} \leq \bar{\varepsilon}_{f-crit}$) signifying that for ($0 < \bar{\varepsilon}_{f-rupture} < \bar{\varepsilon}_{f-crit}$), the resulting design solution will involve FRP rupture and hence will result in an efficient use of the strain rupture capacity of the FRP material.[8]

From Eq. (35) it is clear that the relationship between $\bar{\varepsilon}_{f-rupture}$ and prescribed values of $\varepsilon_{fd-debond}$ depends upon level I and II conservativeness which can be mathematically captured using variables k, $\sigma_{f\varepsilon}$ and γ. The practical range of factor k is [0, 3] and that of $\sigma_{f\varepsilon}$ is [~0 %, 10 %].[8] Based on the values suggested by TR55 and ACI440, the possible ranges for the partial factor of safety on FRP strain is [1.31, 2.93] and [1.05, 2] respectively[8]. It is to be noted that the reciprocal of the factor (C_E) is to be used as the equivalent (γ) in Eq. (35) for ACI440. As mentioned earlier, FIB14 suggests the application of a partial factor of safety on FRP tensile strength and not on FRP rupture strain directly. However, in this study it is believed that the partial factor of safety suggested for FRP tensile strength is equally applicable to FRP rupture strain. This can be justified since FRP can be considered to follow a linear elastic stress-strain relationship right up to the point of rupture. Furthermore it can be assumed that the modulus of elasticity remains unchanged. Under these two assumptions, it can be mathematically demonstrated that the tensile strength and rupture strain of FRP carry the same partial factor of safety.[8] Thus, considering the same partial factor of safety applicable to FRP rupture strain, the possible range of equivalent γ is [1.2, 1.5] based on FIB14 guideline specifications.[8] The total range of γ, covering all the three design guidelines is thus [1.05, 2.93].[8]

5.3 Interpreting level III and IV conservativeness

Since level III and IV conservativeness deal with the flexural contribution of the FRP and the FRP-strengthened RC section respectively, their influence is represented primarily in terms of the normalised flexural capacity (M/bd^2). For demonstration purposes, a typical RC section of overall size (380 × 610 mm) and effective size (380 × 557.5 mm) has been selected and is assessed using the methodology proposed in this paper. A concrete cube strength of 30 MPa is used and the section is reinforced with four 25 mm diameter steel bars having $f_y = 415$ MPa and $E_s = 200{,}000$ MPa. It is assumed that there is no compression steel reinforcement. To work out the initial strain in the FRP (ε_{f0}), the moment under dead load (M_{DL}) is considered to be 100 kN·m. The parameters related to conservativeness as applicable to FRP are varied for these data. Thus, the quantitative inferences presented in this part of the parametric study are for the assumed data only while the qualitative inferences are equally applicable to all design situations.

Figures 6 and 7 respectively present variations in normalised

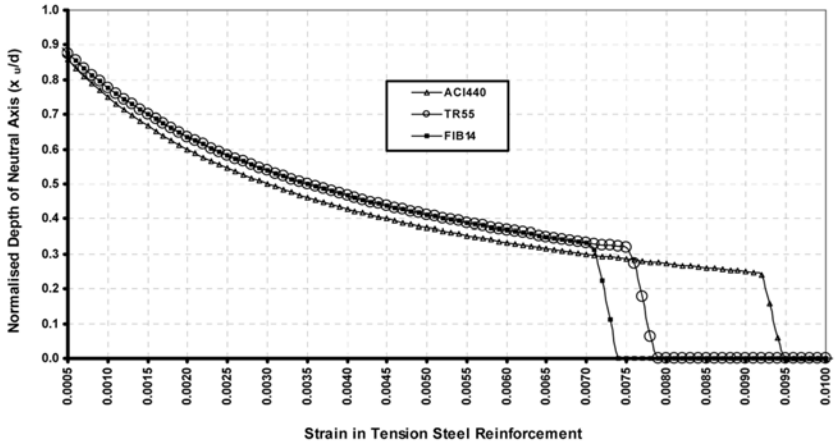

Fig. 6 Variation in normalised depth of neutral axis (x_u / d).

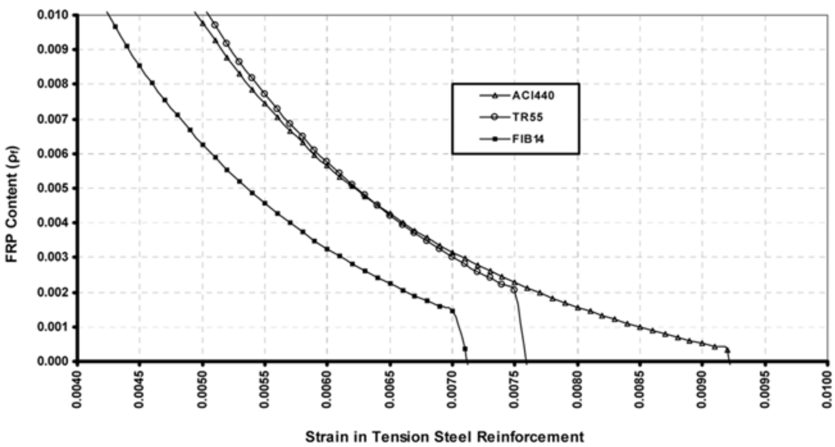

Fig. 7 Variation in FRP content (ρ_f).

depth of neutral axis (x_u / d) required for FRP quantities (ρ_f) for ACI440, TR55 and FIB14. Each point on the curve represents a design solution having a particular level of ductility (in terms of ε_{st}). The point at which these curves show sharp deviation indicates $\varepsilon_{st-crit}$, and all the design solutions falling in the range ($\varepsilon_{st} < \varepsilon_{st-crit}$) do not involve FRP failure at the ultimate condition. It can be seen from Figs. 6 and 7 that a significant proportion of the possible design solutions fall under the strain range ($\varepsilon_{st} < \varepsilon_{st-crit}$). It is shown later that the final level of inherited conservativeness into a design solution is significantly different for the design solutions falling in the range ($\varepsilon_{st} < \varepsilon_{st-crit}$) to those falling in the range ($\varepsilon_{st} > \varepsilon_{st-crit}$). It can be seen from Fig. 7 that the required FRP content (ρ_f) for design solutions in the range ($\varepsilon_{st} < \varepsilon_{st-crit}$) is significantly higher than that in the range ($\varepsilon_{st} > \varepsilon_{st-crit}$).

In order to present a picture of the relative influence of level I to IV conservativeness, the factor M / bd^2 has been calculated for five distinct combinations of identified conservativeness levels as described in Table 2. It is to be noted that in spite of the combination C0 having all of the conservativeness on the FRP dissolved, it still carries all the partial factors of safety on the concrete and steel reinforcement as conventionally prescribed by appropriate RC design codes.[10,12,13] Fig. 8(a)-(c) presents the net sectional flexural capacity in terms of M / bd^2 for these five distinct combinations for ACI440, TR55 and FIB14 respectively. It can be inferred from Fig. 8 that incorporating high ductility in design is achieved at the cost of strength. This simply implies that ductility and flexural strength should be seen together as an optimisation issue while designing, especially for FRP strengthened RC elements where ductility has more significance than conventional RC element design.[8]

It can further be seen from Fig. 8 that M / bd^2 curves for combinations C0, C1 and C2 coincide exactly with one another for the design solutions falling in the range ($\varepsilon_{st} < \varepsilon_{st-crit}$), signifying that design solutions in this range are totally insensitive to level I and II conservativeness. This is clearly obvious as design solutions in this range do not involve FRP failure at the ultimate limit state. Level I and II conservativeness are only inherited into the design for design solutions falling in the range ($\varepsilon_{st} > \varepsilon_{st-crit}$) which involves failure of the FRP. It is thus important to note that in order to maintain the probability of failure (and reliability) of the same order for the design solutions falling in the range ($\varepsilon_{st} < \varepsilon_{st-crit}$) and ($\varepsilon_{st} > \varepsilon_{st-crit}$), different treatment for design solutions in these two ranges is warranted. M / bd^2 curves for

Table 2 Conservativeness combinations.

<table>
<tr><th></th><th>Conservativeness combination</th><th>Active levels of conservativeness</th><th>Dropped levels of conservativeness</th><th>Remarks</th></tr>
<tr><td>1</td><td>C0</td><td>-</td><td>I, II, III, IV</td><td rowspan="5">Partial factors of safety on concrete and steel components are still accounted</td></tr>
<tr><td>2</td><td>C1</td><td>I</td><td>II, III, IV</td></tr>
<tr><td>3</td><td>C2</td><td>I, II</td><td>III, IV</td></tr>
<tr><td>4</td><td>C3</td><td>I, II, III</td><td>IV</td></tr>
<tr><td>5</td><td>C4</td><td>I, II, III, IV</td><td>-</td></tr>
</table>

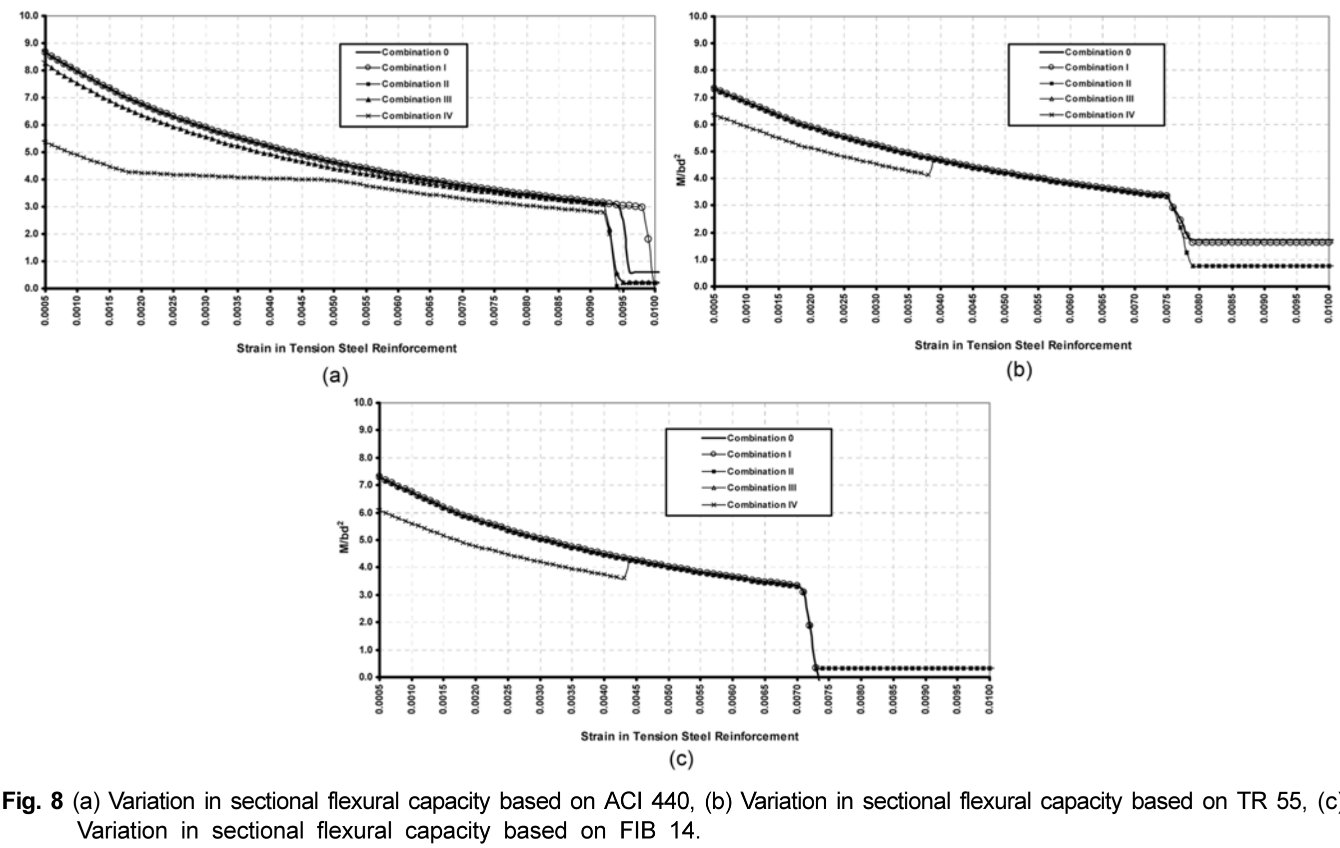

Fig. 8 (a) Variation in sectional flexural capacity based on ACI 440, (b) Variation in sectional flexural capacity based on TR 55, (c) Variation in sectional flexural capacity based on FIB 14.

for combinations C0, C1 and C2 for ACI440 guideline specifications, as evident from Fig. 8(a). For TR55 and FIB14 guideline specifications, since level III conservativeness is absent, M/bd^2 curves for the combination C3 also coincide with the curves for combinations C0, C1 and C2 as seen from Fig. 8(b) and (c). It is observed that even for design solutions in the range ($\varepsilon_{st} > \varepsilon_{st-crit}$), level I conservativeness has minimal effect on section capacity. This is evident from the figures showing M/bd^2 curves for combinations C0 and C1 which sit very close to each other, though not exactly coinciding. Furthermore, level II conservativeness has a sizable influence on section capacity as observed from the M/bd^2 curve for combination C2 taking a sharp and steep deviation from the curves for combinations C0 and C1. There is also an absence of level IV conservativeness for TR55 and FIB14 in the strain range ($\varepsilon_{st} > \varepsilon_{st-adequate}$), and hence M/bd^2 curves for combinations C2, C3 and C4 coincide with one another in this range as seen from Fig. 8(b) and (c). Kansara *et al.*[8] have shown that under an increase in the numerical value of $\bar{\varepsilon}_{f-rupture}$, the design curves show a tendency to shift $\varepsilon_{st-crit}$ to higher values. A little consideration will show that it is advantageous to have a higher value of $\varepsilon_{st-crit}$ for given conditions since $\varepsilon_{st-crit}$ itself is an optimal point in the sense that at this point both the concrete and FRP components of a section are utilised to their maximum strain capacities and a higher numerical value of $\varepsilon_{st-crit}$ signifies more ductility of the section. It is to be noted that for other given conditions, the M/bd^2 curves remain unchanged for the design solutions in the range ($\varepsilon_{st} < \varepsilon_{st-crit}$) even with a change in $\bar{\varepsilon}_{f-rupture}$. Fig. 9 provides a comparative picture of sectional flexural capacity in terms of M/bd^2 for all three design guidelines with respect to bare sectional capacity. M/bd^2 curves for each design guideline in Fig. 9 are for combination C4 (i.e. inheriting all four levels of conservativeness) and the M/bd^2 curve for the bare value is for combination C0 (i.e. it does not inherit any conservativeness on the FRP component). Fig. 10 presents the same picture in terms of residual conservativeness index (*RCI*) for the three design guidelines. *RCI*, with its higher value representing more relative conservativeness, is mathematically described as follows[8]:

$$RCI = 1 - \left[\frac{\left(\frac{M}{bd^2}\right)_{combination-IV}}{\left(\frac{M}{bd^2}\right)_{combination-0}} \right] \tag{42}$$

It is clearly evident from Fig. 10 that TR55 for most part of the possible steel strain range results in the least conservative design solutions in comparison with the other two design guidelines. This is in spite of prescribing stringent specifications for level I and II. Figures 11 and 12 concisely present the influence of the identified levels of conservativeness on flexural capacity in the form of M/bd^2 and *RCI* solution surfaces respectively for ACI440 guidelines for a particular value of $\bar{\varepsilon}_{f-rupture} = 0.015$ and $\varepsilon_{fd-debond} = 0.008$. An

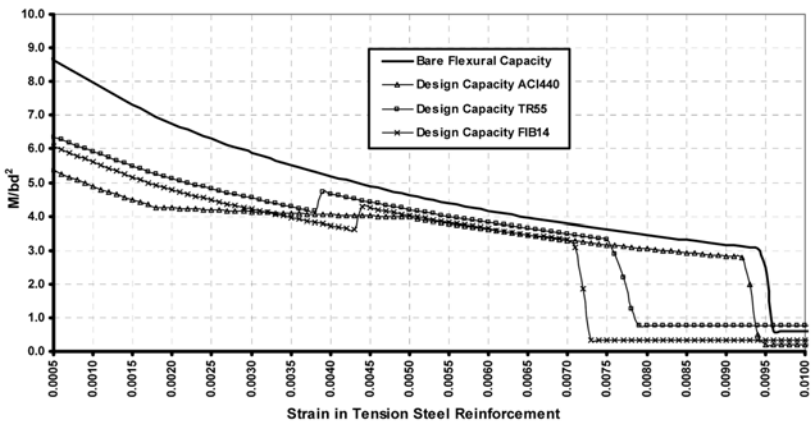

Fig. 9 Comparative sectional flexural capacity under identical conditions.

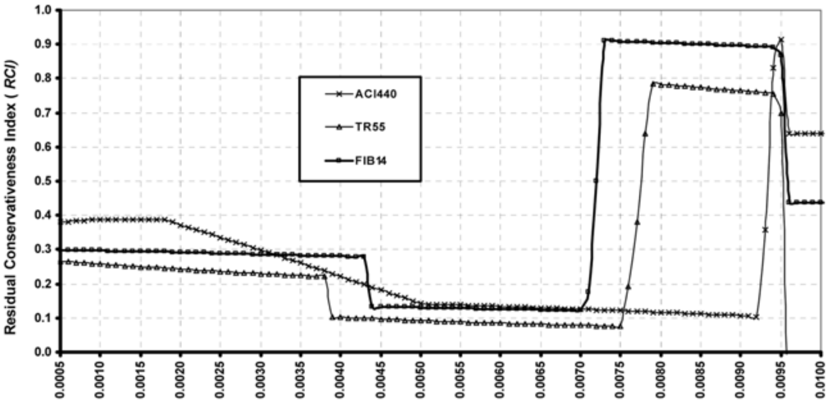

Fig. 10 RCI plot under identical conditions.

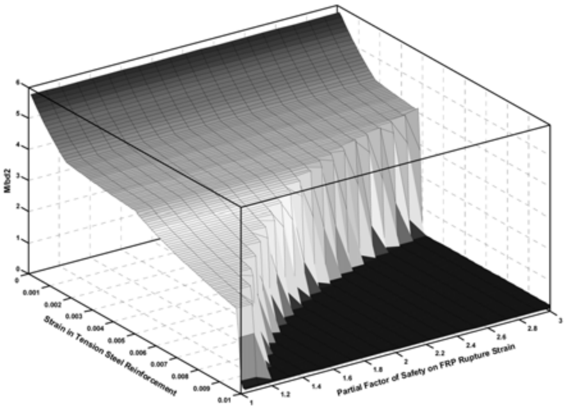

Fig. 11 M / bd^2 solution surface for ACI440.

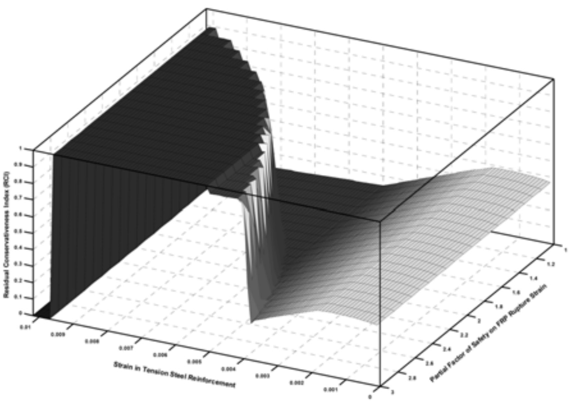

Fig. 12 *RCI* solution surface for ACI440.

important influence of the partial factor of safety (γ) on design solutions for given level I, III and IV conservativeness (i.e. factors k, ψ and ϕ respectively) and the debonding strain limit criterion is to lower the resulting $\varepsilon_{st-crit}$ values with increasing γ, as evident from Fig. 11. Furthermore, from Fig. 11 it is also seen that the M / bd^2 solution surface slopes along γ whilst the ε_{st} axes (before reaching strain level $\varepsilon_{st-crit}$) remain unchanged, which indicates γ has no influence on M / bd^2 in the strain range ($\varepsilon_{st} < \varepsilon_{st-crit}$). As indicated earlier, this is quite obvious since design solutions in this strain range do not involve FRP failure. It is also observed that for the initial part of the M / bd^2 solution surface up to a certain value of γ, there is no change in the M / bd^2 values. This is due to the fact that for lower values of γ, the debonding strain limit will govern FRP failure over FRP rupture strain and hence there is no influence of γ observed on the design solutions. It can be clearly seen from Figs. 10 and 12 that residual conservativeness associated with different design solutions vary quite markedly, which signifies that the resulting probability of failure (and reliability) of various design solutions will be significantly different. Plots similar to Figs. 11 and 12 for TR55 and FIB14 are presented by Kansara *et al.*[8]

6. Conclusions

Based on this investigation, the following broad conclusions can be arrived at:

1) It has been demonstrated in this paper that level I and II conservativeness do not significantly influence the final designs for the case of design solutions not involving FRP failure, or those involving FRP debonding. The reasons for the former case is quite obvious, while for the latter case it is due to the fact that level I and II conservativeness are applied on FRP rupture strain and not on debonding strain and hence level I and II conservativeness do not get inherited into the design solutions involving FRP debonding failure. The result of this is that the widely varying definitions of characteristic FRP rupture strain given by various design guidelines have limited significance in terms of residual conservativeness in the final design solutions.

2) Partial factors of safety representing level II conservativeness have significant influence on the design solutions involving FRP rupture failures. Amongst the three design guidelines considered in this paper, TR55 prescribes the most stringent partial factor of safety on FRP rupture strain followed by ACI440 and FIB14. However, in spite of the stringent criteria, TR55 for most part of the possible steel strain range shows the least residual conservativeness in the design solutions amongst the three design guidelines.

3) It has been demonstrated that conservativeness level I and II gets inherited into the final design solutions only for the design solutions involving FRP failure through rupture. Due to this the residual conservativeness for design solutions involving different failure modes differs considerably, resulting in significantly different values for failure probability and reliability. Hence, in order to maintain the probability of failure and reliability within the same order, different treatment is needed for design solutions falling in different tension steel strain ranges.

Acknowledgments

The authors wish to gratefully acknowledge HEFCE and the University of Bath for financial support in the form of ORS and University of Bath Research Scholarship awards towards this research.

Natations

ε_c	strain in extreme top fibre of concrete component
ε_{cu}	ultimate compressive strain in concrete
ε_{f0}	initial strain level at the time of FRP installation in concrete substrate where FRP is to be placed
ε_{fd}	design failure strain for externally FRP component
$\varepsilon_{fd\text{-}ruptre}$	design rupture strain for externally FRP component
$\varepsilon_{fd\text{-}debond}$	design debonding strain for externally FRP component
ε_{fe}	effective strain for externally FRP component (accounting initial in FRP also)
ε_{fl}	strain in externally FRP component due to live loads
ε_{sc}	strain in compression steel reinforcement
$\varepsilon_{sc\text{-}crit}$	critical strain in compression steel reinforcement
ε_{st}	strain in tension steel reinforcement
$\varepsilon_{st\text{-}adequate}$	adequate strain in tension steel reinforcement
ρ_f	externally bonded FRP content
ρ_{sc}	compression steel reinforcement content
ρ_{st}	tension steel reinforcement content
γ_c	partial factor of safety on concrete material
γ_s	partial factor of safety on steel material

b	width of the section
d	effective depth of the section
d'	depth of the centroid of the compression steel reinforcement from the extreme compression fibre
f_c'	characteristic cylinder compressive stress of concrete (as per ACI318 notations)
f_{ck}	characteristic compressive stress of concrete (cube or cylinder)
f_y	yield stress in steel reinforcement
n	numbers of layers of externally bonded FRP
t_f	thickness of one layer of externally bonded FRP
x_s	depth of neutral axis at service state
x_u	depth of neutral axis at ultimate state
D	overall depth of the section
E_{fd}	design modulus of elasticity of FRP material
E_s	modulus of elasticity of steel material
K	ratio of effective depth to overall depth of the section (d/D)
S	ratio of effective cover to effective depth (d'/d)
M	flexural resistance of the section (in general)
M_n	nominal flexural resistance of the section
M_{u-reqd}	design required ultimate flexural resistance of the strengthened section
$M_{u-strengthened}$	design ultimate flexural resistance of the strengthened section strain)

References

1. Kansara, K. D. and Ramanjaneyulu, K., "Time-dependent Strength Limit-states for Service Life Estimation of Reinforced Concrete Bridges," SERC Research Report No. RCS-MLP12641-RR-2007-2, *Structural Engineering Research Centre*, Chennai, India, 2007.

2. Teng, J. G., Chen, J. F., Smith, S. T., and Lam, L., *FRP-strengthened RC Structures*, John Wiley & Sons, West Sussex, England, 2002.

3. ACI440.2R-08, *Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures*, American Concrete Institute, Michigan, ACI Committee 440, 2008.

4. Concrete Society TR55, "Design Guidance for Strengthening Concrete Structures Using Fibre Composite Materials," Technical Report No. 55 (2/e), The Concrete Society, Surrey, 2004.

5. International Federation of Structural Concrete, "Externally Bonded FRP Reinforcement for RC Structures–Technical Report on the Design and Use of Externally Bonded FRP for RC Structures," *FIB Bulletin 14*, Lausanne, 2001.

6. Japan Society of Civil Engineers, "Recommendations for Upgrading of Concrete Structures with Use of Continuous Fibre Sheets," *JSCE Concrete Engineering Series 41*, Tokyo, 2001.

7. CSA S806-02, "Design and Construction of Building Components with FRP," *Canadian Standards Association*, Toronto, 2002.

8. Kansara, K. D., Ibell, T. J., and Darby, A. P., "Evaluation of Flexural Strengthening Design Criteria for FRP Strengthened RC Structures," Research Report, Department of Architecture and Civil Engineering, University of Bath, Bath, UK, 2009.

9. Razaqpur, G. and Ali, A. M., "A New Concept for Achieving Ductility in FRP-Reinforced Concrete," *Proceedings of the First International Conference on Composites in Infra structures (ICCI 96)*, Tucson, 1996, pp. 401~413.

10. ACI318-08, *Building Code Requirements for Structural Concrete*, American Concrete Institute, Michigan, 2005.

11. Aprile, A. and Feo, L., *Concrete Cover Rip-off of RC Beams Strengthened with FRP Composites*, Composites (Part B: Engineering), Vol. 38, 2006, pp. 759~771.

12. BS 8110, *Structural Use of Concrete–Part 1: Code of Practice for Design and Construction*, British Standards Institution, London, 1997.

13. CEN Eurocode 2 (EN 1992-1-1), *Design of Concrete Structures–Part 1-1: General Rules and Rules for Buildings*, European Committee for Standardisation, Brussels, 1994.