한국항행학회 논문지 제14권 제1호 2010년 2월

균일한 선형 배열의 다중 입출력 레이더 시스템을 위한 압축 센싱

Compressive Sensing for MIMO Radar Systems with Uniform Linear Arrays

임종태*, 유도식*

Jong-Tae Lim*, Do-Sik Yoo*

요 약

압축 센싱 (Compressive Sensing, CS)은 많은 응용분야에서 유망한 기술로 널리 연구되고 있다. 압축 센싱 이론에 의하면 어떤 특별한 기저에서 성긴 신호 (sparse signal)이라는 것이 알려졌다면 이 신호는 전통적인 방법이 사용하는 샘플 수보다 훨씬 적은 샘플로 최적화 과정을 통해 복원이 가능하다는 것이다. 본 논문에서는 이러한 압축 센싱 기술을 균일한 선형 배열로 구성된 다중 입출력 레이더 시스템에 적용하고자 한다. 특별히 압축 센싱 기술을 사용하여 DOA (direction-of-arrival)을 찾는 문제를 고찰하고 그 성능을 전통적인 적응형 다중 입출력 기법의 성능과 비교한다. 모의 실험을 통해 압축 센싱 방법은 전통적인 적응형 다중 입출력 기법에 비해 훨씬 적은 샘플로 비슷한 성능을 보임을 확인할 수 있었다.

Abstract

Compressive Sensing (CS) has been widely studied as a promising technique in many applications. The CS theory tells that a signal that is known to be sparse in a specific basis can be reconstructed using convex optimization from far fewer samples than traditional methods use. In this paper, we apply CS technique to Multiple-input multiple-output (MIMO) radar systems which employ uniform linear arrays (ULA). Especially, we investigate the problem of finding the direction-of-arrival (DOA) using CS technique and compare the performance with the conventional adaptive MIMO techniques. The results suggest the CS method can provide the similar performance with far fewer snapshots than the conventional adaptive techniques.

Key words : MIMO radar, ULA, Compressive Sensing, DOA estimation

I. Introduction

A newly emerging technology, compressive sensing (CS) has been applied in many applications such as imaging systems [1], [2], medical imaging systems [3] and wireless communication systems [4]. CS theory asserts that a signal which is known to be sparse in a specific basis can be exactly reconstructed from far fewer samples than traditional methods use [1], [5], [6]. To obtain the samples, CS uses nonadaptive linear

* 홍익대학교 전기전자공학부(School of Electronic & Electrical Engineering of Hongik University)

· 제1저자 (First Author) : 임종태 · 교신저자 (Corresponding Author) : 유도식

· 투고일자 : 2010년 1월 18일

· 심사(수정)일자 : 2010년 1월 19일 (수정일자 : 2010년 2월 17일)

· 게재일자 : 2010년 2월 28일

projections and the signal can be reconstructed by convex optimization procedure from these projection values.

The application of compressive sensing technique to a sensor array is proposed in [7] They investigated the problem of estimating direction-of-arrival (DOA) by discretizing the angle space. This idea is extended to the problem of DOA estimation for Multple-input multiple-output (MIMO) radar systems in [8]. For a small number of targets, they formulated the DOA estimation as the reconstruction of a sparse signal using compressive sensing. Especially, a distributed MIMO radar scenario is considered in [8].

A MIMO radar system can transmit multiple independent waveforms via its multiple antennas unlike a standard phased-array radar transmitting scaled versions of a single waveform [9], [10], [11]. According to the transmit/receive antenna arraignment, MIMO radar systems are categorized into two types: MIMO radar with widely separated transmit/receive antennas and with colocated antennas. The MIMO radar systems with widely separated tranmit/receive antennas can provide the spatial diversity of the target's radar cross section (RCS) [11]. The MIMO radar systems with colocated antennas can offer higher resolution, higher sensitivity to detecting slowly moving targets, and better parameter identifiability than traditional radar systems [10].

In this paper, we apply CS technology to MIMO radar systems with colocated antennas. In particular, we consider a uniform linear array (ULA) MIMO radar, where the elements of antenas are uniformly spaced in a line as shown in fig. 1. By discretizing the angle space as in [7], [8], we formulated the DOA estimation for MIMO radar system with ULA. For a small number of targets, the DOAs are sparse in the angle sparse and using this sparsity we can obtain DOA estimation using convex optimzation procedure. The performance of CS method is compared to the conventional adaptive MIMO techiques: Capon [12], [13] and Amplitude and phase estimation (APES) [14]. The simulation results suggest the CS method can provide the similar performance with far fewer snapshots the conventional adaptive techniques.

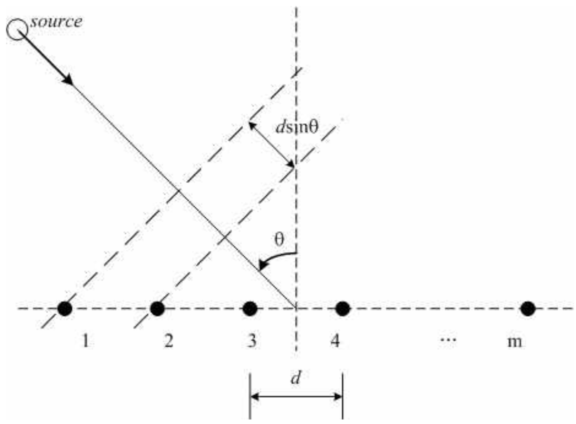

그림 1. 균일한 선형 배열 안테나
Fig. 1. The uniform linear array

Ⅱ. Signal Model

In this section, we briefly introduce the CS theory and formulate the DOA estimation for MIMO radar systems with ULAs.

2-1 Compressive Sensing

Consider a discrete-time signal x, which is represented by an $N \times 1$ column vector. The signal x is represented in terms of a basis of $N \times 1$ vectors , $\psi_1, \psi_2, \cdots, \psi_N$ and thus is expressed as

$$\mathbf{x} = \sum_{i=1}^{N} s_i \psi_i \triangleq \Psi \mathbf{s} \quad (1)$$

where s is the $N \times 1$ column vector of weighting coefficients $s_i = < \mathbf{x}, \psi_i >$ and Ψ is the $N \times N$ basis matrix, $\Psi = [\psi_1 \psi_2 \cdots \psi_N]$. If only K of the si coefficients are nonzero, the signal x is said to be

K-sparse. CS takes non-traditional linear measurement as

$$\mathbf{y} = \Phi\mathbf{x} = \Phi\Psi\mathbf{s} \triangleq \Theta\mathbf{s} \quad (2)$$

where Φ is $M \times N$ measurement matrix (M << N). From the M projection values $y_1, y_2, \cdots, y_M$, i.e., the M measurements, the K-sparse signal x can be reconstructed with high probability by solving a convex optimization problem described by (3)

$$\hat{\mathbf{s}} = \arg\min \|\mathbf{s}'\|_1 \quad subject\ to\ \ \Theta\mathbf{s}' = \mathbf{y}\,. \quad (3)$$

If the measurements are corrupted by an additive Gaussian noise n with zero mean and variance σ^2 and are represented as

$$\mathbf{y} = \Theta\mathbf{s} + \mathbf{n} \quad (4)$$

then s can be recovered by applying the Dantzig selector to the convex optimization problem as in [15]

$$\hat{\mathbf{s}} = \arg\min \|\mathbf{s}'\|_1 \quad s.t.\ \left\|\Theta^H(\mathbf{y} - \Theta\mathbf{s}')\right\|_\infty \le \mu \quad (5)$$

where $\mu = (1 + t^{-1})\sqrt{2\log N}\sigma$ for a positive scalar t.

2-2 Signal model for MIMO radar with ULA

We consider a MIMO radar system with ULAs of Mt transmit antennas and Mr receive antennas. The array transfer vector or steering vector for half-wavelength spacing ULA is given by [12]

$$\mathbf{a}(\theta) = [1\ e^{-j\pi\sin\theta}\ e^{-j2\pi\sin\theta}\ \cdots e^{-j(m-1)\pi\sin\theta}]^T \quad (6)$$

where m is the number of ULA elements, θ denotes the direction of arrival, and [ ]T indicates the transpose operation. Let xi(n) denote the discrete-time baseband signal transmitted by the ith transmit antenna. Assume that the transmitted signals are narrowband and that the propagation is non-dispersive. Then the baseband signal at the kth target is described by

$$y_k(n) = \sum_{i=1}^{M_t} x_i(n)a_i(\theta_k) \triangleq \mathbf{x}^T(n)\mathbf{a}(\theta_k) \quad (7)$$

where θk is the DOA for the kth target, and $\mathbf{x}(n) = [x_1(n)\ x_2(n)\ \cdots x_{M_t}(n)]^T$. Under the simplifying assumption of point targets, the received signal at the lth receiving antenna elements is

$$z_l(n) = \sum_{k=1}^{K} \beta_k b_l(\theta_k)\mathbf{x}^T(n)\mathbf{a}(\theta_k) + \varepsilon_k(n) \quad (8)$$

where K is the number of targets, βk's are the reflection coefficient proportional to the RCSs of the targets, bl(θk) is the lth element of the steering vector $\mathbf{b}(\theta) = [b_1(\theta)\ b_2(\theta)\ \cdots b_{M_r}(\theta)]^T$ for the receive ULA, and εl(n) denotes independent and identically distributed (i.i.d.) Gausian noise with mean zero and variance σ2. If L snapshots are taken at the receiving antenna and zl(n), n =1,2,..., L-1 are formed into a vector, then we have

$$\begin{aligned}\mathbf{z}_l &= \left[z_l(0)\ z_l(1)\ \cdots\ z_l(L-1)\right]^T \\ &= \begin{bmatrix} \sum_{k=1}^{K} \beta_k b_l(\theta_k)\mathbf{x}^T(0)\mathbf{a}(\theta_k) \\ \sum_{k=1}^{K} \beta_k b_l(\theta_k)\mathbf{x}^T(1)\mathbf{a}(\theta_k) \\ \vdots \\ \sum_{k=1}^{K} \beta_k b_l(\theta_k)\mathbf{x}^T(L-1)\mathbf{a}(\theta_k) \end{bmatrix} + \begin{bmatrix} \varepsilon_l(0) \\ \varepsilon_l(1) \\ \vdots \\ \varepsilon_l(L-1) \end{bmatrix} \\ &= \sum_{k=1}^{K} \beta_k b_l(\theta_k)\mathbf{X}\mathbf{a}(\theta_k) + \boldsymbol{\varepsilon}_l \end{aligned} \quad (9)$$

where $\mathbf{X} = \left[\mathbf{x}(0)\ \mathbf{x}(1)\ \cdots\ \mathbf{x}(L-1)\right]^T$ and the noise vector is $\boldsymbol{\varepsilon}_l = [\varepsilon_l(0)\ \varepsilon_l(1)\ \cdots\ \varepsilon_l(L-1)]^T$.

Now, by discretizing the angle space as $\mathbf{a} = [\alpha_1\ \alpha_2\ \cdots\ \alpha_N]$ and defining a vector s as $\mathbf{s} = [s_1\ s_2\ \cdots\ s_N]^T$

where

$$s_n = \begin{cases} \beta_k, & if\ \ \alpha_n = \theta_k \\ 0, & otherwise \end{cases}, \quad (10)$$

the equation (9) can be expressed as

$$\mathbf{z}_l = \sum_{n=1}^{N} b_l(\alpha_n)\mathbf{X}\mathbf{a}(\alpha_n)s_n + \boldsymbol{\varepsilon}_l. \quad (11)$$

The index n of the nonzero element of s indicates that there is a target at the angle αn. If we consider a small number of targets, i.e., K is small, only K elements in s are nonzero. Constructing a basis matrix Ψl for the l-th antenna as

$$\Psi_l = [b_l(\alpha_1)\mathbf{X}\mathbf{a}(\alpha_1) \cdots b_l(\alpha_N)\mathbf{X}\mathbf{a}(\alpha_N)] \quad (12)$$

(11) becomes

$$\mathbf{z}_l = \Psi_l\mathbf{s} + \boldsymbol{\varepsilon}_l. \quad (13)$$

Thus, if we omit the noise term, zl becomes a K-sparse signal. Now, we take M linear projections of the received signal at the l-th antenna as

$$\mathbf{r}_l = \Phi_l\mathbf{z}_l = \Phi_l\Psi_l\mathbf{s} \,\square\, \Theta_l\mathbf{s} \quad (14)$$

where Φl is a measurement matrix and an $M \times N$ random Gaussian matrix. Collecting the projections of Mr receive antennas, we get

$$\mathbf{r} = \begin{bmatrix} \Theta_1 \\ \vdots \\ \Theta_l \end{bmatrix} \mathbf{s} \,\square\, \Theta\mathbf{s}. \quad (15)$$

Therefore, based on the CS theory, the vector s can be recovered by applying the Dantzig selector. The estimated DOA is the solution of the following convex optimization

$$\hat{\mathbf{s}} = \arg\min \|\mathbf{s}'\|_1 \quad s.t.\ \|\Theta^H(\mathbf{y} - \Theta\mathbf{s}')\|_\infty \le \mu \quad (16)$$

where $\mu = (1 + t^{-1})\sqrt{2\log N}\sigma$ for a positive scalar t.

Ⅲ. Simulation Results

In our simulation, we consider MIMO radar systems with uniform linear arrays. For transmitting and receiving the signals the same uniform linear arrays are used. The elements of the uniform linear arrays are arranged with half-wavelength spacing. The orthogonal quadrature phase shift keyed (QPSK) sequences are used as the transmitted waveforms. The received signals are corrupted by zero mean Gaussian noise.

Two scenarios are considered for the target locations and the number of elements in the uniform linear arrays. In the first scenario, K = 3 targets are located at -5°, -10°, -25° with reflection coefficients β1 = 1, β2 = 1, β3 = 1, and the number of the uniform linear array is Mt = Mr = 20. The SNR, which is defined as the ratio of the total transmitting power across the transmitting antennas over the additive noise power, is 20 dB. In the second scenario, the fewer elements are used. Mt = Mr = 10 elements are used and 3 targets are located at -5°, -25°, -40°. The SNR value for this scenario is 10 dB. For both scenarios, we search the direction-of-arrival with 0.5° increments from -90° to 90°. Thus, the angle space is [-90°, -89.5°, ..., 89.5°, 90°]. The maximum number of snapshots in the receiving elements is 256. We compare the performance of the compressive sensing method with those of the adaptive array algorithm: Capon and APES. The positive scalar for the threshold in Dantzig selector, t, is set to 1 for both scenarios, and thus the thresholds are 0.45 and 1.43 respectively.

Fig. 2 shows the moduli of the estimates of the reflection coefficients of the compressive sensing method, Capon and APES for the first scenario. The figures (a) and (b) are corresponding the estimates obtained by Capon and APES using 256 snapshots, respectively. The figure (c) is the result of the compressive sensing method using only 16 snapshots. We can clearly see that the 3 targets are present even using only 16 snapshots. The performance for the second scenario is shown in Fig. 3. The compressive sensing using only 16 snapshots show the similar performance to Capon and APES algorithms using 256 snapshots.

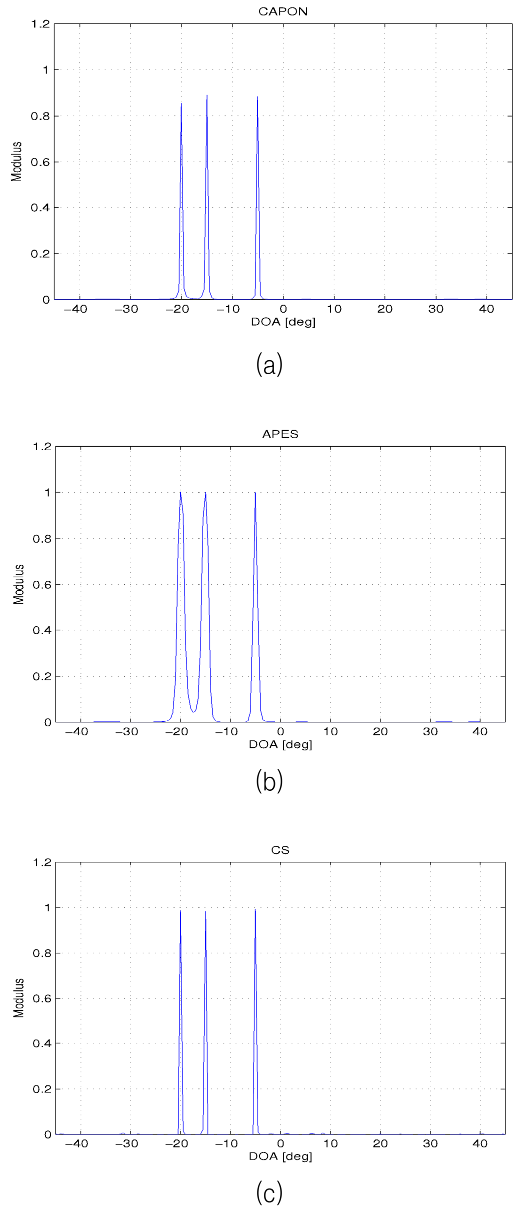

그림 2. 첫번째 시나리오에 대한 DOA 추정치
(a) Capon (b) APES (c) 압축 센싱
Fig. 2. DOA estimates for the first scenario
(a) Capon (b) APES (c) Compressive sensing

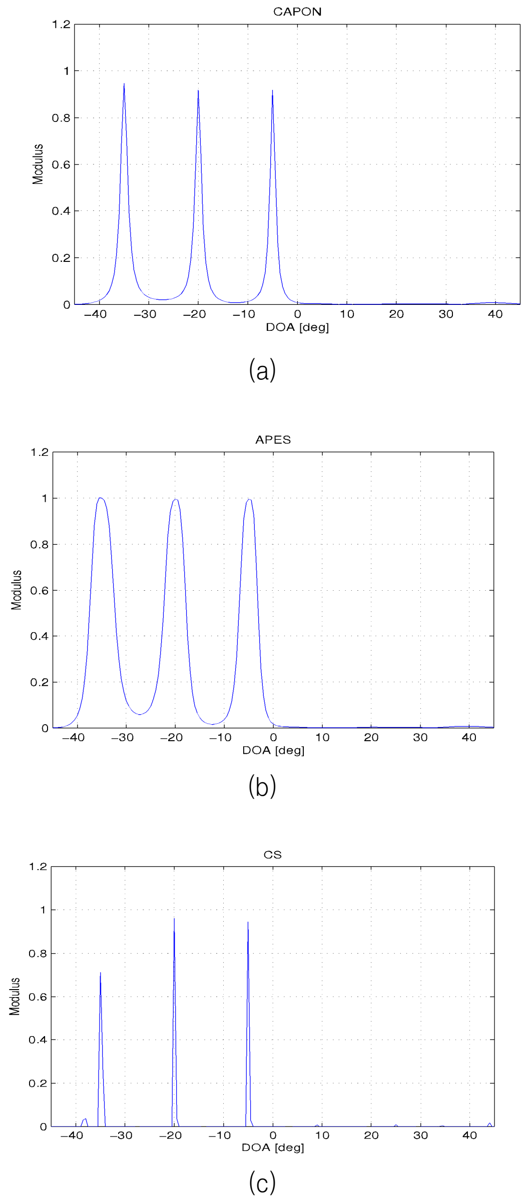

그림 3. 두번째 시나리오에 대한 DOA 추정치
(a) Capon (b) APES (c) 압축 센싱
Fig. 3. DOA estimates for the second scenario
(a) Capon (b) APES (c) Compressive sensing

Ⅳ. Conclusions

In this paper, we investigated the application of compressive sensing for MIMO radar systems with uniform linear arrays. For the small number of targets, we can estimate the DOA of targets using much fewer samples than the conventional adaptive algorithms such as Capon and APES. And the performance of estimating the DOA of targets using compressive sensing method is similar to those of the conventional algorithms. Therefore, the compressive sensing method can provide much benefit in the application where many receive nodes need to transmit the samples to the central collecting center. The effect on quantized measures in compressive sensing is one of the topics to be investigated.

Acknowledgement

The work of J. Lim was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology (2008-0061076) and the work of D.-S. Yoo was supported by National Reserach Foundation of Korea under Grant 2009-0059803.

References

[1] R. G. Baraniuk, "Compressive Sensing," *IEEE Signal Processing Mag.*, pp. 118-124, Jul. 2007.

[2] J. Romberg, "Imaging via compressive sampling," *IEEE Singal Processing Mag., vol. 25, no. 2,* pp. 14-20, Mar. 2008.

[3] M. Lustig, D. Donoho, and J. J. Pauly, "Sparse MRI: The application of compressed sensing for rapid MR imaging," *Magnetic Resonance in Medicine, vol. 58, no. 6*, pp. 1182-1195, Dec. 2007.

[4] W. Bajwa, J. Haupt, A. Sayeed and R. Nowak. "Compressive wireless sensing ," *in Proc. IEEE Information Processing in Sensor Networks, Nashville, TN*, pp. 134-142, Apr. 2006.

[5] E. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inform. Theory, vol. 52, no. 2*, pp. 489-509, Feb. 2006.

[6] D. Donoho, "Compressed sensing," *IEEE Trans. on Information Theory, vol. 52, no.4*, pp. 1289-1306, Apr. 2006.

[7] A. C. Gurbuz, J.H. McClellan, and V.Cevher, "A compressive beamforming method," *in Proc. IEEE Int'l Conf. Acoustics, Speech, and Signal Processing, Las Vegas, NV*, pp. 2617-2620, Mar. - Apr. 2008.

[8] Y. Yu, A. P. Petropulu, and H. V. Poor, "Compressive sensing for MIMO radar," *in Proc. IEEE Int'l Conf. Acoustics, Speech, and Signal Processing*, pp. 3017 - 3020, Apr. 2009.

[9] E. Fishler, A. Haimovich, R.Blum, D. Chizhik, L. Cimini, and R. Valenzuela, "MIMO radar: An idea whose time has come," *in Proc. IEEE Radar Conf., Philadelphia, PA*, pp. 71-78, Apr. 2004.

[10] J. Li and P. Stoica, "MIMO radar with Colocated Anteannas," *IEEE Signal Processing Magazine, vol.24, no. 5*, pp. 106 - 114, Sep. 2007.

[11] A.M. Haimovich, R.S. Blum, and L.J. Cimini, "MIMO Radar with Widely Separated Antennas," *IEEE Signal Processing Magazine, vol. 25, no. 1*, pp. 116 - 129, Jan. 2008.

[12] P. Stoica and R. Moses, Spectral Analysis of Signals, Pearson Prentice Hall, 2005.

[13] P. Stoica, Z. Wang, and J. Li, "Robust Capon Beamforming," *IEEE Signal Processing Letters, vol. 10, no. 6*, pp. 172- 175, Jun. 2003.

[14] J. Li and P. Stoica, "An adaptive filtering approach to spectral estimation and SAR imaging," *IEEE Trans. Signal Processing, vol. 44*, pp. 1469 - 1484, Jun. 1996.

[15] E. Candes and T. Tao, "The Dantzig Selector: Statistical estimation when p is much larger than n," *Ann. Statist., vol. 35*, pp. 2313 - 2351, 2007.

임 종 태 (林鍾泰)

1989년 2월 : 서울대학교 전자공학과 (공학사)
1991년 2월 : 서울대학교 전자공학과 (공학석사)
2001년 8월 : The Univ. of Michigan at Ann Arbor 전기공학과 (공학박사)
1991년 3월 ~ 2004년 8월 (주) 대우일렉트로닉스 영상연구소 및 신호처리연구소 책임 연구원
2004년 9월 ~ 2008년 2월 한국항공대학교 항공전자 및 정보통신공학부 조교수
2008년 3월 ~ 현재 홍익대학교 전자전기공학부 조교수
관심분야 : 디지털 통신 및 방송 시스템, 디지털 신호 처리

유 도 식 (兪堵植)

1990년 8월 : 서울대학교 전기공학과 (공학사)
1994년 2월 : 서울대학교 물리학과 (이학석사)
2002년 4월 :미국 미시간대학교 전기공학과(공학박사)
2002년 2월 ~ 2003년 10월 미시간대학교 박사 후 연구원
2006년 9월 ~ 현재 홍익대학교 전자전기공학부 조교수
관심분야 : 통신이론, 무선통신 시스템 설계, 정보이론, 부호이론, 네트워크 정보이론 등