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DYNAMIC ANALYSIS OF A PERIODICALLY FORCED HOLLING-TYPE II TWO-PREY ONE-PREDATOR SYSTEM WITH IMPULSIVE CONTROL STRATEGIES

HYE KYUNG KIM[1] AND HUNKI BAEK[2†]

[1]DEPT. OF MATHEMATICS EDUCATION, CATHOLIC UNIVERSITY OF DAEGU, KYONGSAN 712-702, SOUTH KOREA
E-mail address: hkkim@cu.ac.kr

[2]DEPT. OF MATHEMATICS, KYUNGPOOK NATIONAL UNIVERSITY, DAEGU 702-701, SOUTH KOREA
E-mail address: hkbaek@knu.ac.kr

ABSTRACT. In this paper, we establish a two-competitive-prey and one-predator Holling type II system by introducing a proportional periodic impulsive harvesting for all species and a constant periodic releasing, or immigrating, for the predator at different fixed time. We show the boundedness of the system and find conditions for the local and global stabilities of two-prey-free periodic solutions by using Floquet theory for the impulsive differential equation, small amplitude perturbation skills and comparison techniques. Also, we prove that the system is permanent under some conditions and give sufficient conditions under which one of the two preys is extinct and the remaining two species are permanent. In addition, we take account of the system with seasonality as a periodic forcing term in the intrinsic growth rate of prey population and then find conditions for the stability of the two-prey-free periodic solutions and for the permanence of this system. We discuss the complex dynamical aspects of these systems via bifurcation diagrams.

1. INTRODUCTION

In population dynamics, it is important to understand the dynamical relationship between predator and prey. Such relationship can be represented by the functional response which refers to the change in the density of prey attached per unit time per predator as the prey density changes. Based on experiments, Holling [14] gave three different kinds of functional responses, which are monotonic in the first quadrant. If we take into account the time a predator uses in handling the prey it has captured, one finds the predator has a type-II functional response. The three kinds of Holling functional response have been studied [5, 6, 15, 25, 27, 28]. According to

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† Corresponding author.

Hassel et al. [13], the Holling type-II functional response is the most common type of functional response among arthropod predators.

In the 1980s, the theories and applications of differential equations with impulse were greatly developed by the efforts of Bainov, Lakshmikantham and others [4, 16], and the theory of impulsive differential equations is being recognized not only to be richer than the corresponding theory of differential equations, but also represent a more natural framework for mathematical modeling of real world problems [22, 34, 37]. Such impulsive systems are found in almost every domain of applied science and have been studied in many investigations: impulsive birth [24, 32], impulsive vaccination [8, 29], chemotherapeutic treatment of disease [17, 21]. In particular, the impulsive prey-predator population models have been discussed by a number of researchers [18, 19, 20, 33, 39, 40, 43, 44] and there are also many literatures on simple multi-species systems consisting of a three-species food chain with impulsive perturbations [1, 2, 3, 12, 35, 36, 38, 41, 42]. Recently, several researchers pay attention to two-prey and one-predator impulsive systems [9, 11, 30, 31, 45, 46].

Now we develop the two-competitive-prey and one-predator system by introducing a proportion that is periodic impulsive harvesting(spraying pesticide) for all species and a constant periodic releasing, or immigrating, for the predator at different fixed time. Thus, we establish a food chain system with Holling type II functional response and impulsive perturbations as follows:

$$\begin{cases} \left.\begin{aligned} x_1'(t) &= x_1(t)\Big(a_1 - b_1x_1(t) - \mu_1 x_2(t) - \frac{e_1 y(t)}{c_1 + x_1(t)}\Big), \\ x_2'(t) &= x_2(t)\Big(a_2 - b_2x_2(t) - \mu_2 x_1(t) - \frac{e_2 y(t)}{c_2 + x_2(t)}\Big), \\ y'(t) &= y(t)\Big(-D + \frac{\beta_1 x_1(t)}{c_1 + x_1(t)} + \frac{\beta_2 x_2(t)}{c_2 + x_2(t)}\Big), \end{aligned}\right\} \\ \qquad\qquad\qquad\qquad t \neq (n+\tau-1)T, t \neq nT, \\ \left.\begin{aligned} \Delta x_1(t) &= -p_1 x_1(t), \\ \Delta x_2(t) &= -p_2 x_2(t), \\ \Delta y(t) &= -p_3 y(t), \end{aligned}\right\} t = (n+\tau-1)T, \\ \left.\begin{aligned} \Delta x_1(t) &= 0, \\ \Delta x_2(t) &= 0, \\ \Delta y(t) &= q. \end{aligned}\right\} t = nT, \\ (x_1(0^+), x_2(0^+), y(0^+)) = (x_{01}, x_{02}, y_0), \end{cases} \tag{1.1}$$

where $x_i(t)(i = 1, 2)$ and $y(t)$ represent the population density of the two preys and the predator at time t, respectively and $\Delta w(t) = w(t^+) - w(t), w = x_i(i = 1, 2)$ and y. Here $a_i(i = 1, 2)$ are intrinsic rate of increase, $b_i(i = 1, 2)$ are the coefficient of intra-specific competition, $\mu_i(i = 1, 2)$ are a parameter representing competitive effects between two preys, $e_i(i = 1, 2)$ are the per-capita rate of predation of the predator, $c_i(i = 1, 2)$ are the half-saturation constant, D denotes the death rate of the predator, $\beta_i(i = 1, 2)$ are the rate of conversing prey into predator, T is the period of the impulsive immigration or stock of the predator, $0 \leq p_1, p_2, p_3 < 1$ present the fraction of the preys and the predator which die due to the harvesting or pesticides etc and q is the size of immigration or stock of the predator.

It is necessary and important to consider systems with periodic ecological parameters which might be quite naturally exposed such as those due to seasonal effects of weather or food supply etc [7]. Indeed, it has been studied that dynamical systems with simple dynamical behavior may display complex dynamical behavior when they have periodic perturbations [10, 23, 26]. Especially, we consider the intrinsic growth rates a_1 and a_2 in system (1.1) as periodically varying function of time due to seasonal variation. Thus, in Section 4, we investigate the seasonal effects on the preys as a periodic forcing term of system (1.1).

In Section 2, we give some notations and lemmas. In Section 3, first, we show the boundedness of the system and take into account the local stability and the global asymptotic stability of two-prey-free periodic solutions by using Floquet theory for the impulsive equation, small amplitude perturbation skills and comparison techniques , and finally, prove that the system is permanent under some conditions. Moreover, we give sufficient conditions under which one of the two preys is extinct and the remaining two species are permanent.

2. Preliminaries

Let $\mathbb{R}_+ = [0, \infty)$ and $\mathbb{R}^3_+ = \{\mathbf{x} = (x(t), y(t), z(t)) \in \mathbb{R}^3 : x(t), y(t), z(t) \geq 0\}$. Denote $\mathbb{N}$ the set of all of positive integers, $\mathbb{R}^*_+ = (0, \infty)$ and $f = (f_1, f_2, f_3)^T$ the right hand of the first three equations in (1.1). Let $V : \mathbb{R}_+ \times \mathbb{R}^3_+ \to \mathbb{R}_+$, then V is said to belong to class V_0 if, for each $\mathbf{x} \in \mathbb{R}^3_+$ and $n \in \mathbb{N}$,

(1)$V$is continuous on$((n-1)T, (n+\tau-1)T] \times \mathbb{R}^3_+ \cup ((n+\tau-1)T, nT] \times \mathbb{R}^3_+$ and

$$\lim_{(t,\mathbf{y}) \to (t_0, \mathbf{x})} V(t, \mathbf{y}) = V(t_0, \mathbf{x}) \text{ exists, where } t_0 = (n+\tau-1)T^+ \text{ and } nT^+,$$

(2)V is locally Lipschitzian in $\mathbf{x}$.

Definition 2.1. For $V \in V_0$, one defines the upper right Dini derivative of V with respect to the impulsive differential system (1.1) at $(t, \mathbf{x}) \in ((n-1)T, (n+\tau-1)T] \times \mathbb{R}^3_+ \cup ((n+\tau-1)T, nT] \times \mathbb{R}^3_+$ by

$$D^+V(t, \mathbf{x}) = \limsup_{h \to 0+} \frac{1}{h}[V(t+h, \mathbf{x} + hf(t, \mathbf{x})) - V(t, \mathbf{x})].$$

The smoothness properties of f guarantee the global existence and uniqueness of solutions of system (1.1) [16].

Definition 2.2. System (1.1) is said to be permanent if there exist two positive constants m and M such that every positive solution $(x_1(t), x_2(t), y(t))$ of system (1.1) with $(x_{01}, x_{02}, y_0) > 0$ satisfies $m \leq x_{0i}(t) \leq M$ and $m \leq y(t) \leq M$ for sufficiently large t, i=1,2.

We will use a comparison result of impulsive differential inequalities. For this, suppose that $g : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ satisfies the following hypotheses:

(H) g is continuous on $((n-1)T,(n+\tau-1)T]\times\mathbb{R}^3_+\cup((n+\tau-1)T,nT]\times\mathbb{R}^3_+$ and the limits $\lim_{(t,y)\to((n+\tau-1)T^+,x)} g(t,y)=g((n+\tau-1)T^+,x)$, $\lim_{(t,y)\to(nT^+,x)} g(t,y)=g(nT^+,x)$ exist and are finite for $x\in\mathbb{R}_+$ and $n\in\mathbb{N}$.

Lemma 2.3. [16] *Suppose $V\in V_0$ and*

$$\begin{cases} D^+V(t,\mathbf{x})\le g(t,V(t,\mathbf{x})),\ t\ne(n+\tau-1)T, t\ne nT,\\ V(t,\mathbf{x}(t^+))\le\psi^1_n(V(t,\mathbf{x})),\ t=(n+\tau-1)T,\\ V(t,\mathbf{x}(t^+))\le\psi^2_n(V(t,\mathbf{x})),\ t=nT,\end{cases}\tag{2.1}$$

where $g:\mathbb{R}_+\times\mathbb{R}_+\to\mathbb{R}$ satisfies (H) and $\psi^1_n,\psi^2_n:\mathbb{R}_+\to\mathbb{R}_+$ are non-decreasing for all $n\in\mathbb{N}$. Let $r(t)$ be the maximal solution for the impulsive Cauchy problem

$$\begin{cases} u'(t)=g(t,u(t)),\ t\ne(n+\tau-1)T, t\ne nT,\\ u(t^+)=\psi^1_n(u(t)),\ t=(n+\tau-1)T,\\ u(t^+)=\psi^2_n(u(t)),\ t=nT,\\ u(0^+)=u_0\ge0\end{cases}\tag{2.2}$$

defined on $[0,\infty)$. Then $V(0^+,\mathbf{x}_0)\le u_0$ implies that $V(t,\mathbf{x}(t))\le r(t), t\ge0$, where $\mathbf{x}(t)$ is any solution of (2.1).

We now indicate a special case of Lemma 2.3 which provides estimations for the solution of a system of differential inequalities. For this, we let $PC(\mathbb{R}_+,\mathbb{R})(PC^1(\mathbb{R}_+,\mathbb{R}))$ denote the class of real piecewise continuous(real piecewise continuously differentiable) functions defined on $\mathbb{R}_+$.

Lemma 2.4. [16] *Let the function $u(t)\in PC^1(\mathbb{R}^+,\mathbb{R})$ satisfy the inequalities*

$$\begin{cases} \dfrac{du}{dt}\le f(t)u(t)+h(t), t\ne\tau_k, t>0,\\ u(\tau_k^+)\le\alpha_k u(\tau_k)+\theta_k, k\ge0,\\ u(0^+)\le u_0,\end{cases}\tag{2.3}$$

where $f,h\in PC(\mathbb{R}_+,\mathbb{R})$ and $\alpha_k\ge0$, θ_k and u_0 are constants and $(\tau_k)_{k\ge0}$ is a strictly increasing sequence of positive real numbers. Then, for $t>0$,

$$\begin{aligned} u(t)\le &u_0\left(\prod_{0<\tau_k<t}\alpha_k\right)\exp\Big(\int_0^t f(s)ds\Big)+\int_0^t\left(\prod_{s\le\tau_k<t}\alpha_k\right)\exp\Big(\int_s^t f(\gamma)d\gamma\Big)h(s)ds\\ &+\sum_{0<\tau_k<t}\left(\prod_{\tau_k<\tau_j<t}\alpha_j\right)\exp\Big(\int_{\tau_k}^t f(\gamma)d\gamma\Big)\theta_k.\end{aligned}$$

Similar result can be obtained when all conditions of the inequalities in the Lemmas 2.3 and 2.4 are reversed.

Using Lemma 2.4, it is easy to prove that the solutions of system (1.1) with strictly positive initial value remain strictly positive as follows:

Lemma 2.5. *The positive octant* $(\mathbb{R}_+^*)^3$ *is an invariant region for system (1.1).*

3. Analysis on system (1.1)

In this section we will perform a global stability analysis of the two-prey-free periodic solution via the Floquet theory. Next, we will establish the conditions for the permanence of the system (1.1), and for the extinction of one of the two preys and permanence of the remaining two species.

Before stating main Theorems, we will show the existence of a two-prey-free periodic solution. In the case in which two preys are eradicated, the system (1.1) is led to the impulsive differential equation (3.1) as a periodically forced linear system:

$$\begin{cases} y'(t) = -Dy(t),\ t \neq (n+\tau-1)T, t \neq nT, \\ \Delta y(t) = -p_3 y(t),\ t = (n+\tau-1)T, \\ \Delta y(t) = q,\ t = nT. \end{cases} \tag{3.1}$$

Let us consider the properties of this impulsive differential equation. Straightforward computation for getting a positive periodic solution $y^*(t)$ of the equation (3.1) yields the analytic form of $y^*(t)$:

$$y^*(t) = \begin{cases} \dfrac{q\exp(-D(t-(n-1)T))}{1-(1-p_3)\exp(-DT)}, (n-1)T < t \leq (n+\tau-1)T, \\ \dfrac{q(1-p_3)\exp(-D(t-(n-1)T))}{1-(1-p_3)\exp(-DT)}, (n+\tau-1)T < t \leq nT, \end{cases} \tag{3.2}$$

$$y^*(0^+) = y^*(nT^+) = \frac{q}{1-(1-p_3)\exp(-DT)}, y^*((n+\tau-1)T^+) = \frac{q(1-p_3)\exp(-D\tau T)}{1-(1-p_3)\exp(-DT)}.$$

Moreover, we obtain that

$$y(t) = \begin{cases} (1-p_3)^{n-1}\left(y(0^+) - \dfrac{q(1-p_3)e^{-T}}{1-(1-p_3)\exp(-DT)}\right)\exp(-Dt) + y^*(t), \\ \qquad (n-1)T < t \leq (n+\tau-1)T, \\ (1-p_3)^{n}\left(y(0^+) - \dfrac{q(1-p_3)e^{-T}}{1-(1-p_3)\exp(-DT)}\right)\exp(-Dt) + y^*(t), \\ \qquad (n+\tau-1)T < t \leq nT \end{cases} \tag{3.3}$$

is a solution of (3.1). Thus the following result is induced from (3.2) and (3.3).

Lemma 3.1. *For every solution* $y(t)$ *and every positive periodic solution* $y^*(t)$ *of system (3.1), it follows that* $y(t)$ *tends to* $y^*(t)$ *as* $t \to \infty$. *Thus, the complete expression for the two-prey free periodic solution of system (1.1) is obtained* $(0, 0, y^*(t))$.

3.1. **Stability of the periodic solution.** In the subsection we will study under what condition we can ensure the two preys are extinct. To achieve our purposes, we theoretically and numerically consider the stability of the periodic solution $(0, 0, y^*(t))$.

Theorem 3.2. *The periodic solution* $(0, 0, y^*(t))$ *of system (1.1) is globally asymptotically stable if*

$$a_iT - \frac{b_i e_i q \Phi(D)}{b_i c_1 + a_i} < \ln \frac{1}{1-p_i}, \tag{3.4}$$

where $i = 1, 2$ *and* $\Phi(D) = \dfrac{1-(1-p_3)\exp(-DT) - p_3\exp(-D\tau T)}{D(1-(1-p_3)\exp(-DT))}$.

Proof. First, we show the local stability of the solution $(0, 0, y^*(t))$. The local stability of the two-pest free periodic solution $(0, 0, y^*(t))$ of system (1.1) may be determined by considering the behavior of small amplitude perturbations of the solution. Let $(x_1(t), x_2(t), y(t))$ be any solution of system (1.1). Define $u(t) = x_1(t), v(t) = x_2(t), w(t) = y(t) - y^*(t)$. Then they may be written as

$$\begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix} = \Psi(t) \begin{pmatrix} u(0) \\ v(0) \\ w(0) \end{pmatrix},$$

where $\Psi(t)$ satisfies

$$\frac{d\Psi}{dt} = \begin{pmatrix} a_1 - \frac{e_1}{c_1}y^*(t) & 0 & 0 \\ 0 & a_2 - \frac{e_2}{c_2}y^*(t) & 0 \\ \frac{\beta_1}{c_1}y^*(t) & \frac{\beta_2}{c_2}y^*(t) & -D \end{pmatrix} \Psi(t)$$

and $\Psi(0) = I$, the identity matrix. So the fundamental solution matrix is

$$\Psi(t) = \begin{pmatrix} \exp(\int_0^t a_1 - \frac{e_1}{c_1}y^*(s)ds) & 0 & 0 \\ 0 & \exp(\int_0^t a_2 - \frac{e_2}{c_2}y^*(s)ds) & 0 \\ \exp(\int_0^t \frac{\beta_1}{c_1}y^*(s)ds & \exp(\int_0^t \frac{\beta_2}{c_2}y^*(s)ds & \exp(\int_0^t -Dds) \end{pmatrix}.$$

The resetting impulsive conditions of system (1.1) become

$$\begin{pmatrix} u((n+\tau-1)T^+) \\ v((n+\tau-1)T^+) \\ u((n+\tau-1)T^+) \end{pmatrix} = \begin{pmatrix} 1-p_1 & 0 & 0 \\ 0 & 1-p_2 & 0 \\ 0 & 0 & 1-p_3 \end{pmatrix} \begin{pmatrix} u((n+\tau-1)T) \\ v((n+\tau-1)T) \\ w((n+\tau-1)T) \end{pmatrix}$$

and

$$\begin{pmatrix} u(nT^+) \\ v(nT^+) \\ w(nT^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v(nT) \\ w(nT) \end{pmatrix}.$$

Note that all eigenvalues of

$$S = \begin{pmatrix} 1-p_1 & 0 & 0 \\ 0 & 1-p_2 & 0 \\ 0 & 0 & 1-p_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Psi(T)$$

are $\lambda_1 = (1-p_1)\exp(\int_0^T a_1 - \frac{e_1}{c_1}y^*(t)dt)$, $\lambda_2 = (1-p_2)\exp(\int_0^T a_2 - \frac{e_2}{c_2}y^*(t)dt)$ and $\lambda_3 = (1-p_3)\exp(-DT) < 1$. Note that

$$\int_0^T y^*(t)dt = \frac{q(1-(1-p_3)\exp(-DT) - p_3\exp(-D\tau T))}{D(1-(1-p_3)\exp(-DT))}. \tag{3.5}$$

It follows from (3.4) that

$$a_1T - \frac{e_1q\Phi(D)}{c_1} < \ln\frac{1}{1-p_1} \text{ and } a_2T - \frac{e_2q\Phi(D)}{c_2} < \ln\frac{1}{1-p_2}. \tag{3.6}$$

Also, we can induce from (3.5) that the conditions $|\lambda_1| < 1$ and $|\lambda_2| < 1$ are equivalent to (3.6). Therefore, from the Floquet theory [4], we obtain $(0, 0, y^*(t))$ is locally stable.

Now, to prove the global stability of the two-prey free periodic solution $(0, 0, y^*(t))$, let $(x_1(t), x_2(t), y(t))$ be a solution of system (1.1). From (3.4), we can select a sufficiently small number $\epsilon_1 > 0$ satisfying

$$\rho = (1-p_1)\exp\Big(a_1T + \frac{b_1e_1(\epsilon_1T - q\Phi(D))}{b_1c_1 + a_1 + b_1\epsilon_1}\Big) < 1.$$

It follows from the first equation in (1.1) that $x_1'(t) \le x_1(t)(a_1 - b_1x_1(t))$ for $t \ne (n+\tau-1)T, t \ne nT$ and $x_1(t^+) = (1-p_1)x_1(t) \le x_1(t)$ for $t = (n+\tau-1)T$. Now, consider the following impulsive differential equation:

$$\begin{cases} u'(t) &= u(t)(a_1 - b_1u(t)), t \ne (n+\tau-1)T, t \ne nT, \\ \Delta u(t) &= 0, t = nT, t = (n+\tau-1)T, \\ u(0^+) &= x_1(0^+). \end{cases} \tag{3.7}$$

From Lemma 2.3, we have $x_1(t) \le u(t)$. Since $u(t) \to \frac{a_1}{b_1}$ as $t \to \infty$, $x_1(t) \le \frac{a_1}{b_1} + \epsilon$ for any $\epsilon > 0$ with t large enough. For simplicity we may assume that $x_1(t) \le \frac{a_1}{b_1} + \epsilon_1$ for all $t > 0$. Similarly, we get $x_2(t) \le \frac{a_2}{b_2} + \epsilon_2$ for any $\epsilon_2 > 0$ and $t > 0$. Consider the following impulsive differential equation:

$$\begin{cases} v'(t) &= -Dv(t), t \ne (n+\tau-1)T, t \ne nT, \\ \Delta v(t) &= -p_3v(t), t = (n+\tau-1)T, \\ \Delta v(t) &= q, t = nT, \\ v(0^+) &= y(0^+). \end{cases} \tag{3.8}$$

Since $y'(t) \ge -Dy(t)$, we can obtain from Lemmas 2.3 and 3.1 that

$$y(t) \ge v(t) > y^*(t) - \epsilon_1 \tag{3.9}$$

for t sufficiently large. Without loss of generality, we may suppose that (3.9) holds for all $t \geq 0$. From (1.1), we obtain

$$\begin{cases} x_1'(t) & \leq x_1(t)\Big(a_1 - \dfrac{b_1e_1(y^*(t)-\epsilon_1)}{b_1c_1+a_1+b_1\epsilon_1}\Big), t \neq (n+\tau-1)T, t \neq nT, \\ \Delta x_1(t) & = -p_1x_1(t), t = (n+\tau-1)T, \\ \Delta x_1(t) & = 0, t = nT. \end{cases} \tag{3.10}$$

Integrating (3.10) on $((n+\tau-1)T, (n+\tau)T]$, we get

$$\begin{aligned} x_1((n+\tau)T) &\leq (1-p_1)x_1((n+\tau-1)T)\exp\Big(\int_{(n+\tau-1)T}^{(n+\tau)T} a_1 - \frac{a_1e_1(y^*(t)-\epsilon_1)}{a_1c_1+b_1+a_1\epsilon_1}dt\Big) \\ &= x_1((n+\tau-1)T)\rho \end{aligned}$$

and hence $x_1((n+\tau)T) \leq x_1(\tau T)\rho^n$ which implies that $x_1((n+\tau)T) \to 0$ as $n \to \infty$. Further, we obtain, for $t \in ((n+\tau-1)T, (n+\tau)T]$,

$$\begin{aligned} x_1(t) &\leq x_1((n+\tau-1)T+)\exp\Big(\int_{(n+\tau-1)T}^{t} a_1 - \frac{b_1e_1(y^*(t)-\epsilon_1)}{b_1c_1+a_1+b_1\epsilon_1}dt\Big) \\ &\leq x_1((n+\tau-1)T)\exp\big((a_1+\frac{e_1}{c_1}\epsilon_1)T\big) \end{aligned}$$

which implies that $x_1(t) \to 0$ as $t \to \infty$. By the same method we can show that $x_2(t) \to 0$ as $t \to \infty$. Now, take sufficiently small positive numbers ϵ_3 and ϵ_4 satisfying $\frac{\beta_1}{c_1}\epsilon_3 + \frac{\beta_2}{c_2}\epsilon_4 < D$ to prove that $y(t) \to y^*(t)$ as $t \to \infty$. Without loss of generality, we may assume that $x_1(t) \leq \epsilon_3$ and $x_2(t) \leq \epsilon_4$ for all $t \geq 0$. It follows from the third equation in (1.1) that, for $t \neq (n+\tau-1)T$ and $t \neq nT$,

$$y'(t) \leq y(t)\Big(-D + \frac{\beta_1}{c_1}\epsilon_3 + \frac{\beta_2}{c_2}\epsilon_4\Big). \tag{3.11}$$

Thus, by Lemma 2.3, we induce that $y(t) \leq \tilde{y}^*(t)$, where $\tilde{y}^*(t)$ is the solution of (3.1) with D changed into $D - \frac{\beta_1}{c_1}\epsilon_3 - \frac{\beta_2}{c_2}\epsilon_4$. Therefore, by letting $\epsilon_3, \epsilon_4 \to \infty$, we obtain from Lemma 3.1 and (3.9) that $y(t)$ tends to $y^*(t)$ as $t \to \infty$. □

From the proof of Theorem 3.2, we obtain the local stability condition of the periodic solution $(0, 0, y^*(t))$.

Corollary 3.3. *The periodic solution* $(0, 0, y^*(t))$ *of system (1.1) is locally stable if*

$$a_iT - \frac{e_iq\Phi(D)}{c_i} < \ln\frac{1}{1-p_i}(i = 1, 2). \tag{3.12}$$

Example 3.4. If we take $a_1 = 1, a_2 = 1, b_1 = 1, b_2 = 1.2, c_1 = 0.9, c_2 = 0.5, e_1 = 0.3, e_2 = 0.2, D = 0.8, \mu_1 = 0.1, \mu_2 = 0.2, \beta_1 = 0.8, \beta_2 = 1, p_1 = 0.7, p_2 = 0.6, p_3 = 0.0001, \tau = 0.6, T = 2$ and $q = 12$, then these parameters satisfy the condition of Theorem 3.2. Thus the periodic solution $(0, 0, y^*(t))$ is globally asymptotically stable.(See Figure 1). In fact, if we fix all parameters as above except q, then the solution $(0, 0, y^*(t))$ becomes a globally asymptotically stable periodic solution when $q > 5.7801$.

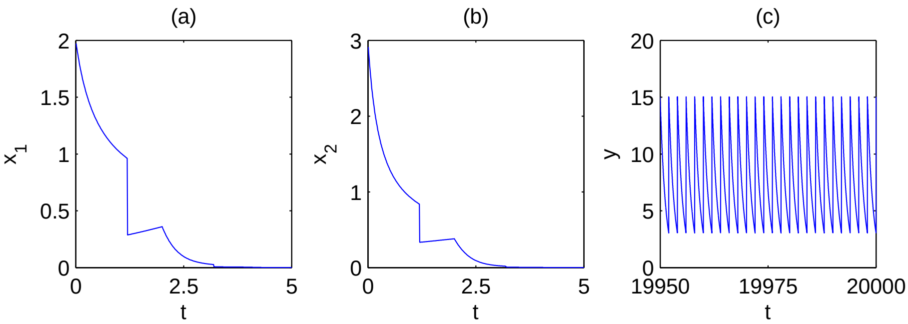

FIGURE 1. (a)-(c) Time series of system (1.1) with an initial value $(2, 3, 1)$.

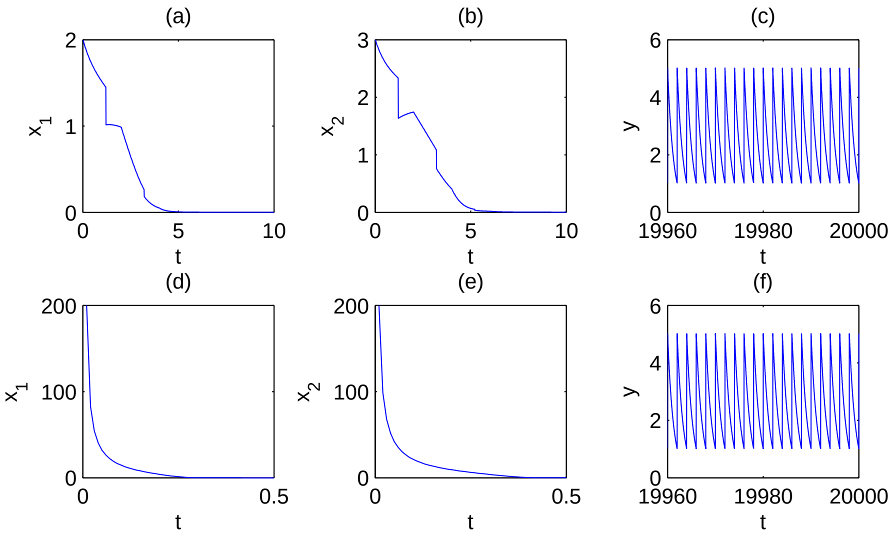

FIGURE 2. (a)-(c) Time series of system (1.1) with an initial value $(2, 3, 1)$ and (d)-(f) Time series of system (1.1) with an initial value $(200, 200, 200)$.

Example 3.5. It follows from Theorem 3.2 and Corollary 3.3 that system (1.1) may not be globally stable if the parameters satisfy the following conditions:

$$a_i T - \frac{e_i q \Phi(D)}{c_i} < \ln \frac{1}{1-p_i} < a_i T - \frac{b_i e_i q \Phi(D)}{b_i c_i + a_i} (i = 1, 2). \tag{3.13}$$

However, Figure 2 exhibits that system (1.1) seems to be globally stable even if the parameters $a_1 = 1, a_2 = 1, b_1 = 0.5, b_2 = 0.3, c_1 = 0.9, c_2 = 0.5, e_1 = 0.3, e_2 = 0.2, D = 0.8, \mu_1 = 0.1, \mu_2 = 0.2, \beta_1 = 0.8, \beta_2 = 1, p_1 = 0.3, p_2 = 0.3, p_3 = 0.0001, \tau = 0.6, T = 2$ and $q = 4$ are satisfied with the condition (3.13).

3.2. **Permanence.** In previous subsection we have shown that the globally asymptotically stable prey-free periodic solution exists under some conditions. Now, we turn our concern to the coexistence of all species. From biological point of view, we need protect animals or plants that are near extinction. In this context, in this subsection, we will discuss when we must harvest or pesticide the preys, and release the predator to maintain ecosystem. For this, we will first show that all solutions of system (1.1) are uniformly bounded.

Theorem 3.6. *There is a $M > 0$ such that $x_1(t) \le M, x_2(t) \le M$ and $y(t) \le M$ for all t large enough, where $(x_1(t), x_2(t), y(t))$ is a solution of system (1.1).*

Proof. Let $(x_1(t), x_2(t), y(t))$ be a solution of (1.1) with $x_{01}, x_{02}, y_0 \ge 0$ and let $F(t) = \frac{\beta_1}{e_1}x_1(t) + \frac{\beta_2}{e_2}x_2(t) + y(t)$ for $t > 0$. Then, if $t \ne (n+\tau-1)T$ and $t \ne nT$, then we obtain that $\frac{dF(t)}{dt} + \delta F(t) = -\frac{b_1\beta_1}{e_1}x_1^2(t) + \frac{\beta_1}{e_1}(a_1+\delta)x_1(t) - \frac{\beta_1\mu_1}{e_1}x_1(t)x_2(t) - \frac{b_2\beta_2}{e_2}x_2^2(t) + \frac{\beta_2}{e_2}(a_2+\delta)x_2(t) - \frac{\beta_2\mu_2}{e_2}x_1(t)x_2(t) + (\delta - D)y(t)$. From choosing $0 < \delta_0 < D$, we have, for $t \ne (n+\tau-1)T, t \ne nT$ and $t > 0$,

$$\frac{dF(t)}{dt} + \delta_0 F(t) \le -\frac{b_1\beta_1}{e_1}x_1^2(t) + \frac{\beta_1}{e_1}(a_1+\delta_0)x_1(t) - \frac{b_2\beta_2}{e_2}x_2^2(t) + \frac{\beta_2}{e_2}(a_2+\delta_0)x_2(t). \quad (3.14)$$

As the right-hand side of (3.14) is bounded from above by $M_0 = \frac{\beta_1(a_1+\delta_0)^2}{4b_1e_1} + \frac{\beta_2(a_2+\delta_0)^2}{4b_2e_2}$, it follows that

$$\frac{dF(t)}{dt} + \delta_0 F(t) \le M_0, t \ne (n+\tau-1)T, t \ne nT, t > 0.$$

If $t = nT$, then $F(t^+) = F(t) + q$ and if $t = (n+\tau-1)T$, then $F(t^+) \le (1-p)F(t)$, where $p = \min\{p_1, p_2, p_3\}$. From Lemma 2.4, we get that

$$\begin{aligned} F(t) &\le F_0(1-p)^{[\frac{t}{kT}]}\exp\Big(\int_0^t -\delta_0 ds\Big) \\ &\quad + \int_0^t (1-p)^{[\frac{t-s}{kT}]}\exp\Big(\int_s^t -\delta_0 d\gamma\Big)M_0 ds \\ &\quad + \sum_{j=1}^{[\frac{t}{kT}]}(1-p)^{[\frac{t-kT}{jT}]}\exp\Big(\int_{kT}^t -\delta_0 d\gamma\Big)q \\ &\le F_0\exp(-\delta_0 t) + \frac{M_0}{\delta_0}(1-\exp(-\delta_0 t)) + \frac{q\exp(\delta_0 T)}{\exp(\delta_0 T)-1}, \end{aligned} \quad (3.15)$$

where $F_0 = \frac{\beta_1}{e_1}x_{01} + \frac{\beta_2}{e_2}x_{02} + y_0$. Since the limit of the right-hand side of (3.15) as $t \to \infty$ is

$$\frac{M_0}{\delta_0} + \frac{q\exp(\delta_0 T)}{\exp(\delta_0 T)-1} < \infty,$$

it easily follows that $F(t)$ is bounded for sufficiently large t. Therefore, $x_1(t), x_2(t)$ and $y(t)$ are bounded by a constant M for sufficiently large t. □

In the following, let us investigate the permanence of system (1.1)

Theorem 3.7. *System (1.1) is permanent if* $D > \max\Big\{\dfrac{a_i\beta_i}{b_ic_i} : i = 1, 2\Big\}$,

$$\begin{aligned}&\big(a_1 - \mu_1\frac{a_2}{b_2}\big)T - \frac{e_1 q\Phi(D - \frac{a_2\beta_2}{b_2c_2})}{c_1} > \ln\frac{1}{1-p_1}\\ \text{and } &\big(a_2 - \mu_2\frac{a_1}{b_1}\big)T - \frac{e_2 q\Phi(D - \frac{a_1\beta_1}{b_1c_1})}{c_2} > \ln\frac{1}{1-p_2}.\end{aligned} \tag{3.16}$$

Proof. Let $(x_1(t), x_2(t), y(t))$ be a solution of system (1.1) with $(x_{01}, x_{02}, y_0) > 0$. From Theorem 3.6, we may assume that $x_1(t), x_2(t), y(t) \le M$ and $M > \max\{\frac{a_1c_1}{e_1}, \frac{a_2c_2}{e_2}\}$. As in the proof of Theorem 3.2, we can assume that $x_1(t) \le \frac{a_1}{b_1} + \epsilon_1$ and $x_2(t) \le \frac{a_2}{b_2} + \epsilon_2$ for $t > 0$. Let $m = \frac{q(1-p_3)\exp(-DT))}{1-(1-p_3)\exp(-DT)} - \epsilon$ for $\epsilon > 0$. Consider the following impulsive differential equation:

$$\begin{cases} u'(t) &= -Du(t), t \ne nT, t \ne (n+\tau-1)T,\\ \Delta u(t) &= -p_3u(t), t = (n+\tau-1)T,\\ \Delta u(t) &= q, t = nT,\\ u(0^+) &= y_0.\end{cases} \tag{3.17}$$

From Lemmas 2.3 and 3.1 we can obtain that $y(t) \ge u(t) > y^*(t) - \epsilon$, hence $y(t) > m$ for sufficiently large t. Thus we only need to find $\bar{m}_1$ and $\bar{m}_2$ such that $x_1(t) \ge \bar{m}_1$ and $x_2(t) \ge \bar{m}_2$ for t large enough. We will do this in the following two steps.

Step 1:Firstly, select sufficiently small numbers m_1 and $m_2 > 0$ such that $m_1 < \frac{c_1}{\beta_1}(D - \frac{\beta_2}{c_2}(\frac{a_2}{b_2} + \epsilon_2))$, $m_2 < \frac{c_2}{\beta_2}(D - \frac{\beta_1}{c_1}(\frac{a_1}{b_1} + \epsilon_1))$ and $\frac{\beta_1}{c_1}m_1 + \frac{\beta_2}{c_2}m_2 < D$. Let $E_1 = -D + \frac{\beta_1}{c_1}m_1 + \frac{\beta_2}{c_2}(\frac{a_2}{b_2} + \epsilon_2) < 0$ and $E_2 = -D + \frac{\beta_1}{c_1}m_1 + \frac{\beta_2}{c_2}m_2 < 0$. We will prove there exist $t_1, t_2 \in (0, \infty)$ such that $x_1(t_1) \ge m_1$ and $x_2(t_2) \ge m_2$. Suppose not. Then we have only consider the following three cases:

(i) There exists a $t_2 > 0$ such that $x_2(t_2) \ge m_2$, but $x_1(t) < m_1$, for all $t > 0$;

(ii) There exists a $t_1 > 0$ such that $x_1(t_1) \ge m_1$, but $x_2(t) < m_2$, for all $t > 0$;

(iii) $x_1(t) < m_1$ and $x_2(t) < m_2$ for all $t > 0$.

Case (i): From (3.16), we can take $\eta_1 > 0$ small enough such that

$$\phi_1 = (1-p_1)\exp\Big(\big(a_1 - b_1m_1 - \mu_1(\frac{a_2}{b_2} + \epsilon_2)\big)T - \frac{e_1}{c_1}(q\Phi(-E_1) + \eta_1T)\Big) > 1. \tag{3.18}$$

We obtain from the conditions of case (i) that $y'(t) \le y(t)(-D + \frac{\beta_1}{c_1}x_1(t) + \frac{\beta_2}{c_2}x_2(t)) \le y(t)(-D + \frac{\beta_1}{c_1}m_1 + \frac{\beta_2}{c_2}(\frac{a_2}{b_2} + \epsilon_2)) = E_1y(t)$ for $t \ne (n+\tau-1)T, t \ne nT$. Thus we have $y(t) \le v(t)$ and $v(t) \to v^*(t)$ as $t \to \infty$, where $v(t)$ is the solution of system

$$\begin{cases} v'(t) &= E_1v(t), t \ne (n+\tau-1)T, t \ne nT,\\ \Delta v(t) &= -p_3v(t), t = (n+\tau-1)T,\\ \Delta v(t) &= q, t = nT,\\ v(0^+) &= y_0\end{cases} \tag{3.19}$$

and

$$v^*(t) = \begin{cases} \dfrac{q\exp(E_1(t-(n-1)T))}{1-(1-p_3)\exp(E_1T)}, (n-1)T < t \leq (n+\tau-1)T, \\ \dfrac{q(1-p_3)\exp(E_1(t-(n-1)T))}{1-(1-p_3)\exp(E_1T)}, (n+\tau-1)T < t \leq nT. \end{cases} \tag{3.20}$$

Therefore, we can take a $T_1 > 0$ such that $y(t) \leq v(t) < v^*(t) + \eta_1$ for $t > T_1$. Thus we get

$$\begin{cases} x_1'(t) & \geq x_1(t)(a_1 - b_1m_1 - \mu_1(\frac{a_2}{b_2} + \epsilon_2) - \frac{e_1}{c_1}(v^*(t) + \eta_1))), \\ & t \neq (n+\tau-1)T, t \neq nT, \\ \Delta x_1(t) & = -p_1x_1(t), t = (n+\tau-1)T, \\ \Delta x_1(t) & = 0, t = nT \end{cases} \tag{3.21}$$

for $t > T_1$. Let $N_1 \in \mathbb{N}$ be such that $(N_1 + \tau - 1)T \geq T_1$. Integrating the equation (3.21) on $((n+\tau-1)T, (n+\tau)T], n \geq N_1$, we can obtain that $x_1((n+\tau)T) \geq x_1((n+\tau-1)T)(1-p_1)\exp(\int_{(n+\tau-1)T}^{(n+\tau)T} a_1 - b_1m_1 - \mu_1(\frac{a_2}{b_2} + \epsilon_2) - \frac{e_1}{c_1}(v^*(t) + \eta_1)dt = x_1((n+\tau-1)T)\phi_1$. Thus $x_1((N_1 + k + \tau)T) \geq x_1((N_1 + \tau)T)\phi_1^k \to \infty$ as $k \to \infty$, which is a contradiction to the boundedness of $x_1(t)$.

Case (ii): The same argument as the case (i) can be applied. So we omit it.

Case (iii): We choose $\eta_2 > 0$ sufficiently small so that

$$\phi_2 = (1-p_1)\exp\Big((a_1 - b_1m_1 - \mu_1m_2)T - \frac{e_1}{c_1}(q\Phi(-E_2) + \eta_2T)\Big) > 1. \tag{3.22}$$

From the assumption of case (iii), we obtain $y'(t) = y(t)(-D + \frac{\beta_1}{c_1}m_1 + \frac{\beta_2}{c_2}m_2) = E_2y(t)$ for $tt \neq (n+\tau-1)T, \neq nT$. It follows from Lemmas 2.3 and 3.1 that $y(t) \leq w(t)$ and $w(t) \to w^*(t)$ as $t \to \infty$, where $w(t)$ is the solution of the following system :

$$\begin{cases} w'(t) & = E_2w(t), t \neq (n+\tau-1)T, t \neq nT, \\ \Delta w(t) & = -p_3w(t), t = (n+\tau-1)T, \\ \Delta w(t) & = q, t = nT, \\ w(0^+) & = y_0 \end{cases} \tag{3.23}$$

and

$$w^*(t) = \begin{cases} \dfrac{q\exp(E_2(t-(n-1)T))}{1-(1-p_3)\exp(E_2T)}, (n-1)T < t \leq (n+\tau-1)T, \\ \dfrac{q(1-p_3)\exp(E_2(t-(n-1)T))}{1-(1-p_3)\exp(E_2T)}, (n+\tau-1)T < t \leq nT. \end{cases} \tag{3.24}$$

Thus there exists a $T_2 > 0$ such that $y(t) \leq w(t) < w^*(t) + \eta_2$ for $t > T_2$ and

$$\begin{cases} x_1'(t) & \geq x_1(t)(a_1 - b_1 m_1 - \mu_1 m_2 - \frac{e_1}{c_1}(w^*(t) + \eta_2)), \\ & t \neq (n+\tau-1)T, t \neq nT, \\ \Delta x_1(t) & = -p_1 x_1(t), t = (n+\tau-1)T, \\ \Delta x_1(t) & = 0, t = nT \end{cases} \tag{3.25}$$

for $t > T_2$. Let $N_2 \in \mathbb{N}$ be such that $(N_2 + \tau - 1)T \geq T_2$. Integrating the equation (3.25) on $((n+\tau-1)T, (n+\tau)T], n \geq N_2$, we can obtain that $x_1((n+\tau)T) \geq x_1((n+\tau-1)T)(1-p_1)\exp(\int_{(n+\tau-1)T}^{(n+\tau)} a_1 - b_1 m_1 - \mu_1 m_2 - \frac{e_1}{c_1}(w^*(t) + \eta_2)dt = x_1((n+\tau-1)T)\phi_2$. Similarly, we have $x_1((N_2+k+\tau)T) \geq x_1((N_2+\tau)T)\phi_2^k \to \infty$ as $k \to \infty$, which is a contradiction to the boundedness of $x_1(t)$. To sum it up , there exist $t_1 > 0$ and $t_2 > 0$ such that $x_1(t_1) \geq m_1$ and $x_2(t_2) \geq m_2$.

Step 2: If $x_1(t) \geq m_1$ for all $t \geq t_1$, then we are done. If not, we may let $t^* = \inf_{t>t_1}\{x_1(t) < m_1\}$. Then $x_1(t) \geq m_1$ for $t \in [t_1, t^*]$ and, by the continuity of $x_1(t)$, we have $x_1(t^*) = m_1$. In this step, we have only to consider two possible cases.

Case (i): Suppose that $t^* = (n_1 + \tau - 1)T$ for some $n_1 \in \mathbb{N}$. Then $(1-p_1)m_1 \leq x_1(t^{*+}) = (1-p_1)x_1(t^*) < m_1$. Select $n_2, n_3 \in \mathbb{N}$ such that $(n_2 - 1)T > \dfrac{\ln(\frac{\eta_1}{M+q})}{E_1}$ and $(1-p_1)^{n_2}\phi_1^{n_3}\exp(n_2\sigma T) > (1-p_1)^{n_2}\phi_1^{n_3}\exp((n_2+1)\sigma T) > 1$, where $\sigma = a_1 - b_1 m_1 - \mu_1(b_2 + \epsilon_2) - \frac{e_1}{c_1}M < 0$. Let $T' = n_2 T + n_3 T$. In this case we will show that there exists $t_3 \in (t^*, t^* + T']$ such that $x_1(t_3) \geq m_1$. Otherwise, by (3.3) and (3.19) with $v(n_1 T^+) = y(n_1 T^+)$, we have

$$v(t) = \begin{cases} (1-p_3)^{n-(n_1+1)}\left(v(n_1 T^+) - \dfrac{q(1-p_3)\exp(-T)}{1-(1-p_3)\exp(E_1 T)}\right) \\ \exp(E_1(t - n_1 T)) + v^*(t), (n-1)T < t \leq (n+\tau-1)T, \\ (1-p_3)^{(n-n_1)}\left(v(n_1 T^+) - \dfrac{q(1-p_3)\exp(-T)}{1-(1-p_3)\exp(E_1 T)}\right) \\ \exp(E_1(t - n_1 T)) + v^*(t), (n+\tau-1)T < t \leq nT, \end{cases}$$

and $n_1 + 1 \leq n \leq n_1 + 1 + n_2 + n_3$. So we get $|v(t) - v^*(t)| \leq (M+q)\exp(E_1(t - n_1 T)) < \eta_1$ and $y(t) \leq v(t) \leq v^*(t) + \eta_1$ for $n_1 T + (n_2 - 1)T \leq t \leq t^* + T'$, which implies (3.21) holds for $t \in [t^* + n_2 T, t^* + T']$. As in step 1, we have

$$x_1(t^* + T') \geq x_1(t^* + n_2 T)\phi_1^{n_3}.$$

Since $y(t) \leq M$, we have

$$\begin{cases} x_1'(t) & \geq x_1(t)(a_1 - b_1 m_1 - \mu_1(\frac{a_2}{b_2} + \epsilon_2) - \frac{e_1}{c_1}M) = \sigma x_1(t), \\ & t \neq nT, t \neq (n+\tau-1)T, \\ \Delta x_1(t) & = -p_1 x_1(t), t = (n+\tau-1)T, \\ \Delta x_1(t) & = 0, t = nT \end{cases} \tag{3.26}$$

for $t \in [t^*, t^* + n_2T]$. Integrating (3.26) on $[t^*, t^* + n_2T]$ we have

$$\begin{aligned} x_1((t^* + n_2T)) &\geq m_1 \exp(\sigma n_2 T) \\ &\geq m_1(1-p_1)^{n_2} \exp(\sigma n_2 T) > m_1. \end{aligned}$$

Thus $x_1(t^* + T') \geq m_1(1-p_1)^{n_2} \exp(\sigma n_2 T)\phi_1^{n_3} > m_1$ which is a contradiction. Now, let $\bar{t}$ = $\inf_{t>t^*}\{x_1(t) \geq m_1\}$. Then $x_1(t) \leq m_1$ for $t^* \leq t < \bar{t}$ and $x_1(\bar{t}) = m_1$. So, we have, for $t \in [t^*, \bar{t})$, $x_1(t) \geq m_1(1-p_1)^{n_2+n_3} \exp(\sigma(n_2+n_3)T) \equiv \bar{m}_1$. For $t > t^*$ the same argument can be continued since $x_1(\bar{t}) \geq m_1$. Hence $x_1(t) \geq \bar{m}_1$ for all $t > t_1$.
Case (ii): $t^* \neq (n + \tau - 1)T, n \in \mathbb{N}$. Suppose that $t^* \in ((n_1' + \tau - 1)T, (n_1' + \tau)T)$ for some $n_1' \in \mathbb{N}$. There are two possible cases for $t \in (t^*, (n_1' + \tau)T)$. Firstly, if $x_1(t) \leq m_1$ for all $t \in (t^*, (n_1' + \tau)T)$, similar to case (i), we can prove there must be a $t_3' \in [(n_1' + \tau)T, (n_1' + \tau)T + T']$ such that $x_1(t_3') \geq m_1$. Here we omit it. Let $\hat{t} = \inf_{t>t^*}\{x_1(t) \geq m_1\}$. Then $x_1(t) \leq m_1$ for $t \in (t^*, \hat{t})$ and $x_1(\hat{t}) = m_1$. For $t \in (t^*, \hat{t})$, we have $x_1(t) \geq m_1(1 - p_1)^{n_2+n_3} \exp(\sigma(n_2 + n_3 + 1)T) = m_1$. So, $m_1 < \bar{m}_1$ and $x_1(t) \geq m_1$ for $t \in (t^*, \hat{t})$. For $t > t^*$ the same argument can be continued since $x_1(\hat{t}) \geq m_1$. Hence $x_1(t) \geq \bar{m}_1$ for all $t > t_1$ Secondly, if there exists a $t \in (t^*, (n_1' + \tau)T)$ such that $x_1(t) \geq m_1$. Let $\check{t} = \inf_{t>t^*}\{x_1(t) \geq m_1\}$. Then $x_1(t) \leq m_1$ for $t \in (t^*, \check{t})$ and $x_1(\check{t}) = m_1$. For $t \in (t^*, \check{t})$, we have $x_1(t) \geq x_1(t^*) \exp(\sigma(t - t^*)) \geq m_1 \exp(\sigma T) > m_1$. This process can be continued since $x_1(\check{t}) \geq m_1$, and have $x_1(t) \geq \bar{m}_1$ for all $t > t_1$. Similarly, we can show that $x_2(t) \geq \bar{m}_2$ for all $t > t_2$. This completes the proof.

□

Example 3.8. Let $a_1 = 2, a_2 = 1, b_1 = 1, b_2 = 0.9, c_1 = 0.9, c_2 = 0.5, e_1 = 0.1, e_2 = 0.2, D = 0.7, \mu_1 = 0.1, \mu_2 = 0.2, \beta_1 = 0.2, \beta_2 = 0.1, p_1 = 0.2, p_2 = 0.1, p_3 = 0.0001, \tau = 0.4, T = 6$ and $q = 2$. Then, from Theorem 3.7, we know that system (1.1) is permanent.(See Figure 3). In this case, if $q < 2.9996$, system (1.1) is permanent.

It follows from Theorems 3.2 and 3.7 that the following Corollaries hold.

Corollary 3.9. *Let $(x_1(t), x_2(t), y(t))$ be any solution of system (1.1). Then $x_1(t)$ and $y(t)$ are permanent, and $x_2(t) \to 0$ as $t \to \infty$ provided that $D > \dfrac{a_2\beta_2}{b_2c_2}$,*

$$(a_1 - \mu_1 \frac{a_2}{b_2})T - \frac{e_1 q \Phi(D - \frac{a_2\beta_2}{b_2c_2})}{c_1} > \ln \frac{1}{1-p_1} \text{ and } a_2T - \frac{b_2e_2q\Phi(D)}{b_2c_2 + a_2} < \ln \frac{1}{1-p_2}.$$

Corollary 3.10. *Let $(x_1(t), x_2(t), y(t))$ be any solution of system (1.1). Then $x_2(t)$ and $y(t)$ are permanent, and $x_1(t) \to 0$ as $t \to \infty$ provided that $D > \dfrac{a_1\beta_1}{b_1c_1}$,*

$$a_1T - \frac{b_1e_1q\Phi(D)}{b_1c_1 + a_1} < \ln \frac{1}{1-p_1} \text{ and } (a_2 - \mu_2 \frac{a_1}{b_1})T - \frac{e_2 q \Phi(D - \frac{a_1\beta_1}{b_1c_1})}{c_2} > \ln \frac{1}{1-p_2}.$$

Example 3.11. Figure 4 is an example that satisfies the condition of the Corollary 3.9. In other words, x_1 and $y(t)$ are permanent, and $x_2(t) \to 0$ as $t \to \infty$, where $(x_1(t), x_2(t), y(t))$ is a

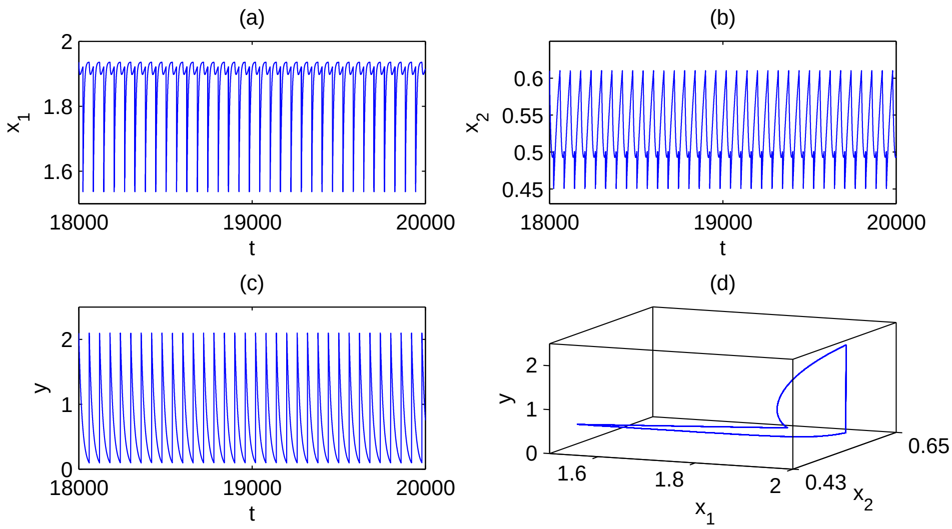

FIGURE 3. (a)-(c) Time series. (d) The trajectory of system (1.1) with an initial value $(2, 3, 1)$.

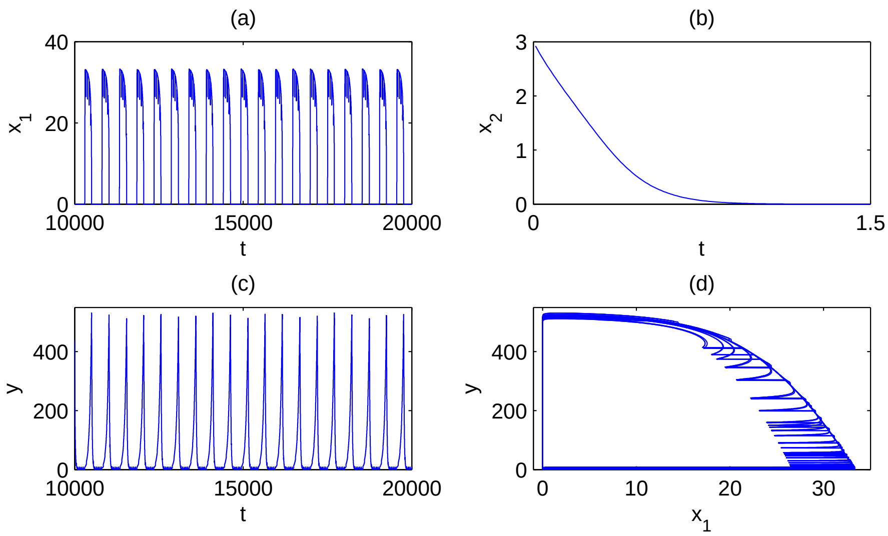

FIGURE 4. (a)-(c) Time series. (d) The trajectory of system (1.1) with an initial value $(2, 3, 1)$.

solution of system (1.1) with $a_1 = 10, a_2 = 1, b_1 = 0.3, b_2 = 1, c_1 = 0.5, c_2 = 0.4, e_1 =$

$0.2, e_2 = 0.8, D = 0.7, \mu_1 = 0.1, \mu_2 = 0.2, \beta_1 = 0.9, \beta_2 = 0.2, p_1 = 0.2, p_2 = 0.7, p_3 = 0.0001, \tau = 0.6, T = 5$ and $q = 8$.

4. Analysis on system (1.1) with sesonality

In this section we consider the intrinsic growth rates a_1 and a_2 in system (1.1) as periodically varying function of time due to seasonal variation. The seasonality is superimposed as follows:

$$a_{01} = a_1(1 + \epsilon_1 \sin(\omega_1 t)) \text{ and } a_{02} = a_2(1 + \epsilon_2 \sin(\omega_2 t)),$$

where the parameter ϵ_i(i=1,2) represent the degree of seasonality; for each $i = 1, 2$, $\lambda_i = a_i\epsilon_i \geq 0$ is the magnitude of the perturbation in a_{0i}, ω_i is the angular frequency of the fluctuation caused by seasonality. With this idea of periodic forcing, we consider the following two-prey and one-predator system with periodic variation in the intrinsic growth rate of the preys.

$$\begin{cases} \left.\begin{aligned} x_1'(t) &= x_1(t)\Big(a_1 - b_1x_1(t) + \lambda_1 \sin(\omega_1 t) - \mu_1 x_2(t) - \frac{e_1 y(t)}{c_1 + x_1(t)}\Big), \\ x_2'(t) &= x_2(t)\Big(a_2 - b_2x_2(t) + \lambda_2 \sin(\omega_2 t) - \mu_2 x_1(t) - \frac{e_2 y(t)}{c_2 + x_2(t)}\Big), \\ y'(t) &= y(t)\Big(-D + \frac{\beta_1 x_1(t)}{c_1 + x_1(t)} + \frac{\beta_2 x_2(t)}{c_2 + x_2(t)}\Big), \end{aligned}\right\} \\ \qquad\qquad\qquad\qquad\qquad\qquad t \neq nT, t \neq (n + \tau - 1)T, \\ \left.\begin{aligned} \Delta x_1(t) &= -p_1 x_1(t), \\ \Delta x_2(t) &= -p_2 x_2(t), \\ \Delta y(t) &= -p_3 y(t), \end{aligned}\right\} t = (n + \tau - 1)T, \\ \left.\begin{aligned} \Delta x_1(t) &= 0, \\ \Delta x_2(t) &= 0, \\ \Delta y(t) &= q. \end{aligned}\right\} t = nT, \\ (x_1(0^+), x_2(0^+), y(0^+)) = (x_{01}, x_{02}, y_0), \end{cases} \tag{4.1}$$

where λ_i and $\omega_i (i = 1, 2)$ represent the magnitude and the frequency of the forcing term, respectively.

Similarly to Lemma 2.5, we obtain that the solution of system (1.1) with a strictly positive initial value remains strictly positive.

Lemma 4.1. *The positive octant $(\mathbb{R}_+^*)^3$ is an invariant region for system (4.1).*

Now, we consider the following impulsive differential equation to prove the boundedness of the solutions to system (4.1) and the stability of the periodic solution $(0, 0, y^*(t))$ of system (4.1) under some conditions.

$$\begin{cases} \left.\begin{aligned} x_{11}'(t) &= x_{11}(t)\Big(a_1+\lambda_1-b_1x_{11}(t)-\mu_1x_{12}(t)-\frac{e_1y_1(t)}{c_1+x_{11}(t)}\Big),\\ x_{12}'(t) &= x_{12}(t)\Big(a_2+\lambda_2-b_2x_{12}(t)-\mu_2x_{11}(t)-\frac{e_2y_1(t)}{c_2+x_{12}(t)}\Big),\\ y_1'(t) &= y_1(t)\Big(-D+\frac{\beta_1x_{11}(t)}{c_1+x_{11}(t)}+\frac{\beta_2x_{12}(t)}{c_2+x_{12}(t)}\Big), \end{aligned}\right\} \\ \qquad\qquad\qquad\qquad\qquad\qquad t\neq(n+\tau-1)T, t\neq nT,\\ \left.\begin{aligned} \Delta x_{11}(t) &= -p_1x_{11}(t),\\ \Delta x_{12}(t) &= -p_2x_{12}(t),\\ \Delta y_1(t) &= -p_3y_1(t), \end{aligned}\right\} t=(n+\tau-1)T,\\ \left.\begin{aligned} \Delta x_{11}(t) &= 0,\\ \Delta x_{12}(t) &= 0,\\ \Delta y_1(t) &= q. \end{aligned}\right\} t=nT,\\ (x_{11}(0^+),x_{12}(0^+),y_1(0^+))=(x_{01},x_{02},y_0). \end{cases} \tag{4.2}$$

It follows from Lemma 2.3 that $x_1(t)\leq x_{11}(t), x_2(t)\leq x_{12}(t)$ and $y(t)\leq y_1(t)$, where $(x_1(t),x_2(t),y(t))$ and $(x_{11}(t),x_{12}(t),y_1(t))$ are any solution to system (4.1) and (4.2), respectively. But, the periodic solutions $(0,0,y^*(t))$ and $(0,0,y_1^*(t))$ of system (1.1) and (4.2), respectively, are the same. Thus, we obtain the following two Theorems by applying Lemma 2.3 and the method used in the proof of Theorems 3.6 and 3.2 to system (4.2).

Theorem 4.2. *There is an $M'>0$ such that $x_1(t)\leq M', x_2(t)\leq M'$ and $y(t)\leq M'$ for all t large enough, where $(x_1(t),x_2(t),y(t))$ is a solution of system (4.1).*

Theorem 4.3. *The periodic solution $(0,0,y^*(t))$ of system (4.1) is globally asymptotically stable if*

$$(a_i+\lambda_i)T-\frac{b_ie_iq\Phi(D)}{b_ic_i+a_i+\lambda_i}<\ln\frac{1}{1-p_i}(i=1,2).$$

Next, we provide the sufficient conditions for the permanence of system (4.1).

Theorem 4.4. *System (4.1) is permanent if* $D>\max\Big\{\frac{(a_i-\lambda_i)\beta_i}{b_ic_i}:i=1,2\Big\}$,
$\Big(a_1-\lambda_1-\frac{(a_2-\lambda_2)\mu_1}{b_2}\Big)T-\frac{e_1q}{c_1}\Phi\Big(D-\frac{(a_2-\lambda_2)\beta_2}{b_2c_2}\Big)>\ln\frac{1}{1-p_1}$
and $\Big(a_2-\lambda_2-\frac{(a_1-\lambda_1)\mu_2}{b_1}\Big)T-\frac{e_2q}{c_2}\Phi\Big(D-\frac{(a_1-\lambda_1)\beta_1}{b_1c_1}\Big)>\ln\frac{1}{1-p_2}$.

Proof. It follows from Theorem 4.2 that we may assume $x_1(t), x_2(t), y(t) \leq M'$ for some $M' > 0$. Consider the following impulsive differential equation:

$$\begin{cases} \left.\begin{aligned} x'_{21}(t) &= x_{21}(t)\Big(a_1 - \lambda_1 - b_1 x_{21}(t) - \mu_1 x_{22}(t) - \frac{e_1 y_2(t)}{c_1 + x_{21}(t)}\Big), \\ x'_{22}(t) &= x_{22}(t)\Big(a_2 - \lambda_2 - b_2 x_{22}(t) - \mu_2 x_{21}(t) - \frac{e_2 y_2(t)}{c_2 + x_{22}(t)}\Big), \\ y'_2(t) &= y_2(t)\Big(-D + \frac{\beta_1 x_{21}(t)}{c_1 + x_{21}(t)} + \frac{\beta_2 x_{22}(t)}{c_2 + x_{22}(t)}\Big), \end{aligned}\right\} \\ \qquad\qquad\qquad\qquad\qquad\qquad t \neq nT, t \neq (n+\tau-1)T, \\ \left.\begin{aligned} \Delta x_{21}(t) &= -p_1 x_{21}(t), \\ \Delta x_{22}(t) &= -p_2 x_{22}(t), \\ \Delta y_2(t) &= -p_3 y_2(t), \end{aligned}\right\} t = (n+\tau-1)T, \\ \left.\begin{aligned} \Delta x_{21}(t) &= 0, \\ \Delta x_{22}(t) &= 0, \\ \Delta y_2(t) &= q \end{aligned}\right\} t = nT, \\ (x_{21}(0^+), x_{22}(0^+), y_2(0^+)) = (x_{01}, x_{02}, y_0). \end{cases} \tag{4.3}$$

From Lemma 2.3, we obtain $x_1(t) \geq x_{21}(t), x_2(t) \geq x_{22}(t)$ and $y(t) \geq y_2(t)$, where $(x_1(t),x_2(t),y(t))$ and $(x_{21}(t), x_{22}(t), y_2(t))$ are any solution to system (4.1) and (4.3), respectively. For system (4.3), we can show the solution $(x_{21}(t), x_{22}(t), y_2(t))$ has a lower bound $m' > 0$ using the method of Theorem 3.7. Thus, system (4.1) is permanent. □

Example 4.5. From Theorem 4.4, we get that system (4.1) with $a_1 = 3, a_2 = 2, b_1 = 0.8, b_2 = 0.6, c_1 = 0.8, c_2 = 0.6, e_1 = 0.8, e_2 = 0.9, D = 0.7, \mu_1 = 0.3, \mu_2 = 0.2, \beta_1 = 0.2, \beta_2 = 0.1, p_1 = 0.1, p_2 = 0.2, p_3 = 0.001, \tau = 0.7, T = 8.0, q = 1, \omega_1 = 2\pi, \omega_2 = \frac{\pi}{4}, \lambda_1 = 2$ and $\lambda_2 = 1$ is permanent. (See Figure 5).

It follows from Theorems 4.3 and 4.4 that the following Corollaries hold.

Corollary 4.6. *Let $(x_1(t), x_2(t), y(t))$ be any solution of system (1.1). Then x_1 and $y(t)$ are permanent, and $x_2(t) \to 0$ as $t \to \infty$ provided that $D > \dfrac{(a_2 - \lambda_2)\beta_2}{b_2 c_2}$,*

$$(a_1 - \lambda_1 - \frac{(a_2 - \lambda_2)\mu_1}{b_2})T - \frac{e_1 q \Phi(D - \frac{(a_2-\lambda_2)\beta_2}{b_2 c_2})}{c_1} > \ln \frac{1}{1 - p_1}$$
$$\text{and } (a_2 + \lambda_2)T - \frac{b_2 e_2 q \Phi(D)}{b_2 c_2 + a_2 + \lambda_2} < \ln \frac{1}{1 - p_2}.$$

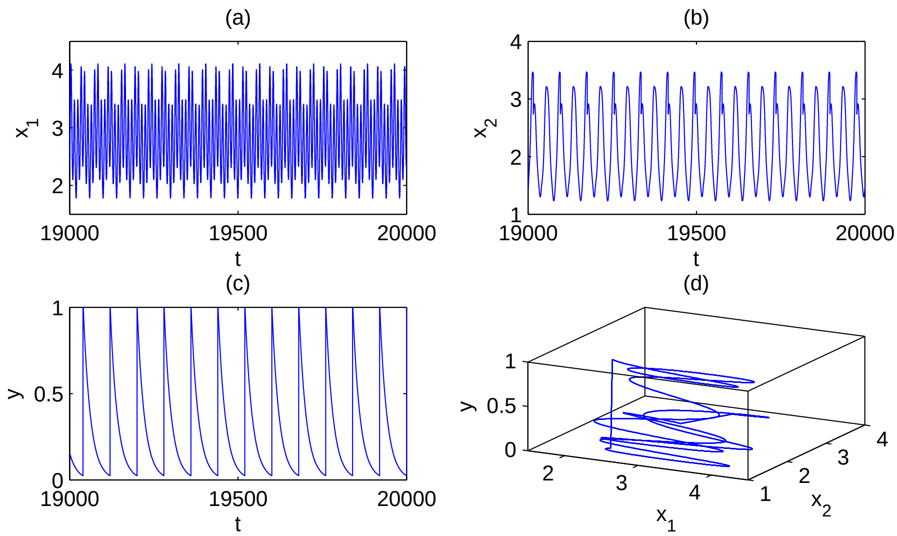

FIGURE 5. (a)-(c) Time series. (d) The trajectory of system (4.1) with an initial value $(2, 3, 1)$.

Corollary 4.7. *Let* $(x_1(t), x_2(t), y(t))$ *be any solution of system (1.1). Then* x_2 *and* $y(t)$ *are permanent, and* $x_1(t) \to 0$ *as* $t \to \infty$ *provided that* $D > \dfrac{(a_1 - \lambda_1)\beta_1}{b_1 c_1}$,

$$(a_1 + \lambda_1)T - \frac{b_1 e_1 q \Phi(D)}{b_1 c_1 + a_1 + \lambda_1} < \ln \frac{1}{1 - p_1}$$

$$\text{and } (a_2 - \lambda_2 - \mu_2 \frac{a_1 - \lambda_1}{b_1})T - \frac{e_2 q \Phi(D - \frac{(a_1 - \lambda_1)\beta_1}{b_1 c_1})}{c_2} > \ln \frac{1}{1 - p_2}.$$

5. DISCUSSION

In this paper, we investigated the effects of impulsive perturbations and seasonality on Holling-type II two-prey one-predator systems. Conditions for system (1.1) and (4.1) to be extinct are given by using the Floquet theory of impulsive differential equation and small amplitude perturbation skills. Also, it is proved that systems (1.1) and (4.1) are permanent under some conditions via the comparison theorem. We gave some examples. We also established the conditions for the extinction of one of two preys and permanence of the remaining two species. These results are utilized to control the population of the designated prey(pest). For example, suppose that x_2 is a harmful pest to be extirpated but x_1 is not. Using Theorems 3.3 and 3.7, one can choose suitable parameters in system (1.1) to eradicate the target prey and to prevent the non-target prey from extinction (see Figure 4). Thus we can get rid of one of two preys selectively by using our results.

Now, to observe the dynamic complexities, we fix the parameters except q in system (4.1) as follows:
$a_1 = 2, a_2 = 3, b_1 = 1, b_2 = 1.5, c_1 = 0.9, c_2 = 0.5, e_1 = 0.25, e_2 = 0.3, D = 0.6, \mu_1 = 0.1, \mu_2 = 0.1, \beta_1 = 0.8, \beta_2 = 0.9, p_1 = 0.5, p_2 = 0.45, p_3 = 0.0001, \tau = 0.6, T = 2, \omega_1 = 2\pi, \omega_2 = \frac{\pi}{4}, \lambda_1 = 0.01$ and $\lambda_2 = 0.02$.

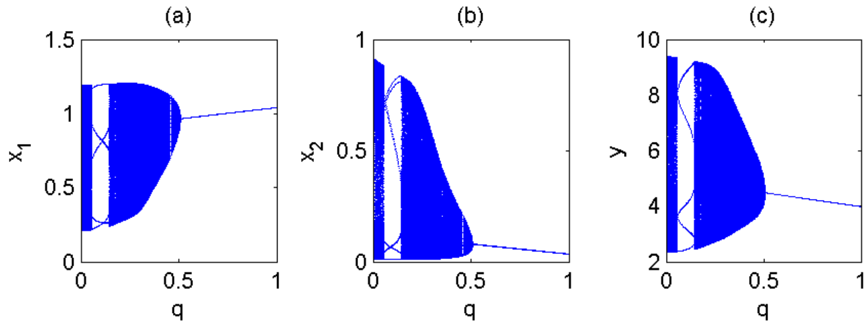

FIGURE 6. Bifurcation diagrams of system (4.1). (a) -(c)) x, y and z are plotted for q.

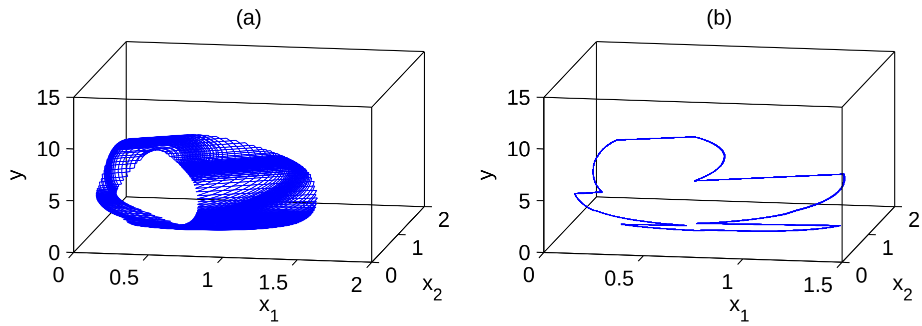

FIGURE 7. Phase portraits of solutions to system (4.1) with an initial condition $(2, 3, 1)$. (a) $q = 0.02$, (b) $q = 0.1$.

Figure 6 displays the bifurcation diagrams of system (4.1) for $0 \leq q \leq 1$. From this Figure, we can see that system (4.1) experiences quasi-periodic oscillation(See Figure 7(a)) when q is very small. However, when $0.06 < q < 0.145$, system (4.1) undergoes periodic window(See Figure 7(b)). Also, system has a chaotic area. Especially, Figure 8 shows two different strange attractors of system (4.1). These numerical simulations point out that the systems dealt in this paper have complex dynamical behaviors including chaotic phase portraits.

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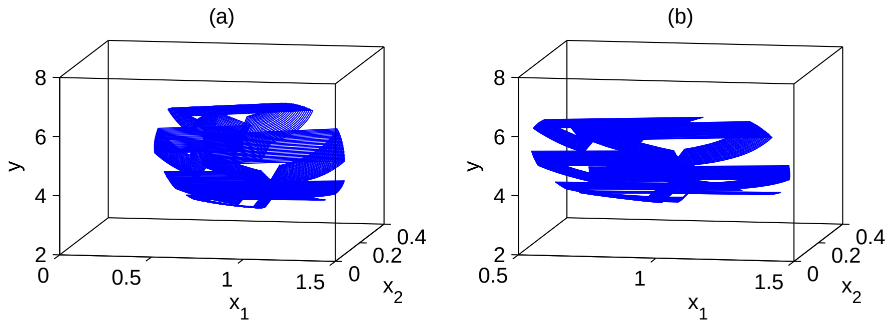

FIGURE 8. Phase portraits of solutions to system (4.1) with an initial condition $(2, 3, 1)$. (a) $q = 0.45$, (b) $q = 0.465$.

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