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MULTI-TYPE FINANCIAL ASSET MODELS FOR PORTFOLIO CONSTRUCTION

JAE-PILL OH

DEPARTMENT OF MATHEMATICS, KANGWEON NATIONAL UNIVERSITY, CHUNCHEON 200-701, KOREA
E-mail address: jpoh@kangwon.ac.kr

ABSTRACT. We define some asset models which are useful for portfolio construction in various terms of time. Our asset models are geometric jump-diffusions defined by the solutions of stochastic differential equations which are decomposed by various terms of time basically. We also can study pricing and hedging strategy of options in our models roughly.

1. INTRODUCTION

The purpose of this paper is to define and introduce several asset models which are distinguished by investing terms of time; intraday, weekly, monthly and quarterly, annual and one year over long-term of time etc. Our asset models are geometric jump-diffusions defined by the solutions of SDE which are decomposed by various terms of time basically. The main purpose of our model is for portfolio construction in various terms of time. But also we can study roughly the pricing and hedging strategy of option in our models.

Since stock market was started in 1531, Antwerp in Belgium, many asset models of stock markets were introduced for hundreds of years. The first attempt using probability notion was due to L. Bachelier in Paris about 1900. Mathematical finance asset models were started from the Black-Scholes model which was studied by F. Black and M. Scholes(1973), and R. Merton(1973). To define this basic asset model, they used Itô stochastic differential equation(SDE) and some martingale theories in key ways even if their main result was option pricing. Development of this area has been closely intertwined with that of the theory of stochastic integration. Nowadays, this area becomes a branch of applied mathematics, and sometimes asset models are called the financial models of stochastic processes defined by SDE. Also we know many kinds of asset models which are represented and defined by the solutions of various SDE. Thus, we also have many kinds of interesting derivatives fortunately, and also have many financial and economical problems sometimes unfortunately nowadays.

Many mathematicians who study mathematical finance may hope their studies are used in real financial markets serviceably. In general, many mathematical finance models are difficult to understand and study for persons who are not mathematicians. It is also difficult to shrink difference between theoretical asset models and usage of it in real markets. We think these are

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caused by many mathematicians are interest in various complicate derivatives theoretically than think about convenience in real markets. Thus, it is important to simplify difficult theoretical asset models and to think the usage of them in financial market, because theoretical study have no meaning if many ordinary persons who are not mathematicians (particularly, do not know martingale theory) can't use them easily.

On the other hand, we know economists and statistician also study many types of asset models, for example, ARCH, GARCH, EWMA, EGARCH, TGARCH, etc. which are a little strange to many mathematicians. Basically, to study pricing and hedging (also Greeks) of option (containing many kind of derivatives) of asset models, we need the expectation of stochastic process. More clearly, to calculate them, we need distribution (or density) function basically. Therefore statistician also made leptokurtic, heavy tail, and use many kinds of difficult time series models. These are also complicated and difficult academical factors to many persons who are interest in asset markets. Thus, as one of alternative measure for these problems, we introduce asset models and explain some usage of them for portfolio construction. Our models are reflective many economical and financial factors. We try and explain our asset models not using deep mathematical theory but use real factors and some data for portfolio construction. We hope many persons make ones best financial portfolio by using our models.

We start from an assumption that asset model can be represented by a geometric jump-diffusion, and can be defined by the solution of SDE derived by jump-type semimartingales basically. Thus, we define our asset models by using several SDE. Also we would like say that many explanations of data in below are not to prove reasonability but to help understanding of our models(c.f. [1], [7]). Our one of main asset model is defined by the following SDE; for $t \in T$, where T is the stopping time of local martingale,

$$
\begin{aligned}
dS_t^i \quad = \quad & S_{t-}^i[(\sigma_t^i dW_t + \frac{1}{2}(\sigma_t^i)^2 dt)I_{\{0 \le t < T_1\}} \\
& +(\mu_t^{(1)i} dt + \sigma_t^i dW_t + \frac{1}{2}(\sigma_t^i)^2 dt + \gamma_t^i dN_t)I_{\{T_1 \le t < T_2\}} \\
& +(\mu_t^{(2)i} dt + \sigma_t^i dW_t + \frac{1}{2}(\sigma_t^i)^2 dt + \delta_t^i dN_t)I_{\{T_2 \le t < T_3\}} \\
& +(\mu_t^{(3)i} dt + dW_t)I_{\{T_3 \le t\}}],
\end{aligned}
$$

where W_t is a Brownian motion and N_t is a Poisson process. We name $\sigma_t^i dW_t$ as noise parts, $\mu_t^{(.)i} dt$ is predictable parts, and $(.)dN_t$ are shocking event parts. For $T_1 \le t < T_2$, $T_2 \le t < T_3$ and $T_3 \le t$, the compositions of functions $\mu_t^{(.)i}$ are multi-variable functions consisted by many kinds of non-shocking financial or non-financial indices. For example, we can use (predictive) correlation level point of interest rate r_t, of quantity increasing rate of currency circulation m_t, of inflation rate f_t, of dividend rate d_t, and of increasing rate of gross domestic product g_t, etc. The factors which are consisting functions γ_t^i and δ_t^i, are determined by shocking financial and non-financial news mainly. We adopt following form of equation as the solution of above SDE as following; for $t \in T$,

$$\begin{aligned} S_t^i &= S_0^i \exp\{\int_0^t \sigma_s^i dW_s\} I_{\{0 \le t < T_1\}} \\ &+ S_{T_1}^i \exp\{\mu_t^{(1)i} t + \int_{T_1}^t (\sigma_s^i dW_s + \gamma_s^i dN_s)\} I_{\{T_1 \le t < T_2\}} \\ &+ S_{T_2}^i \exp\{\mu_t^{(2)i} t + \int_{T_2}^t (\sigma_s^i dW_s + \delta_s^i dN_s)\} I_{\{T_2 \le t < T_3\}} \\ &+ S_{T_3}^i \exp\{\mu_t^{(3)i} t + W_t\} I_{\{T_3 \le t\}}. \end{aligned}$$

As an example, let us think a situation; we have several kinds of assets and want to keep maximum total price of all assets at some time t^*. But we know that the price of special one of assets is decreasing after time $t_0 (< t^*)$, then to keep maximum total price of all assets at time t^* we need to put that asset before t_0 and save up money in bank till we find good possibilities asset, even if t_0 is in today. Thus, for good portfolio, our above multi-dimensional asset model may have some meaning a little.

In Section 2, we analyze some data and define our asset models in various terms of time. In Section 3, we explain several examples for best portfolio. In Section 4, we explain some basic properties of our models and present author's perspective.

2. DATA ANALYSIS AND ASSET MODELS

As we see from tables in below and the Figure 6 of [1], graphs of same asset price movements of Intraday, Weekly, Monthly and Quarterly, Annual and Long-term asset models are maybe different. Therefore, we can predict that asset prices may derive many different asset models for various interesting time intervals. Thus, we define several kinds of asset models. Further, we define so-called "an any time asset model" which is defined in above by using indicator.

Table 1 is showing the real trades of Samsung Electric Co. from 2010, January 21st, A.M.11:46:57 to A.M.11:47:13. As we know in Table 1, there are many trades by same price in short time intervals. Further, for a little long time, there is no tick even if there are many trades. From this fact, we can predict that asset models may not be represented by traditional models. Also, we can image that many different asset models may be got by taking different sampling time intervals.

To predict many different types of asset models, we introduce one more data US Treasury Bond Futures Chicago contract with a maturity of December 1990. This data begin on the 1st of October, 1990, A.M. 07:20:31. The first part of several minutes of real tick data is shown in Table 2. This stream of data resembles what might be seen on a real-time data-feed offered by many vendors.

From this data, we got some figures in [7]. The Figure 2 in [7] is of Table 2. Further, from the Table 6 and Table 7 in [7] which are derived from data of Table 2, we got Figure 4 in [7]. If we compare with Figure 5, Figure 6, Figure 7, and Figure 8 of [7], we can predict that asset

Time hh mm ss	Conclude Price	Ratio	Ups Downs	Put Price	Call Price	Conclude Intensity	Conclude Number
11 47 13	840000	6000	0.72	840000	839000	109.87	1
11 47 09	839000	5000	0.60	840000	839000	109.87	7
11 47 06	840000	6000	0.72	840000	839000	109.88	1
11 47 06	840000	6000	0.72	840000	839000	109.88	1
11 47 03	839000	5000	0.60	840000	839000	109.87	10
11 47 01	839000	5000	0.60	840000	839000	109.89	10
11 46 59	839000	5000	0.60	840000	839000	109.90	16
11 46 59	841000	7000	0.84	841000	839000	109.93	18
11 46 59	841000	7000	0.84	841000	839000	109.90	1
11 46 57	841000	7000	0.84	841000	839000	109.90	153
11 46 57	840000	6000	0.72	840000	839000	109.69	13
11 46 57	840000	6000	0.72	840000	839000	109.67	508
11 46 57	840000	6000	0.72	841000	840000	110.45	10
....	....	...	.....	......	......	......	...

TABLE 1. Real Trades of Samsung Electric Co. Stock. 2010, Jan. 21st A.M.11:46:57 - A.M.11:47:13

Time of Day hh mm ss	Contract Code Month Year	Futures Price US Dollar
07:20:31	US Z 1990	89.59375
07:20:32	US Z 1990	89.56250
07:20:33	US Z 1990	89.59375
07:20:34	US H 1991	89.18750
07:20:38	US H 1991	89.15625
07:20:38	US Z 1990	89.56250
07:20:45	US Z 1990	89.59375
07:20:54	US Z 1990	89.56250
07:21:00	US Z 1990	89.59375
07:21:11	US Z 1990	89.56250
07:21:18	US Z 1990	89.59375
07:21:21	US Z 1990	89.56250
07:21:41	US Z 1990	89.59375
07:21:55	US Z 1990	89.53125
07:21:59	US Z 1990	89.56250
07:21:59	US Z 1990	89.53125
07:22:10	US Z 1990	89.56250
....	....	....

TABLE 2. US Treasury Bond Futures Price Ticks. 1990 October 1st, A.M.07:20:31 - A.M.10:17:20

price models may different for investigating time intervals which we are interesting. Thus, we need several kinds of asset price movement models for our objects.

2.1. **Intraday Asset Model.** As we know from Table 1 which is of Samsung Electric Co., many trades of stock occur in same asset price. But we can predict that asset prices are up and down sometimes frequently and are absorbed in maximum or minimum prices. These up and down, so-called, derive the volatility of asset model, and the (sometimes absorbed) state at maximum and minimum can be represented by using Poisson process because of their scarceness. In general, asset price movement models have smooth part and noise part. The smooth part is linked like the drift parameter of diffusion processes and also is influenced from the volatility defined in below.

On the other hand, we write(represent) a term by using indicator $I_{\{|\mu_t^{(0)}(x)|<\infty\}}$, and scare events by using indicators $I_{\{\mu_t^{(0)}(x)=-\infty\}}$ and $I_{\{\mu_t^{(0)}(x)=\infty\}}$ where $\mu_t^{(0)}(x) = \frac{1}{\Delta t}[\Delta S_t|S_t = x]$, $\Delta S_t = S_t - S_{t-\Delta t}$. We would like use this representation, because this notation is more easy to understand than using jump-stochastic process, Poisson process mainly. Let T be a stopping time which is used in local martingale. For time t which is $t \in T \cap [0, T_1)$, we define an Intraday Asset Model as following;

$$\begin{aligned} dS_t &= S_{t-}[\sigma_t dW_t + \frac{1}{2}\sigma_t^2 dt]I_{\{|\mu_t^{(0)}(x)|<\infty\}} \\ &\quad +(S_0 - 0.15S_0)I_{\{\mu_t^{(0)}(x)=-\infty\}} + (S_0 + 0.15S_0)I_{\{\mu_t^{(0)}(x)=\infty\}}, \end{aligned} \tag{2.1}$$

where T_1 is the close time of asset trading market of a day, W_t is a Brownian motion, and σ_t is the volatility of asset prices defined in prior section. In this model, we know that the price movements of one day are restricted in interval $[0.85S_0, 1.15S_0]$, where index 0 is the starting time of asset trade of a day. We define the solution of above SDE asset model as following; for $0 \le t < T_1$,

$$\begin{aligned} S_t &= S_0 \exp\{\int_0^t \sigma_s dW_s\}I_{\{|\mu_t^{(0)}(x)|<\infty\}} \\ &\quad +(0.85S_0)I_{\{\mu_t^{(0)}(x)=-\infty\}} + (1.15S_0)I_{\{\mu_t^{(0)}(x)=\infty\}}. \end{aligned}$$

2.2. **Weekly, Monthly and Quarterly Asset Model.** When we get asset models of Weekly, Monthly and Quarterly, first we must think about maximum up jump and minimum down jump in prior day. In general, after big jumps, there is a decreasing or increasing wave. These models are almost same with the Black-Scholes asset model if there is no big-jump. But also these models have very complicate non-noise terms. We know that if interest rate is down, asset price is up. Further, the increasing of quantity increasing rate of currency circulation influence to GDP, inflation and the increasing of asset prices. Let Let T be a stopping time, and N_t be a

Poisson process. We define our model as following; for time $t \in T \cap [0, T_2)$,

$$\begin{aligned} dS_t &= S_{t-}[(\sigma_t dW_t + \frac{1}{2}\sigma_t^2 dt)I_{\{0 \le t < T_1\}} \\ &\quad + (\mu_t^{(1)} dt + \sigma_t dW_t + \frac{1}{2}\sigma_t^2 dt + \gamma_t dN_t) I_{\{T_1 \le t < T_2\}}], \end{aligned} \tag{2.2}$$

where $\mu_t^{(1)} = \alpha_t^{(1,1)} + \beta_t^{(1,1)}$ and $\gamma_t = \alpha_t^{(1,2)} + \beta_t^{(1,2)}$. In here, $\alpha_t^{(1,1)}$ is a function made by smooth and non-shocking asset price movement factor points which are in basic financial areas. $\beta_t^{(1,1)}$ is also a function composed by smooth and non-shocking asset price movement factor points which are of basic non-financial areas. In this model, we insist that γ_t may be over 1 relatively. For example, if we want to get good portfolio in haste or by using simple factors, we may put in this model $\alpha^{(1,1)} = -r_t + m_t + d_t$(c.f., Section 3 in below), where r_t is the (predictive) correlation level point of interest rate, m_t is the level point of quantity increasing rate of currency circulation and d_t is the level point of dividend. If we think more for financial factors, we can think the business barometer, the balance of international payment, the exchange rate, the raw materials prices, etc. The function $\beta_t^{(1,1)}$ is composed by non-financial factors; the predictable government policy, and many kinds of predictable social movements etc. Further, $\alpha_t^{(1,2)}$ is a function composed by factors; shocking asset price movement factors which is in basic financial areas and main basic economical factors, for example some inside factors of company(merger and acquisition, new product news etc). The function $\beta_t^{(1,2)}$ is determined by many kinds of shocking non-financial factors, for example, international war, earthquake and the labor management dispute etc. and political and social factors etc. If we think integral form of above model, we can adopt it as

$$\begin{aligned} S_t &= S_0 \exp\{\int_0^t \sigma_t dW_s\} I_{\{0 \le t < T_1\}} \\ &\quad + S_{T_1} \exp\{\mu_t^{(1)} t + \int_{T_1}^t (\sigma_s dW_s + \gamma_s dN_s)\} I_{\{T_1 \le t < T_2\}}. \end{aligned}$$

If we need a model for special restrict term; $T_1 \le t < T_2$, we can get a model

$$dS_t = S_{t-}(\mu_t^{(1)} dt + \sigma_t dW_t + \frac{1}{2}\sigma_t^2 dt + \gamma_t dN_t),$$

and adopt an integral form

$$S_t = S_{T_1} \exp\{\mu_t^{(1)} t + \int_{T_1}^t (\sigma_s dW_s + \gamma_s dN_s)\}.$$

If we want to know asset price movements state for interval $[T_1, T_2)$ mainly at $t = 0$ now, we may use a (similar in result) model;

$$dS_t = S_{t-}(\mu_t^{(1)} dt + \sigma_t dW_t + \frac{1}{2}\sigma_t^2 dt + \gamma_t dN_t), \quad 0 \le t < T_2,$$

and whose integral form

$$S_t = S_0 \exp\{\mu_t^{(1)} t + \int_0^t (\sigma_s dW_s + \gamma_s dN_s)\}, \quad 0 \le t < T_2,$$

because one day movements can be represented by a cluster point in long term.

Note(Sketch of Theoretical Background). We assume a well-defined probability space (Ω, F, P). Let Q be an equivalent martingale measure to P, and M_t be a compensated Poisson process which is a P-martingale $N_t - \lambda t$, where λ is a intensity number of Poisson process N_t. Then $M_t^Q := N_t - \int_0^t \lambda(1 + \Psi_s)ds$ is a Q-local martingale(c.f.[6]). Let T be a stopping time which is used in local martingale. We think an SDE and the solution of it for time $t \in T \cap [0, T_2)$;

$$\begin{aligned} dS_t &= S_{t-}[(\mu_t^{(0)} dt + \sigma_t dW_t) I_{\{0 \le t < T_1\}} \\ &\quad + (\mu_t^{(1)} dt + \sigma_t dW_t + \gamma_t dM_t^Q) I_{\{T_1 \le t < T_2\}}], \end{aligned}$$

where σ_t is the volatility of asset prices defined by an SDE

$$d\sigma_t = \gamma(\sqrt{\sigma_t}, t)\sigma_t dt + \rho(\sqrt{\sigma_t}, t)\sigma_t d\tilde{W}_t,$$

where $\tilde{W}_t$ is another Brownian motion which is independent with W_t. Then we can get a solution as following;

$$\begin{aligned} S_t &= S_0 \exp\{\mu^{(0)} t + \int_0^t (\sigma_t dW_s - \frac{1}{2}\sigma_s^2)\} I_{\{0 \le t < T_1\}} \\ &\quad + S_{T_1} \exp\{\mu_t^{(1)} t + \int_{T_1}^t (\sigma_s dW_s - \frac{1}{2}\sigma_s^2 - \gamma_s \lambda [1 + \Psi_s] ds)\} \\ &\quad (\Pi_{T_1 \le s \le t}(1 + \gamma_s)\Delta N_s) I_{\{T_1 \le t < T_2\}}. \end{aligned}$$

From this if we put for simplicity $N_t := M_t^Q + \int_0^t \lambda(1 + \Psi_s)ds$ and $\Pi_{T_1 \le s \le t}(1 + \gamma_s)\Delta N_s \risingdotseq \exp\{\gamma_t dN_t\}$ roughly, we can derive above our equations and solutions(c.f. [6]).

2.3. **Annual Asset Model.** For this model, we must think many factors which influence to movements of asset prices. First we define an asset model as following; for $t \in T \cap [0, T_3)$,

$$\begin{aligned} dS_t &= S_{t-}[(\sigma_t dW_t + \frac{1}{2}\sigma_t^2) I_{\{0 \le t < T_1\}} \\ &\quad + (\mu_t^{(1)} dt + \sigma_t dW_t + \frac{1}{2}\sigma_t^2 + \gamma_t dN_t) I_{\{T_1 \le t < T_2\}} \\ &\quad + (\mu_t^{(2)} dt + \sigma_t dW_t + \frac{1}{2}\sigma_t^2 + \delta_t dN_t) I_{\{T_2 \le t < T_3\}}]. \end{aligned} \tag{2.3}$$

In this model, functions $\mu_t^{(2)}$ and δ_t are consisted by several delicate factors. These factors can be distinguished roughly by functions consisted by non-shocking financial factors $\alpha_t^{(2,1)}$, non-shocking non-financial factors $\beta_t^{(2,1)}$, shocking financial factors $\alpha_t^{(2,2)}$, and shocking non-financial factors $\beta_t^{(2,2)}$. Thus, we put $\mu_t^{(2)} := \alpha_t^{(2,1)} + \beta_t^{(2,1)}$ and $\delta_t := \alpha_t^{(2,2)} + \beta_t^{(2,2)}$. In this model, δ_t can be restricted in $[-1,1]$ because, for a little long term, shocking jumps look like small relatively. Non-financial factors are consisted by political factors and social factors etc. As an example, if we think simply, we may define as $\alpha_t^{(2,1)} = r_t - m_t + d_t$(c.f., Section 3 in below). In general, in prior model, if the interest rate is down, then asset price is increased. But, for a little long time model $\alpha_t^{(2,1)}$, it is changed to direct ratio with the interest rate and asset price. Further, the excessive increasing of quantity increasing rate of currency circulation derive inflation, and the value of asset is decreased and asset price is also decreasing. The integral form of this model is following;

$$\begin{aligned} S_t &= S_0 \exp\{\int_0^t \sigma_s dW_s\} I_{\{0 \le t < T_1\}} \\ &\quad + S_{T_1} \exp\{\mu_t^{(1)} t + \int_{T_1}^t (\sigma_s dW_s + \gamma_s dN_s)\} I_{\{T_1 \le t < T_2\}} \\ &\quad + S_{T_2} \exp\{\mu_t^{(2)} t + \int_{T_2}^t (\sigma_s dW_s + \delta_s dN_s)\} I_{\{T_2 \le t < T_3\}}. \end{aligned}$$

In this model, as we know, there is no term $\mu_t^{(0)}(x)$ for $0 \le t < T_1$, because the graph of one day movements looks like a cluster point only in this model. If we need a model for special restrict term; $T_2 \le t < T_3$, we can get a model

$$dS_t = S_{t-}(\mu_t^{(2)} dt + \sigma_t dW_t + \frac{1}{2}\sigma_t^2 + \delta_t dN_t),$$

and an integral form

$$S_t = S_{T_2} \exp\{\mu_t^{(2)} t + \int_{T_2}^t (\sigma_s dW_s \delta_s dN_s)\}.$$

If we want to know the asset price movements state of time interval $[T_2, T_3)$ at $t = 0$, we may use a model;

$$dS_t = S_{t-}(\mu_t^{(2)} dt + \sigma_t dW_t + \delta_t dN_t), \quad 0 \le t < T_3,$$

and whose integral form;

$$S_t = S_0 \exp\{\mu_t^{(2)} t + \int_0^t (\sigma_s dW_s + \delta_s dN_s)\}, \quad 0 \le t < T_3.$$

2.4. **One Year Or More Asset Model.** This model is defined by as following; for $t \in T \cap [0, \infty)$,

$$\begin{aligned} dS_t &= S_{t-}[(\sigma_t dW_t + \frac{1}{2}\sigma_t^2) I_{\{0 \le t < T_1\}} \\ &\quad + (\mu_t^{(1)} dt + \sigma_t dW_t + \frac{1}{2}\sigma_t^2 + \gamma_t dN_t) I_{\{T_1 \le t < T_2\}} \\ &\quad + (\mu_t^{(2)} dt + \sigma_t dW_t + \frac{1}{2}\sigma_t^2 + \delta_t dN_t) I_{\{T_2 \le t < T_3\}} \\ &\quad + (\mu_t^{(3)} dt + dW_t) I_{\{T_3 \le t\}}]. \end{aligned} \tag{2.4}$$

where $\mu_t^{(3)} = \alpha_t^{(3)} + \beta_t^{(3)}$. For example, if we want to get good portfolio, we may put $\alpha_t^{(3)} = r_t - m_t + f_t + g_t + d_t$(c.f. the meaning of this combination refer to Section 3 in below). Because, after one year, the correlation level point of inflation increasing rate f_t, the point of increasing rate of gross domestic product g_t and the d_t are positive for asset prices increasing. Particularly, also r_t is in direct proportion to asset prices, because the accumulated interest for a little long time is influence to asset prices, not to value of asset. In this model, the term $\delta_t dM_t$ is not needed because the influence of many shocking news are soaked into Brownian motion term $\sigma_t dW_t$ which is named as a noise part. The integral form is

$$\begin{aligned} S_t &= S_0 \exp\{\int_0^t \sigma_s dW_s\} I_{\{0\le t<T_1\}} \\ &+S_{T_1} \exp\{\mu_t^{(1)} t + \int_{T_1}^t (\sigma_s dW_s) + \gamma_s dN_s\} I_{\{T_1\le t<T_2\}} \\ &+S_{T_2} \exp\{\mu_t^{(2)} t + \int_{T_2}^t (\sigma_s dW_s) + \delta_s dN_s\} I_{\{T_2\le t<T_3\}} \\ &+S_{T_3} \exp\{\mu_t^{(3)} t + W_t\} I_{\{T_3\le t\}}. \end{aligned}$$

If we need a model for special restrict term; $T_3 \le t$, we can get a model

$$dS_t = S_{t-}(\mu_t^{(3)} dt + dW_t),$$

and an integral form

$$S_t = S_{T_3} \exp\{\mu_t^{(3)} t + W_t\}.$$

If we need to know the asset price movement state of time t in interval $[T_3, \infty)$ at $t = 0$, we may use a (similar) model in result;

$$dS_t = S_{t-}(\mu_t^{(3)} dt + dW_t), \quad 0 \le t,$$

and whose integral form

$$S_t = S_0 \exp\{\mu_t^{(3)} t + W_t\}, \quad 0 \le t.$$

2.5. **Asset Model for Portfolio.** We can see in below Table 3, in increasing mood, some asset prices are increased and some are decreased. This means that some factor give positive influence to some assets, but may give negative influence to another assets. Thus, for portfolio, we need a multi-dimensional asset price model even if it is a little simple, rough and a little coarse one.

Asset	Final Price	Up, Down	increment	Total Trade
KT	50,000	$\triangledown$	-600	3,012,590
SK	86,100	$\triangledown$	-700	200,403
Hynix	23,600	$\triangle$	350	10,048,500
POSCO	559,000	$\triangle$	4,000	296,957
Samsung El.	808,000	$\triangle$	8,000	273,960
LG El.	110,000	$\triangle$	4,000	1,517,998
Kor.El.Pow.	39,350	$\triangledown$	-500	2,366,905
Korean Air.	57,500	$\triangle$	1,200	514,561
Hyundai Motor	113,500	$\triangle$	4,500	2,003,889
....	....	.....	...	...

TABLE 3. Final Prices of KOSPI, Jan. 28, 2010 (Compared With The Day Before)

By using above models, we combine them as one and define an asset model to get best portfolio as following; for $t \in T$, where T is the stopping time of local martingale.

$$\begin{aligned} dS_t^i &= S_{t-}^i[(\sigma_t^i dW_t + \frac{1}{2}(\sigma_t^i)^2 dt) I_{\{0 \le t < T_1\}} \\ &\quad + (\mu_t^{(1)i} dt + \sigma_t^i dW_t + \frac{1}{2}(\sigma_t^i)^2 dt + \gamma_t^i dN_t) I_{\{T_1 \le t < T_2\}} \\ &\quad + (\mu_t^{(2)i} dt + \sigma_t^i dW_t + \frac{1}{2}(\sigma_t^i)^2 dt + \delta_t^i dN_t) I_{\{T_2 \le t < T_3\}} \\ &\quad + (\mu_t^{(3)i} dt + dW_t) I_{\{T_3 \le t\}}], \end{aligned} \tag{2.5}$$

The integral form of above SDE model for portfolio is following.

$$\begin{aligned} S_t^i &= S_0^i \exp\{\int_0^t \sigma_s^i dW_s\} I_{\{0 \le t < T_1\}} \\ &\quad + S_{T_1}^i \exp\{\mu_t^{(1)i} t + \int_{T_1}^t (\sigma_s^i dW_s + \gamma_s^i dN_s)\} I_{\{T_1 \le t < T_2\}} \\ &\quad + S_{T_2}^i \exp\{\mu_t^{(2)i} t + \int_{T_2}^t (\sigma_s^i dW_s + \delta_s^i dN_s)\} I_{\{T_2 \le t < T_3\}} \\ &\quad + S_{T_3}^i \exp\{\mu_t^{(3)i} t + W_t)\} I_{\{T_3 \le t\}}. \end{aligned}$$

Further we may use models; for $0 \le t < T_1$,

$$dS_t^i = S_{t-}^i(\sigma_t^i dW_t + \frac{1}{2}(\sigma_t^i)^2 dt),$$

whose integral form is

$$S_t^i = S_0^i \exp\{\int_0^t \sigma_s^i dW_s\},$$

and particularly, for the state in $t \in [T_1, T_2)$, we may use a model

$$dS_t^i = S_{t-}^i(\mu_t^{(1)i}dt + \sigma_t^i dW_t + \frac{1}{2}(\sigma_t^i)^2 dt + \gamma_t^i dN_t), \quad 0 \le t < T_2,$$

whose integral form is

$$S_t^i = S_0^i \exp\{\mu_t^{(1)i}t + \int_0^t (\sigma_s^i dW_s + \gamma_s^i dN_s)\}, \quad 0 \le t < T_2,$$

and particularly, for the state in $t \in [T_2, T_3)$, we may use a model

$$dS_t^i = S_{t-}^i(\mu_t^{(2)i}dt + \sigma_t^i dW_t + \frac{1}{2}(\sigma_t^i)^2 dt + \delta_t^i dN_t), \quad 0 \le t < T_3,$$

whose integral form is

$$S_t^i =_0^i \exp\{\mu_t^{(2)i}t + \int_0^t (\sigma_s^i dW_s 1 + \delta_s^i dN_s)\}, \quad 0 \le t < T_3.$$

Finally, for the state in $T_3 \le t$, we may use a model

$$dS_t^i = S_{t-}^i(\mu_t^{(3)i}dt + dW_t), \quad 0 \le t,$$

whose integral form is

$$S_t^i = S_0^i \exp\{\mu_t^{(3)i}t + W_t\}, \quad 0 \le t.$$

3. SIMULATED EXAMPLES FOR BEST PORTFOLIO

We know a financial proverb ”There is no free lunch in the world”. As we know this means that, if possible, any investor must check almost all factors which influence to financial asset prices, and do not desire luck and not rely on chance vaguely. Another proverb ”Don’t be eggs in one basket” means that do not invest in only one financial asset even if it looks like best. Thus, we must check many influence level points and need portfolio strategy. As an example, these predictable points maybe made by factor’s index values $\times$ correlation rates.

Kind of Asset (i)	$\alpha_t^{(1,1)}$ $-r_t$	$\alpha_t^{(1,1)}$ m_t	$\alpha_t^{(1,1)}$ d_t	... ...	$\beta_t^{(1,1)}$ a_t	... ...	$\alpha_t^{(1,2)}$ b_t	... ...	$\beta_t^{(1,2)}$ c_t	... ...	Total Point
Asset A	-0.25	0.3	0.17	0.6	-0.1	0.2	-0.2	0.5	-0.2	-0.1	0.92
Asset B	-0.25	0.3	0.00	0.6	-0.1	-0.7	0.1	-0.3	0.2	0.1	-0.15
Asset C	-0.25	0.3	0.30	0.4	0.0	0.3	-0.2	0.2	0.4	0.3	1.75
Asset D	-0.25	0.3	0.27	0.3	0.1	0.1	-0.2	0.2	0.4	-0.1	1.12
...	...	...	...	...	...	...	...	...	...	...	...

TABLE 4. Best Portfolio Example Chart For $T_1 \le t < T_2$ Model At $t = 0$. Each Predictive Points Is In $[-1, +1]$

In Table 4, Asset C has bigger than others. But this asset is dangerous, because $\beta_t^{(1,2)}$ which is consisted by shocking non-financial factor points are $+0.4$ and $+0.3$ bigger than another assets. Thus, it is not recommendable. Also, Asset D is dangelous.

Kind of Asset i	$\alpha_t^{(2,1)}$ r_t	$\alpha_t^{(2,1)}$ m_t	$\alpha_t^{(2,1)}$ d_t	... ...	$\beta_t^{(2,1)}$ a_t	... ...	$\alpha_t^{(2,2)}$ b_t	... ...	$\beta_t^{(2,2)}$ c_t	... ...	Total Point
Asset A	0.15	-0.2	0.17	-0.1	0.2	-0.2	0.7	0.5	-0.2	-0.1	0.92
Asset B	0.15	-0.2	0.00	-0.1	-0.7	0.1	0.5	-0.3	0.2	0.1	-0.25
Asset C	0.15	-0.2	0.30	0.0	0.3	-0.3	0.4	0.2	-0.2	0.3	0.95
Asset D	0.15	-0.2	0.27	0.1	0.1	-0.3	-0.2	0.4	0.2	-0.1	0.42
...	...	...	...	...	...	...	...	...	...	...	...

TABLE 5. Best Portfolio Example Chart For $T_2 \le t < T_3$ Model At $t = 0$. Each Predictive Points Is In $[-1, +1]$

In above Table 5, Asset A and Asset C have almost same total points $+0.92$ and $+0.95$. But Asset A is more dangerous than Asset C, because $\alpha_t^{(2,2)}$ of Asset A has point $+0.7$ and of Asset C is $+0.4$, which are points of shocking financial factors. These shocking financial factor is occur occasionally.

Kind of Asset i	$\alpha_t^{(3)}$ r_t	$\alpha_t^{(3)}$ $-m_t$	$\alpha_t^{(3)}$ d_t	$\alpha_t^{(3)}$ f_t	$\alpha_t^{(3)}$ g_t	... ...	$\beta_t^{(3,1)}$ a_t	$\beta_t^{(3,1)}$ b_t	$\beta_t^{(3,1)}$ c_t	... ...	Total Point
Asset A	0.35	0.3	-0.1	-0.1	0.2	-0.2	0.7	0.5	-0.2	-0.1	1.35
Asset B	0.35	0.3	0.0	-0.1	-0.7	0.1	0.5	-0.3	0.2	0.1	0.45
Asset C	0.35	0.3	0.3	0.0	0.3	-0.3	0.4	0.2	-0.2	-0.3	1.05
Asset D	0.35	0.3	0.2	0.1	0.1	-0.2	-0.2	0.4	0.2	-0.1	1.15
...	...	...	...	...	...	...	...	...	...	...	...

TABLE 6. Best Portfolio Example Chart For $T_3 \le t$ Model At $t = 0$ Each Predictive Points Is In $[-1, +1]$

In above Table 6, Asset A has the largest total point $+1.35$. But it has a big shocking factor point $+0.7$. If we know the source of this dangerous factor point, the price of Asset A may not dangerous than Asset D. On the other hand, Asset C has good high point $+1.05$. Further, it has good contents because non-financial factors points $\beta_t^{(3,1)}$ are low points than others.

Postscript. If we say plainly, many mathematicians may not knowledgable in economics. The deterministic functions $\mu_t^{(\cdot)}$, γ_t and δ_t, which are defined by many factors in above, may not absolute truths. But we would like to insist in the facts; the represented asset models in above are different according to the terms of investing time intervals and to the kinds of given data of indices etc. To get more concrete and detailed models, to know positive and negative skill of factors, to know the combinations of financial factors, and to get logical points of factors, we need much helping from economists. For example, to understand the role and analysis of Fisher Effect, the velocity of money, Cobb-Douglas production function, and the formula of security market line in capital asset pricing models etc., we need many kinds of meeting with each other.

4. SUMMARY

Our main object of this paper, as we mentioned in Introduction, is to introduce and define some asset models which are influenced from many factors for portfolio. As we know there are very many factors which influence to the price of assets (c.f. Table 7, and data in Korean Statistical Information Service), and also are many papers and articles which study the influence (level) of some factors to the price of assets. We may would like to say and point out one; interest rate and the quantity increasing rate of currency circulation are main factors which influence to the price movement of assets. If we think about the quantity increasing rate of currency circulation, it is in direct proportion with asset prices in short-term. But it is inverse proportion to long-term asset price in general. We know also the interest rate is one of main factors. But in general, it also is not strong than season factors, sudden change of exchange rate, international financial panic, many political factors(for examples, the general election, the major election, international agreement etc). Form these, we also need new asset models which reflect these many above factors for portfolio. We also have many open problems in the coefficients $\mu_t^{(\cdot)}$, γ_t and δ_t of SDE which must be studied.

Index	...	2009.1	2009.2	2009.3	2009.4	2009.5	2009.6	...
r_t A	...	2.43	2.06	1.77	1.79	1.91	1.93	...
r_t B	...	3.05	2.59	2.38	2.35	2.33	2.32	...
...	...	...	...	...	...	...	...	...
m_t A	...	33,506.0	30,803.4	30,320.7	30,678.2	30,356.4	32,152.2	...
m_t B	...	63,424.0	70,559.2	71,710.1	59,752.4	59,916.4	65,406.1	...
...	...	...	...	...	...	...	...	...
KOSPI A	...	1,162.11	1,063.03	1,206.26	1,369.40	1,395.89	1,390.07	...
KOSPI B	...	1,156.37	1,139.75	1,140.45	1,322,10	1,400.50	1,395.24	...
...	...	...	...	...	...	...	...	...

TABLE 7. Index in Korean Statistical Information Service

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