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STABILITY AND THE EFFECT OF HARVESTING IN A BUDWORM POPULATION MODEL

GUL ZAMAN[1], YONG HAN KANG[2], AND IL HYO JUNG[2†]

[1]CENTRE FOR ADVANCED MATHEMATICS AND PHYSICS, NATIONAL UNIVERSITY OF SCIENCES AND TECHNOLOGY, RAWALPINDI 46000, PAKISTAN
E-mail address: `gzaman@camp.nust.edu.pk`

[2]DEPARTMENT OF MATHEMATICS, PUSAN NATIONAL UNIVERSITY, PUSAN 609-735, SOUTH KOREA
E-mail address: `yonghann@pusan.ac.kr, ilhjung@pusan.ac.kr`

ABSTRACT. In this work, we consider a nonlinear budworm model by a system of three ordinary differential equations originally created by Ludwig et al. in 1978. The nonlinear system describes the dynamics of the interaction between a budworm and a fir forest. We introduce stability techniques to analyze the dynamical behavior of this nonlinear system. Then we use constant effort harvesting techniques to control the budworm population. We also give numerical simulations of the population model with harvest and without harvest.

1. INTRODUCTION

Forest entomology is one of the major areas for application of population models. Several types of models have been used for simulation of forest insect population dynamics. A series of theoretical models of forest insects has been developed by Berryman and Millstein [1]. These models are modifications of the discrete-time analog of the logistic model. Parameters of these models can be adjusted to fit the available data. Other examples of theoretical models are: a spruce budworm model, a gypsy moth model, a southern pine beetle model, and a generic model of tree conquest by bark beetles. The spruce budworm is the most serious pest affecting the spruce fir forest of North America and Canada [13, 5]. There are three major spruce budworm outbreaks triggered in North America reported in this century, beginning in 1910 and the late 1960s. After an outbreak, populations decline simultaneously over very large areas, even where defoliation is not significant. Populations tend to increase steadily again and spread to younger trees [8]. The heavy budworm predator destroyed about 40 percent of a forest during six to eight years. The historical and spatial characteristics of spruce budworm outbreaks in northeastern British Columbia can be found in [4]. For more information concerning the most recent data and complete defoliated area, we refer the reader to [9, 10, 14]. The rise and

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fall of these budworm populations, in terms of intrinsic factors and predation, can in fact be modeled by a logistic model [3].

This work is based on the work of three researchers; a mathematician, Ludwig, and two biologists, Jones and Holling. These researchers proposed an ingenious model for the interaction of these insects in 1978 [7]. They introduced a nonlinear system consisting of three first order ordinary differential equations (ODEs) having some parameters(see [2]). These three ODEs represent the rate of change in the budworm population density I, the surface of the forest F, and the energy E, respectively. In this model, the heavy budworm exhibits sudden outbreaks from low to high density. These outbreaks cause defoliation of the forest, that is a small change in any variable in the system affects other variables in the system. However this model has a defect in that there is only one generation of forest, that is, if the forest dies, then it does not recover. To overcome this defect, the model should be improved by modifying some terms of the system.

In this paper, we consider an improved nonlinear population model with three variables, which are divided into two categories, that is, the budworm (insect) is a fast variable and the forest and the energy are slow variables. We first investigate the stability in the nonlinear system and then using the constant effort harvesting techniques, we study the effect of harvesting on the budworm population model. We lastly give the numerical simulations of both systems; a system without harvesting and a system with harvesting. Note that the qualitative behavior of this nonlinear system helps us to understand the budworm disturbance in a forest.

2. A POPULATION MODEL HAVING ONE FAST AND TWO SLOW VARIABLES

Each variable has an associated time interval where change occurs. Some variables, such as the budworm density, can change dramatically in a few years. Therefore, the budworm density is considered as a fast variable in an appropriate time interval of the population on the order of months. The forest is assigned as a slow variable since it cannot alter its area in a short time. An appropriate time scale is on the order of tens of years. The forest is divided into two variables: one variable describing the energy reserve of the forest and the other, the total surface area of branches.

From these assumptions of three variables, a budworm density model is given by the system of three ordinary differential equations

$$\begin{aligned}\frac{dI}{dt} &= r_I I(1-\frac{I}{KF}) - \omega\frac{I^2}{(\sigma F)^2+I^2},\\ \frac{dF}{dt} &= r_F F(1-\frac{F}{K_F}\frac{K_E}{E}),\\ \frac{dE}{dt} &= r_E E(1-\frac{E}{K_E}) - \epsilon\frac{I}{F}.\end{aligned} \tag{2.1}$$

Here, I is the population density of budworms per acre, F is the area of the branches per acre, and E is the energy level of the forest. In the first equation of the system (2.1), K is the maximum number of budworms per unit surface of a tree, r_I is the intrinsic growth rate of

TABLE 1. The parameters value

Parameter	Description	Value
r_I	The intrinsic growth rate of the budworm	1.52
r_F	The intrinsic branch growth rate of the fir forest	0.095
r_E	The energy growth rate	0.92
K_F	The maximum surface of the trees	25440
K_E	The maximum energy level	1.0
ϵ	The energy consumption rate	0.00195
ω	The maximum predation rate of the budworm	43190
K	The maximum number of budworms per unit of tree	335
σ	The budworm number that is half of the maximum predation	1.11

the budworm, ω is the maximum predation rate, and σ is the budworm population where the predation is half the maximum. The second equation of the system (2.1) has the logistic form in where F identifies the total surface of the forest. In this model, F is used as a slow variable, r_F is the intrinsic branch growth rate, and K_F is the maximum amount of the surface area. The factor K_E/E reflects the fact that F does not increase under stress conditions. The energy factor E is always close to the maximum energy level K_E and the surface area F always near to K_F. The third equation of the system (2.1) determines the rate of change of the energy. r_E is the intrinsic growth rate of the energy, K_E is the maximum energy level, and ϵ is the rate of energy consumption. If the density of budworms is small, then the energy E approaches its maximum level K_E. Moreover I/F means the number of budworms per branch.

First, we consider a system (2.1) with a constant budworm population density [6]. For equilibria, we consider $dF/dt = 0$ that implies that $F = 0$ or $F = EK_F/K_E$ [16]. Also from $dE/dt = 0$, we obtain $F = (\epsilon I K_E)/(r_E E(K_E - E))$, which has vertical asymptotes at $E = 0$ and has a minimum at $K_E/2$. With a comparison of both values of F we get

$$E^3 - K_E E^2 + \frac{\epsilon I K_E^2}{r_E K_F} = 0. \tag{2.2}$$

The equilibria of the second and third equations of the system (2.1) are $(F, E) = (25440, 1)$ and $(89.0400, 0.0035)$, respectively, which intersect both equations.

If $(I_\infty, F_\infty, E_\infty)$ is an equilibrium point of the system (2.1) with $I_\infty > 0$, $F_\infty > 0$, and $E_\infty > 0$, then from the first equation of the system (2.1) we obtain

$$r_I(1 - \frac{I_\infty}{KF_\infty}) - \omega \frac{I_\infty}{(\sigma F_\infty)^2 + {I_\infty}^2} = 0. \tag{2.3}$$

For $F_\infty > 0$, and by using the second and third equations of the system (2.1) we get

$$E_\infty = \frac{K_E}{K_F} F_\infty, \tag{2.4}$$

TABLE 2. The polynomial roots

No	F	E	I
1	4713	0.1853	2.1721×10^{14}
2	-3332	-0.1310	1.5071×10^{14}
3	4 complex roots	4 complex roots	4 complex roots

and

$$I_\infty = \frac{r_E K_E}{\epsilon K_F} F_\infty^2 (1 - \frac{F_\infty}{K_F}). \tag{2.5}$$

Then, substituting (2.5) into (2.3), we get

$$\begin{aligned} &F_\infty^6 - 3K_F F_\infty^5 + (3r_E K_E + \epsilon K)\tfrac{K_F^2}{r_E K_E} F_\infty^4 - (2\epsilon K + r_E K_E)\tfrac{K_F^3}{r_E K_E} F_\infty^3 \\ &+(\epsilon\sigma^2 + K r_E K_E)\tfrac{\epsilon K_F^4}{r_E^2 K_E^2} F_\infty^2 + (\omega K - \sigma^2 r_I K_F)\tfrac{\epsilon^2 K_F^4}{r_I r_E^2 K_E^2} F_\infty \\ &+(\epsilon\sigma^2 r_I K K_F - \omega K r_E K_E)\tfrac{\epsilon^2 K_F^5}{r_I r_E^3 K_E^3} = 0, \end{aligned} \tag{2.6}$$

which is a polynomial of order six. Considering values of the parameters given in Table 1, we can find the roots of this polynomial represented in Table 2. The appropriate values for the parameters in Table 1 have been estimated in [7], first, base on general knowledge of the biology of the situation and then with more refinement from the extensive field study of a forest in New Brunswick [11].

In order to investigate that the system is stable (unstable) we use the possible equilibria $(I_\infty, F_\infty, E_\infty)$ and the community matrix of the system (2.1) at $(I_\infty, F_\infty, E_\infty)$. We consider the equilibrium point $(I_\infty = 2.1721 \times 10^{14}, F_\infty = 4713, E_\infty = 0.1853)$ of the system (2.1). The community matrix of the system (2.1) at $(2.1721 \times 10^{14}, 4713, 0.1853)$ yields

$$\begin{bmatrix} -9.2898 \times 10^{15} & 9.635 \times 10^{18} & 0 \\ 0 & 0.0885 & 2.8481 \\ -2.128 \times 10^{-4} & 9.7788 \times 10^{6} & 0.5790 \end{bmatrix}. \tag{2.7}$$

The characteristic equation of this matrix is the following :

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{2.8}$$

where the coefficients $a_i\,(i = 1, 2, 3)$ are given by $a_1 = 5.6228 \times 10^{15}$, $a_2 = -6.2008 \times 10^{15}$, $a_3 = -8.4065 \times 10^{23}$.

However the coefficients $a_i (i = 1, 2, 3)$ in (2.8) do not satisfy Routh-Hurwitz conditions [2, 15] :

$$a_1 > 0,\ a_3 > 0,\ a_1 a_2 > a_3.$$

Hence the equilibrium point of the system is not asymptotically stable represented in Figure 1. In Figure 1, the energy E and F decline sharply during a budworm outbreak and eventually E becomes negative. Negative values of E are unrealistic. Our model is not valid when extensive tree deaths occur. In fact, we note that this model has a defect in that there is only one generation of forest, that is, if the forest dies, then it does not recover. To overcome this defect, the model should be improved by modifying some terms of the system so that we have to make the system more realistic. To do this, we use the energy factor $E^2/(T_E^2+E^2)$, where T_E is a constant threshold value. In the first equation of the system (2.1), the fraction is multiplied by the energy factor. This factor means that if the energy E of the forest falls below a certain threshold value, then the energy factor becomes a small number. In fact, the low energy level of the forest does not totally support the budworm population. By providing large energy E, the energy factor approaches one and the population further increases. However, when the energy E is decreasing, the energy factor also sharply decreases. Therefore, a small rise in energy factor sharply increases the budworm population. Moreover, in the third equation of the system (2.1), the predator term $\epsilon I/F$ is replaced by $(\epsilon I/F)E^2/(T_E^2+E^2)$. The energy factor changes the first and third equations of the system (2.1) so that we have

$$\begin{aligned}\frac{dI}{dt} &= r_I I\left(1-\frac{I}{KF}\frac{E^2+T_E^2}{E^2}\right)-\omega\frac{I^2}{(\sigma F)^2+I^2},\\ \frac{dF}{dt} &= r_F F\left(1-\frac{F}{K_F}\frac{K_E}{E}\right),\\ \frac{dE}{dt} &= r_E E\left(1-\frac{E}{K_E}\right)-\epsilon\frac{I}{F}\frac{E^2}{E^2+T_E^2}.\end{aligned}\tag{2.9}$$

A consequence of such a small energy means that there is a very small stress put on the forest from budworms during the time when they are feeding. The low stress condition, in fact, lowers the energy level determining that the forest is not able to support a large budworm density and the predation term almost vanishes. The change of energy creates a logistic growth. Conversely, if the energy E approaches to T_E, then the level of stress on the forest increases. When more energy is available the budworm consumes more trees and the forest declines. The energy factor tends to unity if the budworm density reaches a large energy level E.

In order to find the stable position of the system (2.9), first, we set $dF/dt=0$ with $K_E=1$ to obtain

$$F=K_F E.\tag{2.10}$$

Also using $dE/dt=0$, we obtain

$$\epsilon I=r_E K_F(E^2-T_E^2)(1-E).\tag{2.11}$$

Similarly from the first equation of the system (2.9) we get

$$r_I(KFE^2\sigma^2F^2+KFE^2I^2-(E^2+T_E)I(\sigma F)^2-(E^2+T_E^2)I^3)-\omega IKFE^2=0.\tag{2.12}$$

Substituting (2.10) and (2.11) in this equation with some manipulation yields

$$\begin{aligned}
g(E) \quad := \quad & \frac{r_I r_E^3 K_F^3}{\epsilon^3} E^{11} - 3\frac{r_I r_E^3 K_F^3}{\epsilon^3} E^{10} + (\frac{r_I r_E^3 K_F^3 (3+4T_E^2)}{\epsilon^3} + \frac{r_I r_E^2 K K_F^3}{\epsilon^2}) E^9 \\
& - \Bigg(\frac{r_I r_E^3 K_F^3 (1+12T_E^2)}{\epsilon^3} + \frac{2 r_I r_E^2 K K_F^3}{\epsilon^2} \Bigg) E^8 + \Bigg(\frac{6 r_I r_E^3 K_F^3 T_E^2 (2+T_E^2)}{\epsilon^3} \\
& + \frac{r_I r_E^2 K K_F^3 (1+2T_E^2)}{\epsilon^2} + \frac{\sigma^2 r_I r_E K_F^3}{\epsilon} \Bigg) E^7 - \Bigg(\frac{\sigma^2 r_I r_E K_F^3}{\epsilon} \\
& + \frac{2 r_I r_E^3 K_F^3 T_E^2 (2+9T_E^2)}{\epsilon^3} + \frac{4 r_I r_E^2 K K_F^3 T_E^2}{\epsilon^2} - \frac{\omega R_E K K_F^2}{\epsilon} \Bigg) E^6 \\
& + \Bigg(\frac{2 r_I r_E^3 K_F^3 T_E^4 (9+2T_E^2)}{\epsilon^3} + \frac{r_I r_E^2 K K_F^3 (2T_E^2 + T_E^4)}{\epsilon^2} + \frac{2\sigma^2 r_I r_E K_F^3 T_E^2}{\epsilon} \\
& - \frac{\omega K R_E K_F^2}{\epsilon} + r_I \sigma^2 K K_F^3 \Bigg) E^5 - \Bigg(\frac{6 r_I r_E^3 K_F^3 T_E^4 (1+2T_E^2)}{\epsilon^3} + \frac{2 r_I r_E^2 K K_F^3 T_E^4}{\epsilon^2} \\
& + \frac{2\sigma^2 r_I r_E K_F^3 T_E^2}{\epsilon} - \frac{\omega K R_E K_F^2 T_E^2}{\epsilon} \Bigg) E^4 + \Bigg(\frac{r_I r_E^3 K_F^3 T_E^6 (12+T_E^2)}{\epsilon^3} \\
& + \frac{r_I r_E^2 K K_F^3 T_E^4}{\epsilon^2} + \frac{\sigma^2 r_I r_E K_F^3 T_E^4}{\epsilon} - \frac{\omega K R_E K_F^2 T_E^2}{\epsilon} \Bigg) E^3 \\
& - \Bigg(\frac{r_I r_E^3 K_F^3 T_E^6 (4+3T_E^2)}{\epsilon^3} + \frac{\sigma^2 r_I r_E K_F^3 T_E^4}{\epsilon} \Bigg) E^2 + (\frac{3 r_I r_E^3 K_F^3 T_E^8}{\epsilon^3}) E \\
& - \frac{r_I r_E^3 K_F^3 T_E^8}{\epsilon^3} = 0. \qquad (2.13)
\end{aligned}$$

Here the function g is a polynomial function with respect to E of order 11 and so is continuous. Since $g(0) = -\frac{r_I r_E^3 K_F^3 T_E^8}{\epsilon^3} < 0$ and $g(1) = r_I \sigma^2 K K_F^3 > 0$, g has a root between $E = 0$ and $E = 1$. In fact, the polynomial of (2.13) has ten complex and one real root, $E_\infty = 0.5127$, where we considered $T_E = 0.09$, $r_I = 0.52$, $K_F = 250$ and $K = 750$ and all the others parameters values represented in Table 1.

From $E_\infty = 0.5127$ we obtain $I_\infty = 3005.6$, and $F_\infty = 128.1750$. We consider the community matrix of the system (2.9) at the equilibrium point ($I_\infty = 3005.6, F_\infty = 128.1750, E_\infty = 0.5127$) given by

$$\begin{bmatrix} -37.5562 & 82.0555 & 1.7527e+003 \\ 0 & 0.0451 & 23.7500 \\ -7.1902e-005 & 5.6097e-007 & -0.0486 \end{bmatrix}. \qquad (2.14)$$

To find the stability of the system, we investigate eigenvalues of this matrix. The characteristic equation of the matrix (2.14) is

$$\lambda^3 + 37.5597\lambda^2 + 0.2557\lambda + 0.0518 = 0. \qquad (2.15)$$

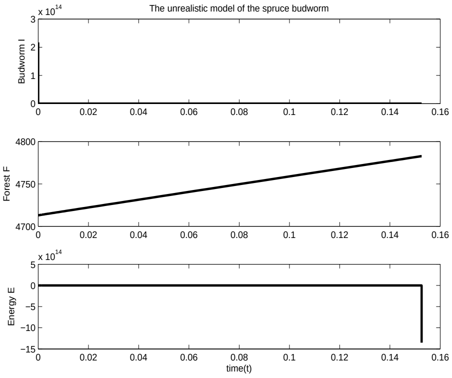

FIGURE 1. The unstable model of the population dynamics.

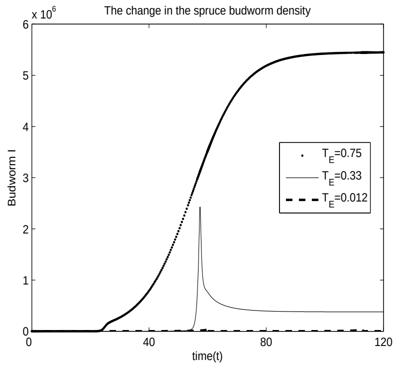

FIGURE 2. The stable model of the spruce budworm.

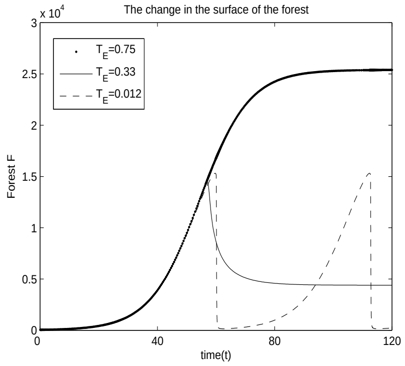

FIGURE 3. A stable model of the forest F.

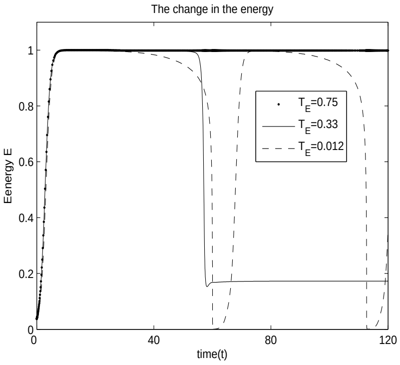

FIGURE 4. A stable model of energy E.

Thus the condition $a_1a_2 - a_3 > 0$ ensure that all of three eigenvalues found from (2.15) have negative real parts, which shows that the equilibrium point of the system (2.9) is asymptotically stable. Furthermore we investigated that

$$\begin{cases} r_I = 0.52,\ \sigma = 2.11,\ \omega = 5125 \text{ (asympotatically stable)}, \\ r_I = 0.72,\ \sigma \geq 1.7,\ \omega = 5325 \text{ (asympotatically stable)}, \\ r_I = 1.32,\ \sigma = 1.11,\ \omega = 4325 \text{ (unstable)}. \end{cases}$$

The numerical results of the asymptotically stable and unstable models are represented in Figures 1, 2, 3, and 4. For the numerical simulation of the system (2.1) and (2.11), we use **ode45** solver from MATLAB which is based on an explicit Runge-Kutta method [12]. In general, **ode45** is the best function for applying as a first try for most problems. First, we consider initial values $I(0) = 2.1721 \times 10^{14}$, $F(0) = 4713$, and $E(0) = 0.1853$ for the numerical simulation of the system (2.1) represented in Figure 4. Then, we make a change in the initial guess and use $I(0) = 0.5701$, $F(0) = 26.5476$, and $E(0) = 0.000037$ for the system (2.9) to obtain Figures 2, 3, and 4. In Figures 2, 3, and 4 the set of constant threshold values $T_E = 0.75$ is chosen to illustrate a stable model. In Figures 2, 3, and 4, the budworm density, the surface of the forest and the energy are plotted as function of time for three different values of T_E: 0.75, 0.33, and 0.012. The system (2.9) for the associated total energy $T_E = 0.19$ is also stable. Note that, if T_E is small compared with K_E, then the budworm shows a sharp decline near E=T_E. Furthermore, we note here that both systems are not asymptotically stable for the same values. Therefore for very small amounts of energy, the energy factor support makes the model stable; while without the energy factor, the model is not stable everywhere.

3. THE EFFECT OF HARVESTING AND NUMERICAL RESULTS

In this section, we apply constant effort harvesting to reduce the number of the budworm population. The effort harvesting analysis in the budworm population can be found in [17]. To control the budworm population, we apply the harvesting term $\hbar(t)$ to the stable model (2.9). The harvesting term $\hbar(t)$ is a linear function of the population size $\hbar(t) = H_T I$, where H_T is some constant number. Then, we obtain the harvested model

$$\begin{aligned} \frac{dI}{dt} &= r_I I\left(1 - \frac{I}{KF}\frac{E^2 + T_E^2}{E^2}\right) - \omega\frac{I^2}{(\sigma F)^2 + I^2} - H_T I, \\ \frac{dF}{dt} &= r_F F\left(1 - \frac{F}{K_F}\frac{K_E}{E}\right), \\ \frac{dE}{dt} &= r_E E\left(1 - \frac{E}{K_E}\right) - \epsilon\frac{I}{F}\frac{E^2}{E^2 + T_E^2}. \end{aligned} \tag{3.1}$$

In order to find the numerical solution of the system (3.1), we use **ode45** solver from MATLAB [12]. To determine the change in each population, first, we find the numerical solution of the system (2.9) without harvesting and then solve the system (3.1) with harvesting for the initial guess:

$$I(0) = 9,\ F(0) = 40,\ E(0) = 0.037. \tag{3.2}$$

If a harvesting term is introduced to the competing species model, plots of the trajectories in the phase plane can be used to test the effect of the harvesting plan before the budworms are actually killed. We seek a suitable value for the harvesting term $\hbar(t)$ so that the population has a stable equilibrium level is acceptable for the fir forest. The coordinates of this point should not exceed the carrying capacities K_F of the tree surface and should allow for normal population growth and a possible error in budworm numbers. When viewing the graphs, remember that each density without harvesting, is marked by the undashed lines. The harvesting densities are marked by dash-dotted lines. In each of the figures, the biological parameters in Table 1 are chosen according to the conditions for stability or instability of the equilibriums given in the previous section.

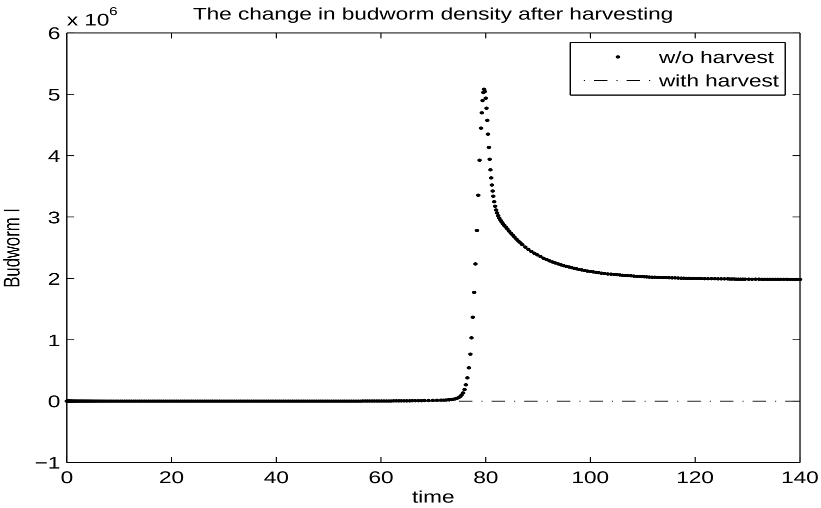

FIGURE 5. The stable model of the budworm population with harvesting.

In the case of the large population of budworms, we need a large number for the harvesting terms to reduce the budworm population. We used, in the one variable model, three different harvesting terms, representing that a very small change in harvesting, bringing a significant change in the budworm population. In the system (3.1), we used one value of the harvesting term for numerical simulation. Figure 5 illustrates how constant effort harvesting depends on the budworm population. In order to find a stable harvesting model, we choose $H_T = 3.2$ and a constant threshold value $T_E = 0.75$. If the value of H_T, increases then the population of the budworm becomes zero while the forest surface F and the energy E are increasing. In the case of a very small value of H_T, the population of the budworm increases rapidly and destroys the fir forest. Figure 6 represents the surface of the forest in the harvesting model (3.1) and without the harvesting model (2.9). The number of budworms is removed by harvesting meaning that the predation rate is very small or no predation if the budworm population vanishes. Actually, the surface of the forest grows about F=14000 from the absence of budworms. The dotted line in Figure 7 is the numerical result of the system (3.1) showing an increase in energy after harvesting. The observed pattern, denoted by the dotted lines in Figures 5, 6, and 7, is a result of a reduction in the predation rate due to constant yield harvesting when the budworm is exploited at a high rate.

Remark 3.1. In this work, we used one set of parameters for the equilibria of both dynamical systems (2.1) and (2.9) with three different values of ω, T_E, and ϵ. Simulations with different sets of parameter values can be used to obtain a sampling of possible behaviors of a dynamical system, but only analysis can guarantee that all possible behaviors have been found. It may be interested in the mathematical analysis to understand the behavior of the model system (3.1).

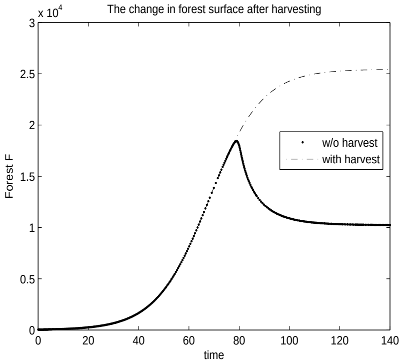

FIGURE 6. The stable model of the forest surface with harvesting.

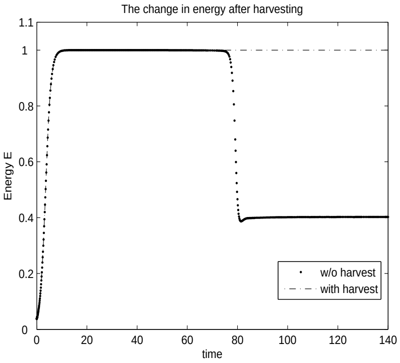

FIGURE 7. The stable model of the energy for the forest with harvesting.

4. CONCLUSIONS

In this work, we proposed a mathematical model of one predator (budworm) and two preys (forest and energy). We discussed the properties of this model and introduced a stability analysis. These techniques together with the numerical results, indicated that the budworm population model exhibited very interesting properties. The stability strategy emphasized the fact that a detailed monitoring of population size is continuously required.

To control the budworm population, we used a constant effort of harvesting as a control parameter in the system and developed new mathematical models. We concluded that when a particular species is harvested in an interacting multi species system, the equilibrium state is drastically changed. For instance, if the harvesting rate of the budworm increases beyond a particular limit, the equilibrium state becomes unstable and has an effect on each variable in the system. The models, developed in this work, predicted the change in a whole population after harvesting. We found that there was a positive relationship between the budworm population and the harvest rate. The out comes of such models constituted epidemiology indicators to develop pest management plans that make the most efficient use of the resources available in a different forest. We hope that the stability techniques and modeling approaches of this paper will be useful for other insects population models.

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