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An empirical study on the material distribution decision making†

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Abstract

This paper addresses a mathematical approach to decision making in a real-world material distribution situation. The problem is characterized by a low-volume and highly-varied mix of products, therefore there is a lot of material movement between the facilities. This study focuses especially on the transportation scheduler with a tool that can be used to quantitatively analyze the volume of material moved, the type of truck to be used, production schedules, and due dates. In this research, we have developed a mixed integer programming problem using the minimum cost, multi-period, multi-commodity network flow approach that minimizes the overall material movement costs. The results suggest that the optimization approach provides a set of feasible solution routes with the objective of reducing the overall fleet cost.

Keywords: Empirical study, mathematical approach, realistic logistics management.

1. Introduction

Transit Systems is a leading supplier of the commercial avionics and military electronics systems, as well as the service and support solution for more than 400 of its products. Headquartered in Melbourne, Australia, Transit Systems employs more than 6,000 people at principal locations in Melbourne Victoria, Adelaide South Australia, Canberra Australian Capital Territory, Sydney New South Wales, and additional Victoria operations in Geelong, Bendigo, Shepparton and Morwell.

Most of the printed circuit boards (PCBs) required for the avionics and communications products are manufactured at the Melbourne facility. Once the PCBs are manufactured and tested, they are sent to other manufacturing facilities in Victoria, e.g., Geelong, Bendigo and Shepparton, for further operations and completion of the final product. The final product is brought back to Melbourne for testing, packing and dispatch. The Transit Systems operations is characterized by a low-volume and highly-varied mix of products. Therefore, there is a lot of material movement between the facilities. On the average, it is estimated that

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Transit Systems spends about AU$780 thousand every year on material movement between plants.

There is therefore an identifiable opportunity to improve the logistics of material movement within the Victoria plants of Transit Systems. This project will provide the transportation scheduler with a tool that can be used to quantitatively analyze the volume of material moved, the type of truck to be used, production schedules, and due dates so as to minimize the overall material movement cost.

The tool that is being developed in this project is a general-purpose tool for logistics management and not specific to Transit Systems operations alone. The aim of this paper, as a preliminary study, is to show the possibility of solving a complicated real-world logistics issue based on a optimization technique. A search of the literature indicates that the methodology chosen for optimization of the mathematical approach has not been used in this context before and hence is of immense research interest and value (Holmberg, 1994; Holmberg, 2000; Holmberg, 2002; Lee, 1993; Van Roy, 1996).

2. Problem description

In this manufacturing system, there are a number of factories (typically, a main factory and several satellite factories). Each factory manufactures parts, which are used for assembly of products at the same or another factory. Parts are transferred between factories by using several kinds of vehicles (trucks) on demand. The approach assumes that material need not be delivered to its final destination during the period in which it is first loaded onto a truck at its source. Rather, it may be unloaded at another factory on the truck's route, be stored there for an arbitrary length of time, and be reloaded onto another truck for delivery to its final destination, in other words, each factory may act as a transsshipment point. Each vehicle has a specified capacity and cost of usage. The objective is to find the most cost-effective way to move material between the factories so as to meet production schedules for the assembled products, minimize the sum of transportation and storage costs, and minimize penalties for late delivery of parts.

It is assumed that for a fixed planning horizon, a list of requests for shipments is given, each specifying a part, its quantity, its source and destination, the date available for shipment and the date required at the destination, penalty for lateness, etc. We assume that the quantity of material to be shipped may be spilt into several smaller shipments if desired.

The shipping cost may include a fixed cost component for the use of the vehicle, a cost proportional to the weight and the distance shipped, a cost differential for various times of the day or week, etc. Some of the truck schedules might be already determined, while others may be selected in the optimization. Some of the truck schedules might necessarily be regular, e.g., the same schedule daily or weekly, while others may be more variable. Constraints may be imposed on the schedules, such as time windows for a dock at a factory, restricting the times at which shipments are sent or received.

3. The solution approach

The logistics system will be handled as a network in time/space, i.e., a node in the network will represent a factory at a given point in time. Time will be discretized, e.g., measured in hours. Flow in the network will represent physical movement of material and/or movement in time. The truck schedules determine the links in the network, e.g., a trip from factory 1 to factory 2, leaving at 9 am and arriving at 10 am is a link from (F1, 9) to (F2, 10), with the flow capacity equal to the capacity of the truck (Van Roy, 1993).

Once the network structure has been determined by selecting the truck schedules, the material flow (parts) is assigned to the links of the network between source-destination pairs. Thus there are two categories of decisions to be made: the vehicle schedules and the shipping schedules of the parts.

A vehicle schedule may be thought of as a list of stops, with corresponding times, together with a specification of the vehicle type. We refer to such a list as a route. (In our terminology, a route is time-specific.) The possible number of such routes is, of course, generally very large.

The solution approach is considered by the following variables:

X_l^{kt} = quantity of shipment k traversing link (i, j) during period t (i.e., leaving node i at time t, destined for j) (continuous variable)

Y_{vm} = 1 if a vehicle of type v is assigned to run m; otherwise 0 (binary variable)

Z_i^{kt} = quantity of shipment k residing at node i (inventory) during period t (continuous variable)

The objective function of this approach is to minimize the number (set) of possible routes. In practice, smoothness may not even be an objective. The machinery, however, is there to limit the number of classes scheduled for each period.

4. Discussion of the approach

The cost function includes the following three components:

a. For each vehicle v, the cost for the use of the vehicle assigned to each route, i.e., $f_{vm}Y_{vm}$, where f_{vm} would typically include (i) a fixed cost per use of the vehicle v and (ii) a cost/km times the length of route m.

b. The variable cost/km per unit of product k times the distance shipped, i.e., $c_k p_l X_l^{kt}$.

c. For each shipment k, each node i, and each period t, a term $h_i^{kt} Z_i^{kt}$ which represents either (i) the holding cost per unit of that shipment at that node, where t<r_k, i.e., the shipment is not yet due at the destination d_k, or (ii) the late penalty per unit per period for a delay in delivery of the shipment, where i $\neq d_k$ and t $\geq r_k$, i.e., the due date r_k has arrived and the shipment is not yet at its destination dk.

The approach assumes that the quantity of material to be shipped, i.e., X, is a continuous variable. The approach allows for the manager to predetermine some or all of the truck schedules (the values of the Y variables) if desired and to let the approach schedule the material shipments and any remaining trucks.

It is assumed by the approach that truck schedules may vary in every period, although it can easily be modified to require that the schedules be repeated (weekly, for example). This would reduce the number of integer variables (Y).

While there may be thousands of parts in the system, variables need to be defined only for those that are to be shipped during the planning period 1 through T. That is, the number of values of the subscript k will not be the total number of parts in the system, but just the number of parts to be shipped during the planning period [1,T].

Implementation might be in a' rolling horizon' fashion. For example, T might represent a two-week planning horizon, while only the truck schedules for the week # 1 are put firmly in place and the schedules for week # 2 are treated as tentative. The approach could then be run again during the following to determine the final schedule for week # 2 (and tentative schedule for week # 3), etc. As another alternative, the approach might treat the truck schedules (Y) for week # 1 of the planning period as predetermined by the previous week's optimization run and treated as constants, while the schedules (Y) for week # 2 found by the approach are made firm. In either case, the X variables for both weeks would be treated as variables.

The number of constraints may be quite large. Suppose we have the following numbers:
5 factories (variables of i)
15 links in the network (variables of l)
40 periods (hours) in the planning period (values of t)
200 possible routes to be considered (e.g., 10 routes, each of which may be begun in any one of 20 morning hours)
2 types of trucks (values of v)
50 shipments to be scheduled (values of k)
Then there will be
$20\times40 = 800$ constraints
$7\times50\times40 = 14000$ constraints
$2\times40 = 80$ constraints
200 constraints
(a total of 15080 constraints)
and
$12\times50 = 600$ variables of type X
$2\times200 = 400$ variables of type Y
$7\times50\times40 = 14000$ variables of type Z
(a total of 14600 continuous variables and 400 discrete variables)

This illustrates the desirability of decomposing the problem into more manageable subproblems, which will be discussed next.

5. Decomposition of the approach

The mathematical approach is a mixed integer linear problem, which might be costly to solve, even for a relatively small network (Van Roy, 1996). But the properties of the problem make it amenable to solution by decomposition in several ways, as discussed below.

As mentioned earlier, there are two types of flows in the underlying space-time network:
·material flow in space and time (represented by X and Z, respectively) and
·flow of vehicles (represented by Y).

(i) Fixing the runs Y by scheduling trucks on the routes in each time period defines a smaller network through which the material is allowed to flow. That is, if no truck is scheduled to travel across a link between two factories, the capacity of that link is zero; otherwise the capacity of that link is the sum of the capacities of all trucks traversing that link. If Y is tentatively fixed, then the problem of selecting X (shipments) and Z (inventories) is a minimum-cost multiple-commodity network flow problem. This is a linear programming problem with a special structure allowing for more efficient solution (Kim, 1989).

In such situations, it is appropriate to use the Benders decomposition (sometimes referred to as the Benders partitioning) algorithm, which would use a master problem to select 'trial' truck schedules, i.e., to assign values to Y. Then, with Y temporarily fixed, the multi-commodity flow sub-problem would be solved for X and Z in order to schedule the shipments (and storage) of materials. The dual variables (shadow prices) for the constraints of this multi-commodity network flow problem would be used to generate a new constraint for the next Benders's master problem (which is a pure integer programming problem, and therefore more costly to solve), which would then select a new set of truck schedules, i.e., a new trial value of Y. Thus the Benders algorithm alternates between the master problem, which tentatively schedules the vehicles on the routes (Y), and the sub-problem, which schedules the shipments of materials (X and Z). The master problem, namely selecting the values of the integer variables, is generally costly to solve, while the sub-problems are linear programming problems in continuous variables, which are relatively efficient to solve. The network structure may be exploited to permit these sub-problems (multi-commodity flow problems) to be solved even more efficiently (McBride, 2007).

(ii) If, on the other hand, the variables X and Y, were to be relaxed, the problem is separated into several sub-problems:

·for each shipment k, a single-commodity network flow problem with variables X and Z.

·for each vehicle type, a set of assignment problems with variables Y.

This suggests the use of Lagrangian relaxation, a price-directed decomposition technique in which a price $\lambda_1^t \geq 0$ is assigned to each link l=(i, j) for each period t. For each link l in each period t, the quantity is included in the cost function.

In this optimization problem, shipments can be sent over the link even though vehicles might not be assigned to traverse the link, but an additional cost, represented by λ , is imposed upon these shipments. If total shipments made across a link exceed the capacity of the vehicles traversing the link, the price λ_l^t for using the link is increased. On the other hand, if there is excess capacity of vehicles traversing the link, the price $\lambda$of using the link is reduced in order to encourage increased utilization of those vehicles. In Lagrangian relaxation, the problem of adjusting the prices λ is assigned to a Lagrangian master problem.

Solving the problem via Lagrangian relaxation then requires alternately assigning values to the prices for using link capacities (the master problem) and solving the sub-problems (De Souza and Armentano, 1999). Unlike the Benders decomposition, each Lagrangian sub-problem generally does not give us feasible solutions to the original problem, but it can be shown that it gives us a lower bound on the optimal solution. The master problem is, in effect, trying to maximize this lower bound by adjusting the multipliers in such a way as to increase the minimum (with respect to X, Y and Z) value of the Lagrangian objective function.

Whether using the Benders decomposition or Lagrangian relaxation, most of the computational effort lies in the respective master problem, i.e., selecting trial assignments of vehicles to routes or assigning prices for the use of the links in each period.

For problems, which lend themselves to solution by either the Benders decomposition or Lagrangian relaxation, a relatively new technique known as cross-decomposition has been developed (Garcia and Proth, 2006). In this approach, the sub-problems of the Benders decomposition and Lagrangian relaxation are used, and Benders's and Lagrangian master problems are, for the most part, avoided. The trial values needed by Benders's sub-problems are provided by the Lagrangian sub-problems (the knapsack problems) while the prices for the use of links needed in the Lagrangian sub-problems are provided by the dual variables (shadow prices) of the linear programming which is Benders's sub-problem.

In practice, it is likely that the cross-decomposition algorithm would be terminated before optimality is proved. At that stage, several 'good' if not optimal solutions should have been generated by Benders's sub-problem. The best of these solutions, if not optimal, does provide an upper bound on the optimum. As mentioned above, each Lagrangian sub-problem provides a lower bound on the optimal solution. Thus the algorithm, when terminated, provides a bound on the error.

6. Conclusions

Though there have been several approaches developed to analyze material movement between plants based on certain criteria, literature review suggests that there have not been attempts to develop an optimization approach using cross-decomposition methods with the objective of reducing overall material movement costs. Even with very few plants, types of trucks and type of material to be distributed, this project could explode into a very complex problem with several thousand variables. In this project, we have developed a mixed integer programming problem using a minimum cost, muti-period, multi-commodity network flow approach that minimizes the overall material movement costs. Due to a confidential issue which is demanded by the company in the outset of the study, some information (such as formulas and a model) are not shown in this paper. The approach applies cross-decomposition methods that use the Benders decomposition procedures and Lagrangian relaxation. The outcome of this optimization approach is a set of feasible solution routes that minimizes the overall fleet cost. The output from the approach will help the fleet scheduler to visualize the impact of decision variables and also quantitatively analyze the volume of material moved, the type of truck to be used, production schedules and due dates.

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