

Clinical Article

The Variable Ellipsoid Modeling Technique as a Verification Method for the Treatment Planning System of Gamma Knife Radiosurgery

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Objective : The secondary verification of Leksell Gamma Knife treatment planning system (LGP) (which is the primary verification system) is extremely important in order to minimize the risk of treatment errors. Although prior methods have been developed to verify maximum dose and treatment time, none have studied maximum dose coordinates and treatment volume.

Methods : We simulated the skull shape as an ellipsoid with its center at the junction between the mammillary bodies and the brain stem. The radiation depths of the beamlets emitted from 201 collimators were calculated based on the relationship between this ellipsoid and a single beamlet expressed as a straight line. A computer program was coded to execute the algorithm. A database system was adopted to log the doses for $31 \times 31 \times 31$ or 29,791 matrix points allowing for future queries to be made of the matrix of interest.

Results : When we compared the parameters in seven patients, all parameters showed good correlation. The number of matrix points with a dose higher than 30% of the maximal dose was within $\pm 2\%$ of LGP. The 50% dose volume, which is generally the target volume, differs maximally by 4.2%. The difference of the maximal dose ranges from 0.7% to 7%.

Conclusion : Based on the results, the variable ellipsoid modeling technique or variable ellipsoid modeling technique (VEMT) can be a useful and independent tool to verify the important parameters of LGP and make up for LGP.

KEY WORDS : Gamma knife radiosurgery · Treatment planning system · Quality assurance.

INTRODUCTION

The typical treatment plan in routine administration of Gamma Knife Radiosurgery (GKRS) requires high dose radiation in order to treat the deep seated intracranial skull lesions in a single session. In Leksell Gamma Knife treatment planning system (LGP), the treatment design is based upon $31 \times 31 \times 31$ or 29,791 matrix points. As a part of the quality assurance (QA) in GKRS, the verification of this LGP design is important to assure patients' safety.³⁾ To be truly independent from LGP, a secondary verification method requires another computer program which is run

in the independent computer system. Also, this computer program should have the independent algorithms to test the validity of primary LGP treatment designs. Several efforts to determine the dose from the primary LGP designs were made in the literature.^{1,2,4,6,7)} However, prior studies calculated the dose for a single matrix point out of 29,791 matrix points. The matrix point of interest was either given by the LGP^{4,6,7)} or the coordinate of the maximal dose was not available.^{1,2)} Furthermore, no study has been done to prove its accuracy on the effective treatment volume. These parameters are also important to minimize the injury to the critical structure, such as the optic pathway, optic lenses and the brain stem.

We designed a new algorithm that determines the radiation depth from an equation, where the skull geometry is expressed as a simple ellipsoid. Based upon the new ellipsoid algorithm or variable ellipsoid modeling technique (VEMT), we developed a computer program to test its accuracy on the maximal dose and the coordinate of the maximal dose

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as well as the volume receiving a dose greater than 50% of the maximal dose. To seek the number of matrix points of interest, we adopted a database system to log in each calculated dose for the entire 29,791 matrix points. We compared its results with the parameters from LGP for seven patients.

MATERIALS AND METHODS

We simulated the skull shape by the ellipsoid with the center at the junction between the mammillary bodies and the brain stem (JMBS) using the patient's preexisting MR images (Fig. 1).

There are two coordinate systems in the gamma knife radiosurgery. One of them is the Frame Coordinate System (FCS) that LGP expresses with reference to the stereotactic frame (Fig. 2). Any point could be expressed as (x_f, y_f, z_f) in FCS. One specific point of FCS is (S_x, S_y, S_z) . This coordinate represents the isocenter expressed in FCS, which means all beamlets from 201 collimator focus on this point.

The other coordinate system is the Shot Center Coordinate System (SCCS) with the origin at the shot isocenter during treatment (Fig. 3). The shot isocenter is the center of collimator helmet. Any point in the helmet could be expressed as (x_s, y_s, z_s) in SCCS. The coordinates of two systems are mathematically related by the following transformation relation :

$$\begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = \begin{bmatrix} x_f - S_x \\ -(y_f - S_y) \cos \alpha - (z_f - S_z) \sin \alpha \\ + (y_f - S_y) \sin \alpha - (z_f - S_z) \cos \alpha \end{bmatrix} \dots\dots\dots(1)$$

where α is the angle related to gamma angle γ of LGP. The two angles are related by $\alpha = \gamma - 90^\circ$. This angle represents the angular relationship of the stereotactic frame and the collimator helmet when the frame is setup for the gamma knife treatment.

The introduced ellipsoid and straight lines in the SCCS are as follows :

$$\frac{(x_s - l)^2}{a^2} + \frac{[(y_s - m)^2 \cos \alpha + (z_s - n)^2 \sin \alpha]^2}{b^2} - \frac{[(y_s - m)^2 \sin \alpha + (z_s - n)^2 \cos \alpha]^2}{c^2} = 1 \dots\dots\dots(2)$$

$$y_s = \frac{y_{p,i}}{x_{p,i}} x_s, z_s = \frac{z_{p,i}}{x_{p,i}} x_s, (i=1, \dots, 201) \dots\dots\dots(3)$$

where the points of 201 collimators are given as $(x_{p,i}, y_{p,i}, z_{p,i})$ in SCCS. The coordinate (l, m, n) of the JMBS is also a coordinate in the SCCS coordinate system.

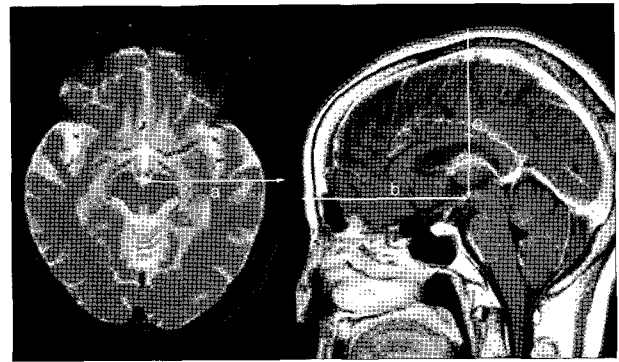


Fig. 1. The a, b and c which are required for equation (2) can be measured from the preexisting MRI. The center of the head is the junction between the mammillary bodies and the brain stem, which is the landmark for measurements.

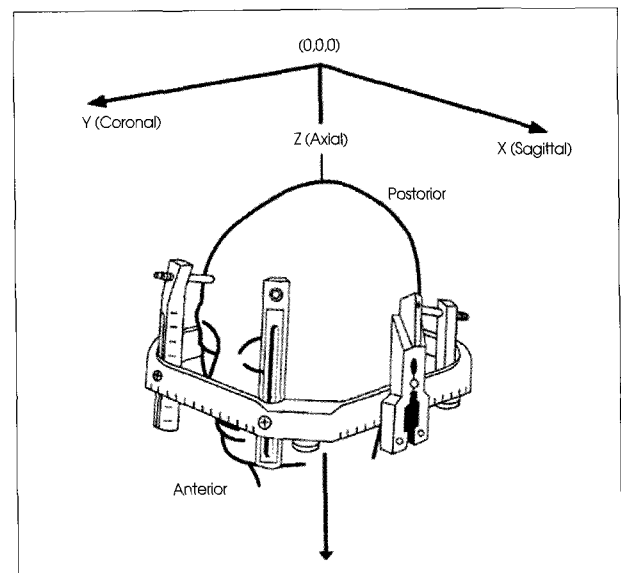


Fig. 2. The Frame coordinate system (x, y, z) with the origin at the posterior, superior, right corner of the head.

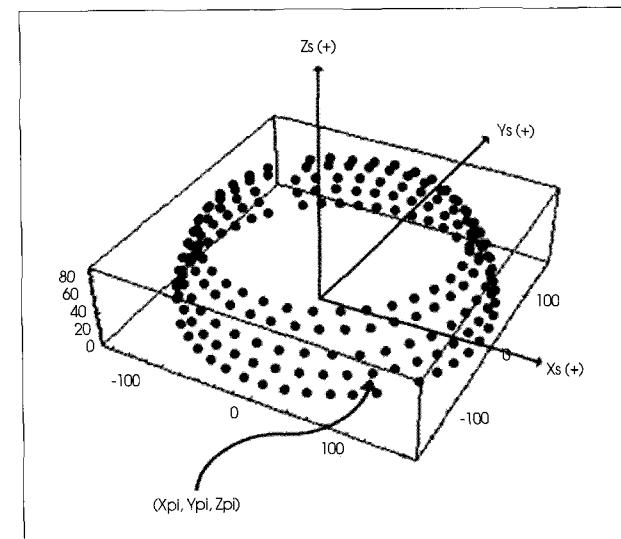


Fig. 3. The Shot Center coordinate system (x_s, y_s, z_s) and helmet collimators illustrated by Mathematica 5.2.

The dose rate at the collimator helmet center (focal point) for single beamlet expressed as the straight line is given as follows :

$$\dot{D}_i(t) = \frac{\dot{D}_{cal}(0) \cdot e^{-\ln 2 \cdot t/t_{1/2}}}{201} \times e^{-\mu(d_i(f,p)-80)} \dots\dots\dots(4)$$

where $t_{1/2}$ is the half life of Gamma Knife source ^{60}Co , $d_i(f,p)$ is the distance from the scalp to the focal point (iso-center) on which 201 beamlets focus. $\dot{D}_{cal}(0)$ is the dose rate at the calibration time, and μ , the linear attenuation coefficient, $\mu = 0.0063\text{mm}^{-1}$. $d_i(f,p)$ is determined from the point of intersection of the ellipsoid and the straight lines.

Then, the dose rate at the arbitrary point P corresponding to one shot is calculated based on the depth and single-beam dose profiles in addition to the above relation as follows (Fig. 4) :

$$\dot{D}_s(P; C_f; t) = \sum_{i=1}^{201} \dot{D}_i(t) \cdot C_f \cdot e^{\mu \cdot r_i} \cdot PW_i \cdot F(C_f; r_i) \dots\dots(5)$$

where C_f is the collimator factor and PW_i is 1 or 0 according to the plug existence.

$F(C_f; r_i)$ in the above relation is the function related to single-beam dose profiles and depends on the collimator factor and the transverse radial distance (Fig. 5). Then the dose at the arbitrary point for the multiple shots is calculated using LGP shot times as follows :

$$D(p) = \sum_{s=1}^n D_s(p; C_f; t) \cdot t_s(LGP) \dots\dots\dots(6)$$

where $t_s(LGP)$ is the shot time determined according to the treatment planning of LGP.

Leksell GammaPlan, V5.34, and Gamma Knife treat-

ment unit, model B (Elekta Instruments, Inc., Norcross, GA, USA) were used in this study. To calculate these formalisms, we used the PHP: Hypertext Preprocessor language (<http://www.php.net>). The Apache web server program (<http://www.apache.org>), PHP scripting language and MySQL softwares were used to carry out the mathematical calculations. For the entire 29,791 matrix set of points, the doses were calculated from the 201 beamlets. For the blocked collimators, the dose calculation was skipped leaving the dose as zero. The calculated dose along with its associated matrix coordinate point was stored in the database (MySQL, <http://www.mysql.com>). 29,791 records are made for a single case of Gamma Knife treatment. The database fields consist of the coordinate of a matrix, radiation doses for each shot and the sum of doses from all shots. After completing this procedure for a designated number of shots, the maximal dose point is queried as the highest sum of the doses. When the maximal dose is determined, the number of matrix points whose dose exceeded 50% of the maximal dose is queried in order to obtain the 50% dose volume. The 50% volume is then calculated by multiplying the unit matrix volume (voxel) and the number of matrix points. The number of matrix points with higher than 30% is searched in a similar procedure.

RESULTS

Results from the seven patients using our new verification algorithm are summarized in Table 1. For each shot, the calculation of all 29,791 matrix points took around 25 minutes(0.05 second per matrix point) to be performed. These seven cases include both single and multiple shot treatments. All the collimators (4, 8, 14 and 18 mm) were

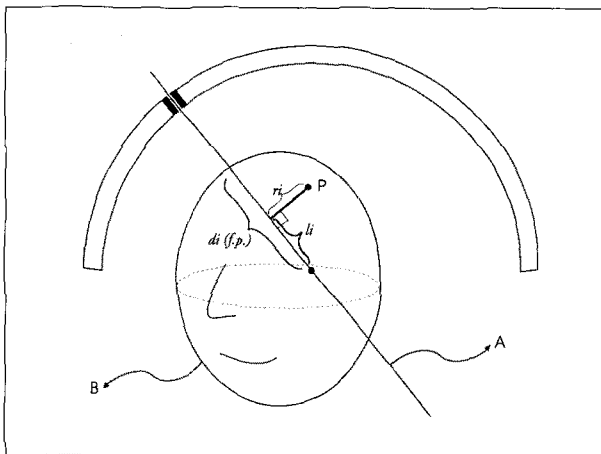


Fig. 4. A single beamlet was taken as the straight line (A). The skull is simulated as an ellipsoid (B). The beam depth $d_i(f,p)$, the geometrical relation between the arbitrary point P and the focal point(-) are illustrated.

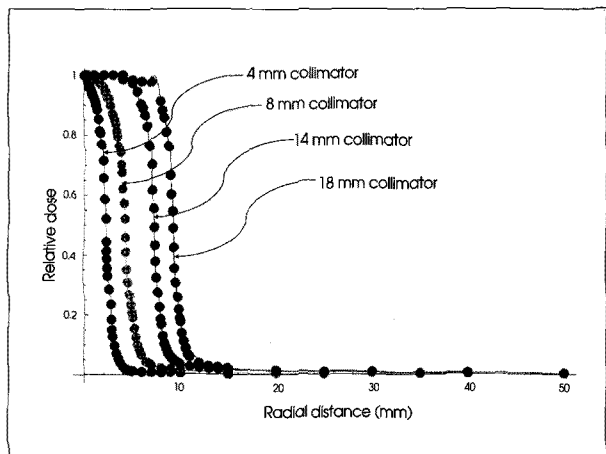


Fig. 5. Single-beam dose profiles for the four available helmet collimators drawn by Mathematica 5.2 using the dose profiles data provided by Elekta. Single-beam dose profiles expressed by Linear Interpolating Function using Mathematica 5.2.

Table 1. The comparison of the parameters from LGP and VEMT for seven cases

Patient	LGP	VEMT	Difference	
A	Number of matrix points			
	With dose > 30%	1,661	1,693	1.9%
	Volume with dose > 50%	96.9 mm ³	98.5 mm ³	1.6%
	(Point of) Maximum dose	(115.1,100.3,106.0) 80.0 Gy	(107.1,92.3,101.5) 80.6 Gy	12.1 mm 0.7%
B	Number of matrix points			
	With dose > 30%	9,511	9,625	1.1%
	Volume with dose > 50%	592.3 mm ³	589.5 mm ³	0.4%
	(Point of) Maximum dose	(64.8,106.9,81.1) 44.0 Gy	(64.8,106.9,81.6) 45.8 Gy	0.5 mm 4.2%
C	Number of matrix points			
	With dose > 30%	10,854	11,007	1.4%
	Volume with dose > 50%	2,900.0 mm ³	2,896.8 mm ³	-0.1%
	(Point of) Maximum dose	(63.2,106.6,79.0) 44.0 Gy	(64.0,108.2,79.0) 45.5 Gy	1.7 mm 3.5%
D	Number of matrix points			
	With dose > 30%	14,729	14,768	0.2%
	Volume with dose > 50%	5,800.0 mm ³	5,880.0 mm ³	1.3%
	(Point of) Maximum dose	(63.0,106.5,77.9) 44.0 Gy	(63.9,106.5,77.9) 44.6 Gy	0.9 mm 3.8%
E	Number of matrix points			
	With dose > 30%	3,564	3,570	0.1%
	Volume with dose > 50%	100.1 mm ³	104.3 mm ³	4.2%
	(Point of) Maximum dose	(107.5,89.7,67.3) 24.0 Gy	(107.5,89.7,67.3) 25.0 Gy	0.0 mm 4.1%
F	Number of matrix points			
	With dose > 30%	11,808	11,684	-1.0%
	Volume with dose > 50%	689.8 mm ³	711.2 mm ³	3.0%
	(Point of) Maximum dose	(64.6,109.1,83.1) 44.0 Gy	(64.6,109.1,83.1) 47.1 Gy	0.0 mm 7.0%
G	Number of matrix points			
	With dose > 30%	12,333	12,373	0.3%
	Volume with dose > 50%	7,900.0 mm ³	7,920.0 mm ³	0.2%
	(Point of) Maximum dose	(106.1,69.2,118.4) 40.0 Gy	(106.1,69.2,118.4) 41.1 Gy	0.0 mm 2.9%

LGP : Leksell GammaPlan, VEMT : variable ellipsoid modeling technique

included as the single shot treatment.

Number of matrix points with the dose higher than 30% of the maximal dose

We calculated the number of matrix points which was exposed to more than 30% of the maximal dose. In case E, which showed the best result, 3,564 matrix points had the dose more than 30% in LGP. With VEMT, there were 3,570 matrix points which received the higher dose than 30%. In case A which has the worst result, LGP showed 1,661 matrix points and there were 1,693 matrix points under VEMT calculation. The average deviation between

LGP and our new method was 0.6% with the standard deviation of 0.87.

50% dose volume

The 50% dose volume represents the number of matrix points receiving greater than 50 % of maximal dose. In case C, we found the best calculation results which were 2,900.0 mm³ in LGP and 2,896.8 mm³ in VEMT. The average deviation between LGP and VEMT was 1.5% with the standard deviation of 1.5.

The point of maximum dose

The good results were seen in patients E, F and G. In these cases, both LGP and VEMT indicated the same maximal points. However, in patient A, we obtained the worst result which showed the deviation from the maximal dose point to be 12.1 mm. The average deviation between LGP and VEMT was 2.1 mm with a standard deviation 4.1.

The maximum dose

The discrepancy between our calculation of the maximum dose and that calculated by LGP was within 0.7% to 7%. In case A, LGP calculated the maximal output as 80.0 Gy while the result of VEMT was 80.6 Gy. The average deviation between LGP and VEMT was 3.7% with the standard deviation of 1.8.

DISCUSSION

To accomplish an exact, safe and rigorous treatment, independent verification methods have been developed by many scientists^{1,2,4,6,7}. One of the key issues not addressed by the previous methods is how to calculate the radiation depth from 201 beamlets. To acquire each radiation depth, the patient's skull geometry has to be simulated. All of the prior efforts were made based upon the 24 point measurements using the bubble plastic helmet^{1,2,4,6,7}. This measurement is a part of the routine Gamma Knife procedure to simulate the skull geometry. LGP system simulates the skull shape by the cubic spline interpolation based upon eight

equal angular spaced points along four latitudes. To calculate the radiation depth from 201 beamlets, Zhang et al.⁷ and Beck et al.¹ used the linear interpolation of the three skull points selected nearby. The penetration depth is calculated by performing a weighted average of three nearby skull points. Jin et al.² and Tsai.⁶ simplified all scalp depths as single representative beam path length or the average target depth. Tsai applied only one radiation depth to represent the one maximal dose for the given matrix point. Because of the simplicity, the accuracy decreases when it is off-center⁶. Jin et al.² added the concept of the correction factor to the average penetration depth or the representative path length to compensate this off-center deviation. Marcu et al.⁴ also used average radius R from scalp measurements. They modeled the skull geometry as a sphere. They acquired the radiation depth of a matrix point calculating the relationship between a sphere and straight line. All prior reports produced excellent results with low error rates. However, they only focused on the dose at specific coordinates or the radiation time. Along with the verification of dose, the maximal dose point is also considered to be critical as a quality assurance program⁹. No prior studies have determined the maximal dose point. Furthermore, the volume receiving over 50% of the maximum dose along with the number of points receiving greater than 30% of the maximum dose has never been analyzed. To calculate these parameters, we included the entire 29,791 set of matrix points, derived from the 201 beamlets, and adopted a database system to store these values. Calculations were carried out by querying this database. Calculating the dose of 29,791 matrix points took approximately 25 minutes for each shot. This computation time is slightly increased secondary to the fact that PHP is not the most efficient programming language for mathematical calculations but was used because of the author's familiarity with it. We believe the entire calculation time could be shortened if a faster programming language is adopted. However, for a known single coordinate, the dose can be calculated in seconds with the VEMT program, which can be used as a routine pre-treatment dose verification method for a specific matrix point.

We simulated the skull as an ellipsoid with the equation (2). The variable ellipsoid modeling technique or VEMT uses the transverse width, anterior to posterior length, height of the skull from the junction between the brain stem and mammillary body on the MRI or CT scan. Together with the optic chiasm, the mammillary bodies are not surrounded by the hemispheres, which is readily identifiable on the MRI image. We also utilized the picture archiving and communication system (PACS) and designated a landmark to decrease person to person bias. Measuring these param-

eters in a computer program can increase its accuracy and reproductivity. One of the benefits of the current method is that it does not take parameters from 24 scalp measurements using a plastic helmet. This can be performed independently from the routine LGP procedures. The relationship between a specific matrix point in an ellipsoid and a straight line can be simultaneously calculated with the equations (2) and (3). Instead of an ellipsoid, Marcu et al.⁴ adopted the sphere of radius R . They calculated the radiation depth with the relationship between this sphere and a beamlet. Although they reported an excellent result, one of the drawbacks of their model is that the difference in skull width, length and height cannot be taken into account to the algorithm. They acquired the radius R by averaging the scalp measurements from the bubble plastic helmet. The concept of the ellipsoid model to simulate the skull geometry has been used by previous authors. The equation has been used to measure the fetal head circumference by Shields et al.¹. Zhang et al.⁷ used an ellipsoid phantom that has the fixed width and length and height to test their dose verification method. However, their radiation depth was calculated based on the linear interpolation from the data supplied by the LGP program. Jin et al.² used an ellipsoid model to formulate the correction factor to compensate for the average path length for off-center point.

In our study, all parameters showed good correlation with those from LGP. The point of maximal dose expressed as one coordinate is for verifying the target coordinate designed by LGP program. In three cases in this study, they correspond perfectly with the LGP calculation. In one case, the distance of the point of VEMT from the point by LGP was 12.1 mm. Although they were 12.1 mm apart, the doses differed by only 0.7%. The 50% dose volume is important because the tumor boundary is designed by this parameter on LGP. Hence, it is essential that this value should be determined. By this volume, we can also compare the volumetric accuracy of the treatment design along with the number of points higher than 30%. This parameter was included in this study because it is reported routinely after a gamma knife treatment is designed by the LGP program.

CONCLUSION

We provide a method of quality assurance program based upon an ellipsoid modeling technique which uses the width, length and height of skull measured in the MRI or CT. Unlike prior studies, we calculated the doses for 29,791 matrix points to calculate the maximal dose point with the dose, radiation volume receiving greater than 50% of the maximum dose and the number of matrix points

with higher than 30% of maximal dose. The VEMT is a useful and accurate independent tool to verify important parameters of LGP providing additional essential information important in assuring patient safety.

References

1. Beck J, Berndt A : An algorithm for independent verification of Gamma Knife treatment plans. *Med Phys* 31 : 2780-2784, 2004
2. Jin JY, Drzymala R, Li Z : A simple method of independent treatment time verification in gamma knife radiosurgery using integral dose. *Med Phys* 31 : 3337-3344, 2004
3. Maitz AH, Wu A, Lunsford LD, Flickinger JC, Kondziolka D, Bloomer WD : Quality assurance for gamma knife stereotactic radiosurgery. *Int J Radiat Oncol Biol Phys* 32 : 1465-1471, 1995
4. Marcu SM, Wu QJ, Pillai K, Weinhaus MS : GammaPlan-Leksell Gamma Knife radiosurgery treatment planning verification method. *Med Phys* 27 : 2146-2149, 2000
5. Shields JR, Medearis AL, Bear MB : Fetal head and abdominal circumferences : effect of profile shape on the accuracy of ellipse equations. *J Clin Ultrasound* 15 : 241-244, 1987
6. Tsai JS, Engler MJ, Rivard MJ, Mahajan A, Borden JA, Zheng Z : A formalism for independent checking of Gamma Knife dose calculations. *Med Phys* 28 : 1842-1849, 2001
7. Zhang P, Dean D, Wu QJ, Sibata C : Fast verification of Gamma Knife treatment plans. *J Appl Clin Med Phys* 1 : 158-164, 2000