Journal of the Korea Society of Mathematical Education Series D:
Research in Mathematical Education
Vol. 14, No. 1, March 2010, 79–98

韓國數學敎育學會誌 시리즈 D:
<數學敎育硏究>
제 14 권 제 1 호 2010 년 3 월, 79–98

Research of the Relationships between Self-control, Thinking Quality and Mathematical Academic Achievement for Senior School Students[1,2]

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(Received December 15, 2009. Accepted February 10, 2010)

To analyze the relationships between self-control, thinking quality and mathematical academic achievement, 197 senior school students were asked to complete questionnaires called "self-control ability on mathematics for middle school students" and "thinking quality for senior school students." The results were as follows:

(1) There was strongly positive relevance between self-control ability, thinking quality and mathematical academic achievement.
(2) A model was presented in which self-control ability had a direct impact on mathematical academic achievement, meanwhile had indirectly influenced mathematical academic achievement by thinking quality which acted as the intermediate variable. Thinking quality had a direct impact on mathematical academic achievement, too.
(3)There's no significant difference between the two groups of boys and girls on the structural weights.

Keywords: self-control, thinking quality, mathematical academic achievement
MESC Classification: C43, D63
MSC2010 Classification: 97C40, 97D60

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[1] This work is supported by Natural Science Foundation of Shandong Province under Grant No. Y2006F28.
[2] This paper has been presented at the 15th International Seminar on Education of Talented Children and Creativity Development in Mathematics at Woosuk University, Samrye, Jeonbuk, Korea;February 19–20, 2010.
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1. INTRODUCTION

Self-control as an ingredient of meta-cognition refers to the process that to meet the pre-concerted goal, one plan, supervise, check-up, evaluate, feedback, control and adjust the practical activity itself. In recent years, psychologists made many studies about the relationship between self-control and learning, including the structure of self-control, relationship between self-control and thinking quality, self-control in reading and comprehension, memory activity and writing. What was concerned in this study was the relationship between self-control, thinking quality and achievements. As we know, self-control and thinking quality were important factors of achievements, but what was the relationship among the three things like, especially in the subject of mathematics?

LITERATURE REVIEW

Yu (2004) found that self-control ability of mathematics had a significant effect on mathematics achievements. Psychological literature of Soviet Russia relatively early reflected the study of thinking quality. In west psychology group, Guilford (1967) regarded the thinking quality as: sensibility to problem, fluency (including three factors named as associative fluency, idea fluency and language fluency), flexibility, originality, elaboration and ability of redefine. On this basis, psychologists of Europe and America made more and more deeply studies on thinking quality, mainly performed on the following three aspects: Firstly, the importance of thinking quality was emphasized, especially the velocity, difficulty, profundity and density of thinking; Secondly, experiment researches were further deepened; Thirdly, experiments on cultivating were coming to be emphasized. Zhu (1984) regarded thinking quality, especially its profundity, originality and criticality as an important aspect of thinking development, and represented the age character and expression of these thinking development. Furthermore, Zhu (1993) elaborated the essential of thinking quality, the significance of creativity of thinking quality, the factors meaningful to thinking quality development, and the difference on thinking quality development between individuals and between age-stages. Lin (1999) thought that the difference of thinking ability was embodied by the difference of thinking quality, and the ingredients and manifestations included five aspects of profundity, flexibility, originality, criticality and agility. Dong (1990) found that there existed significant or exceptionally significant relationship between meta-cognition and thinking quality in a study with Chinese language reading in elementary school. As was found in the following studies, the essential of this relationship was causational relation, which means that change of meta-cognition brought about change of thinking quality. That was to say, difference of meta-cognition was the fundamental cause of difference of thinking quality among students.

Above all, there have been many valuable researches about self-control, thinking quality, achievements and combination of two variables above. However, there are rare studies combining these three variables together to probing their relationships; Furthermore, with regard to the subject of mathematics which particularly needs self-control and thinking quality in solving problems, there lacks substantive studies.

RESEARCH HYPOTHESES AND CONSTRUCTION OF THE MODEL

In this study, we hypothesized that self-control was the important factor inducing difference of achievements and effect of self-control making on achievements partly by means of thinking quality. Senior school students were regarded as participants for the relatively stability of their self-control and thinking quality. Relying on subject of mathematics and utilizing Structural Equation Modeling, relationship among three variables were explored and validated.

The variables are shown in Table 1, where the Self-control was defined as exogenous latent variable and ξ was used as the symbol. Other latent variables were defined as endogenous variables using η as the symbol. Plan, adjust, test, manage, and evaluate were defined as exogenous manifest variables and x was used as the symbol. Other manifest variables used y as the symbol.

Table 1. Latent variables and Manifest variables

Latent variable	Manifest variable
Self-control (ξ)	Plan (x_1)
	Adjust (x_2)
	Test (x_3)
	Manage (x_4)
	Evaluate (x_5)
Thinking Quality (η_1)	Profundity (y_1)
	Flexibility (y_2)
	Originality (y_3)
	Criticality (y_4)
	Agility (y_5)
Achievements (η_2)	Achievement 1(y_6)
	Achievement 2(y_7)

Thus, we developed the following hypotheses and conceptual framework.

H1: Self-control had a positive effect on achievements.
H2: Thinking quality had a positive effect on achievements.
H3: Self-control had a positive effect on thinking quality.

The framework is shown in Figure 1.

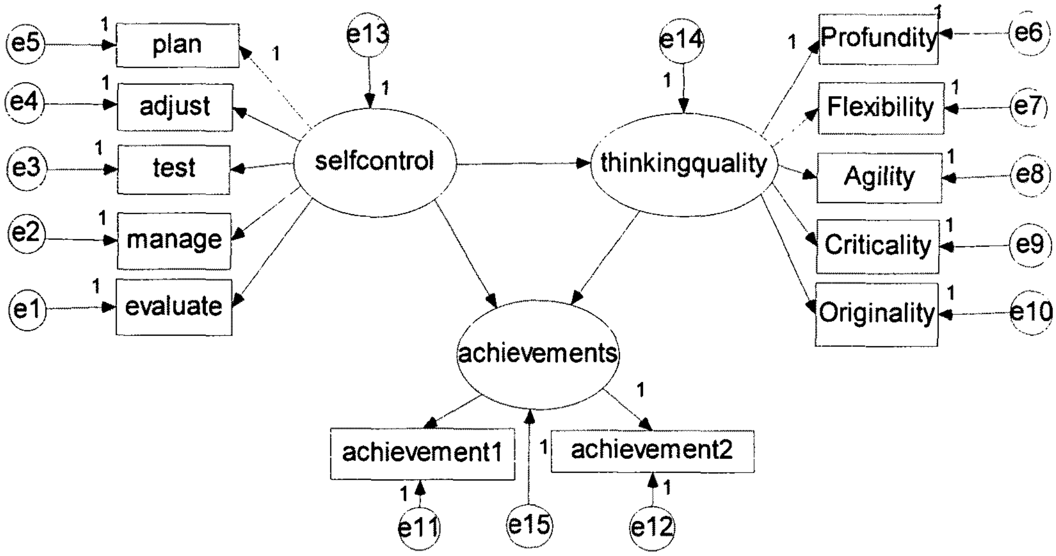

Figure 1. Model of Self-control, Thinking Quality and Mathematics Achievements

METHODOLOGY

Participants

The participants were 212 students came from 4 Senior Grade classes of three schools in Jinan, Shandong province. 114 of them were female and the others were male students. Their performance was at the average level of all the students in Grade Three.

Questionnaire Design

Questionnaire 1

With the method of confirmatory factor analysis, Zhang (2003) founded a second-order-factor model with five first-order factors which was the best-fit to the mathematical self-control ability of middle school students. This model had high construct validity. The

"Questionnaire of Mathematical Self-Control Ability for Middle School Students" he drew up was composed of 47 items divided into five dimensions: plan, adjust, test, manage and evaluate. All the items were measured with a 5-point Likert rating scales, ranging from 1 (completely disagree) to 5 (completely agree). This questionnaire had a total homogeneity reliability of 0.8871, and five factor homogeneity reliability of 0.6646, 0.7405, 0.7853, 0.7800, and 0.6814.

Considering participants' response to the items of this questionnaire apt to take on relatively strong central-tendency error, this study adopted even-level of 6-point Likert rating scales. Items numbered 10,20,24,27,38,41,42,43,44,45,46 were reverse items, respectively valued A~F with score of 1~5. While the others were opposite, respectively valued A~F with score of 5~1. The more scores one attained from this questionnaire, the higher self-control ability he (she) had.

That questionnaire was utilized in this study to collect and analyze mathematical self-control of participants. In actual testing, to reduce students' psychological carry-over effect, we renamed it as "mathematical study questionnaire". In this study, internal consistency α coefficient and test-retest reliability of 10-day-interval were both more than 0.7, which meant this questionnaire had good reliability.

Questionnaire 2

In this study, we adopted "Questionnaire of Thinking Quality for the Senior Grade Three Student" drawn up by ourselves. In actual testing, to reduce students' psychological carry-over effect, we renamed it as "mathematical study test".

Considering the validity of questionnaire, the process of drawing up the questionnaire was as follows:

(1) Literature retrieval, theory research and questionnaire preliminary drawn up

In the process of drawing up the questionnaire, we consulted many mature theoretic opinions, which provided the questionnaire with theoretical validity. According to the study of Lin (1999), mathematics thinking quality was divided into five ingredients: profundity, flexibility, originality, criticality and agility. Regarding with measurement of these five mathematical thinking quality ingredients, we drew on the idea of measuring reading thinking quality in the study of Dong (1990). On that basis, six items on each ingredient were formed through filtering and induction according to the measurement purpose.

(2) Specialist interview and questionnaire revision

Discusses with some specialists in mathematical psychology and one specialist teacher in senior school about the validity of the questionnaire were made several times. Four items on each ingredient were formed finally through comparison, filtering and modification one by one. Scoring details were made through discuss with several graduates. In the process of prediction, we asked for the opinions of the teachers and participants and made serious revision on the contents and instructions of the questionnaire. So the questionnaire had relatively high expert validity.

(3) Prediction and re-revision

Two classes of Senior Grade Three in Zibo (Shandong province) were chosen to be predicted. In the prediction, the items were out of order and arranged randomly. The total time was determined to 2 hours through estimating and discussing, which served as the reference standard of formal testing. Seeking the opinions of the teachers and participants, with which we revised the representation, instructions and scoring details of the items. Through statistical analysis of pre-test results and opinions of specialists, the questionnaire was revised again. So, this questionnaire had validity in tool structure and testing form.

(4) Meta-analysis: expert validity analysis

According to the study of Zhu (1991) about expert validity, all the 20 items were out of order and arranged randomly, and 6 research staffs unrelated to this study were asked to categorize the items to 5 dimensions. Among 6 research staffs, 3 were randomly selected to not be told about concrete definition or explanation of each dimension, while the others were told in advance. On that basis, the items having much divergence with up-front design were reconsidered and reorganized.

Then, "Questionnaire of Thinking Quality of the Senior Grade Three Student" was finally determined, which was composed of 5 ingredients: profundity, flexibility, originality, criticality and agility. 4 items on each ingredient were made.

We had a testing with formal participants by the revised questionnaire. In the testing, for each class, an experimenter trained in advance and the class tutor both showed up to hand out and reclaim questionnaires. 212 questionnaires were let out and 208 were reclaimed. The rate of reclaiming was 98.11%. Among these 208 questionnaires, 11 were invalid questionnaires and 197 were valid. The rate of valid reclaiming was 94.7%.

Confirmatory factor analysis was done with the questionnaire utilizing the software Amos7.0 (Bentler, 1995; Hou, 2004; Joreskog. & Sorbom, 1993). Analysis of reliability indicated that internal consistency α coefficient of mathematics thinking quality questionnaire was 0.88, and test-retest reliability of 10-day-internal was 0.84. For validity, confirmatory factor analysis showed that the value of GFI was up to 0.82, which meant closeness of fit of the model was acceptable, which was to say that the data didn't exclude the model. So, the questionnaire had good construct validity.

Questionnaire 3

Mathematics achievement at the end of the Second Semester in academic year 2008–2009 (named achievement 1) and that on the beginning of the First Semester in academic year 2009–2010 (named achievement 2) were chosen, and the average score of them was regarded as the mathematics achievement of each participant.

DATA ANALYSIS AND RESULTS

Data Analysis

All the data were analyzed with the software SPSS17.0 and Amos7.0, and the main statistics methods were correlation analysis and structural equation modeling.

Analysis of Correlation between Self-control, Thinking Quality and Mathematics Achievements

The results were as Table 2.

	Self-control	Thinking quality	Achievements
Self-control	0.81^{**}	0.54^{*}	
Thinking quality	0.76^{**}		
Achievements			

Note: $^{**}p<0.01$, $^{*}p<0.05$

As indicated in table 2, self-control, thinking quality and mathematics achievements all had significantly positive correlation with each other.

Model of Self-control and Thinking Quality affecting Mathematics Achievements

To further explore a model of self-control and thinking quality affecting mathematics achievements, we utilized structural equation modeling to explore the relationship between the factors. Then we got the model of self-control and thinking quality affecting mathematics achievements as shown in figure 2.

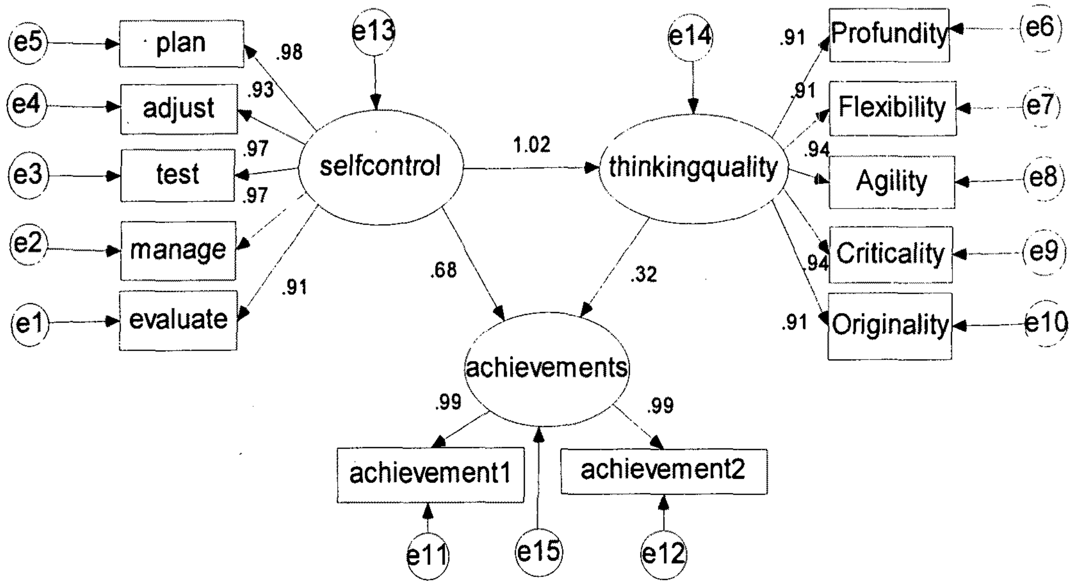

Figure 2. Structure Model of Self-control, Thinking Quality and Mathematics Achievements (standardized estimates)

As was shown in figure 2, among the path coefficients of the model of self-control and thinking quality affecting mathematics achievements, self-control affected mathematics achievements directly, and the value of direct-effect was 0.68. Self-control also affected mathematics achievements indirectly through thinking quality and the value of indirect-effect was 0.33. So the gross effect of Self-control affecting Mathematics Achievements was 1.01. The ratio of mesmeric effect of thinking quality to mathematics achievements to direct-effect of self-control affecting mathematics achievements was 1∶ 2.06. Thinking quality affected mathematics achievements directly with the direct-effect of 0.32.

Model Fit

Model of self-control and thinking quality affecting mathematics achievements had fit indicators as shown in table 3.

Table 3. Fit Indicators of the Model of Self-control, Thinking Quality and Mathematics Achievements

	χ^2	df	χ^2/df	GFI	AGFI	IFI	TLI	CFI	*RMSEA*
Model	172.1	51	3.37	0.82	.89	.91	.87	.96	.038

As was shown in table 3, the indicators GFI, CFI and AGFI were all close to or bigger than 0.90 and indicator RMSEA was smaller than 0.05, which indicated that the closeness of fit was acceptable. That was to say data didn't exclude the model.

Difference between Groups on Models of Relationship between Self-control, Thinking Quality and Mathematics Achievements

Data was divided into two groups namely female group and male group, and the two corresponding models of self-control and thinking quality affecting mathematics achievements were tested to be different or not. The result was that $p = 0.43 > 0.05$, showing there was not significant difference between groups.

CONCLUSION AND DISCUSSION

Conclusion

(1) Self-control, thinking quality and mathematics achievements all had significantly positive correlation with each other.
(2) In the model of self-control and thinking quality affecting mathematics achievements, self-control affected mathematics achievements directly and indirectly through thinking quality. Thinking quality affected mathematics achievements directly.
(3) There was no significant difference between female group and male group.

Self-control Affecting Thinking Quality

We found in this study that self-control affected thinking quality significantly, which was in accordance with what had been found by Dong (1990). To interpret it, Dong (1990) considered that self-control and thinking quality were two aspects of the same thing. The former was the intrinsic organized form of thinking structure and it was the deep-level structure of thinking. The latter was the extrinsic representation form of thinking structure and it was the surface-layer structure of thinking.

To analysis from the angle of concrete thinking activity, it was not difficult to find that thing. For example, in the representation process of flexibility, it had to include the ingredient of adjust and manage. Just like what Zheng & Liang (1998) had inferred, these qualities of thinking had direct relationship with the development level of self-control. In particular, self consciousness of intrinsic thinking (including cognitive structure) and the ability of self evaluate and self adjust should be regarded as the important qualifications of the qualities of thinking. So, in this sense, we could say that cultivating and practice of self-control was the crux of improving thinking quality.

Thinking Quality as the Medium

Self-control affected mathematics achievements indirectly through thinking quality, which hinted us that improving students' self-control through effective ways such as active guiding and practice could optimize students' mathematical thinking quality to a certain extent. The direct effect of self-control on mathematics achievements was bigger than the indirect effect through thinking quality, which told us that self-control was still important to mathematics achievements.

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APPENDIX 1

QUESTIONNAIRE 1: QUESTIONNAIRE OF MATHEMATICAL SELF-CONTROL ABILITY FOR MIDDLE SCHOOL STUDENTS

学号： 性别：男 女 年龄：

学校： 班级：

亲爱的同学们，下面是你在学习数学是可能出现的一些做法或想法。请你根据自己的实际情况比较你与这些想法或做法之间的相像程度，并在适合你的答案的字母上打“√”。请你认真回答每一个问题。注意，每个问题只能选一个答案。答案之间无对错之分。本试卷只为科学研究之用，和你的学习成绩、鉴定无关。

请你一定按照自己的平时的习惯来回答问题。谢谢你的合作。

1.在开始解答数学问题之前，我会告诫自己：“首先应该认真理解题意”。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

2. 在具体解答之前我会对问题的答案进行猜想。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

3.在解题过程中，我会经常估计一下前景和成功的可能性。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

4.在证明一个问题时，我会不断提醒自己：“这就是要证明的结论”。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

5.在证明几何题时，我会尽量准确地画出图形，感觉一下结论的正确性。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

6.在求解问题时，我会先问自己:“已知条件是什么，结论是什么，要获得结论还需要哪些条件，如何才能得到这些条件。”

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

7.我头脑中有许多数学基本题型（模型），解题时我总是把题目与这些模型相对照。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

8.做数学题时，我喜欢先做容易的，再做稍难的，最后做难题。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

9.证明数学问题时，我一般是先将条件与结论用明确的数学符号表示出来，然后再寻找沟通条件与结论的桥梁。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

10.数学学习中，老师怎么说我就怎么做。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

11.在解答数学问题的过程中，我对自己采取的解题方法的有效性是心中有数的。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

12.在解答数学问题的过程中，我会经常问自己:“这一解题方法正确吗？”

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

13.在解答数学问题的过程中，我能清楚的感觉到自己所采用的解题方法的优劣性。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

14.在解答数学问题的过程中，我经常提醒自己要注意问题的关键。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

15.如果对一个数学概念不理解，我会分析一下概念的一个实际例子。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

16.在解数学题的过程中，我会问自己：“要获得结论，还缺少哪些条件？”

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

17.在分析题意时，我会问自己：“哪些知识与本题是有联系的？”

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

18.如果感到用一种方法难以理解某一数学内容时，我会尝试换一种方法来学习它。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

19.如果解题发生困难，我就考虑这个问题的特例或最简单的情况。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候

这样 F 从不这样

20.学习完某一数学知识后，我没有想过要用自己的语言来表述它。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

21.看数学书时，我读完一段就用自己的语言重新叙述一下所读的内容。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

22.学习了数学概念以后，我的脑子里就会有一些概念的具体例子。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

23.学习完一节数学课后，我会写出这节课的重点内容，并与课本进行对照，已确定自己是否已经理解相应的内容。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

24. 如果解不出某个数学题目，我一般不会怀疑题目本身的错误。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

25. 学习完新的数学知识后，我以用它来解决一些相关问题的方法检验自己是否已经理解所学内容。

26.解完数学题后，我会想：“还有更好的方法吗？”

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

27. 解完数学题后，我一般不去总结解题的关键。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

28. 解完数学题后，我会考虑：“这个解题方法能够用来解决类似的问题吗？”

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

29. 解完数学题后，我会问自己：“这个问题能够推广吗？”

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

30. 解完数学题后，我一般会采用另一种解题方法来检验答案的正确性。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

31.数学学习过程中，如果忘记了要用的公式，我就重新推导它。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

32.遇到对某一数学知识不太理解的情况时，我会在不理解的地方做个记号。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

33.在学习代数时，我重点分析问题的数量关系。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

34.在解决数学问题时，我会把题目中的重要信息提取出来，并按他们的相互关系做出示意图。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

35.在解答数学题时，我会用等式、图像或图形以及表格等各种方式来组织、解释问题中的信息。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

36.如果解决某个数学问题有几种方法，而我对其中的任何一种方法都不是十分有把握时，我会对每一种方法都尝试一下，感觉一下哪一种方法最适合自己。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

37.在记忆数学知识时，我会按照概念、定理的相互关系作出相应的知识表。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

38.我没有想过在解数学题过程中要对定理进行重新理解。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

39.在证明数学问题时，我会先考虑一下自己的理由是否充分，再根据自己的感觉做出选择的决定。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

40.在解决数学问题时，我会先通读一遍题目，对问题有一个整体把握，然后再细节化。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

41.当我非常顺利的完成考试时，我会想："那是因为我运气好。"

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

42.解数学题目时，只要想出一种解题方法我就心满意足了。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

43.订正数学作业就是再解一遍题目。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

44.在数学学习中，粗心大意是我产生错误的主要原因。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

45.我常常感到自己在数学学习中存在问题，但不能确定问题究竟在哪里。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

46.我能够听懂数学课，但常常出现解题时用不上的情况。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

47.数学考试后，我对自己成绩的估计与实际获得的分数基本一致。

A.总是这样 B 大部分时候这样 C 一半时候这样 D 小部分时候这样 E 极少时候这样 F 从不这样

APPENDIX 2

QUESTIONNAIRE 2: QUESTIONNAIRE OF THINKING QUALITY FOR THE SENIOR GRADE THREE STUDENT *

学号：　　　　　　　　性别：男　女　　　　　　年龄：

学校：　　　　　　　　　　　　　　　　　　　　班级：

亲爱的同学们，下面是一些数学测题，不要把它想的太难呀，有些是很简单的，甚至是初中或小学的内容。相信你能努力完成它们。本试卷只为科学研究之用，和你的学习成绩、鉴定无关，不记名字，请一定独立完成。谢谢你的合作。

一. 填充题

1. 若函数$f(x)$ 的定义域是$[0,1]$，则函数$f(x^2)$的定义域是_______.

2. 设集合$A=\{5,\log_2(a+3)\}$，$B=\{a,b\}$，若$A\cap B=\{2\}$,则$A\cup B=$_____.

3. 从下列数列里，找出14后面的三个数:$1,3,2,6,5,15,14,$___,___,___$,122,\ldots$

二. 以简短的步骤作出以下各题

1.有一个人从市场上用60元钱买回一只羊，接着又以70元卖给别人，然后又以80元买回来，最后以90元钱卖给别人。请问此人在这次买卖中是赔还是赚？赔多少还是赚多少？

2. 若方程$x^2\lg m-2x+1=0$有两个不等实根，试求常数m的范围。

3. 比较$\frac{6}{11},\frac{10}{17},\frac{12}{19},\frac{15}{23},\frac{20}{33},\frac{60}{37}$的大小

4. 已知$\frac{1+sinx}{\cos x}=-\frac{1}{2}$，则$\frac{\cos x}{\sin x-1}$的值是______

* 深刻性：（一、1、2、二、1、2、）灵活性：（一、3、二、3、4、5、）
敏捷性：（二、6、7、8、9、）批判性：（三、）独创性：（四、）

5. 已知$\sqrt{-x^2-4x} \geq \frac{4}{3}x+11-a$的解集为$\{x \mid -4 \leq x \leq -2\}$，求实数$a$的值.

6.若α是锐角，$sin(\alpha-\frac{\pi}{6})=\frac{1}{3}$, 求$cos\alpha$的值。

7. 解方程$\left(x-\frac{6}{x}\right)(x-1)+1=x$.

8已知$sinxcosx=\frac{1}{8}$，且$\frac{\pi}{4} \leq \alpha \leq \frac{\pi}{2}$，则$sinx-cosx$的值等于______.

9. 计算$\frac{\sqrt{3+2\sqrt{2}}-\sqrt{2}}{\sqrt{2}-\sqrt{3-2\sqrt{2}}}$

三. 判断下列各题的问题及证明或解答有错误吗？

请说出正确或错误的理由，如果错误，请在题目旁边修正解答。

1.设$f(x)=\frac{1+x^2}{1-x^2}$，求证$f\left(\frac{1}{x}\right)=-f(x)$

证明：$f\left(\frac{1}{x}\right)=\frac{1+\left(\frac{1}{x}\right)^2}{1-\left(\frac{1}{x}\right)^2}=-\frac{1+x^2}{1-x^2}=-f(x)$

2已知双曲线$x^2-\frac{y^2}{2}=1$，过点$B(1,1)$能否作直线L，使L被此双曲线所截得的线段被B点平分。

解：设L与双曲线交于$(x_1,y_1),(x_2,y_2)$两点，则

$x_1^2-\frac{y_1^2}{2}=1(1)$ 且$x_2^2-\frac{y_2^2}{2}=1$ (2)

$(1)-(2)$得$(x_1+x_2)(x_1-x_2)-\frac{1}{2}(y_1+y_2)(y_1-y_2)=0(3)$

由中点坐标公式得$x_1+x_2=2$，$y_1+y_2=2$，代入(3)得$\frac{y_2-y_1}{x_2-x_1}=2$

即直线L的斜率为2，所以直线L的方程为$y-1=2(x-1)$，即$y=2x-1$.

3.若$3sin^2\alpha + 2sin^2\beta = 2sin\alpha$,则$sin^2\alpha + sin^2\beta$的取值范围是________.

解：由$sin^2\beta = sin\alpha - \frac{3}{2}sin^2\alpha$得

$sin^2\alpha + sin^2\beta = sin\alpha - \frac{1}{2}sin^2\alpha = -\frac{1}{2}(sin\alpha - 1)^2 + \frac{1}{2}$

$\because |sin\alpha| \le 1 \therefore$ 当$sin\alpha = 1$时，$sin^2\alpha + sin^2\beta$取得最大值$\frac{1}{2}$

当$sin\alpha = -1$时，$sin^2\alpha + sin^2\beta$取得最小值-$\frac{3}{2}$.

4. 解方程$\sqrt{x^2+6x+10} + \sqrt{x^2-6x+10} = 10$

解：原式可化为$\sqrt{(x+3)^2+1} + \sqrt{(x-3)^2+1} = 10$

令$1 = y^2$,得$\sqrt{(x+3)^2+y^2} + \sqrt{(x-3)^2+y^2} = 10(*)$

根据椭圆的定义，此为焦点在$(-3,0),(3,0),2a=10$的椭圆，

即$a=5, c=3$则$b^2 = a^2 - c^2 = 25 - 9 = 16$

则$(*)$式就是$\frac{x^2}{25} + \frac{y^2}{16} = 1$

将$y^2 = 1$代入上式，得$x = \pm\frac{5}{4}\sqrt{15}$.

四. 下面的问题稍有些难度，努力试试看吧！（解答可以写在本页反面）

1.若不等式$2x-1 > m(x^2-1)$对满足$-2 \le m \le 2$的所有m均成立，求x的取值范围。

2. 求以$\sqrt{a^2+b^2}, \sqrt{a^2+4b^2}, \sqrt{4a^2+b^2}$为三边的三角形的面积。

3. 解方程$x^4 - 10x^3 - 2(a-11)x^2 + 2(5a+6)x + 2a + a^2 = 0$.

4.八个人参加某次会议，如果每两个人握一次手，那么共握手多少次？