Staged Finite Element Modeling with Coupled Seepage and Stress Analysis

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Abstract

This paper proposes an approach for staged finite element modeling with coupled seepage and stress analysis. The stage modeling is based on the predefined inter-relationship between the base model and the unit stage models. A unit stage constitutes a complete finite element model, of which the geometries and attributes are subject to changes from stage to stage. The seepage analysis precedes the mechanical stress analysis at every stage. Division of the wet and dry zone and the pore pressures are evaluated from the seepage analysis and used in determining input data for the stress analysis. The results of the stress analysis may also be associated with the pore water pressures. For consolidation analysis, the pore pressure and the displacement variables are mixed in a coupled matrix equation. The time marching solution produces the dissipation of excess pore pressure and variation of stresses with passage of time. For undrained analysis, the excess pore pressures are computed from the stress increment due to loading applied in the unit stage and are used in revising the hydraulic head. The solution results of a unit stage are inherited and accumulated to the subsequent stages through the relationship of the base model and the individual unit stages. Implementation of the proposed approach is outlined on the basis of the core procedures, and numerical examples are presented for demonstration of its application.

Keywords : *finite element method, staged modeling, coupled analysis, seepage analysis*

1. Introduction

Staged finite element modeling is devised for analysis of objects with changing geometry, material properties and load conditions over a given period of time. Such a modeling is widely used in analyzing structures such as dams, tunnels and bridges under construction, and thus is termed "staged construction modeling" in some finite element analysis programs. However, staged modeling is not necessarily related to construction stages, but is also useful in simulating stepwise loading and unloading processes, time dependent material effects, or sequentially changing support conditions. Staged modeling is most widely adopted in analyzing soil mechanical problems which involve a number of factors that distinguish them from other mechanical problems. The interaction of water and soil is the most unique factor that should be considered in soil mechanical analysis. The saturation of soil by pore water affects the unit weight and the elastic properties of the modeling material. The effective stresses are also concerned with the pore water pressures. While soil consolidation can be regarded as a gradual dissipation of excessive pore pressure, the stress increment due to loading increases the pore pressures under the undrained condition. In short, the water within soil becomes a critical factor involved in soil mechanical stress analysis.

The problem becomes more complex when the existence of water is not uniform throughout the whole analysis region. One part of the region may be saturated while the other part may be dry. This situation is sometimes handled simply by assuming a ground water table. In most cases, however, a horizontal plane of the water table cannot properly represent the actual state, and thus it is necessary

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Table 1 Comparison of the proposed approach with others

	Approaches in other programs	Present approach
Stage modeling concept	Each stage is defined by modeling data independent of other stages.	Stages are defined in terms of the base model, and inter-related through data sharing and inheriting.
Data referencing	Data are referenced or recovered by the data identity for each stage.	Data are referenced or recovered by data pointer of the base model.
Control of coupling in solution process through stages	Oriented for text-based scripts. Coupling is not carried over to the next stage.	Oriented for graphical user interface. Coupled analysis results are carried over to subsequent stages automatically.
Coupling of seepage and stress	Either sequential coupling or equation coupling, but not both.	Both sequential coupling and equation coupling are available. Two types of coupling can be combined.
Evaluation of seepage force	Only the hydraulic pressure is considered.	The hydraulic pressure and seepage gradient force are considered.
Modification of hydraulic head due to load increment	Not available.	Increment of hydraulic head is computed by Henkel's formula.

to evaluate a more accurate phreatic surface of the ground water flow through seepage analysis. This is the reason why the seepage analysis need be coupled with stress analysis in modeling soil mechanical problems. There have been many research works related to coupling seepage with displacement and stress analysis (Li, 1983; Lai, 2008; Rice, 2010).

Under the same situation, it is desired to also couple the seepage analysis in the staged modeling of soil mechanical problems. The intention of this paper is to suggest a new procedure of coupling the seepage and stress analysis in staged modeling. The coupled analysis seeks the solution through the inter-relation between the seepage and mechanical behaviors. The excess pore pressures and the displacements can be coupled directly in the system equations, and can be obtained from the solution of the equations. Britto and Gunn (1987) suggested this procedure for analysis of soil consolidation. However, the seepage and displacement variables cannot be coupled directly in equations, but should be inter-related through two independent sets of finite element solutions, i.e., one for seepage analysis and the other for mechanical stress analysis. Thus, coupling the seepage and stress analyses can be achieved by associating the results of one analysis with the input parameters of the other analysis. The coupled analysis may be implicated with different options such as consolidation, undrained analysis and transient analysis. For transient or consolidation analysis, it may be assumed that each stage has a specified time span, and the seepage flow and the phreatic surface are time dependent. For such a case, this paper also suggests the procedure of staged modeling with coupled transient seepage and stress analysis, for which a stage is divided into a number of time steps with preset intervals.

The algorithm of staged modeling with coupled seepage and stress analysis has not been much published in the form of research papers, but implemented in a number of geotechnical finite element programs such as Plaxis (Brinkgrev *et al*., 2008), Crisp (Woods and Rahim, 2007), etc. The major differences between the approaches generally adopted by these programs and the one proposed by this paper are summarized in Table 1.

The concept of a base model and unit stage model is introduced to relate the stages systematically and to simplify the data sharing and inheriting between stages. The results of stress analysis and seepage analysis such as stresses, strains, displacements, pore pressures, hydraulic heads, etc. are accumulated or inherited from one stage to the next through the data structures of the base model. The data for the unit stage model can be referenced efficiently by using the data pointer of the base model.

The method and procedure of the staged modeling with coupled analysis suggested in this paper were implemented as part of a finite element analysis program, VisualFEA (Cook and Lee, 2001).

2. Basic mechanism of staged modeling

The feature of stage modeling is described here on the basis of the implementation in VisualFEA. A stage is the basic unit of the staged analysis. The finite element modeling and solutions are carried out sequentially in terms of stages. A staged model consists of two or more stages. Each stage independently has all the ingredients of finite element analysis. The entire model is designed to simulate the actual situation subject to stepwise changes in geometry, material properties, boundary constraints and load conditions.

2.1 Base model and unit stage model

The finite element analysis model for one stage is termed here the unit stage model. The staged analysis is completed through serially connected solutions of the unit stage models from the first stage to the last. Unit stage models share the common data with each other. The model comprising the whole data shared by all unit stages is termed the base model. Every unit stage model is a subset of the base model. Staged modeling always involves one base model and two or more unit stage models. The unit stage model is constructed using the mechanism of extracting necessary components from the base model. Alternatively, a new stage is defined by modifying the unit stage model descended from the previous stage. New attributes involved in the modification are automatically built up in the base model. Each unit stage model is accessible as a single finite element model for display, inspection and modification.

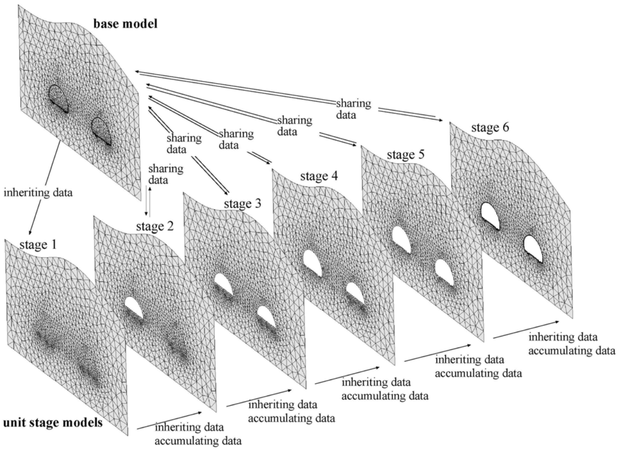

Fig. 1 Relationship between the base model and the unit stage models

The conceptual relationship between the base model and the unit stage model is shown in Fig. 1.

2.2 Data sharing and inheriting

All the unit models share the data of the base model. In other words, the base model comprises all the data included in the stage models. The first unit stage model created initially (i.e., unit model for stage 1) inherits its data from the base model, and is later modified to keep only necessary data by using exclusion, assignment or clearing actions. Likewise, a stage model initially inherits its data from the preceding stage model, and is modified later through various operations.

Creation or modification of any geometric objects such as finite element meshes is applied to the base model. The geometric objects created for the base model are automatically shared by all the currently existing stages. On the other hand, material properties, boundary constraints and load conditions can be assigned either to the base model or directly to individual unit stage models. Any assignment performed at a stage level is applied to the corresponding unit stage model, and is also included in the base model. However, such an assignment is not applied to the other unit stage models. Nevertheless, any assignment to the base model is automatically inherited to all the unit stage models.

The modeling data as well as the solved results of all the stages are saved in a single data file using the data structure of the base model.

2.3 Solution process of stage models

A complete processing of the finite element solver is performed for each of the unit stage models, and is repeated sequentially from the first stage to the last. The solution cycle of each unit stage forms an independent patch of finite element processing isolated from other stages. The coupling of seepage and structural mechanics is also localized within a unit stage model. In the case of time dependent analysis, the time stepping is limited to the time span of a unit stage. However, the modeling data and the solution results for a unit stage model are not separate or independent from other unit stages. The inter-relationship between the unit stages is maintained through data sharing and inheriting in terms of the base model. The unit stage model for a stage is processed in association with its preceding and subsequent stage. The solution results of a stage are reflected in the processing of the next stage. For example, unbalanced forces resulting from excavation at a stage are relaxed over the next few stages with prescribed rates. The history of loading and unloading in preceding stages affects the stress paths of the succeeding stages. The solution results such as displacements, stresses, plastic strains, etc. are accumulated stage by stage.

3. Coupling of seepage flow in staged analysis

The mechanical behavior of soil is influenced by seepage flow due to the interaction between soil and pore water, moist property variation and seepage force. Thus, it is often required to couple the seepage flow with soil mechanical analysis. As in the case of consolidation analysis, the pore water pressure and the displacement can be coupled in one matrix equation and obtained simultaneously from its solution. However, other effects of seepage flow cannot be coupled with mechanical variables at the equation level, but can be involved in mechanical analysis as input factors. Thus, the seepage flow is first analyzed, and the solution outputs are used in determining the input data for mechanical analysis. Reversely, the hydraulic head can also be modified using stresses computed from the mechanical analysis under an undrained condition.

In staged modeling, the seepage analysis and the mechanical stress analysis are performed in sequence for every unit stage model. The coupled analysis results are inherited or accumulated to each of the seepage and mechanical models in the subsequent stages.

3.1 Association of seepage flow in stress analysis

The existence of pore water creates differences in the mechanical behavior of soil, and should be taken into account in soil stress analysis. The seepage flow can be associated with stress analysis as follows.

3.1.1 Assignment of different material properties for saturated and unsaturated zone

The water content of the soil is determined by the existence of seepage flow. For simplicity, it is assumed that soil is fully saturated below the phreatic surface and is dry above the surface. The state of saturation creates the differences in application of material properties such as the unit weight, angle of internal friction, cohesion, etc. In this regard, the seepage analysis is used in classifying the modeling region into the saturated and unsaturated zone by the phreatic surface.

3.1.2 Consolidation

The consolidation of soil results from the gradual reduction of excessive pore pressure with the passage of time in the saturated zone. The pore pressures and displacements are coupled as time dependent variables in the system equations for the saturated zone, and are obtained by solving the equations. The initial state of the pore pressures is determined by using the depth from the phreatic surface of the seepage flow. The loading increases the pore pressures which constitute excess pore pressures.

3.1.3 Pore pressure and effective stress analysis

The effective stress of soil is obtained from the total stress by subtracting the pore pressure. The effective stress increases as the excess pore pressure decreases, while the total stress remains at a constant level.

3.1.4 Application of seepage force

The flow of water through soil mass results in seepage force being exerted on the soil itself (Bierawski and Maeno, 2006). The seepage force per unit volume of soil mass is obtained by

$$f_x = \gamma_w i \tag{1}$$

where i is the hydraulic gradient of seepage flow, and γ_w is the unit weight of water.

3.2 Method of coupling seepage flow and stresses in staged analysis

Coupled solutions of seepage and soil mechanics are obtained for every stage. The coupling of two different fields is isolated within each unit stage model. Each stage starts with the seepage analysis, which produces information on the state of saturation in soil. This information is used in evaluating the input parameters for the stress analysis which follows in the next step. There are two levels of coupling at this step. One level involves coupling by equations in which the variables of two different fields are mixed. The matrix equations are assembled and solved for mixed variables of pore pressure and displacement. The interaction between the two different types of variables is embedded in equations. The equation level coupling is applied in the stress analysis if long term displacement behavior with dissipation of pore pressures i.e., consolidation, is to be evaluated. The other coupling at this step is achieved through estimation of the change in pore pressure due to stress increments. This estimation is valid under the undrained condition. Instant loading or unloading for the unit stage results in a change of pore pressures which can be converted into a change of hydraulic head and eventually into modification of the phreatic surface.

The coupling of seepage analysis and stress analysis may be pursued under different assumptions and conditions. The interaction between the two different fields can be taken into account by the following options of coupling in staged analysis.

3.2.1 Coupling of seepage analysis

The most important aspect of seepage coupling is the implication of the soil moisture with mechanical behavior. Therefore, the seepage analysis can be omitted if the phreatic surface is already available or can be assumed arbitrarily. The state of soil saturation can be implicated in the staged modeling either with or without seepage analysis. The following three options can be considered in associating the pore pressure with mechanical analysis.

- Assumption of ground water table: The phreatic surface is assumed to be flat and horizontal, and a certain level of ground water table is assumed without seepage flow analysis. The initial pore pressure and the material properties are determined using the level of water table set for each stage. The seepage forces due to hydraulic gradient are considered to be zero in this case.
- Predetermined phreatic surface: The seepage surface may be imported from an external source or prescribed as input data in the case where they are predetermined by independent analysis.
- Coupled seepage analysis: The phreatic surface and the pore pressures are determined through seepage analysis. The solved results are reflected in determining the input data for mechanical analysis.

3.2.2 Time dependence of seepage flow

Whether to apply steady state or transient analysis is determined by the variation of seepage boundary conditions with the passage of time within a unit stage.

- Steady state analysis: It is assumed that the seepage boundary conditions remain constant throughout the duration of the unit stage. The solution of a unit stage model is obtained by one cycle of seepage analysis and stress analysis.

- Transient analysis: It is assumed that the seepage boundary conditions change significantly with the passage of time throughout the time span of the unit stage. A unit stage is divided into a number of time steps. The stepping through time proceeds with alternating seepage analysis and stress analysis at every step. For each step, the seepage equations are first solved, and the displacement equations are assembled and solved with application of the seepage analysis results.

3.2.3 Evaluation of pore pressure due to loading

The pore pressures are also influenced by the stresses due to loading. The change of pore pressures due to stresses can be estimated for the following two situations.

- Consolidation: The dissipation of pore pressure is evaluated using the coupled matrix equations with a vector of mixed pore pressure and displacement variables. The solution is marched forward in time along the time steps predefined for the unit stage. The consolidation process is obtained as a function of time.
- Undrained condition: The loading increases the pore pressure under the undrained condition. The excess pore pressure associated with the loading condition can be predicted by applying the Henkel's pore pressure parameters (Henkel, 1960), and converting them into an increment of the hydraulic head.

The solution results of a unit stage model are inherited or accumulated in the subsequent stages. The next stage starts from the state of seepage and mechanical stress evaluated in the preceding stage.

4. Implementation of staged analysis with coupling of seepage and stresses

Processing of the staged analysis with coupling of seepage and stresses has been implemented as a specialized feature of Visual FEA. The user-interface for handling of the staged model takes large size of program codes, but is not described here. Only the computational aspects directly related to solution processing are detailed in this section.

4.1 Overall procedures

The solution of the staged analysis proceeds automatically without requiring any user interaction following the procedures set for the chosen options. The staged analysis is accomplished chiefly by employing the following four core procedures alternately.

- Procedure A: Seepage analysis
- Procedure B: Mechanical stress analysis
- Procedure C: Coupled equation analysis
- Procedure D: Processing of a unit stage model

Each of the procedures A, B and C performs a complete finite element processing from the assembly of element equations to the solution of the assembled equations and hydraulic head or stress evaluation. The paths for steady state analysis and transient analysis are branched within procedure A. Coupling of seepage and mechanical behavior is realized through the sequential processing of procedures A and B. The variables of water pore pressures and displacements are mixed in a coupled matrix equation of procedure C. In this case, the pore pressures are evaluated either using the assumed water table or using the phreatic surface from the seepage analysis. The time factor is already included in the coupled equation in procedure C for which the transient analysis option is not applicable in procedure A. Procedure D carries out a unit stage analysis by repeated use of procedure A and either procedure B or C as scheduled for the option of coupling seepage flow. The entire staged analysis is accomplished by proceeding with unit stage analyses using procedure D from stage 1 to the last stage. There are many other computational procedures involved in the staged modeling with coupled analysis, e.g., computation of in-situ stresses, stepwise relaxation of unbalanced forces, etc., which are omitted from the detailed description.

The overall procedures are represented by the

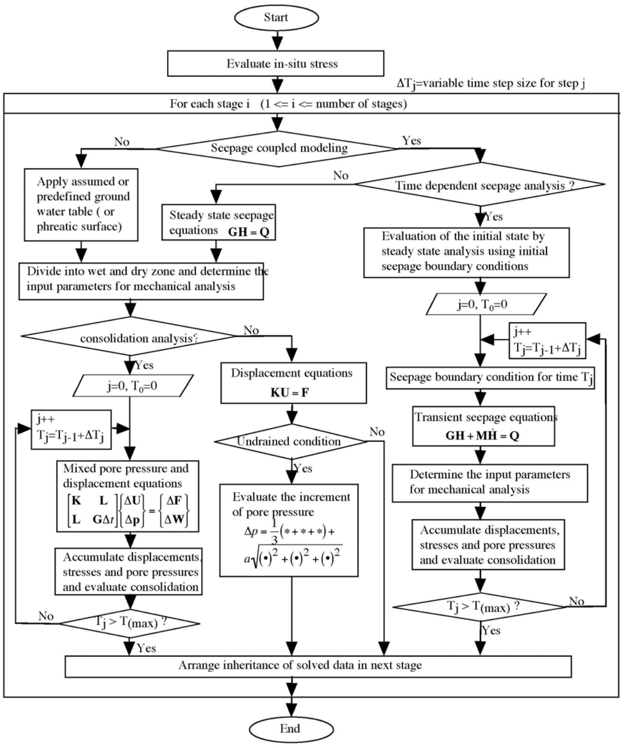

Fig. 2 Flow of staged analysis with coupling of seepage and mechanical behavior

schematic flow chart in Fig. 2.

4.2 Seepage analysis

The finite element equation for steady state seepage analysis can be written as

$$\mathbf{GH}=\mathbf{Q} \tag{2}$$

where **H** is a nodal head vector, and **Q** is the applied flux vector. The conductance matrix **G** is obtained by the equation,

$$\mathbf{G}=\int_V \mathbf{S}^T\mathbf{kS}dV \tag{3}$$

where **S** is a matrix relating the gradient of the hydraulic head to the nodal values of the heads. The terms of the matrix **S** are obtained by differentiating the shape functions $\overline{\mathbf{N}}$ which is used in interpolating the hydraulic head h from the element nodal values $\mathbf{H}^e$.

$$h=\overline{\mathbf{N}}\mathbf{H}^e \tag{4}$$

In Equation (4), **k** is the permeability matrix,

$$\mathbf{k}=\begin{bmatrix}k_x & 0\\ 0 & k_y\end{bmatrix} \tag{5}$$

for plane seepage models, and

$$\mathbf{k}=\begin{bmatrix}k_x & 0 & 0\\ 0 & k_y & 0\\ 0 & 0 & k_z\end{bmatrix} \tag{6}$$

for 3-dimensional seepage models. The values k_x, k_y and k_z are the permeability coefficients in x, y and z directions, respectively.

The equation for a transient seepage analysis can be written in a general form as follows,

$$\mathbf{GH}+\mathbf{M}\dot{\mathbf{H}}=\mathbf{Q} \tag{7}$$

$\dot{\mathbf{H}}$ is a vector representing the change in head with time, and **M** is the storage matrix computed by

$$\mathbf{M}=\int_V \frac{S}{T}\overline{\mathbf{N}}^T\overline{\mathbf{N}}dV \tag{8}$$

where S is the storage coefficient and T is the transmissivity.

4.3 Mechanical stress analysis

Stress analysis is based on the finite element procedure for solid mechanics.

The force and displacement relationship can be expressed simply by the finite element equations,

$$\mathbf{KU}=\mathbf{F} \tag{9}$$

where **U** and **F** are respectively the nodal displacement vector and the force vector, and

$$\mathbf{K}=\int_V \mathbf{B}^T\mathbf{DB}dV \tag{10}$$

is the stiffness matrix. Strains **ε** and the nodal displacements $\mathbf{U}^e$ are related within an element by

$$\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{U}^e \tag{11}$$

where matrix **B** is computed from derivatives of the shape function **N** by which the displacements *u* are assumed to vary over the element.

$$u = \mathbf{N}\mathbf{U}^e \tag{12}$$

The matrix **D** relates the strains $\boldsymbol{\varepsilon}$ to the stresses $\boldsymbol{\sigma}$.

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon} \tag{13}$$

4.4 Modification of pore pressure under undrained condition

The increment of pore pressure due to increment of stresses can be estimated by Equation (14)

$$\Delta p = \frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) + a\sqrt{(\Delta\sigma_1 - \Delta\sigma_2)^2 + (\Delta\sigma_2 - \Delta\sigma_3)^2 + (\Delta\sigma_3 - \Delta\sigma_1)^2} \tag{14}$$

In the equation, Δp is the increment of pore pressure, and $\Delta\sigma_1$, $\Delta\sigma_2$ and $\Delta\sigma_3$ represent the increment of principal stresses. The coefficient *a* is Henkel's pore pressure parameter (Henkel, 1960) which is obtained from

$$a = \frac{1}{\sqrt{2}}\left(A - \frac{1}{3}\right) \tag{15}$$

The value of *A* can be determined from standard tri-axial tests. The pore pressure increment can be regarded either as excess pore pressure or as an increment of the hydraulic head (Das, 1997).

4.5 Consolidation analysis using coupled equations

The excess pore pressures are assumed to vary over a finite element according to

$$e = \overline{\mathbf{N}}\mathbf{e}^e \tag{16}$$

The shape function $\overline{\mathbf{N}}$ for the excess pore pressures is not necessarily identical to the shape function **N** for displacements. Britto and Gunn (1987) suggested the stiffness equations in (17) with coupled displacement and pore pressure variables for which the shape function $\overline{\mathbf{N}}$ is one order less than **N**. For example, if **N** is quadratic, $\overline{\mathbf{N}}$ is linear.

$$\begin{bmatrix} \mathbf{K} & \mathbf{L} \\ \mathbf{L} & \mathbf{G}\Delta t \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{U} \\ \Delta\mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \Delta\mathbf{F} \\ \Delta\mathbf{W} \end{Bmatrix} \tag{17}$$

Here, $\Delta\mathbf{U}$ and $\Delta\mathbf{p}$ represent the increment of displacements and increment of excess pore pressure respectively. On the right hand side, $\Delta\mathbf{F}$ and $\Delta\mathbf{W}$ are the incremental load vector and the incremental seepage force due to the seepage gradient on the boundary, respectively. The matrix L is computed by

$$\mathbf{L} = \int_V \mathbf{B}^T \mathbf{m} \overline{\mathbf{N}} dV \tag{18}$$

where **m** is a vector for addition of the excess pore pressure *p* to the stress vector.

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + \mathbf{m}p \tag{19}$$

For plane strain models, the relationship between the total stress vector $\boldsymbol{\sigma}$ and the effective stress vector $\boldsymbol{\sigma}'$ can be written using the vector, $\mathbf{m}^T = \lfloor 1 \quad 1 \quad 0 \rfloor$. That is,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} \sigma'_x \\ \sigma'_y \\ \tau'_{xy} \end{Bmatrix} + \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} p \tag{20}$$

For 3D solid models, the vector can be extended to $\mathbf{m}^T = \lfloor 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \rfloor$ for the stress vector

$$\boldsymbol{\sigma}^T = \lfloor \sigma_x \quad \sigma_y \quad \sigma_z \quad \tau_{xy} \quad \tau_{yz} \quad \tau_{zx} \rfloor \tag{21}$$

The solution of Equation (17) is obtained by stepping through in time with predefined time intervals.

4.6 Recording of solved data

The analysis results are obtained and saved for every stage. The data records are arranged in terms of the data structure in the base model. For example, the computed stresses are rearranged following the order of elements and nodes in the base model. The data for any unit stage model can be accessed simply by referencing the pointers in the base model. The solution output data are written in a single data file using the data structure of the base model.

The data for a unit stage model can also be positioned using the file pointer recorded in the file header. Such a data file structure is intended to maintain the consistent data indexing within each stage throughout the entire staged modeling, and to facilitate systematic data recovery and inspection. The solved data from seepage analysis and the mechanical analysis are recorded in sequence for every stage. If transient analysis is applied, the seepage and mechanical analysis data may be saved either for all time steps or for the last single step as desired.

5. Examples of application

Two examples are presented here to demonstrate the application of the staged modeling with coupled analysis suggested in this paper. They are given in different spaces, i.e., one in two dimensional and the other in three dimensional space. In each model, only 4 stages are included for simplicity, although more stages are desired in practical engineering analysis. The modeling data applied here may not represent realistic values, because the intension of the examples is to emphasize the applicability of the proposed approach rather than to analyze the numerical results.

5.1 Braced excavation model

The first example is a plane strain modeling of braced excavation problem. Nogueria *et al.* (2009) suggested procedures of coupled excavation analysis. Kishnani and Boja (1993) studied the soil-structure interaction in braced interaction through seepage coupled finite element analysis. The modeling in this example is based on the method similar to those in the previous studies. However, this example can be distinguished from others by its inclusion of staged modeling concept.

Only half of the problem region is modeled due to its symmetry. The finite element mesh is shown with its modeling dimensions in Fig. 3. It is assumed that steel sheet piles are inserted into the ground prior to the excavation and three horizontal struts are installed to support the piles when excavation reaches the corresponding depth. The sheet piles are modeled by frame elements, and the struts are modeled by truss elements. The truss elements are turned on at the stages of their installation. Interface elements are adopted in simulating the slip behavior between the sheet piles and the surrounding ground. For the seepage analysis, the frame and truss elements are ignored, and the interface elements are used as impermeable elements cutting off the seepage flow. An interface element is automatically turned off at the stages the interfacing plane strain element is excluded. Gravitational body forces are applied to the stress analysis model. The initial water table is assumed to be 1m below the ground surface. The material properties are assigned to the model as follows.

- Modulus of elasticity: 7×10^4kN/m^2
- Poison's ratio: 0.3
- Cohesion: 10kN/m^2(dry), 15kN/m^2(wet)
- Angle of internal friction: 10°(dry), 15°(wet)
- Unit weight: 18kN/m^3(dry), 22kN/m^3(wet)
- Permeability: 0.01m/sec
- Axial rigidity of strut: 3×10^4kN/m
- Axial rigidity of sheet pile: 5×10^5kN/m
- Flexural rigidity of sheet pile: 6×10^4kN-m^2/m
- Yield criterion: Mohr-coulomb for the ground and linear elastic for others

The solution of the coupled stage model produces various data related to seepage and mechanical behavior. The pore pressure and the stresses in

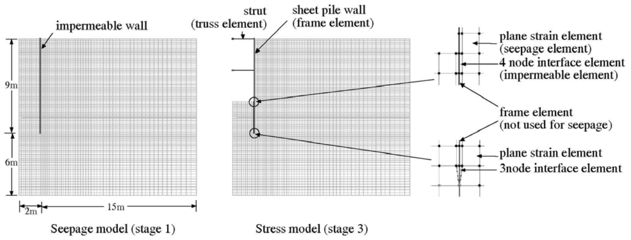

Fig. 3 Finite element meshes for the excavation model

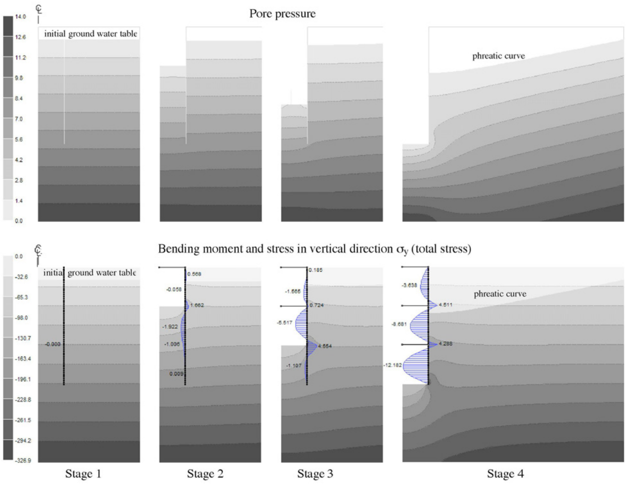

Fig. 4 Analysis results of the excavation model

vertical direction are represented by contours, and the bending moments induced in the sheet pile are represented as bending moment diagrams in Fig. 4. The image of the phreatic surface is overlaid over the contour images of the pore pressures and the stresses.

5.2 Tunneling model

The second example is a three dimensional solid modeling of underground tunnel. Yoo (2005) investigated the interaction between tunneling and groundwater using stress-pore pressure coupled analysis of a tunnel model similar to the one in this example. However, the tunneling analysis in this example is based on the staged modeling, differently from the examples in the reference.

Only half of the problem region is modeled due to its symmetry about the center line of the tunnel. The horseshoe shaped tunnel is 10m wide and constructed 26m below the ground surface. The grouting zone is constructed to surround the tunnel section as shown in Fig. 5, and is included in the model at the stage of inside tunneling by replacing the ground properties of the region with the grouting material properties. The ground water table is placed initially at the ground surface, and is assumed to be under a drawdown condition without recharge afterward. The pore pressure is assumed to be zero on the surface of the tunnel.

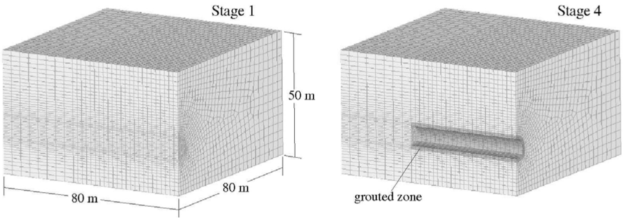

Fig. 5 Finite element meshes for the tunneling model

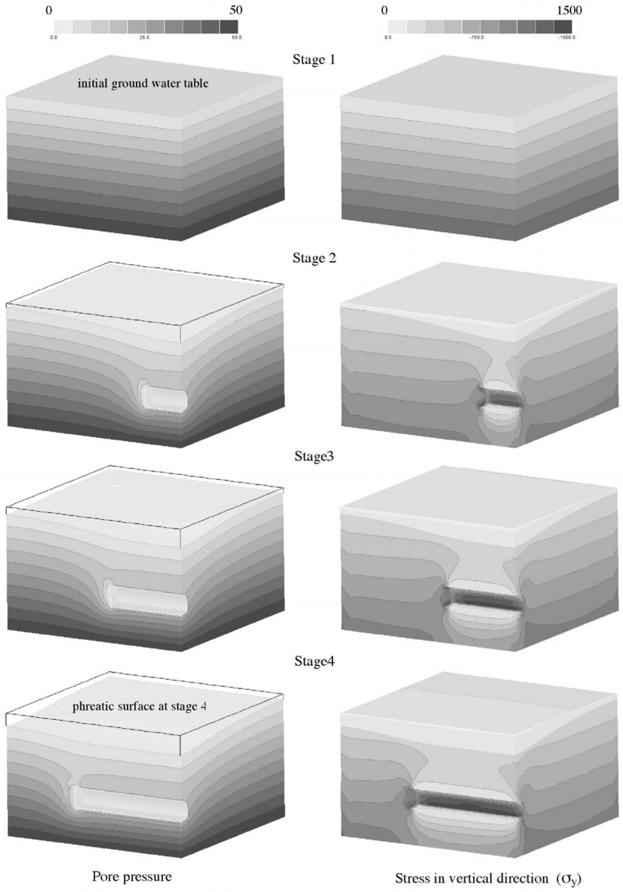

Fig. 6 Analysis results of the tunneling model

The following material properties are assigned to the model.

- Modulus of elasticity of the surrounding ground: 2×10^5kN/m^2
- Modulus of elasticity of the grouted zone:

3×10^5kN/m^2

- Poison's ratio: 0.3
- Cohesion of surrounding ground: 50kN/m^2(dry), 60kN/m^2(wet)
- Angle of internal friction: 30°(dry), 40°(wet)
- Unit weight: 22kN/m^3(dry), 27kN/m^3(wet)
- Yield criterion: Mohr-coulomb for the ground and linear elastic model for grouted zone
- Permeability of the surrounding ground: 10^{-5}m/sec
- Permeability of the grouted zone: 10^{-6}m/sec

The solution of the coupled stage model produces various information about seepage and mechanical behavior at all stages. The pore pressure distribution and the stress in vertical direction are graphically visualized in Fig. 6.

The pore pressure image shows the drawdown of the phreatic surface which is also overlaid over the stress contour image.

6. Conclusions

Staged modeling is an effective means of analyzing the soil mechanical models subject to stepwise changes of geometries and attributes. The state of saturation and the pore water pressures are important factors affecting the mechanical behavior of soil. Accordingly, the staged modeling with coupling of seepage and mechanical stress analysis is justifiable. The intention of this paper is to propose a finite element analysis procedure with new systematic and logical modeling concepts as a standard of staged modeling with coupled analysis. The suggested concept of a base model and unit stage model simplifies and systematizes the processing and data handling of staged modeling. The association of the seepage and stress analyses can be achieved efficiently through conceptual isolation of every unit stage model. The interaction of pore pressure and mechanical behavior can be implicated bilaterally between the seepage analysis and the mechanical analysis within a unit stage model. The inter-relationship between stages is maintained through data sharing, inheritance and accumulation. The modeling data and solution results of unit stage models can be accessed efficiently by referencing the base model.

The proposed procedure of staged modeling has been implemented in a finite element analysis program, VisualFEA.

The implementation could be achieved efficiently by repeated use of the core procedures. The validity and usefulness of the method have been proved through many applications of the program to actual design and analysis in geotechnical engineering. The examples demonstrate the applicability of the proposed method to practical problems frequently encountered in soil mechanics and geotechnical engineering.

References

Li, G.C., Desai, C.S. (1983) Stress and Seepage Analysis of Earth Dam, *J. Geotechnical and Geoenvironmental Engineering*, 109(7), pp.946 ~960.

Britto,A.M., Gunn, M.J. (1987) *Critical State Soil Mechanics via Finite Elements*, Ellis Horwood, Chichester, p.115.

Brinkgreve, R.B.J., Broere, W., Waterman, D. (2008), *Plaxis 2D-Version 9.0*, Plaxis bv.

Woods, R., Rahim, A. (2007), CRISP2D, Geotechnical Finite Element Analysis Software,, Crisp Consortium Ltd. p.223.

Cook, R.D., Lee, J.Y. (2001) *VisualFEA and General User Manual*, John Wiley & Son's, New York, p. 521.

Lai, D., Liang, R. (2008) Coupled Creep and Seepage Model for Hybrid Media, *J. Engineering Mechanics*, 134(3), pp.217~223.

Rice, J.D., Duncan, J.M. (2010) Deformation and Cracking of Seepage Barriers in Dams Due to Changes in the Pore Pressure Regime, *J. Geotechnical and Geoenvironmental Engineering*, 136(1), pp.16~25.

Bierawski, L.G., Maeno, S. (2006) DEM-FEM Model of Highly Saturated Soil Motion Due to Seepage Force, *J. Waterway, Port, Coastal, and Ocean Engineering*, 132(5), pp.401~409.

Henkel, D.J. (1960) The Shear Strength of Saturated Remolded Clays, *Proceeding of Research Conference in Shear Strength of Cohesive Soils*, ASCE, pp.533

~554.

Das, B.M. (1997) *Advanced Soil Mechanics*,Taylor & Francis, New York, p.133.

Nogueria, C.L., Azevedo, R.F., Zornberg, J.G. (2009) Coupled Analysis of Excavations in Saturated Soil, *I.J. Geomechanics*, 9(2), pp.73~81.

Kishnani, S.S., Boja, R. (1993) Seepage and Soil-Structure Interactions Effects in Braced Excavation, *J. Geotechnical Engineering*, 119(5), pp.912~928.

Yoo,C (2005) Interaction Between Tunneling and Groundwater-Numerical Investigation using Three Dimensional Stress-Pore Pressure Coupled Analysis, *J. Geotechnical and Geoenvironmental Engineering*, 131(2), pp.240~250.

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