The Mixed Finite Element Analysis for Saturated Porous Media using FETI Method

Lee, Kyungjae* Tak, Moonho** Park, Taehyo†

Abstract

In this paper, FETI(Finite Element Tearing and Interconnecting) method is introduced in order to improve numerical efficiency of Staggered method. The porous media theory, the Staggered method and the FETI method are briefly introduced in this paper. In addition, we account for the MPI(Message Passing Interface) library for parallel analysis, and the proposed combined Staggered method with FETI method. Finally Lagrange multipliers and CG(Conjugate Gradient) algorithm to solve decomposed domain are proposed, and then the proposed method is verified to be numerically efficient by MPI library.

Keywords : *porous media, mixed finite element method, domain decomposition, FETI, MPI*

1. Introduction

The saturated porous media are composed of solid and fluid phases. Solid and fluid have different properties, so it is difficult to analyze by the continuum theory therefore, it is necessary to use porous theory considering solid mechanics and fluid flow theory. Theoretical formulation for the porous media was introduced by Biot in 1941. Biot proposed three dimensional governing equations by Poroelasticity that applied continuum mechanics to Soil

Consolidation Theory by Terzaghi (1925). In order to perform numerical analysis model using Biot's governing equations to be considered behavior of solid and fluid, the finite element method using hybrid element has been studied by many researchers. The mixed formulations based upon a variational theorem were firstly introduced by Hermann (1967) to solve a plate bending. In addition, the mixed formulation was extended by Ghaboussi and Wilson (1973), Zienkiewicz *et al.* (1980), Borja (1986), Voyiadjis and Abu-Farsakh (1997), Park *et al.* (2005a, 2005b, 2005c) in the soil scope for the instantaneous or consolidation analysis. The governing equations are represented into a mixed form in terms of displacement and pore pressure formulation. However, the mixed finite element analysis cannot be solved easily by direct or monolithic solution because the continuity requirements for the shape functions of each constituent are different. Especially, if solid and fluid are nearly incompressible and the fluid is nearly impermeable, then solving for the equations becomes a difficult task due to element locking. Indeed, finite element should be satisfied Baduska-Brezzi Condition (Baduska, 1973; Zienkiewicz, *et al.*, 2000) or pass to the patch test in which different high order interpolation is required for an element. In order to solve this problem, staggered method is proposed by Park and Tak (2010). The nearly incompressible and impermeable saturated porous media using equal order element and staggered method. This result was much close to one dimensional and two dimensional consolidation theory of value. But Numerical efficiency is reduced because staggered method performs iteration more than one

† 책임저자, 종신회원・한양대학교 토목공학과 교수
Tel: 02-2220-0321 ; Fax: 02-2220-4322
E-mail: cepark@hanyang.ac.kr
* 학생회원・한양대학교 토목공학과 석사과정
** 학생회원・한양대학교 토목공학과 박사과정

• 이 논문에 대한 토론을 2011년 2월 28일까지 본 학회에 보내주시면 2011년 4월호에 그 결과를 게재하겠습니다.

time. Thus, we in order to improve the numerical efficiency, propose the method through graft FETI method onto staggered method for saturated porous media.

The FETI method is an effective domain decomposition method for parallel analysis that was introduced by C. Farhat (1991). FETI method requires fewer interprocessor communications than traditional domain decomposition algorithms, while it still offers the same amount of parallelism. Typically, the spatial domain is decomposed onto a set of subdomains and each of these is assigned to an individual processor. Moreover, subdomains analyses are progressed by direct and iterative method. Continuities of subdomains are defined by Lagrange multiplier of among the subdomains. Here is how to obtain Lagrange multiplier using PCPG (Preconditioned Conjugate Projected Gradient) algorithm. Lee *et al.* (2010) proposed porous media analysis using FETI on solid phase. But this method is applied to solid phase only. For such a reason, as more increasing number of subdomain running time did not change because pore pressure computation time. In this paper, we present through combine staggered method with FETI method implemented analysis of porous media and then, the proposed method is verified numerical efficiency by point to point MPI library.

2. Governing Equation

Governing equation for porous media was derived for the first time by Biot in 1941. We present the mass balance equation for fluid flow and the momentum balance equation for displacement and assume that fluid flow and solid are in linear elastic field behavior in macroscopic.

2.1 Governing Equation for Porous Media and Finite Element Formulation

Generally, the governing equation for porous media is derived from the momentum balance equation for displacement and the mass balance equation for pore pressure. The mass balance and momentum balance equations are represented by continuum mechanics theory, as follows;

$$\left(\frac{\alpha-n}{K^s}+\frac{n}{K^f}\right)\frac{\partial p^f(x,t)}{\partial t}+\alpha\nabla\cdot\mathbf{v}^s(x,t)+\frac{1}{\rho^f(x,t)}\nabla\cdot\left(\frac{\mathbf{k}(x,t)}{\mu^f}\left(-\nabla p^f(x,t)+\rho^f(x,t)\mathbf{g}(t)\right)\right)=0 \quad (1)$$

where, superscript s and f is the solid phase and fluid phase respectively, α is the Biot's constant, n is the porosity, t is time, K^s is the bulk modulus of solid phase, K^f is the bulk modulus of fluid phase, $p^f(x,t)$ is the pressure of fluid phase, $\mathbf{v}^s(x,t)$ is the velocity vector of solid phase, $\rho^f(x,t)$ is the density of the fluid particle, $\mathbf{k}(x,t)$ is the permeability tensor, μ^f is the dynamic viscosity for fluid, and $\mathbf{g}(t)$ is the gravity vector.

The momentum balance equation for porous media is presented in a total phase including solid and fluid phases. If saturated porous media is assumed to be continuum, equilibrium equation of continuum is represented as follows;

$$\nabla\cdot\left(\boldsymbol{\sigma}''(x,t)+\mathbf{I}p^f(x,t)\right)+\rho(x,t)\mathbf{b}(x,t)=0 \quad (2)$$

where, $\boldsymbol{\sigma}''(x,t)$ is the Cauchy stress tensor, is the identity tensor, is the density of total phase and is the body force vector.

Mass balance equation for porous media is represented fluid flow in the porous media using Darcy law and material time derivative. Momentum balance equation is expressed as displacement in the solid phase by the relative equation of Cauchy stress tensor and pore pressure represented deformation of porous media through constitutive equation. For the finite element analysis, initial condition and boundary condition for displacement and fluid flow are applied to the mass balance equation and the momentum balance equation, respectively. Then transformation weak form is represented as follows;

$$\mathbf{C}_2\frac{\partial\tilde{\boldsymbol{u}}^s}{\partial t}+\mathbf{S}\frac{\partial\tilde{\boldsymbol{p}}^f}{\partial t}+\boldsymbol{D}\tilde{\boldsymbol{p}}^f=\mathbf{f}^f \quad (3)$$

$$\mathrm{K}_t\tilde{u}^s - \mathrm{C}_1\tilde{p}^f = \mathrm{f}^s \tag{4}$$

where, is stiffness matrix, is drainable matrix, is compressible matrix and is coupled matrix. Then, above equations are applied to the Backward Difference Method for time analysis. The displacement and pore pressure are given by;

$$\mathrm{u}^s_{n+1} = \mathrm{K}^{-1}_{t,n+1}\{\mathrm{f}^s_{n+1} + \mathrm{C}_{1,n+1}\mathrm{p}^f_{n+1}\} \tag{5}$$

$$\mathrm{p}^f_{n+1} = \left[\mathrm{D}_{n+1} + \frac{1}{\triangle t_{2,n+1}}\mathrm{S}_{n+1}\right]^{-1}[\mathrm{f}^f_{n+1} + \frac{1}{\triangle t_{2,n+1}}\mathrm{S}_{n+1}\mathrm{p}^f_n + \frac{1}{\triangle t_{1,n+1}}(\mathrm{C}_{2,n+1}(\mathrm{u}^s_n - \mathrm{u}^s_{n+1}))] \tag{6}$$

2.2 Staggered Method for Porous media

Saturated porous media analysis is considered to be of different properties of solid and fluid phases. Thus, in order to analyze the equation (3) and (4) should be used Direct Method or Iterative Method. But the shape function for continuity problems of solid and fluid should be resolved in the coupling matrices. To solve the above problems, order of element has to be satisfied with the Babuska-Brezzi conditions (1973, 1974) or to pass the patch test.

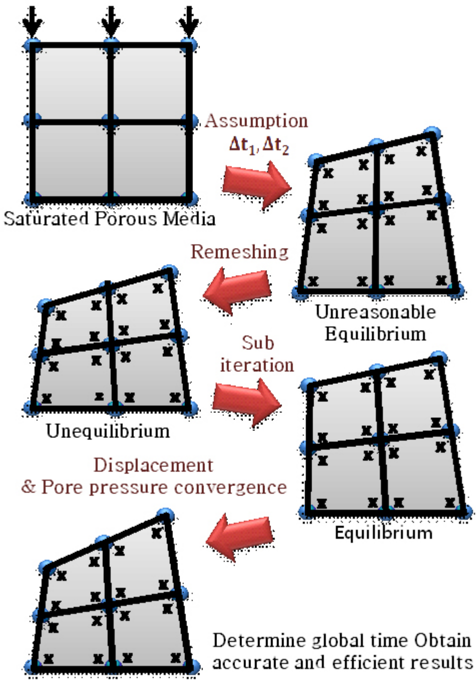


Fig. 1 Procedure of Staggered method (Park and Tak, 2010)

It is difficult to secure convergences in state variables such as displacements and pore pressures for the transient analysis of porous media that is nearly incompressible and fluid flow that is very slow and nearly impermeable. Namely, when the compressible matrix and the drainable matrix have very small values, a standard implicit process using the Gaussian elimination procedure cannot be solved by the simple time step analysis. Therefore, partitioned analyses such as explicit-explicit and explicit-implicit are used. However, the restrictions on the determination of the time step are quite severe. In order to remedy these problems, Park and Tak (2010) proposed Staggered method with multi time step method, remeshing scheme and sub-iteration (Fig. 1). At the multi time step and the sub-iteration step, compatibility and the convergence problems are solved with equal order shape functions. Namely, multi time step, remeshing and sub-iteration is applied to Eq. (6) that is expressed as follows;

$$\begin{aligned}&\tilde{\mathrm{p}}^f_{n+(i+2)}(t_2) = \\ &[\mathrm{D}_{\mathrm{n+(i+2)}} + \frac{1}{\triangle t_{2,n+(i+2)}}\mathrm{S}_{\mathrm{n+(i+2)}}]^{-1} \\ &[\mathrm{f}^{\mathrm{f}}_{\mathrm{n+(i+2)}} + \frac{1}{\triangle t_{1,n+(i+2)}}(\mathrm{C}_{\mathrm{2,n+(i+2)}}(\tilde{\mathrm{u}}^s_n(t_1) \\ &- \tilde{\mathrm{u}}^s_{n+(i+1)}(t_1) + \tilde{\mathrm{u}}^s_n(t_1) - \tilde{\mathrm{u}}^s_{n+(i+2)}(t_1))) \\ &+ \frac{1}{\triangle t_{2,n+(i+2)}}\mathrm{S}_{\mathrm{n+(i+2)}}\tilde{\mathrm{p}}^f_{n+(i+1)}(t_2)]\end{aligned} \tag{7}$$

where, subscript is defined iteration number.

3. FETI (Finite Element Tearing and Interconnecting) Method

FETI Method is proposed by C. Farhat (1991) that is domain decomposition method for a parallel finite element computational method for the solution of static problems. FETI method requires fewer interprocessor communications than traditional domain decomposition algorithms, while it still offers the same amount of parallelism. Traditional domain decomposition is realized directly about continuity

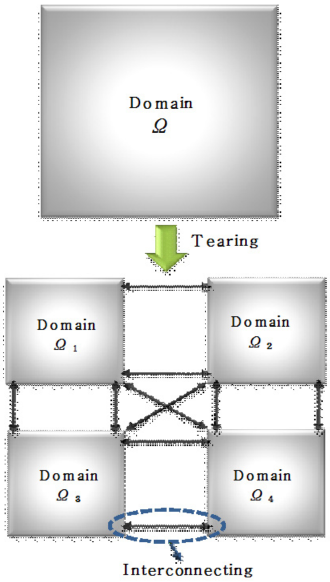


Fig. 2 Finite Element Tearing and Interconnecting

in subdomain boundary. But FETI is solved using Lagrange multiplier.

3.1 Review of the FETI Method

The starting point in the derivation of the FETI method is the subdomain energy ;

$$J^s = \mathrm{u}^{s^T}\mathrm{f}^s - \frac{1}{2}\mathrm{u}^{s^T}u^s \tag{8}$$

where, superscript s denotes the number of subdomain, superscript T is transpose, and are the displacement and force vectors of subdomain and is stiffness matrix.

In order for the subdomain displacement to yield the required global displacement, must satisfy the interface condition;

$$\sum_{s=1}^{N_s}\boldsymbol{B}^s\boldsymbol{u}^s = 0 \tag{9}$$

where, connectivity matrix, the signed matrix with entries -1, 0, 1 describing the subdomain interconnectivity. The energy expression for the global domain is just the sum of subdomain level energy with the constraint condition Eq. (9) augment via Lagrange multiplier as shown below;

$$J_{total} = \sum_{s=1}^{n_s} J^s - \lambda_b^s \sum_{s=1}^{n_s}\boldsymbol{B}^s\boldsymbol{u}^s \tag{10}$$

It is observed that the constraint equation (9) produces no work. The stationary of (10) yield the following subdomain level governing equation:

$$\boldsymbol{K}^s\boldsymbol{u}^s = \boldsymbol{f}^s - \boldsymbol{B}^{s^T}\lambda, \qquad (s=1,......,n) \tag{11}$$

$$\sum_{s=1}^{N_s}\boldsymbol{B}^s\boldsymbol{u}^s = 0 \tag{12}$$

Observe that, Eq. (11) is strictly local for each subdomain provided is known, the constraint equation (12) extends over several subdomains that are connected. Nevertheless, the subdomain displacement vector is of local nature.

The differentially partitioned FETI method begins with the solution of from Eq. (11)

$$u^s = K^{s^+}(f^s - B^{s^T}\lambda) + \boldsymbol{R}^s\alpha^s \tag{13}$$

where, is an inverse of and is the null space matrix. Null space matrix is satisfying as below;

$$\mathrm{R}^{s^T}\boldsymbol{K}^s\boldsymbol{u}^s = 0 \tag{14}$$

is the complementary displacement vector that accounts for the rigid body motions for floating subdomain. Thus, the solution of by the FETI method is reduced to the solution of the interface force and the complementary displacement vector.

In order to obtain the appropriate equation for the and, substitute Eq. (11) into Eq. (14) and Eq. (13) into (12) to arrive at

$$\begin{bmatrix} F & -G \\ -G^T & 0 \end{bmatrix}\begin{bmatrix}\lambda \\ \alpha\end{bmatrix} = \begin{bmatrix} d \\ -e\end{bmatrix} \tag{15}$$

where,

$$F = \sum_{s=1}^{n_s}\boldsymbol{B}^s\boldsymbol{K}^{-1}\boldsymbol{B}^{s^T}$$

$$G = \left[\boldsymbol{B}^1\boldsymbol{R}^1.....\boldsymbol{B}^{n_f}\boldsymbol{R}^{n_f}\right]$$

$$a = \left[a^{1^T} \ldots\ldots a^{n_f^T}\right]$$

$$d = \sum_{s=1}^{n_s} B^s K^{s^{-1}} f^s$$

$$e = \left[f^{1^T} R^1 \ldots\ldots f^{n_f^T} R^{n_f}\right]^T$$

In order to solve the above equation, PCPG (Preconditioned Conjugate Projected Gradient) algorithm is used. PCPG algorithm has been extended from the CG (Conjugate Gradient) algorithm. Precondition and Project is applied to CG algorithm for fast convergence that is PCPG algorithm. PCPG algorithm performed by iteration is applied to projector P. P is denoted as follow;

$$P = I - G(G^T G)^{-1} G^T \tag{16}$$

Detail PCPG algorithm for obtaining Lagrange multiplier is progressed as below;

4. FETI method within Staggered method

In order to improve for the numerical efficiency of Staggered method, Lee *el al.* (2010) proposed porous media analysis using domain decomposition method. However the proposed method is applied into only solid phase consequently, as the number of subdomains increased total running time did not change because running time in fluid phase (pore pressure) is nearly constant. Thus, in this paper, we proposed Staggered method using FETI method applied to both solid and fluid phase.

Staggered method for the porous media is solved with pore pressure after obtained the displacement. Displacement running time is longer than pore pressure running time. Thus, in order to improve numerical efficiency, FETI method applied to calculation of the displacement is proposed by Lee *el al.* (2010). But, proposed method is applied to only solid phase. Consequently, the total analysis time did not change as increasing number of the subdomain due to pore pressure analysis time. In this paper, after displacement for subdomain was calculated on many CPU, analysis of pore pressure is applied to domain decomposition and calculated on multiple CPU for obtain numerical efficiency. FETI method is applied to Staggered method, vector d and e is represented in the FETI interface problem as below;

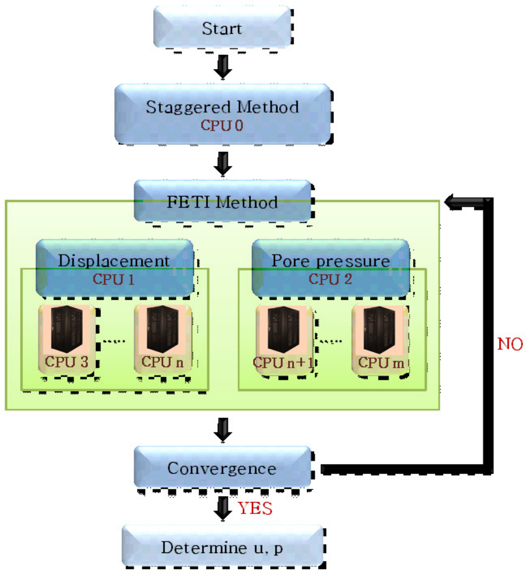


Fig. 3 FETI method applied into Staggered method

$$d = \sum_{s=1}^{n_s} \left(B^s K^{s^{-1}} (f^s + Cp^w)\right) \tag{17}$$

$$e = \left[(f^{1^T} + C^1 p^{w^1}) R^1, \ldots\ldots, (f^{n_f^T} + C^n p^{w^n}) R^{n_f}\right]^T \tag{18}$$

Also, vector d and e is expressed in the FETI interface problem for the fluid phase as follows;

$$d = \sum_{s=1}^{n_s} \left(B^s [D + \frac{1}{\Delta t} S]^{-1} (f^s + \frac{1}{\Delta t} Sp^f + \frac{1}{\Delta t} (C_2 (u^s_n - u^s_{n+1})))\right) \tag{19}$$

$$e = [(f^{1^T} + C^1 \frac{1}{\Delta t} Sp^f + \frac{1}{\Delta t} (C_2 (u^s_n - u^s_{n+1}))^{f^1}) R^1 \ldots$$
$$(f^{n_s^T} + C^1 \frac{1}{\Delta t} Sp^f + \frac{1}{\Delta t} (C_2 (u^s_n - u^s_{n+1}))^{f^1}) R^{n_s}]^T \tag{20}$$

FETI method applied to Staggered method for all phase is used parallel analysis using MPI library (Fig. 3).

4.1 MPI Communication

Staggered method and FETI method are progressed by the MPI communication protocol as shown in Fig. 3. In this program, Main MPI library are used and CPU is allocated for parallel analysis at the same time in start. Generally, MPI library is used to Non-blocking Communication (MPI_Isend, MPI_Irecv *et al.*) and Collective Communications (MPI_Scatter, MPI_Reduce *et al.*) for efficiency and reduce communication deadlock. But staggered method is composed of after calculating displacement, pore pressure is calculated. Thus we use Point to Point communication protocol using MPI_Send and MPI_Recv (Blocking communication).

4.2 Computation of Staggered Method Using FETI (CPU 0)

In the Staggered method, CPU 0 (main CPU) is responsible to input date, send data and receive data from FETI module accurate remeshing method. Input data in the FETI module are transferred into other CPUs. Moreover, calculated results such as displacements and pore pressures at each CPUs are assembled in this CPU. After that, CPU 0 performs convergence check and determines displacements and pore pressure during sub iteration. Moreover, remeshing scheme is also progressed.

4.3 Computation of Displacement Using FETI (CPU 1~CPU n)

From CPU 1 to CPU n, displacements are calculated with FETI method. CPU 1 has duty to control including other CPUs, and send the calculated data to CPU 0. Parallel analysis is calculated d and e expressed to sum of the subdomain in Eq. (15),(17) and (18). At this calculate, inverse matrix and pseudo-inverse matrix are obtained from each CPU. Namely, pseudo-inverse matrix is obtained by SVD (Singular Value Decomposition), and null space is calculated in Eq. (15). In the CPU 1, Lagrange Multiplier (Table 1) and Eq. (15) are calculated. Finally, displacement of each subdomain is calculated by Eq. (13) and displacement vector is sent by MPI_Send communication.

Table 1 Preconditioned Conjugate Projected Gradient Algorithm

Initialization

$\lambda_0 = G(G^T G)^{-1} e$

$r_0 = d - F_I \lambda_0$

Iterations n=1,2, . . . until convergence

$w_{n-1} = P r_{n-1}$ $\quad (projected)$

$z_{n-1} = \widetilde{F_I^{-1}} w_{n-1}$ $\quad (precondition)$

$y_{n-1} = P z_{n-1}$ $\quad (re-projected)$

$\beta_n = \dfrac{y_{n-1}^T w_{n-1}}{y_{n-2}^T w_{n-2}}$ $\quad (\beta_1 = 0)$

$p_n = y_{n-1} + \beta_n p_{n-1}$ $\quad (p_1 = y_0)$

$\alpha_n = \dfrac{y_{n-1}^t w_{n-1}}{p_n^T F_I p_n}$

$\lambda_n = \lambda_{n-1} + \alpha_n p_n$

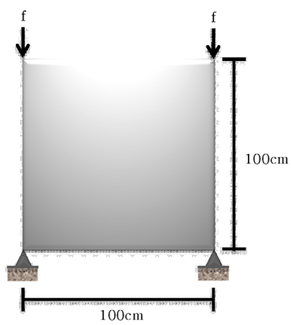
$r_n = r_{n-1} - \alpha_n F_I p_n$

4.4 Computation of pore pressure using FETI (CPU n+1~CPU m)

After the computation of displacement is finished, the computation of pore pressure is started immediately. To calculate the pore pressure, we apply the parallel analysis to the calculation of pore pressure such as the process of the computation of displacement. In this case, the value of Eq. (6) about stiffness matrix of displacement is very small. Thus, it is necessary to use very small value of the time step. The pore pressure is finally calculated by Eq. (6) like the case of the computation of displacement and sent by MIP_Send.

5. Numerical Verification

In order to verify of proposed method for FETI applied to Staggered method, we are used to decompose



Material Property	
E	3.0×107Pa
ν	0.3
ρ	1000kg/m^3
n	0.3
α	0.1
f	100kN

Fig. 4 Numerical Analysis Model

porous media in two dimensional and finite element analysis commercial programs (Abaqus Ver. 6.9). In this paper, porous media model is used width 100cm and length 100cm. Porous media is analyzed under the assumption of fully saturated and linear elastic behavior of 4 nodes (Fig. 4).

5.1 System Constitutions

In order to the numerical verification of parallel analysis for porous media, it is used for Linux cluster of the Neahlem CPU (2.4GHz, 8Core) of 33nodes and QRD transmission speed network. Also, MPI communication for parallel analysis is used library that is offered MPICH2 (1999) and GCC complier.

5.2 Numerical Accuracy Verification of FETI Method

Basically, this numerical simulation is performed by the modified program developed by Tak and Park (2010). The Staggered method is compared with the explicit-implicit method (Pastor *et al.*,2000), the implicit-implicit method (Huang *et al.*,2004), Terzaghi' sone-dimensional consolidation theory (1925), and ABAQUS (2007). This numerical verification is used to porous media of size of width 100cm and length 100cm. in order to verify of numerical accuracy, pore pressure is compared with in the staggered method and the FETI method applied to staggered method. Pore pressure of proposed method by Lee *et al.* (2010) is compared with pore pressure in this research as follows; (Fig. 5)

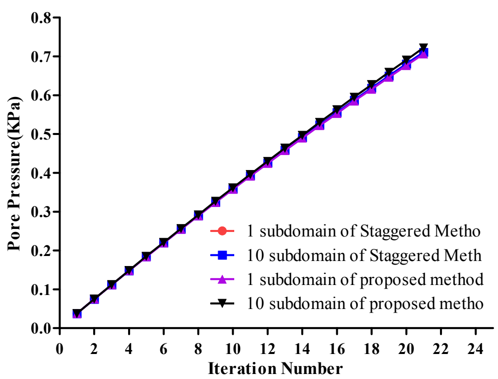


Fig. 5 Accuracy Verification

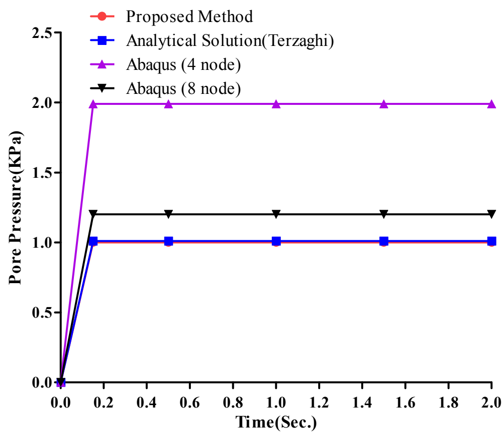


Fig. 6 Comparison of Pore Pressure at Node 110 (100 Elements)

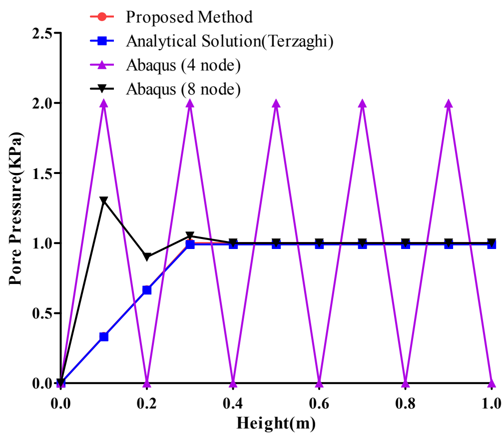
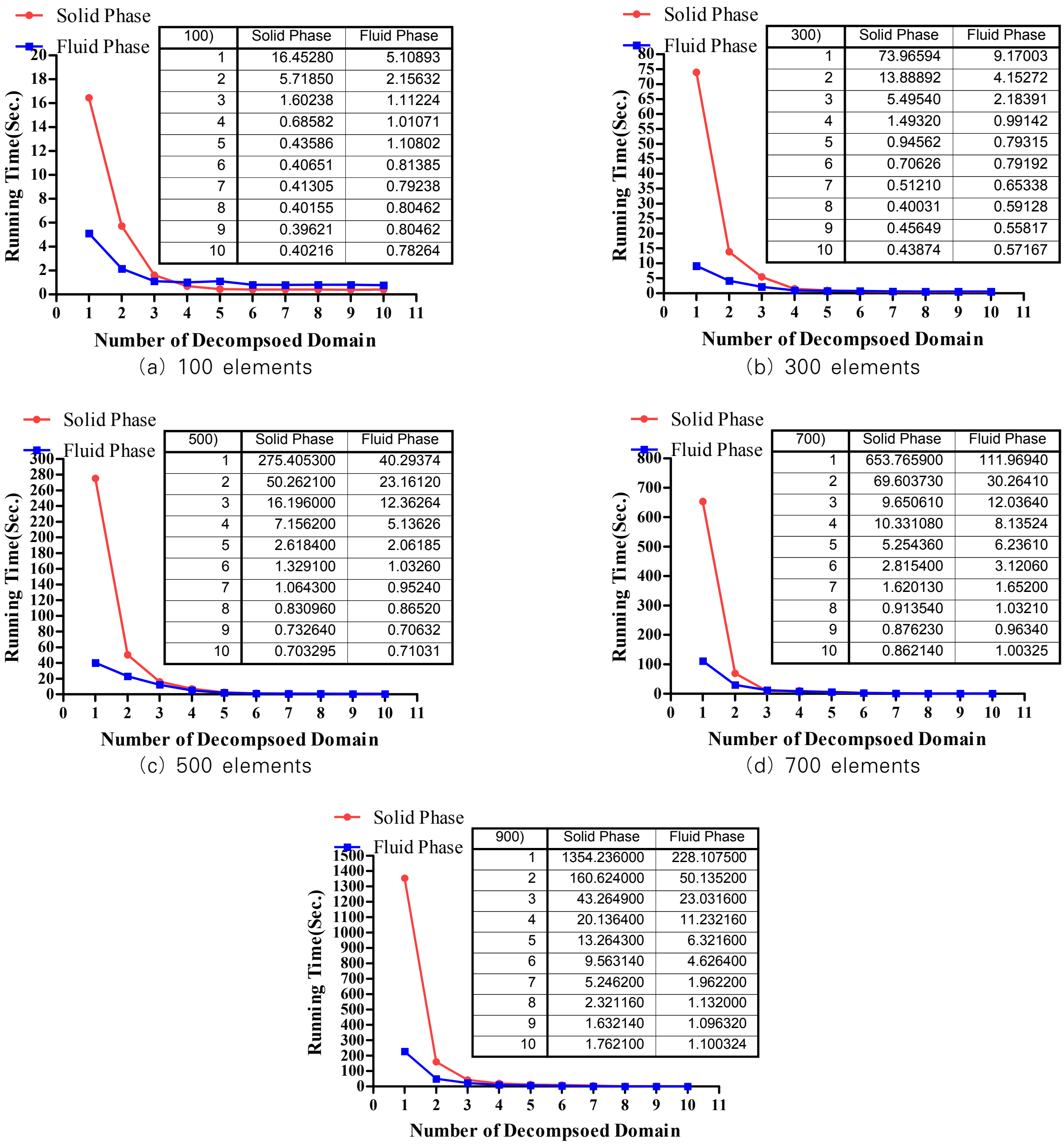


Fig. 7 Comparison of Pore Pressure with Elapsed Time (100 Elements)

In Fig. 5 pore pressure is represented in the decomposed porous media and the proposed method by Lee *et al.* (2010). The accuracy improvement of pore pressure has no difference as shown in Fig. 6 and Fig. 7. Thus in this paper, proposed method is presented to considered only numerical efficiency.

5.3 Numerical Efficiency Verification of FETI Method

In the numerical test, two dimensional equal order elements of 100, 300, 500, 700, 900 are used for porous media analysis. Also, elements are divided to one of subdomain from 10 of subdomain. In order to express numerical efficiency, solid and fluid phases running time in porous media are represented by each



100)	Solid Phase	Fluid Phase
1	16.45280	5.10893
2	5.71850	2.15632
3	1.60238	1.11224
4	0.68582	1.01071
5	0.43586	1.10802
6	0.40651	0.81385
7	0.41305	0.79238
8	0.40155	0.80462
9	0.39621	0.80462
10	0.40216	0.78264

(a) 100 elements

300)	Solid Phase	Fluid Phase
1	73.96594	9.17003
2	13.88892	4.15272
3	5.49540	2.18391
4	1.49320	0.99142
5	0.94562	0.79315
6	0.70626	0.79192
7	0.51210	0.65338
8	0.40031	0.59128
9	0.45649	0.55817
10	0.43874	0.57167

(b) 300 elements

500)	Solid Phase	Fluid Phase
1	275.405300	40.29374
2	50.262100	23.16120
3	16.196000	12.36264
4	7.156200	5.13626
5	2.618400	2.06185
6	1.329100	1.03260
7	1.064300	0.95240
8	0.830960	0.86520
9	0.732640	0.70632
10	0.703295	0.71031

(c) 500 elements

700)	Solid Phase	Fluid Phase
1	653.765900	111.96940
2	69.603730	30.26410
3	9.650610	12.03640
4	10.331080	8.13524
5	5.254360	6.23610
6	2.815400	3.12060
7	1.620130	1.65200
8	0.913540	1.03210
9	0.876230	0.96340
10	0.862140	1.00325

(d) 700 elements

900)	Solid Phase	Fluid Phase
1	1354.236000	228.107500
2	160.624000	50.135200
3	43.264900	23.031600
4	20.136400	11.232160
5	13.264300	6.321600
6	9.563140	4.626400
7	5.246200	1.962200
8	2.321160	1.132000
9	1.632140	1.096320
10	1.762100	1.100324

(e) 900 elements

Fig. 8 Running Time by the Number of Decomposed Domain

element. Running time was reduced as increasing subdomain in each element. Also, total running time of porous media was decreased because running time of fluid phase is reduced. Running time of solid and fluid phases of each element is represented in Fig. 8.

6. Conclusion

In this paper, in order to improve numerical efficiency, we propose parallel analysis method using FETI method applied to Staggered method. For the proposed method that is utilized with porous media theory, finite element formulation and FETI method. Also FETI method is applied to porous media consists of solid and fluid phases. Finally, in order to verify the proposed method, we used to finite element analysis of 2 dimensional saturated porous media model. Saturated porous media is decomposed into many subdomains and subdomains are computed displacement and pore pressure, respectively. Also numerical accuracy of the proposed method is compared with Staggered method by Lee *et al*. (2010). Consequently, as increasing number of subdomain, numerical efficiency of porous media analysis is improved than serial analysis of porous media.

We have the work for comparing blocking with non-blocking MPI communication and more improved algorithms for numerical efficiency under the nonlinear analysis are presented in forthcoming works.

Acknowledgement

This research was supported by WCU (World Class University) program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (R32-2008-000-20042-0).

References

ABAQUS (2010) Analysis User's Manual, *ABAQUS 6.9 Documentation*, ABAQUS, Inc.

Babuska, I. (1973) The Finite Element Method with Lagrange Multipliers, *Numerische Mathematik*, 20(3), pp.179~192.

Brezzi, F. (1974) On the Existence, Uniqueness and Approximation of Saddle-Point Problems Arising from Lagrangian Multipliers, *RAIRD*, 8(2), pp.129~151.

Biot, M. (1941) General Theory of Three-Dimensional Consolidation, *Journal of Applied Physics*, 12(1), pp.155~164.

Borja, M. (1986) Finite Element Formulation for Transient Pore Pressure Dissipation: A Variational Approach, *International Journal of Solid and Structures*, 22(11), pp.1201~1211.

Farhat, C. (1991) A Method of Finite Element Tearing and Interconnecting and Its Parallel Solution Algorithm, *International Journal for Numerical Methods in Engineering*, 32, pp.1205~1277.

Ghaboussi, J., Wilson, E.L. (1973) Flow of Compressible Fluid in Porous Elastic Media, *International Journal for Numerical Methods in Engineering*, 5(3), pp.419~442.

Gropp, W., Lusk, E., Thakur, R. (1999) Using MPI-2: Advanced Features of the Message-Passing Interface, *MIT Press*, p. 382.

Herrmann, L.R. (1967) Finite Element Bending Analysis for Plates, *Journal of The Engineering Mechanics Division, ASCE*, 93(1), pp.13~26.

Huang, M., Yue, Z.Q., Tham, L.G., Zienkiewicz, O.C. (2004). On the Stable Finite Element Procedures for Dynamic Problems of Saturated Porous Media, *International Journal for Numerical Methods in Engineering*, 61(9), pp.1421~1450.

Lee, K., Tak, M., Kang, Y., Park, T. (2010) The Mixed Finite Element Analysis for Porous Media Using Domain Decomposition Method, *Journal of the Computational Structural Engineering Institute of Korea*, 23(4), pp.369~378.

Park, T., Tak, M. (2010) A New Coupled Analysis for Nearly Incompressible and Impermeable Saturated Porous Media on Mixed Finite Element Method: I. Proposed Method, *KSCE*, 14(1), pp.7~16.

Park, T., Tak, M., Kim, H. (2005a) Analysis of Saturated Porous Media Using Arbitrary Lagrangian Eulerian Method: I. Theoretical Formulation, *KSCE Journal of Civil Engineering, KSCE*, 9(3), pp.233~239.

Park, T., Tak, M., Kim, H. (2005b) Analysis of Saturated Porous Media Using Arbitrary Lagrangian Eulerian Method: II. Finite Element Formulation, *KSCE Journal of Civil Engineering, KSCE*, 9(3), pp.241~246.

Park, T., Tak, M., Kim, H. (2005c) Analysis of Saturated Porous Media Using Arbitrary Lagrangian Eulerian Method: III.Numerical Examples, *KSCE Journal of Civil Engineering, KSCE*, 9(3), pp.247~254.

Pastor, M., Li, T., Zienkiewicz, O.C., Quecedo, M. (2000) A Fractional Step Algorithm Allowing Equal Order of Interpolation for Coupled Analysis of Saturated Soil Problems, *Mechanics of Cohesive-frictional Materials*, 5(7), pp.511~534.

Tak, M., Park, T. (2010) A New Coupled Analysis for Nearly Incompressible and Impermeable Saturated Porous Media on Mixed Finite Element Method: II. Verifications, *KSCE*, 14(1), pp.17~24.

Terzaghi, K. (1925) Principles of Soil Mechanics, Engineering News-Record.

Voyiadjis, G.Z., Abu-Farsakh, M.Y. (1997) Coupled Theory of Mixtures for Clayey Soils, *Journal of Computers and Geotechnics*, 20(3/4), pp.195~222.

Zienkiewicz, O.C., Chang, C.T., Bettess, P. (1980) Drained, Undrained, Consolidating and Dynamic Behavior Assumptions in Soils, *Geothchnique*, 30 (4), pp.385~395.

Zienkiewicz, O.C., Taylor, R.L., Zhu, J.Z. (2000) The Finite Element Method Sixth edition, *Elsevier Butterworth-Heinemann*, pp.334.

● 논문접수일 2010년 11월 1일
● 논문심사일 2010년 11월 4일
● 게재확정일 2010년 11월 29일