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논 문
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A New Robust Discrete Integral Variable Structure Controller with Disturbance Observer for Uncertain Discrete Systems

이 정 훈*
(Jung-Hoon Lee)

Abstract - In this paper, a new discrete integral variable structure controller based on the a new sliding surface and discrete version of the disturbance observer is suggested for the control of uncertain linear systems. The reaching phase is completely removed by introducing a new proposed integral sliding surface. The discrete version of disturbance observer is derived for effective compensation of uncertainties and disturbance. A corresponding control with disturbance compensation is selected to guarantee the quasi sliding mode on the predetermined integral sliding surface for guaranteeing the designed output in the integral sliding surface from any initial condition for all the parameter variations and disturbances. Using Lyapunov function, the closed loop stability and the existence condition of the quasi sliding mode is proved. Finally, an illustrative example is presented to show the effectiveness of the algorithm.

Key Words : Discrete integral variable structure system, Digital sliding mode control, Disturbance observer

1. Introduction

The theory of the variable structure system (VSS) or sliding mode control (SMC) can provide the effective means to the problem of controlling uncertain dynamical systems under parameter variations and external disturbances in case of the continuous[1]-[4] and discrete time system[5]-[18]. One of its essential advantages is the robustness of the controlled system to variations of parameters and external disturbances in the quasi sliding mode on the predetermined sliding surface, $s(k)=0$[5][6]. The proper design of the sliding surface can determine the almost output dynamics and its performances. In the SMC for discrete time systems, a few issue is considered in the design of VSS controller, i.e. the stable design of the sliding surface[10][11], reacheability from a given initial state to the fixed sliding surface[12][15][17], existence of quasi sliding mode[5][7][8][15] to gather with closed loop stability[6], robustness against uncertainties and disturbance[6][15][16][18], etc.[19]. In 1985, Milosavljevic defined the quasi sliding mode and presented the condition for the existence of the quasi sliding mode in discrete VSS[5]. The sliding and convergence condition for controlling discrete-time system is suggested by Sarpturk et. al[7] which is modified from that of Milosavljevic's where an absolute value condition for the reaching and the existence condition of the quasi sliding mode is imposed. Furuta in 1990 proposed the design methodology of discrete VSS by using the transformation matrix, and using Lyapunov function the quasi sliding and convergence condition is proposed and the sliding sector concept is introduced to design sliding mode controller for linear single-input discrete-time systems[8]. In [9], the problems of robust model following control of discrete-time uncertain systems is considered. Using equivalent control of the discrete VSS, the sliding surface is designed in [8] and [13], both are different. Using a candidate Lyapunov function, the coefficient of the sliding surface is designed[10]. By means of optimal theory to minimize the cost function, the optimal sliding surface is chosen with selection of the optimal switching gain[11]. Wang[12] designed the a simple sliding mode such that the robust stability of the uncertain system and reduced the chattering along the sliding mode. However a counterexample showing the instability of the control scheme proposed by Wang et al. was given in [13]. The band of the quasi sliding mode is rigorously defined and a new reaching condition is established in [14]. For multivariable system, Koshkouei and Zinober suggested a new condition for the existence of the discrete-time sliding mode and presented a design procedure such that the robust stability of the sliding motion is achieved in [15]. The fixed and adaptive sliding mode control in the presence of an unknown disturbance were in [16]. Hui and Zak compared the difference in the requirements for

* 정 회 원 : 공학연구원, 경상대학교 공대 전기전자공학부 제어계측공학과 교수・공박
E-mail : jhleew@gnu.ac.kr
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the sliding mode behavior for continuous- and discrete-time systems and discussed the limitations of discrete-time variable structure sliding mode control[17]. Cheng et al provided a simple design technique of sliding mode controllers for a class of multi input uncertain discrete-time system with matching conditions[18]. For uncertain nonlinear system, the discrete-time implementation of a second-order sliding mode control scheme is analyzed in [20]. In [23], for time delay discrete singular uncertain system, a variable structure controller is designed. In [24], for the same system as in [23], a quasi-sliding mode variable structure controller is presented. In [25], a static output feedback integral variable structure controller for uncertain systems with unmatched system matrix uncertainty is suggested.

Until now in most of discrete VSSs except [25], the used sliding surface is only the linear combination of the full state $s(k)=c^T x(k)$ and fixed in state space. Because of this, the closed loop system has the reaching phase for the initial state far from the sliding surface and the quasi sliding mode for the robustness is not guaranteed during this phase. Except [25] the integral action is not introduced to the discrete VSS to improve the output performance so far.

In this paper, a new discrete integral variable structure controller based on the a new integral sliding surface and discrete version of the disturbance observer is suggested for the control of uncertain linear systems. The reaching phase is completely removed by introducing a new proposed integral sliding surface which stems from [22] in continuous time. The discrete version of disturbance observer is introduced to the effective compensation of uncertainties and disturbance. A corresponding control with disturbance compensation is selected to guarantee the quasi sliding mode on the predetermined sliding surface for guaranteeing the designed output in the integral sliding surface from any initial condition for all the parameter variations and disturbances. The advantages obtained after removing the reaching phase are discussed. Finally, an illustrative example is presented to show the effectiveness of the algorithm.

2. A Discrete Integral Variable Structure Systems

2.1 Descriptions of Plants

Let the uncertain linear time invariant discrete plant to be controlled be given in the state space representation by

$$\overline{X_{k+1}} = (\Lambda+\triangle\Lambda)\overline{X_k} + (\Gamma+\triangle\Gamma)u_k + F_k \\ = \Lambda\overline{X_k} + \Gamma u_k + \overline{T_{Lk}}, \quad \overline{X_0} \tag{1}$$

$$\overline{T_{Lk}} = \triangle\Lambda\overline{X_k} + \triangle\Gamma u_k + F_k \tag{2}$$

where $k \ge 0$ is an integer, $\overline{X_k} \in R^n$ is the state, $\overline{X_0} \in R^n$ is its initial condition of the state,$u_k \in R^1$ is the input control to be determined, nominal matrices $\Lambda \in R^{n\times n}$, $\Gamma \in R^{n\times 1}$ is full rank, and $\overline{T_{Lk}}$ is the unknown lumped uncertainty to be estimated.

Assumption:

A1: (Λ, Γ) is completely controllable

A2: The lumped uncertainty $\overline{T_{Lk}}$ is piecewise smooth and, bounded and satisfies the matching conditions[18]

Now, the integral of the state is augmented as follows:

$$X_{0k+1} = X_{0k} + T\overline{X_k}, \qquad X_{00} \tag{3}$$

where T is a sampling time and X_{00} is the initial condition of the integral. Now, let define a new augmented state as

$$X_k = [X_{0k}^T \quad \overline{X_k^T}]^T \tag{4}$$

then the augmented discrete state equation is obtained as

$$X_{k+1} = \begin{bmatrix} I & T\cdot I \\ 0 & \Lambda+\triangle\Lambda \end{bmatrix} X_k + \begin{bmatrix} 0 \\ \Gamma+\triangle\Gamma \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \overline{T_{Lk}} \tag{5}$$

$$= (A+\triangle A)X_k + (B+\triangle B)u_k + B_1\overline{T_{Lk}}$$

$$= AX_k + Bu_k + \triangle AX_k + \triangle Bu_k + B_1\overline{T_{Lk}}$$

$$= AX_k + Bu_k + T_{Lk} \tag{6}$$

$$T_{Lk} = \triangle AX_k + \triangle Bu_k + B_1\overline{T_{Lk}} \tag{7}$$

2.2 Integral Sliding Surface

For the system (6), an integral augmented sliding surface is proposed as follows:

$$S_k = C^T X_k \\ = \left[C_0 \ \ \overline{C}\right]\left[X_{0k}^T \quad \overline{X_k^T}\right]^T (=0) \tag{8}$$

where

$$X_{00} = -C_0^{-}\overline{C}\overline{X_0} \tag{9}$$

so that the sliding surface is zero at initial time $k=0$ for any initial condition $\overline{X_0}$ in state space. Therefore the reaching phase to the sliding surface is removed completely and there is no need of the consideration of the reaching condition in this suggested discrete VSS. Except [25], this point is not considered in the previous discrete VSS works mentioned in Introduction. The following definitions are introduced.

Definition 1: *Discrete Ideal Sliding Mode*[8][15]

If $S_k=0, \ \ k\ge 0$ is satisfied, then it is called as an *discrete ideal sliding mode.*

Definition 2: *Quasi Sliding Mode*[5]

For any real number $\epsilon>0$, if $\|S_k\|<\epsilon$ for all k is satisfied because of the finite sampling time, then it is called as *quasi sliding mode.*

Substituting (6) in $S_{k+1}=0$ yields the equivalent control[8][10]

$$S_{k+1} = C^T X_{k+1} \\ = C^T\left[AX_k + Bu_k + T_{Lk}\right] \tag{10}$$

$$U_{eqk} = -\left(C^T B\right)^{-1} C^T AX_k - \left(C^T B\right)^{-1} C^T T_{Lk} \tag{11}$$

which can not be implemented because of the unknown lumped disturbance. The closed loop system by equivalent control is obtained as

$$X_{k+1} = \left[I - B(C^T B)^{-1} C^T\right] A X_k = A_{cs} \cdot X_k, \quad X_0 \tag{12}$$

where

$$A_{cs} = \left[I - B(C^T B)^{-1} C^T\right] A . \tag{12a}$$

The solution of (12) defines the surface in *discrete ideal sliding mode* of the proposed integral sliding surface.

2.3 Control Input

Now, to estimate the lumped uncertainty (2) for compensation by the control input, one step delay nonlinear disturbance observer of discrete version[25] is obtained as follows from (9)

$$\begin{aligned}\widehat{T_{Lk}} &= X_k - A X_{k-1} - B u_{k-1} \\ &= T_{Lk-1}\end{aligned} \tag{13}$$

which stems from continuous version in [22] and is identical to that of [3].

Assumption:

A3: It is assumed that the disturbance observer error from the real value is bounded as

$|T_L(k) - T_L(k-1)| < \triangle$,

$\triangle = samll\ positve\, constant > 0$

Now to stabilize the (6) with the chosen integral sliding surface and compensation by means of disturbance observer, a following discrete control input is presented

$$u_k = -K X_k - B^- \widehat{T_{Lk}} - G S_k \tag{14}$$

where K and G are the design parameters in the control input and satisfy the following condition

$$\begin{aligned}&Q' < 0, \quad Q' = Q - CC^T, \quad Q = M^T M, \\ &M = C^T\left[(A - BK) - BGC^T\right]\end{aligned} \tag{15}$$

Using the control input (14) with the integral sliding surface (8) and disturbance observer (13), the existence of the quasi sliding mode and closed loop stability is investigated in text theorem.

Theorem 1: If the sliding surface is designed in the stable i,e, stable A_{cs}, the proposed input with disturbance observer satisfies the quasi sliding mode on the predetermined sliding surface from the initial state and stability in the sense of Lyapunov.

Proof; Take a discrete candidate Lyapunov function as

$$V_k = S_k^T S_k \tag{17}$$

then

$$\begin{aligned}V_{k+1} &= S_{k+1}^T S_{k+1} \\ &= S_{k+1}^T C^T\left[A X_k + B u_k + T_{Lk}\right] \\ &= S_{k+1}^T C^T\left[A X_k + B(-K X_k - B^- \widehat{T_{Lk}} - G S_k) + T_{Lk}\right] \\ &= S_{k+1}^T C^T\left[(A - BK) X_k - BG S_k + \triangle\right] \\ &\simeq S_{k+1}^T C^T\left[(A - BK) - BGC^T\right] X_k \\ &= S_{k+1}^T M X_k = X_k^T M^T M X_k \\ &= X_k^T Q X_k\end{aligned} \tag{18}$$

then

$$\begin{aligned}\triangle V_k = V_{k+1} - V_k &= S_{k+1}^T S_{k+1} - S_k^T S_k \\ &= X_k^T\left[Q - CC^T\right] X_k \\ &= X_k^T Q' X_k < 0\end{aligned} \tag{19}$$

which implies that

$$(i)\ \|S_{k+1}\| < \|S_k\| \tag{20}$$

$$(ii)\ S_k^T \triangle S_{K+1} < -\frac{1}{2}\|\triangle S_{k+1}\|^2 \tag{21}$$

$$\triangle S_{k+1} = S_{k+1} - S_k$$

$$(iii)\ |S_k^T S_{k+1}| < \|S_k\|^2 \tag{22}$$

are satisfied[15] which completes the proof of Theorem 1. Except [25], for the discrete VSS, the stability of the closed loop system is hardly investigated in the previous VSS works. However, in this paper by the results of Theorem 1, closed loop stability is proved and the quasi sliding mode on the integral sliding surface for all $k \ge 0$ is guaranteed. So the performance designed in the sliding surface is almost also guaranteed.

3. Design Example and Simulation Studies

Consider a position control problem of the following direct drive motor plant with disturbances

$$\overline{X}_{k+1} = \begin{bmatrix} 1 & 0.01 \\ 0 & 0.4575 \end{bmatrix} \overline{X_k} + \begin{bmatrix} 0 \\ 12.246 \end{bmatrix} u_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} T_{Lk} \tag{23}$$

where $\overline{X_k} = \left[\overline{X_{1k}} \ \ \overline{X_{2k}}\right]^T$, $\overline{X_{1k}}$ and $\overline{X_{2k}}$ are the position and its speed of the direct drive motor.

The sampling time is selected as $T = 10[mSec]$. An integral state is as follows

$$X_{ok+1} = X_{ok} + T X_{1k} \tag{24}$$

Then, the augmented state equation is obtained as

$$X_{k+1} = \begin{bmatrix} 1 & 0.01 & 0 \\ 0 & 1 & 0.01 \\ 0 & 0 & 0.4575 \end{bmatrix} X_k + \begin{bmatrix} 0 \\ 0 \\ 12.246 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T_{LK} \tag{25}$$

To design the integral sliding surface, the closed loop system matrix in (12) is selected as

$$A_{cs} = \begin{bmatrix} 1 & 0.01 & 0 \\ 0 & 1 & 0.01 \\ -1 & -2.51 & -0.025 \end{bmatrix} \tag{26}$$

in order to assign the stable poles at 0.0, 0.995, and 0.98. By (12a), C is selected as

$$C^T = [1 \ \ 2.5 \ \ 1] \tag{27}$$

From (26), the constant feedback gain K in (14) is designed as

$$K = [0.0817 \ \ 0.2050 \ \ 0.0394] \tag{28}$$

and G is selected as $G = 0.01$ to satisfy the condition (15).

As a results of the systematic design, M, and Q' are as follows:

$$M = [-0.1225 \ \ -0.3061 \ \ -0.1225] \tag{29}$$

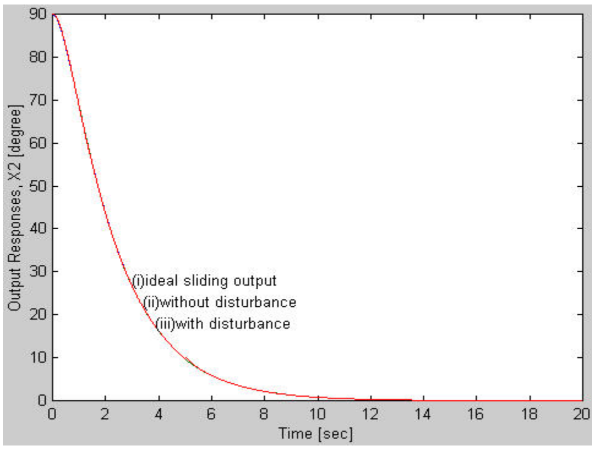

Fig. 1 Three output positions of motor (i) ideal sliding output (ii) without disturbance, and (iii) with disturbance

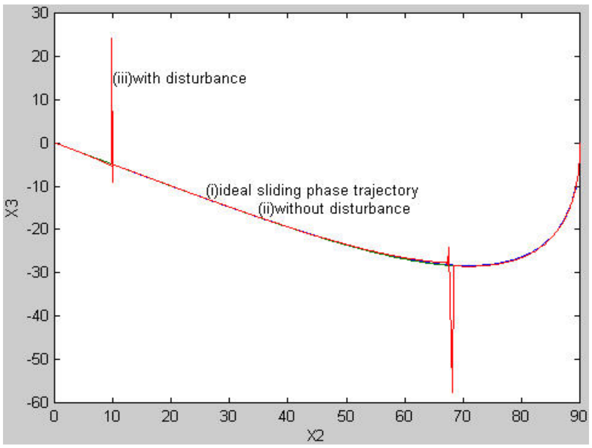

Fig. 2 Phase trajectories for the three cases (i) ideal liding trajectory (ii) without disturbance, and (iii) with disturbance.

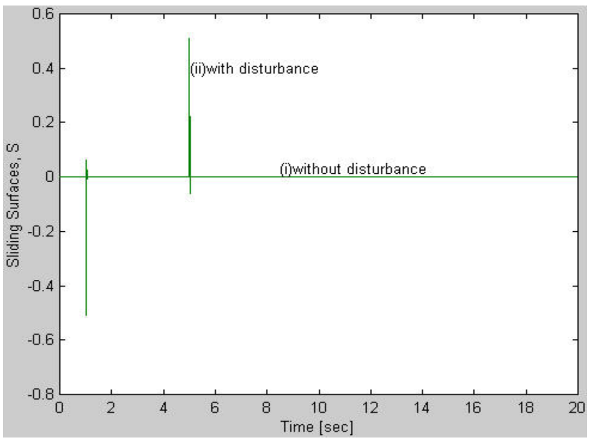

Fig. 3 Two sliding surfaces for the two cases (i) without disturbance and (ii) with disturbance

$$Q=\begin{bmatrix}-0.9850 & -2.4625 & -0.9850\\ -2.4625 & -6.1563 & -2.4625\\ -0.9850 & -2.4625 & -0.9850\end{bmatrix} \tag{30}$$

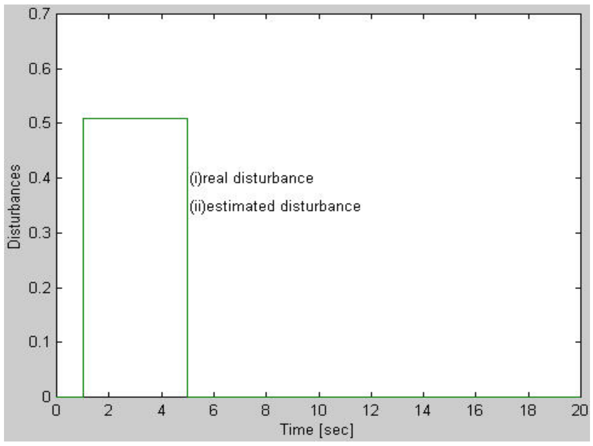

Fig. 4 Load variation of disturbance from 1[sec] to 5[sec] and its estimated value by disturbance observer

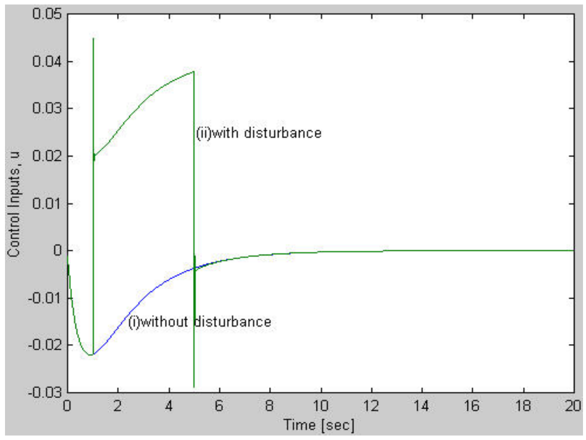

Fig. 5 Control inputs for the two cases (i) without disturbance and (ii) with disturbance

The eigenvalues of Q are 0, 0, -8.1263 which satisfy the relationship of (15) and the stability condition (19). An initial condition for (20) is given as $\overline{X_0}=[90 \quad 0]^T$ $[degree \ \ degree/\sec]^T$ and $X_{00}=-225.0$ by (9). The simulation is carried out under $T_{Lk}=0.5087$ load variation of disturbance from 1[sec] to 5[sec]. Fig. 1 shows three output positions of motor (i) ideal sliding output, i.e. solution of (12), (ii) without disturbance, (iii) with disturbance. As can be seen, the three outputs are almost identical. The phase trajectories for the three cases (i) ideal sliding trajectory, (ii) without disturbance, (iii) with disturbance are depicted in Fig. 2. There is no reaching phase in these figures. However the phase trajectory under disturbance is disturbed because of one step delay estimation of disturbance observer and the quasi sliding mode of the discrete VSS, however fastly recovered by the suggested control input. Fig. 3 shows the two sliding surfaces for the two cases (i) without disturbance and (ii) with disturbance. The load variation of disturbance from 1[sec] to 5[sec] and its estimated value by means of the discrete one step delay disturbance observer are shown in Fig. 4. The control inputs for the two cases (i) without disturbance and (ii) with disturbance are depicted in Fig. 5. From the simulation studies, the effectiveness of the proposed discrete VSS is proven.

4. Conclusions

In this paper, a design of a new robust discrete integral VSS with disturbance observer is presented for the control of uncertain linear discrete systems under lumped uncertainties. To successfully remove the reaching phase problems, a discrete integral sliding surface is suggested to define the hyper plane from any given initial condition. For the design of its sliding surface, the ideal sliding dynamics is obtained. After choosing the desired performance by means of any well developed linear discrete regulator theories, the integral sliding surface is determined to have exactly that performance from a given initial condition to the origin. The discrete version of disturbance observer is presented to effectively estimate the lumped uncertainties. A corresponding control input with disturbance observer is also designed to almost guarantee the performance pre-determined in the integral sliding surface. The robustness of the pre-determined output for all the lumped uncertainties is investigated in Theorem 1 together with the existence condition of the quasi sliding mode of the discrete VSS and the stability of the closed loop system in the sense of Lyapunov. Through simulation studies, the usefulness of the proposed controller is verified.

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저 자 소 개

이 정 훈 (李 政 勳)

1966년 2월 1일생. 1988년 경북대학교 전자공학과 졸업(공학사), 1990년 한국과학기술원 전기 및 전자공학과 졸업(석사). 1995년 한국과학기술원 전기 및 전자공학과 졸업(공박). 현재 2010년 현재 경상대학교 제어계측공학과 교수. 1997-1999 경상대학교 제어계측공학과 학과장. 마르퀘스사의 Who's Who in the world 2000년 판에 등재. American Biograhpical Institute(ABI)의 500 Leaders of Influence에 선정.

Tel : +82-55-751-5368

Fax : +82-55-757-3974

E-mail : jhleew@gnu.ac.kr