# Optimal Location of FACTS Devices Using Adaptive Particle Swarm Optimization Hybrid with Simulated Annealing

**Ali Ajami†, Gh. Aghajani* and M. Pourmahmood****

**Abstract –** This paper describes a new stochastic heuristic algorithm in engineering problem optimization especially in power system applications. An improved particle swarm optimization (PSO) called adaptive particle swarm optimization (APSO), mixed with simulated annealing (SA), is introduced and referred to as APSO-SA. This algorithm uses a novel PSO algorithm (APSO) to increase the convergence rate and incorporate the ability of SA to avoid being trapped in a local optimum. The APSO-SA algorithm efficiency is verified using some benchmark functions. This paper presents the application of APSO-SA to find the optimal location, type and size of flexible AC transmission system devices. Two types of FACTS devices, the thyristor controlled series capacitor (TCSC) and the static VAR compensator (SVC), are considered. The main objectives of the presented method are increasing the voltage stability index and over load factor, decreasing the cost of investment and total real power losses in the power system. In this regard, two cases are considered: single-type devices (same type of FACTS devices) and multi-type devices (combination of TCSC, SVC). Using the proposed method, the locations, type and sizes of FACTS devices are obtained to reach the optimal objective function. The APSO-SA is used to solve the above non–linear programming optimization problem for better accuracy and fast convergence and its results are compared with results of conventional PSO. The presented method expands the search space, improves performance and accelerates to the speed convergence, in comparison with the conventional PSO algorithm. The optimization results are compared with the standard PSO method. This comparison confirms the efficiency and validity of the proposed method. The proposed approach is examined and tested on IEEE 14 bus systems by MATLAB software. Numerical results demonstrate that the APSO-SA is fast and has a much lower computational cost.

**Keywords:** FACTS devices, optimal location, PSO algorithm, SA algorithm, APSO-SA algorithm

## 1. Introduction

The flexible AC transmission system (FACTS) has received much attention over the last few decades. It uses high current power electronic devices to control the voltage, power flow, stability, etc., of a transmission system. FACTS devices can be connected to a transmission line in various ways, such as in series, shunt, or a combination of series and shunt. The term and definition of various FACTS devices are described in references [1], [2]. FACTS devices are very effective and capable of increasing the power transfer capability of a line, insofar as thermal limits permit, while maintaining the same degree of stability [3], [4].

In recent years, with the deregulation of the electricity market and due to competition between utilities, the amount of unplanned delivered power increases. If these exchanges are not controlled, some lines may become overloaded. These devices control the power flow in the network, reduce the flow in overloaded lines, thereby resulting in an increase in system loadability (SL), low system losses, improved network stability and reduced cost of production [1], [5]-[6]. It is important to find the location, type and size of these devices because of their significant costs. Studies and realizations have shown their capabilities in steady state or dynamic conditions.

The reference [7] provides an idea regarding the optimal locations of FACTS devices, without considering the investment cost of FACTS devices and their impact on generation costs. The optimal location taking into consideration the generation cost of the power plants and investment cost of the FACTS devices as studied in [8]. The reference [9] discusses the optimal location problem by power loss reduction.

The main objective of this paper is to develop an algorithm to find and choose the optimal location, type and size of FACTS devices based on the economic saving function, which is obtained by energy loss reduction. This paper presents the PSO and APSO-SA methods for ascertaining the optimal location of FACTS devices to achieve the minimum cost of FACTS devices, total real power losses in the power system and to improve system loadability, while satisfying the power system constraints for single and multi-type FACTS devices. In the single type case, the variables for the optimization of each device are its location in

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the network and its setting. In the case of multi-type devices, the type of device is taken as an additional variable for optimization.

This paper is organized as follows. Following the introduction, mathematical models are described in section 2. Then, in section 3, objective function is described. In section 4, the proposed method for optimal location of FACTS devices is discussed in detail, and in section 5 the implemented algorithm is described. The simulation results are given in section 6. Finally, a brief conclusion appears in section 7.

## 2. Mathematical Models

### 2.1 Steady State Models of FACTS Devices

For static applications, FACTS devices can be modeled by the power injection model (PIM) [7]-[8], [10]-[11]. The injection model describes the FACTS as a device that injects a certain amount of active and reactive power to a node, so that the FACTS device is represented as PQ elements. The PIM doesn't destroy the symmetrical characteristic of the admittance matrix and allows efficient and convenient integration of FACTS devices in to existing power system analytical tools. This is the main advantage of PIM.

***TCSC:***

Fig.1 shows the model of a transmission line with a TCSC connected between buses $i$ and $j$. The transmission line is represented by its lumped $\pi$ equivalent parameters.

During the steady state condition, the TCSC can act as capacitive or inductive mode, respectively, to decrease or increase the impedance of the branch. The TCSC is modeled with variable series reactance. Its value is a function of the reactance of line, $X_L$, where the device is located. The upper and lower limit of the TCSC reactance is given in (1).

$$-0.8X_L \le X_{TCSC} \le 0.2X_L \quad \text{p.u.} \tag{1}$$

The corresponding power injection model of the TCSC incorporated within the transmission line is shown in Fig.2 [12-14].

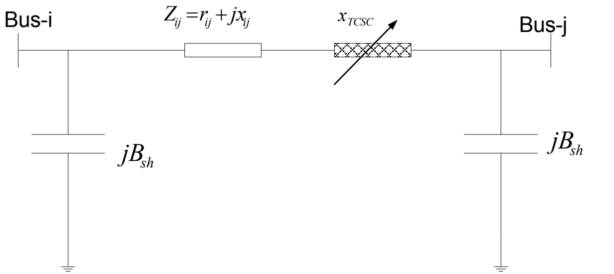


**Fig. 1.** Single line diagram of transmission line with TCSC.

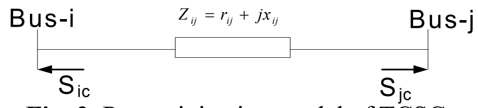


**Fig. 2.** Power injection model of TCSC.

The difference of line admittance, before and after installation of TCSC is given in (2)

$$\Delta y_{ij} = y'_{ij} - y_{ij} = (g'_{ij} + j b'_{ij}) - (g_{ij} + j b_{ij}) \tag{2}$$

Where,

$$\begin{aligned} g_{ij} &= \frac{r_{ij}}{\sqrt{r_{ij}^2 + x_{ij}^2}} \\ b_{ij} &= \frac{-x_{ij}}{\sqrt{r_{ij}^2 + x_{ij}^2}} \\ g'_{ij} &= \frac{r_{ij}}{\sqrt{r_{ij}^2 + (x_{ij} + x_{TCSC})^2}} \\ b'_{ij} &= \frac{-(x_{ij} + x_{TCSC})}{\sqrt{r_{ij}^2 + (x_{ij} + x_{TCSC})^2}} \end{aligned} \tag{3}$$

When a TCSC is installed in the line between buses $i$ and $j$, the reformed admittance matrix is obtained from (4).

$$Y'_{Bus} = Y_{Bus} + \begin{array}{cc} \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & \Delta y_{ij} & 0 & \dots & -\Delta y_{ij} & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & -\Delta y_{ij} & 0 & \dots & \Delta y_{ij} & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} & \begin{array}{c} \\ i \\ \\ \\ j \\ \\ \end{array} \\ \begin{array}{cccccc} \quad & col-i & \quad & \quad & col-j & \quad \end{array} & \end{array} \tag{4}$$

***SVC:***

The main purpose of the SVC is voltage controlling at weak points in the network. Fig. 3 shows the single line diagram of the compensated transmission line with an SVC at bus j and its power injection model is represented in Fig. 4. In this study, the SVC is treated as a variable capacitance, where $I_{SVC}$ is the complex injected current of the SVC at node $j$ [14].

It can be expressed as follows:

$$I_{SVC} = j\, B_{SVC} * V_j \tag{5}$$

The SVC can behave as a capacitive or inductive mode to absorb or inject reactive power, respectively. The SVC

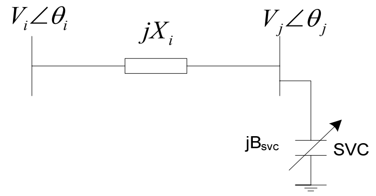


**Fig. 3.** The single line diagram of compensated transmission line with SVC.

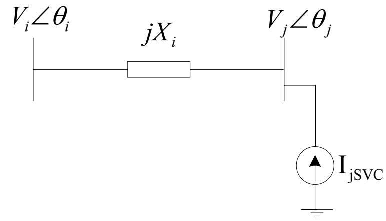


**Fig. 4.** Power injection model of SVC.

can be represented by a shunt variable susceptance inserted in the bus or at the mid point of the transmission line. The SVC is a voltage controlling device and its susceptance must be determined for regulation of bus voltage at the desired value. The SVC nominal values are corresponding to the power system. In this paper, we considered as below:

$$-100 \le Q_{SVC} \le +100 \qquad \text{Mvar} \tag{6}$$

When the SVC is installed at node $j$, the reformed admittance matrix can be expressed as (7).

$$Y'_{Bus} = Y_{Bus} + \begin{array}{cc} \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & Y_{SVC} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} & \begin{array}{c} \\ \\ j \\ \\ \\ \\ \end{array} \\ col-j & \end{array} \tag{7}$$

### 2.2 Power System Model

The power flow equations with FACTS devices are given as below:

$$P_{Gi} - P_{Di} - \sum_{j=1}^{n} |V_i||V_j| \begin{Bmatrix} G_{ij\_FACTS} Cos\,\delta_{ij} + \\ B_{ij\_FACTS} Sin\,\delta_{ij} \end{Bmatrix} = 0 \tag{8}$$

$$Q_{Gi} - Q_{Di} - \sum_{j=1}^{n} |V_i||V_j| \begin{Bmatrix} G_{ij\_FACTS} Sin\,\delta_{ij} - \\ B_{ij\_FACTS} Cos\,\delta_{ij} \end{Bmatrix} = 0 \tag{9}$$

$$|V_i|_{min} \le |V_i| \le |V_i|_{max} \tag{10}$$

$$\delta_{ij} \le \delta_{ij}^{max} \tag{11}$$

Where,

$P_{Gi}, Q_{Gi}$ : Generated real and reactive power at bus i.

$P_{Di}, Q_{Di}$ : Real and reactive power of load at bus i.

$n$ : Number of buses

$G_{ij\_FACTS}$ : Real part of (i, j)$^{th}$ element of network admittance matrix included FACTS devices.

$B_{ij\_FACTS}$ : Imaginary part of (i, j)$^{th}$ element of network admittance matrix included FACTS devices.

$\delta_{ij}$ **:** Difference of phase angle between buses $i$ and $j$

$|V_i|_{min}, |V_i|_{max}$ **:** Maximum and minimum voltage magnitude at bus

## 3. APSO-SA Algorithm

In recent years, many optimization algorithms have been introduced. Some of these algorithms are traditional optimization algorithms which use exact methods to find the best solution. The idea is that if a problem can be solved, then the algorithm should find the best global solution. As the search space increases, the cost of these algorithms increases. Therefore, when the search space complexity increases, the exact algorithms can be slow to find the global optimum.

There are several stochastic algorithms such as: genetic algorithms (GA) (Holland, 1975), guided local search (GLS) (Voudouris, 1997), tabu search (TS) (Glover, 1989, 1990), variable neighbourhood search (VNS) (Mladenovic and Hansen, 1997), iterated local search (ILS) (Stützle, 1999), simulated annealing (SA) (Kirkpatrick et al. 1983), greedy randomized adaptive search procedure (GRASP) (Feo and Resende, 1995), memetic algorithms (MA) (Moscato, 1989), scatter search (SS) (Cung et al. 1997), ant colony optimization (ACO) (Marco Dorigo et al. 1999), particle swarm optimization (PSO) (Kennedi and Eberhart 1995) and shuffled frog leaping algorithm (SFL) (Eusuff, Lansey 2003). Each of these algorithms has its unique characteristics. Particle swarm optimization (PSO) and simulated annealing (SA) are two efficient and well known stochastic algorithms.

### 3.1 The Standard PSO Algorithm

A particle swarm optimizer is a population based stochastic optimization algorithm modeled based on the simulation of the social behavior of bird flocks. PSO is a population-based search process where individuals initialized with a population of random solutions, referred to as particles, are grouped into a swarm. Each particle in the swarm represents a candidate solution to the optimization problem, and if the solution is a combination of variables, the particle can correspondingly be a vector of variables. In a PSO system each particle is "flown" through the multidimensional search space, adjusting its position in the search space according to its own experience and that of neighboring particles. The particle therefore makes use of the best position encountered by itself and that of its neighbors to position itself toward and optimal solution. The performance of each particle is evaluated using a predefined fitness function, which encapsulates the characteristics of the optimization problem.

Generally, a numerical optimization problem can be described as follows [15]:

$$\begin{aligned} &min\, F(X), \quad X = [x_1, x_2, \ldots, x_N]^T \\ &St. \quad x_i \in [a_i, b_i], \quad i = 1,2,\ldots,N \end{aligned} \tag{12}$$

The core operation of a PSO is the updating formulae of the particles, i.e. the velocity updating equation and position updating equation. The global optimizing model proposed by Shi and Eberhart (1999) is as follows:

$$\begin{aligned} v_{id}(t+1) = W * v_{id}(t) + &\\ C_1 * rand_1 * (p_{id} - x_{id}(t)) + &\\ C_2 * rand_2 * (p_{id} - x_{id}(t)) & \end{aligned} \tag{13}$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \tag{14}$$

Where, $v_{id}$ is the velocity of particle $i^{th}$ in dimension $d^{th}$ (is the particle position), $W$ is the inertia weight factor, $C_1$ and $C_2$ are two positive constant parameters called acceleration coefficients. Rand1 and rand2 are the random functions in the range [0, 1], $p_{id}$ is the best position of the $i^{th}$ particle in dimension $d^{th}$ and $p_{gd}$ is the best position among all particles in the swarm.

### 3.2 Simulated Annealing

Simulated Annealing (Metropolis et al. 1956, Kirkpatrick et al. 1983) [16], [17] is a metastrategy local search method that attempts to avoid producing the poor local maximum inherent in the steepest ascent method. It is a metaheuristic algorithm used to navigate through the space of solutions containing many local minimum and has been applied to many combinatorial optimization problems. The main idea behind simulated annealing is an analogy with the way in which liquids freeze and crystallize. When liquids are at a high temperature their molecules can move freely in relation to each other. As the liquid's temperature is lowered, this freedom of movement is lost and the liquid begins to solidify. If the liquid is cooled slowly enough, the molecules may become arranged in a crystalline structure. The molecules making up the crystalline structure will be in a minimum energy state. If the liquid is cooled very rapidly it does not form such a crystalline structure, but instead forms a solid whose molecules will not be in a minimum energy state. The fundamental idea of simulated annealing is therefore that the moves made by an iterative improvement algorithm are like the rearrangements of the molecules in a liquid that occur as it is cooled and that the energy of those molecules corresponds to the cost function which is being optimized by the iterative improvement algorithm. Thus, the simulated annealing algorithm aims to achieve a global optimum by slowly converging to a final solution, making downwards moves with occasional "upwards" moves (the probability of these occurring decreasing with the "temperature") and thus hopefully ending up in a global optimum. This is in contrast to the greedy approach of only considering a move which results in the largest possible decrease (if minimizing) in the objective function, which resembles a rapid cooling of a liquid to a solid, and thus according to the hypothesis, resulting in a local optimum rather than a global optimum.

In the SA algorithm, the improvements are obtained by choosing another solution ($x'$) that belongs to the neighborhood of the current solution ($x_0$). When the current solution changes from $x_0$ to $x'$, the objective function will also change, namely, $\Delta = f(x') - f(x_0)$, where $f(x')$ is the value of the objective function at $x'$. For the minimization problem, if $\Delta < 0$, the new solution $x'$ will be accepted. If $\Delta \geq 0$, the new solution will be accepted with the probability $exp(-\Delta / T)$, where T is the temperature (this is simply implemented by choosing a random number in the range from 0 to 1) and comparing this with the probability; if it is less, the new solution will be accepted otherwise it will be rejected. Generally, the algorithm starts from a high temperature, and then the temperature gradually decreases. At each temperature, a search will be performed for a certain number of iterations, which is called the temperature length. When the termination condition is satisfied, the algorithm will stop.

The most significant character of SA is the probabilistic jumping property, i.e. a worse solution has a probability to be accepted as a new solution. Moreover, by adjusting the temperature, such a jumping probability can be controlled. In particular, probability is rather high when temperature is high and decreases as the temperature decreases; and when the temperature tends toward zero the probability approaches zero so that only a better solution can be accepted. It has been theoretically proven that under certain conditions, SA is globally convergent in probability 1.

### 3.3 APSO-SA Algorithm

Slow convergence of the PSO before it provides an accurate solution is a drawback, closely related to its lack of any adaptive accelerators in the velocity updating formulae. In (13), $C_1$ and $C_2$ determine the step size of the particles movements through the $p_{id}$ and $p_{gd}$, respectively. In the original PSO, these step sizes are constant and all particles are the same. For doing more sensitive and faster movements, new step sizes can be modified which should accelerate the convergence rate. In each iteration, the value of the objective function is a criterion that presents the relative improvement of this movement in respect to the previous iteration movement. Thus, the difference between the values of the objective function in the different iterations can select the accelerators. Adding two additional coefficients to the original step sizes in (13), it causes adaptive movements. Therefore, the velocity updating formula turns to the following form:

$$\begin{aligned} v_{id}(t+1) &= W*v_{id}(t)+ \\ & C_1*rand_1*(f(p_{id}(t))-f(x_{id}(t)))*(p_{id}-x_{id}(t))+ \\ & C_2*rand_2*(f(p_{gd}(t))-f(x_{id}(t)))*(p_{gd}-x_{id}(t)) \end{aligned} \quad (15)$$

Where, $f(p_{id}(t))$ is the best fitness function that is found by ith particle and $f(p_{gd}(t))$ is the best fitness function that is found by swarm up to now and other parameters are chosen as in section 3.1. Globally optimizing an objective function in a given search domain consists in finding its global optimum without being trapped in any local optimum. When strongly multi-modal problems are being optimized, the PSO algorithm usually suffers from premature suboptimal convergence (simply premature convergence or stagnation) which occurs when some poor particles attract the swarm, due to a local optimum or bad initialization, preventing further exploration of the search space. According to [18], although the PSO finds good solutions much faster than other evolutionary algorithms, it usually cannot improve the quality of the solutions as the number of iterations is increased. The rationale behind this problem is that particles converge to a single point, which is on the line between the global best and personal best positions. This point is not guaranteed to be even a local optimum. Proof can be found in [19]. Another reason for this problem is the fast rate of information flow between particles, resulting in the creation of similar particles (with a loss in diversity) which increases the possibility of being trapped in local minima [20]. This feature prevents a standard PSO from being of any practical interest for many of applications. In general, any mechanism that can increase diversity will help in preventing premature convergence. In fact, to overcome this issue, a "hybrid" method can be proposed. By combining APSO with an SA algorithm, we can get a new mixed optimization approach, called APSO-SA.

Using the jumping property of SA can help create greater diversification that causes the algorithm to escape from the local optimum. Fast and adaptive properties of APSO will help with rapid convergence, when SA combines with APSO. As mentioned in section 3.2., SA accepts worse solutions with a probability of $\exp(-\Delta/T)$. When the algorithm becomes trapped in a local optimum valley it can jump from the valley with a probability leading to greater diversity. So, by employing both SA and APSO algorithms to develop a new mixed algorithm (APSO-SA), we can make full use of the strong quick convergence ability of APSO and the strong local search ability of SA and offsets each others' weaknesses.. In fact, APSO-SA has rapid convergence without premature convergence.

In the APSO-SA algorithm, we name every point which is found by (16), the temporary point $x_{id}(p)$ ($x_{id}(p)=x_{id}(t+1)$). If $x_{id}(p)$ is better than $x_{id}(t)$, it will be accepted and if it is worse than $x_{id}(t)$, we will accept it with a probability of $\exp(-\Delta/T)$, ($\Delta = f(x_{id}(p))-f(x_{id}(t))$). This process is performed for all particles. When a temporary point is rejected, it is that we named it a detoured particle $x_{id}(d)$, and it is given back in the opposite direction of the previous movement. These descriptions are formulated by the following equations:.

$$\begin{aligned} & x_{id}(p)=x_{id}(t)+v_{id}(t) \\ & \Delta = f(x_{id}(p))-f(x_{id}(t)) \\ & If \quad \Delta \prec 0 \quad then \\ & \qquad x_{id}(t+1)=x_{id}(p) \\ & If \quad \Delta \geq 0 \quad then \\ & \qquad x_{id}(d)=x_{id}(p)+\alpha * v_{id}(t) \\ & \qquad x_{id}(t+1)=x_{id}(d) \end{aligned} \quad (16)$$

Where,

$$\alpha = \begin{cases} +1 & probability = e^{(-\Delta/T)} \\ -1 & otherwise \end{cases} \quad (17)$$

In general, the proposed APSO-SA algorithm works as follows. First, the algorithm parameters such as number of particles, initial particles and velocities, constants of $C_1$ and $C_2$, $T_0$ and annealing schedule and any other parameters, are initialized. Then, the algorithm starts with the initial swarm as initial solutions. Computing new velocities using the APSO algorithm, temporary positions are calculated. For each particle, $\Delta$ is calculated, if $\Delta \prec 0$ then the solution will be accepted as a better solution, otherwise a worse solution will be accepted with a probability of $exp(-\Delta/T)$ and the detoured particle is turned back to the opposite direction of the traveled route (see (16) and (17)). This procedure causes diversification and escape from the local optimum. This process is iterated for all particles in the swarm. Afterwards, the annealing schedule is performed. If one of the termination conditions is satisfied then the algorithm stops otherwise the proposed procedure is iterated.

The general pseudo-code for the APSO-SA algorithm is given in Appendix A.

***Remarks:***

1. The term $(f(p_{id}(t))-f(x_{id}(t)))$ and $(f(p_{gd}(t))-f(x_{id}(t)))$ are named local and global adaptive coefficients, respectively. In each iteration, the former term defines the

movement step size in the direction of the best position which is found by $i^{th}$ particle in dimension $d^{th}$ and the later term defines the movement step size in the direction of the best optimum point, whichever have been found by the swarm, adaptively. In other words, the adaptive coefficients decrease or increase the movement step size relative to being close or far from the optimum point, respectively. By means of this method, velocity can be updated adaptively instead of being fixed or changed linearly. Therefore, using the adaptive coefficients, the convergence rate of the algorithm will be increased if it is performed by the proportional large or short steps. Here, this fast version of the PSO algorithm is called the fast PSO (APSO).

2. Stochastic optimization approaches have problem dependent performance. This dependency usually results from the parameter setting of each algorithm. Thus, using different parameter settings for the APSO algorithm, which is a stochastic optimization algorithm, will result in high performance variances. In general, no single parameter setting exists which can be applied to all problems. Therefore, all parameters of the APSO should be determined optimally, by trial and error.
3. There are three stopping criteria. The first criterion is related to the maximal number of iterations of the algorithm, the second is when no improvement has been made for a certain number of iterations in the best solution, and the third is when a satisfactory solution is found.
4. The adaptive version of PSO is proposed for continuous variable functions. Moreover, the main idea of fasting can be applied to the discrete form of the PSO [21].
5. Increasing the value of the inertia weight, w, will increase the speed of the particles resulting in more exploration (global search) and less exploitation (local search). On the other hand, decreasing the value of w will decrease the speed of the particle resulting in more exploitation and less exploration. Thus, an iteration-dependent weight factor often outperforms a fixed factor. The most common functional form for this weight factor is linear, and changes with step i as follows:

$$W_{t+1} = W_{max} - \frac{W_{max} - W_{min}}{N_{iter}} \times t \tag{18}$$

Where, $N_{iter}$ is the maximum number of iterations and $W_{max}$ and $W_{min}$ are selected to be 0.9 and 0.1, respectively.

6. The initial temperature $T_0$ and the annealing way play important roles in SA and may affect the performance of the APSO-SA. In the following simulations, the initial temperature is set by the following empirical formula [22]:

$$T_0 = \frac{f(p_{gd})}{ln(0.2)} \tag{19}$$

The $p_{gd}$ is the best position between all particles in the swarm. As for the annealing way, an exponential annealing function, i.e. $T(t+1) = \theta * T(t)$, is employed, where $0 < \theta < 1$ denotes the annealing rate.

7. A stop condition typically can happen, when no improvement has been made for a certain number of iterations or the maximum number of iterations has been reached or when $T_0$ is lower than the smallest typical temperature $(T_{min})$.
8. Lastly, the proposed APSO is still a general optimization algorithm that can be applied to any real world continuous optimization problems.

In this paper, we will apply such an approach for a multi objective function and we will compare the obtained results from the APSO-SA with a standard PSO algorithm for the IEEE 14 bus system.

## 4. Implementation of Suggested Algorithm

For the implemented algorithm we used two cases:

### 4.1 Single –Type Case

In this case, the goal of optimization is to find the best location of the TCSC or SVC in the power system. Therefore, a configuration is represented with two variables as below:

1. The first variable corresponds to the location of the device and contains the numbers of the nodes or branches where the FACTS device (TCSC or SVC) is located. The possible values are identified in Table 1.

**Table 1.** Numbering of the power system elements

| Values | Elements |
|---|---|
| 1 | bus 1 |
| ⋮ | ⋮ |
| $n_n$ | bus $n_n$ |
| $n_{n+1}$ | branch 1 |
| ⋮ | ⋮ |
| $n_n + n_b$ | branch $n_b$ |

2. The second variable indicates the size of the FACTS device. Its value will be normalized in the range of 0 to 1. According to the FACTS devices model, those real values, $z_{\text{realF}}$, can be calculated as below:

$$z_{realF} = z_{min\,F} + (z_{max\,F} - z_{min\,F})z_F \tag{20}$$

Where, $z_{min\,F}$ and $z_{max\,F}$ are respectively minimum and maximum setting value of device and $z_F$ is its normalized value.

Fig. 5(a) shows the single line diagram of the IEEE 14 bus system with a FACTS device. Fig. 5(b) illustrates the configuration of a coded solution for single type FACTS devices.

### 4.2 Multi-Type Case

In this case, the goal of optimization is to find the best location for two FACTS devices (TCSC and SVC). Hence, a configuration is represented with three variables as below [7]:

1. The first element corresponds to the location of the devices and contains the numbers of the elements (nodes and branches) where the FACTS devices are located.
2. The second string indicates the type of devices. A value is assigned to each type of FACTS device: 0 for no devices, 1 for TCSC and 2 for SVC.
3. The third element shows the size of the FACTS devices. It may take discrete values normalized to be in the range of 0 to 1.

Fig. 6 illustrates the configuration of the coded solution for multi type FACTS devices with three coded strings.

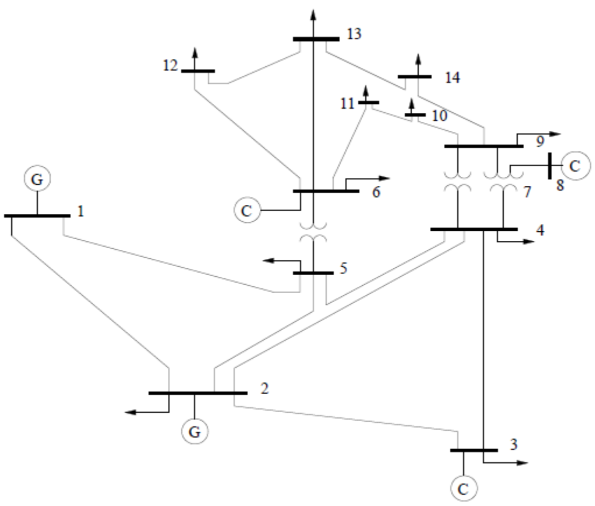


**Fig. 5(a).** Single line diagram of IEEE 14 bus system with FACTS device.


**Fig. 5(b).** Configuration of coded solution for single type FACTS devices.

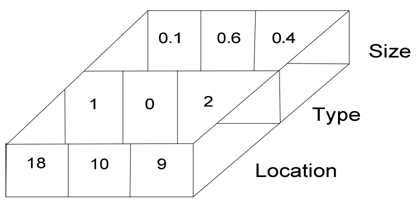


**Fig. 6.** Configuration of coded solution for multi FACTS devices.

## 5. Proposed Objective Function

The main goal of optimization is to minimize the installation costs of FACTS devices and real power losses in power systems and to improve the system's load ability.

The objective function is defined as the sum of three terms with individual criteria. The first one is related to installation cost, the second part of the objective function concerns total real power losses in the power system, and the third term corresponds to increasing load ability.

$$F = IC + P_{LK} + |J - 1| \tag{21}$$

The optimal installation cost of FACTS devices in US$ is given as below:

$$IC = C * S \tag{22}$$

Where, $C$ is the installation cost of FACTS devices in US$/KVAR. The installation cost of the TCSC and SVC are taken from the Siemens database and are reported in [23]. The installation cost of various FACTS devices are given by (23).

$$\begin{aligned} C_{TCSC} &= 1.5S^2 - 713S + 153750 \\ C_{SVC} &= 0.3S^2 - 305S + 127380 \end{aligned} \tag{23}$$

Where, the S is the operating range of FACTS devices in the MVAR.

$$S = |Q_2| - |Q_1| \tag{24}$$

Where, $Q_2$ and $Q_1$ are the reactive power flow in the line after and before installing FACTS device in the MVAR, respectively.

The exact loss formula of a power system with N buses is [24]:

$$P'_{LT} = \sum_{j=1}^{N} \sum_{K=1}^{N} \begin{bmatrix} \alpha_{jK}(P_j P_K + Q_j Q_K) + \\ \beta_{jK}(Q_j P_K - P_j Q_K) \end{bmatrix} \tag{25}$$

Where $P_j$, $P_K$ and $Q_j$, $Q_K$, respectively, are the real and reactive powers injected at buses j and K. The $\alpha_{jK}$, $\beta_{jK}$ are the loss coefficients defined by:

$$\alpha_{jK} = \frac{r_{jK}}{V_j V_K} cos(\delta_j - \delta_K) \tag{26}$$

$$\beta_{jK} = \frac{r_{jK}}{V_j V_K} sin(\delta_j - \delta_K) \tag{27}$$

Where, $r_{jK}$ is the real part of the $jK$ element of impedance matrix ($[Z_{\text{bus}}]$). If single type FACTS devices are used, the total loss can be written as follows [24]:

$$P_{LK} = P_{LT}^{'} - [P_{ic} - P_{jc}] \tag{28}$$

Where more than one device is used at a time, the total loss can be expressed as:

$$P_{LK} = P_{LT}^{'} - \sum [P_{ic} - P_{jc}] \tag{29}$$

$P_{ic}, P_{jc}$ are injected real powers by installed FACTS devices.

In (30) J includes the indicating violation factor of line power flow limits and bus voltage limits [25]:

$$J = \prod_{Line} OVL_{line} \times \prod_{Bus} VS_{Bus} \tag{30}$$

Where, the OVL and VS denote the line over load factor and bus voltage stability index respectively.

$$OVL = \begin{cases} 1 & P_{pq} \le P_{pq}^{max} \\ exp\left(\lambda\left|1 - \dfrac{P_{pq}}{P_{pq}^{max}}\right|\right) & P_{pq} \succ P_{pq}^{max} \end{cases} \tag{31}$$

$$VS = \begin{cases} 1 & 0.9 \le V_b \le 1.1 \\ exp(\beta|1 - V_b|) & otherwise \end{cases} \tag{32}$$

Where, $P_{pq}$ is the real power flow between buses $p$ and $q$, $P_{pq}^{max}$ the thermal limit for the line between $p$ and $q$, $V_b$ the voltage at bus $b$, and $\lambda$ & $\beta$ are the small positive constants both equal to 0.1.

### 5.1 Constraints

The objective function is optimized with the following constraints:

1. Line flow and bus voltage constraints.
   This constraint is defined by (30).
2. FACTS device's constraints

$$(i) \quad -0.8X_L \le X_{TCSC} \le 0.2X_L \tag{33}$$

$$(ii) \quad -100MVAR \le Q_{TCSC} \le 100MVAR \tag{34}$$

Where, $X_{\text{TCSC}}$ and $X_L$ are injected reactance by the TCSC and reactance of the line where the TCSC is installed. The injected reactive power by SVC at the connected bus is $Q_{\text{SVC}}$.

1. Power flow constraints

$$g(V,\theta) = 0 \tag{35}$$

Where,

$$g(V,\theta) = \begin{cases} \left.\begin{matrix} P_t(V,\theta) - P_t^{net} \\ Q_t(V,\theta) - Q_t^{net} \end{matrix}\right\} PQ \quad Bus \\ \left.P_m(V,\theta) - P_m^{net}\right\} PV \quad Bus \end{cases}$$

Where, $P_t$ and $Q_t$ are the calculated real and reactive power for $PQ$ bus, $P_m$ is the calculated real power for $PV$ bus, $P_t^{net}$ and $Q_t^{net}$ are the specified real and reactive power for $PQ$ bus, $V$ the voltage magnitude at different buses and $\theta$ is the voltage phase angle at different buses.

### 5.2 Finding Maximum System Loadability (MSL)

After the maximum numbers of iterations, the value of $J$ is checked for the $p_{gd}$ particle. If it is equal to 1 then the current value of the SL can be met without violating the line flow and bus voltage limit constraints and the $p_{gd}$ particle is saved with its cost of installation and SL. Then, the SL is increased by 1% and again the PSO or APSO-SA algorithms are run. If the value of $J$ for the $p_{gd}$ particle is not equal to 1 then the $p_{gd}$ particle is unable to meet the current SL and the $p_{gd}$ particle with $J = 1$, obtained in the previous run is considered as the best optimal settings and the SL corresponding to that $p_{gd}$ particle is considered as the MSL. The step by step procedure to find the optimal installation cost of FACTS devices and the MSL is shown in Fig. 7.

## 6. Simulation Results

The solutions for optimal location of FACTS devices to minimize the objective function for the IEEE 14 bus system was obtained and discussed below. The test system data are taken from [26]. The location, setting of FACTS devices and optimal objective function value, total real power losses of the power system and maximum system loadability (MSL) are obtained using the PSO and APSO-SA techniques for single-and multi-type devices and are given in Table 2. Before installing the FACTS device the total real power losses of the system is 118kW. In this table, the $P_{pqb}$ , $Q_{pqb}$ and $P_{pqa}$ , $Q_{pqa}$ are real and reactive power flow in the line $p-q$, before and after placing the FACTS device, respectively. These powers are given in P.U. and Sbase is 260MW.

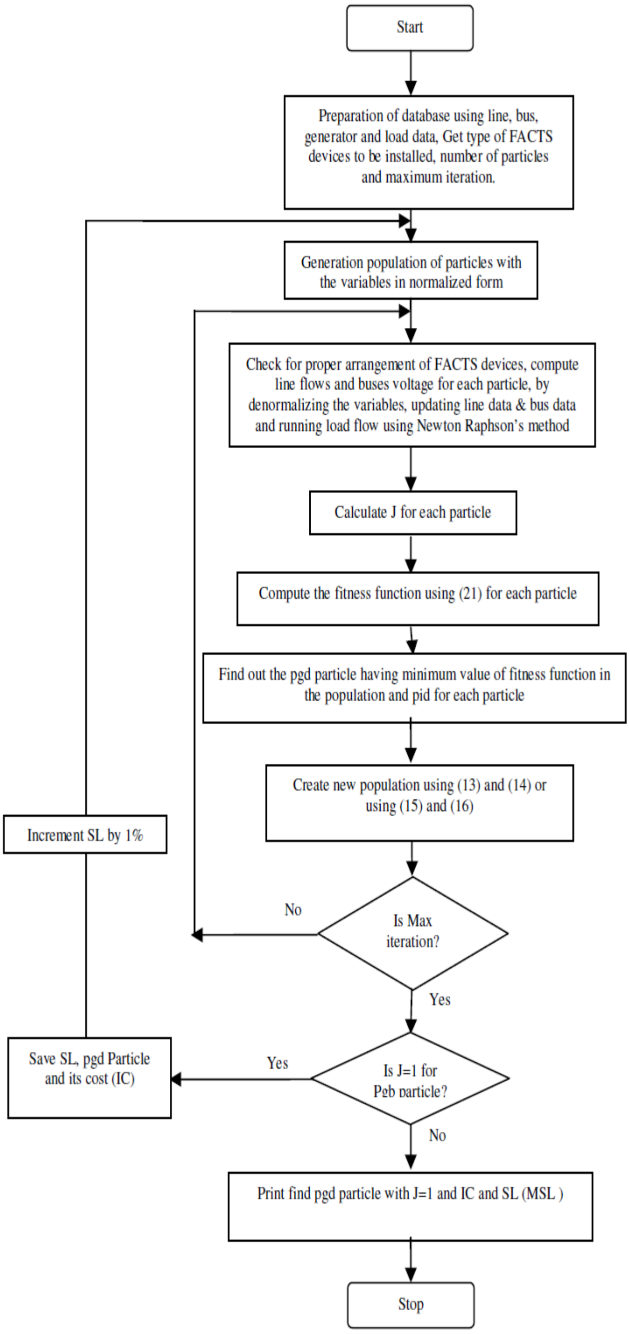


**Fig. 7.** The Flowchart of PSO or APSO-SA algorithm for optimal location of FACTS devices.

It is seen from Table 2 that in the case of the TCSC installation:

1. The suitable place for the TCSC is obtained between 5 and 6 buses using the PSO and PSO-SA methods.
2. The normalized objective function value is reduced from 0.258 (in the PSO method) to 0.237 (in the APSO-SA method).
3. Total real power loss is reduced from 118KW in cases before FACTS device installation, to 59.208KW and 58.8KW in cases after installation with the PSO and APSO-SA methods, respectively.
4. The size of the TCSC is increased from 0.47 pu (in PSO method) to 0.51 pu (in APSO-SA method). Therefore, the cost of installation (IC) is increased from $0.0034*10^6 US\$$ (in the PSO method) to $0.0041*10^6 US\$$ (in the APSO-SA method). In general, the objective function is reduced.
5. The TCSC installation does not effect the MSL.

In the case of SVC installation:

1. The suitable place for an SVC is obtained with bus 13 by using the PSO and PSO-SA methods.
2. The normalized objective function value is reduced from 0.21(in the PSO method) to 0.193 (in the APSO-SA method).
3. Total real power loss is reduced from 118KW in cases before FACTS device installation, to 59.62KW and 58KW in cases after installation with the PSO and APSO-SA method, respectively.
4. The size of the SVC is increased from 0.34 pu (in the PSO method) to 0.41 pu (in the APSO-SA method). Therefore, the cost of installation (IC) is increased from $0.031*10^6 US\$$ (in the PSO method) to $0.032*10^6 US\$$ (in the APSO-SA method).
5. The maximum system loadability is increased from 1.025 (in the PSO method) to 1.029 (in the APSO-SA method).

In the case of a multi type (TCSC and SVC) installation:

1. The suitable place for the TCSC is between bus 5 and bus 6 and the SVC is bus 13 using the PSO and PSO-SA methods.
2. The normalized objective function value is reduced from 0.2215(in the PSO method) to 0.2057 (in the APSO-SA method).
3. Total real power loss is reduced from 118KW in cases before FACTS device installation, to 55.96KW and 53.95KW in cases after installation with the PSO and APSO-SA method, respectively.
4. The sizes of the TCSC and SVC are increased from 0.6 pu and 0.36 (in the PSO method) to 0.64 pu and 0.42 pu (in the APSO-SA method), respectively. Therefore, the cost of installation (IC) in this case is increased from $0.04801*10^6 US\$$ (in the PSO method) to $0.0504*10^6 US\$$ (in the APSO-SA method).
5. The maximum system loadability is increased from 1.08 (in the PSO method) to 1.1 (in the APSO-SA method).

The convergence speed of the objective function using the PSO and APSO-SA techniques for single and multi-type devices are shown in Figs. 8 to 10, respectively. It is seen from these figures that the speed convergence of the objective function is increased in the case of the APSO-SA method.

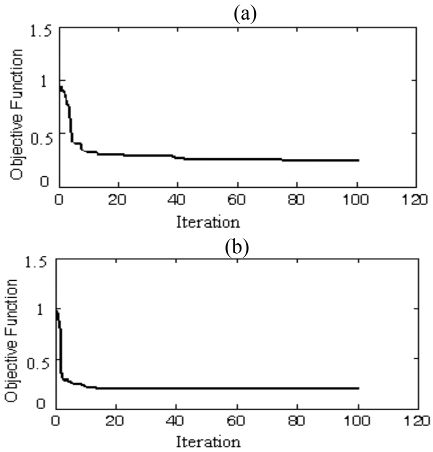


**Fig. 8.** Objective function convergence when TCSC is installed: a) PSO b) APSO-SA.

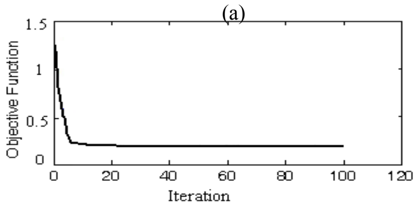


**Fig. 9(a).** Objective function convergence when SVC is installed based PSO algorithm.

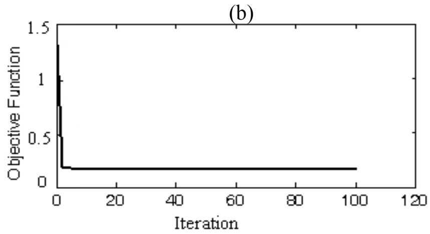


**Fig. 9(b).** Objective function convergence when SVC is installed based APSO-SA.

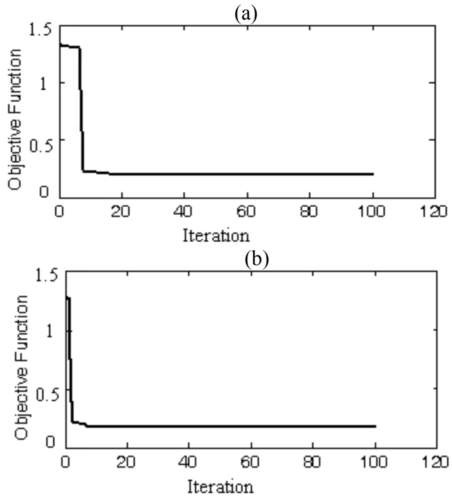


**Fig. 10.** Objective function convergence when TCSC & SVC are installed: a) PSO b) APSO-SA.

## 7. Conclusion

In this paper, the optimal location of FACTS devices is found to minimize the cost of installation and total real power losses of power systems and improve system loadability, for single and multi-type FACTS devices using APSO-SA techniques and its results compared with standard PSO techniques. Simulations were performed on the IEEE 14 bus system. Optimizations were performed on a variety of parameters, namely location of the FACTS devices, and their settings in the line for single-type FACTS devices. In the case of multi-type FACTS devices, the type of device to be placed is also considered as a variable in the optimization. In both single-and multi-type devices, it is observed that:

**Table 2.** Line flows before and after installing single- and multi-type FACTS devices, optimal setting and optimal objective function value, cost of installation, total real power losses and MSL with PSO and APSO-SA algorithm

| MSL | IC (*$10^6$ US$) | Total power loss(KW) | Normalized objective function | Device Setting (pu) | Qpqa ( pu) | Ppqa (pu) | Qpqb (pu) | Ppqb (pu) | To Bus q | From Bus p | Type of device used | Case |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0034 | 59.208 | 0.258 | -0.47 | 1.01 | 1.45 | 1.026 | 1.385 | 6 | 5 | TCSC (PSO) | |
| 1 | 0.0041 | 58.8 | 0.237 | -0.51 | 0.979 | 1.5 | 1.026 | 1.385 | 6 | 5 | TCSC (APSO-SA) | Single Type |
| 1.025 | 0.031 | 59.62 | 0.21 | -0.34 | -0.037 | -0.161 | -0.058 | -0.135 | 13 | | SVC (PSO) | |
| 1.029 | 0.032 | 58 | 0.193 | -0.41 | 0.01 | -0.164 | -0.058 | -0.135 | 13 | | SVC (APSO-SA) | |
| 1.08 | 0.04801 | 55.96 | 0.2215 | -0.6<br>-0.36 | 1.001<br>-0.01 | 1.817<br>-0.158 | 1.026<br>-0.058 | 1.385<br>-0.135 | 6<br>13 | 5 | TCSC<br>SVC (PSO) | Multi Type |
| 1.1 | 0.0504 | 53.95 | 0.2057 | -0.64<br>-0.42 | 0.985<br>-0.056 | 1.304<br>-0.142 | 1.026<br>-0.058 | 1.385<br>-0.153 | 6<br>13 | 5 | TCSC<br>SVC (APSO-SA) | |

1. The APSO-SA algorithm improves acceleration of the convergence speed, in comparison with the standard PSO algorithm.
2. The APSO-SA algorithm expands the search space, in comparison with the standard PSO algorithm.
3. Decreasing the real power losses of power system with the optimal location of FACTS devices.
4. System loadability can be improved further after optimal placing of the FACTS devices.
5. In the case of multi-type FACTS devices, improved objective function was experienced in comparison with single-type cases.

## Appendix A

The pseudo code for the APSO-SA algorithm is:

Initialize APSO-SA parameters.

(APSO procedure)

Loop:

REPEAT

For each particle i;

Evaluate the objective function of the particle i, i.e. $f(x_{id}(t))$;

Update the global and local best positions and their objective function values;

Calculate the velocity by (15);

(SA procedure)

Calculate the temporary position $x_{id}(p)$ by (16);

Using (17) calculate the new position;

Perform annealing schedule;

END of Loop

If stop condition is true then stop else go to Loop;

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