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Discussion: On 'Negative apparent resistivity in dipole–dipole electrical surveys' (Jung, H.K., Min, D.J., Lee, H.S., Oh, S.H., and Chung, H., *Exploration Geophysics*, 40, 33–40)

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The DC resistivity method is a well known geophysical exploration technique and is widely applied in mining exploration, groundwater surveys, and engineering and environmental investigations. Negative apparent resistivities, especially in 2D surface dipole–dipole surveys, are often observed and generally have been regarded as measurement errors or the effect of noise. Cho et al. (2002) insisted that if the earth is isotropic and flat, and the survey line is straight, apparent resistivity is always positive regardless of subsurface structures. On the contrary, Jung et al. (2009) insisted that negative apparent resistivity could be caused by a geological structure. In this discussion, we re-examine the possibility of negative apparent resistivity since the conclusions of these two papers are entirely contrary. We assume that a 2D resistivity survey is performed along a straight line on the flat surface of an isotropic earth, as Jung et al. similarly adopted. We first describe the behaviour of surface charge, electric field, potential, and current inside and outside an inhomogeneity. We carry out 3D finite-element modelling for the same U-shaped model presented by Jung et al. as a typical model that yields negative apparent resistivity. Then, we compare our result with that of Jung et al. and discuss whether negative apparent resistivity is really possible under the stated conditions.

First, let us explain the behaviour of electric field and potential difference. Suppose that an inhomogeneity with resistivity ρ is surrounded by a uniform medium having resistivity ρ_b, and $\mathbf{E}_P$ describes the primary field behaviour in the absence of any inhomogeneity. This primary field generates charges on the surface of the inhomogeneity in such a way that the total amount of these charges equals zero, by the principle of charge conservation. These surface charges in turn generate the secondary electric field $\mathbf{E}_S$ in accordance with Coulomb's law. These fields, $\mathbf{E}_P$ and $\mathbf{E}_S$, have different magnitudes and directions (Kaufman, 1992). Correspondingly, the total field $\mathbf{E}$ is given by the sum of the primary and the secondary electric field:

$$\mathbf{E} = \mathbf{E}_P + \mathbf{E}_S \tag{1}$$

Similarly, the potential field, V, can be represented as the sum of the primary potential, V_P, and secondary potential fields, V_S,

$$V = V_P + V_S \tag{2}$$

for the points inside and outside the inhomogeneity. Note that only the secondary field contains information about the resistivity, shape, dimensions, and location of the inhomogeneity.

Let ΔV be the potential difference measured between the two points M and N on the earth's surface. The apparent resistivity is calculated from the measured potential difference and a geometric factor, and is defined as

$$\rho_a = \rho_b \frac{\Delta V}{\Delta V_P} = \rho_b \frac{\Delta V_P + \Delta V_S}{\Delta V_P} = \rho_b \left(1 + \frac{\Delta V_S}{\Delta V_P}\right), \tag{3}$$

where ΔV_P is the primary potential difference in the absence of any inhomogeneity and $\rho_b/\Delta V_P$ can be regarded as a geometric factor. Thus, conversion to apparent resistivity can be thought of as simple compensation for the primary potential difference or electric field, which has no information about the inhomogeneity. The measured potential difference can be expressed as follows:

$$\Delta V = \Delta V_P + \Delta V_S = \int_M^N (\mathbf{E}_P + \mathbf{E}_S) \cdot d\mathbf{L}. \tag{4}$$

In a surface 2D survey, the locations of the potential electrodes M and N are usually arranged so that ΔV_P must be positive. Then, a negative apparent resistivity appears only when the secondary potential difference is negative and greater in magnitude than the primary potential difference.

Let us assume that a current source I is located on the surface of a homogeneous and isotropic half-space. Due to the semi-spherical symmetry, the direction of the primary field is perfectly radial and equipotential lines appear as concentric circles on the surface. Accordingly, the primary potential shows maximum gradient along the radial direction; thus, the primary potential difference in the direction of a survey line also becomes maximum. However, the secondary potential does not show a maximum gradient along the direction of the survey line, unless there is an extraordinary symmetry between the survey line and the inhomogeneity. This means that the direction of the secondary electric field usually does not coincide with the direction of the survey line, and the secondary field component measured in the direction of the survey line is always less than or at most equal to the secondary electric field itself in magnitude. Consequently, the secondary potential difference in the direction of the survey line should be less than or equal to the maximum secondary potential difference. Thus we can write

$$\mathbf{E}_P^{\text{measure}} = |\mathbf{E}_P| \quad \text{and} \quad \mathbf{E}_S^{\text{measure}} \le |\mathbf{E}_S|, \tag{5}$$

where the superscript measure indicates the field component in the direction of the survey line.

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From equation 4, we can see that the apparent resistivity becomes negative if the primary and secondary electric fields are in opposite directions and the magnitude of the secondary field in the direction of the survey line is greater than that of the primary electric field:

$$\mathbf{E}_{\mathrm{P}}^{\mathrm{measure}} < \mathbf{E}_{\mathrm{S}}^{\mathrm{measure}}. \quad (6)$$

In general, the primary and secondary fields are different in magnitudes and directions, but the secondary electric field cannot be greater than the primary field in magnitude. When the inhomogeneity is conductive, the direction of the secondary field inside the inhomogeneity is opposite to that of the primary field; thus, the total field decreases. As the inhomogeneity becomes more conductive, the secondary field inside the inhomogeneity increases. In the limit of a perfect conductor, the secondary field becomes equal to the primary field in magnitude and the total electric field vanishes within the inhomogeneity since the potential remains constant at all points within the perfect conductor. The secondary field outside the inhomogeneity depends on the distance from surface charges but cannot exceed the primary field in magnitude. Thus we can say

$$|\mathbf{E}_{\mathrm{P}}| \geq |\mathbf{E}_{\mathrm{S}}| \quad (7)$$

at all points inside and outside the inhomogeneity. By equations 5 and 7, therefore, equation 6 is invalid if the survey line is straight and the earth is flat and isotropic; the negative apparent resistivity cannot be measured under the given conditions.

Another important issue, which is ignored by Jung et al., is the possibility of numerical error in 3D numerical modelling. There are two basic approaches to 3D numerical modelling: differential equation and integral equation methods. Both methods are useful and necessary. Differential equation solutions are easier to set up, and are preferable for modelling complex geology because the entire earth is discretized. Integral equation methods involve more difficult mathematics, but they are less expensive because only anomalous regions are discretized (Ting and Hohmann, 1981). Here, we have used the finite-element method for 3D resistivity modelling and compared its result with that of Jung et al., who adopted the finite-difference method.

It is an important first step to check the validity and accuracy of the numerical modelling, particularly when the material property contrast is fairly high, such as in case of the 3D model of Jung et al., where the resistivity contrast is the fourth power of 10. The analytic solution of a simple subsurface model such as a 1D earth is usually used to check the validity of the numerical modelling adopted. Convergence testing of the numerical calculation is also a good measure of its stability, since the calculated value should converge to a constant value as the model is discretized finer. Another important criterion for the validity of numerical calculations is reciprocity in the DC resistivity problem: interchanging the source and receiver locations must produce the same results. Satisfying reciprocity is a necessary but insufficient condition for the validity of the adopted numerical technique. Thus, the degree to which reciprocity is satisfied is closely related to the accuracy of the finite-difference and finite-element solutions (Snyder, 1976; Okabe, 1978; Pridmore et al., 1981).

To check the validity of our 3D finite-element modelling (FEM) program, the responses of two-layered models were calculated using our FEM code and compared with the analytic solutions. The resistivity and thickness of the top layer were assumed to be 100 Ω.m and 10 m, respectively. The resistivity of the bottom layer was in the range of 0.01–10 Ω.m. A dipole–dipole array was deployed, with dipole length a = 10 m. The numerical results are shown in Figure 1 by solid circles and

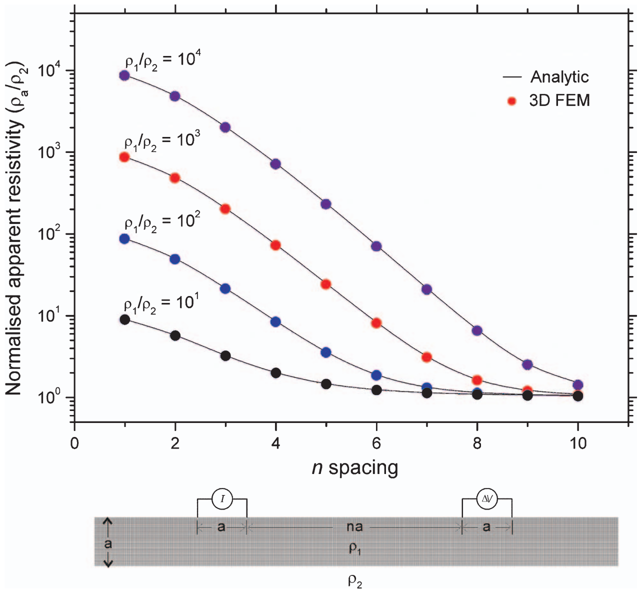


Fig. 1. Comparison of the analytic and numerical solution with finite element discretization over a two-layered earth model.

the analytic solutions for the model by solid lines. The numerical results approach the analytic solution with an absolute accuracy of better than 3 percent. Further tests (not shown) made with the integral equation solution (Beasley and Ward, 1986; Cho, 1989) for a 3D cube generally indicated good agreement with an absolute accuracy within 3 percent. The results obtained with the finite-element method for the 3D cube model satisfy reciprocity to within ~1 percent. Thus, we can be sure of the validity of our 3D finite-element modelling program.

Jung et al. pointed out that negative apparent resistivity could be measured if the potential increases with distance from a current electrode, which is obviously true. Our major consideration, however, is whether this phenomenon could occur along a straight line or not since the main issue is the negative apparent resistivity in 2D dipole–dipole measurements. As a typical model that can show this phenomenon and thus produce negative apparent resistivity, they presented a U-shaped conductive model shown in Figure 2. The conductivities of the anomalous body and background medium are 10 S/m and 0.001 S/m, respectively. Because of the extraordinarily high conductivity contrast, 10^4, it is basically very difficult to get accurate calculations for this model, no matter what kind of numerical modelling algorithm is adopted. In this case, discretization should be very fine to obtain accurate solutions, but such fine discretization result in matrices that are rather unwieldy to handle, especially on a personal computer.

Figure 3*a* is the apparent resistivity pseudo-section for the model shown in Figure 2, presented by Jung et al. They used a 3D resistivity modelling program (Spitzer, 1995) based on the finite-difference method, and used very fine node spacing, 1/20 of the dipole length, to provide higher accuracy. Many negative apparent resistivity data appear in the pseudo-section, which apparently supports their conclusion. Carefully examining the modelling results, however, we could find some problems that cannot be neglected. First, the results do not satisfy reciprocity. As depicted in Figure 2, the survey line is perfectly symmetrical with respect to the centre of the U-shaped body in the direction of the survey line. Thus, apparent resistivity must be symmetric about the centre of the U-shaped body to satisfy reciprocity. Let us illustrate this problem with two apparent resistivity data:

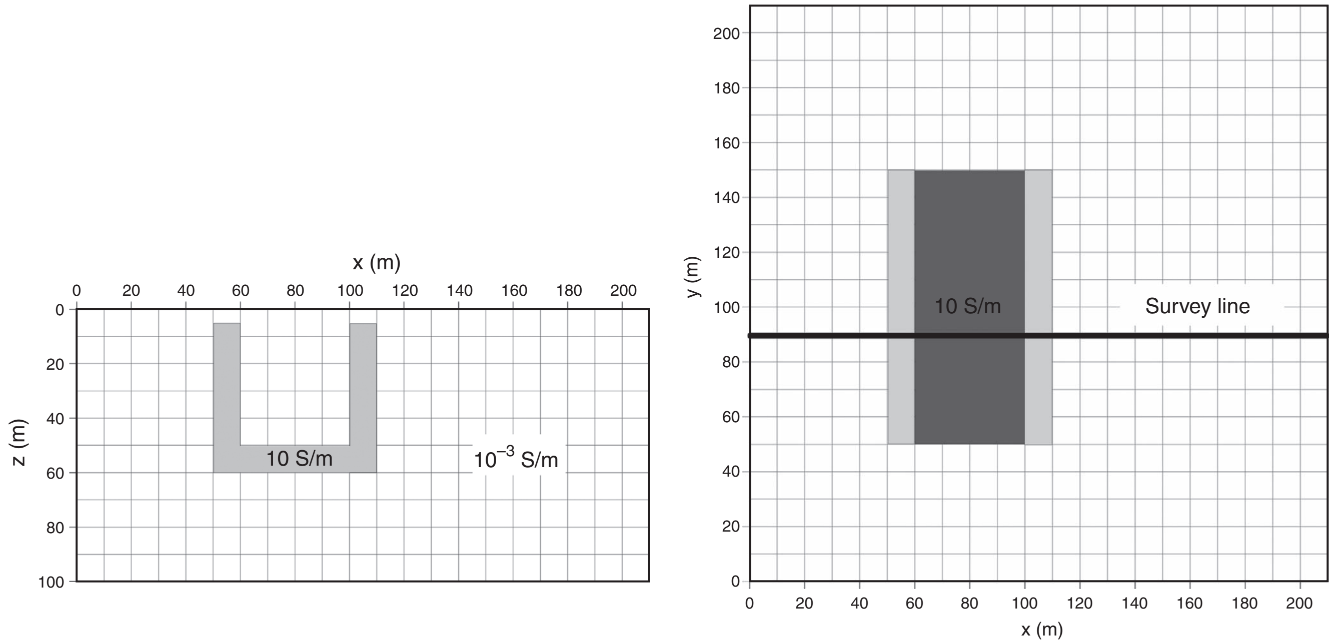
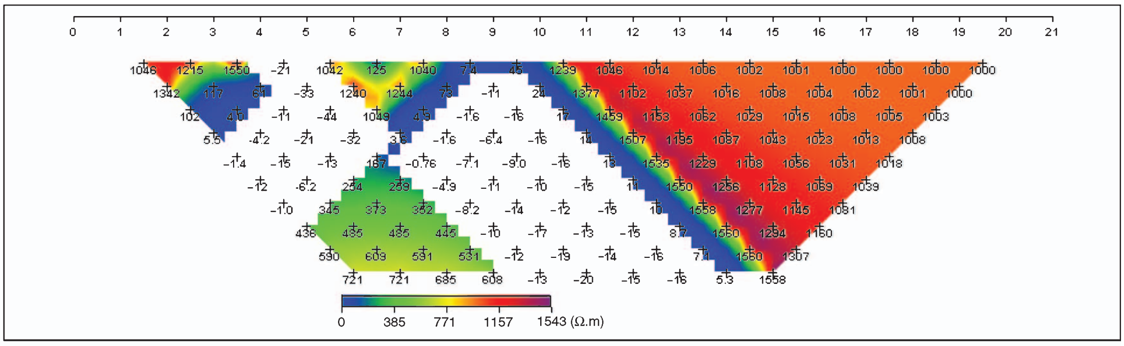
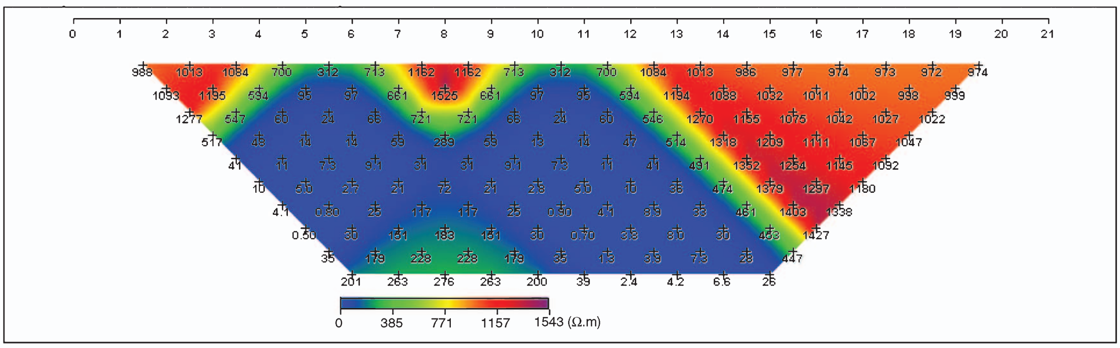


Fig. 2. The U-shaped low-resistivity model presented by Jung et al. (2009).



(*a*) Finite difference method (Jung et al., 2009)



(*b*) Finite element method (this study)

Fig. 3. Apparent resistivity pseudo-sections from the finite difference method (FDM), and finite element method (FEM).

1. the datum when the four electrodes are located at station numbers 4, 5, 6, and 7, and
2. the datum when the four electrodes are located at the station number 9, 10, 11, and 12.

Because of symmetry, the two apparent resistivity values must be identical, but they are quite different from each other; they do not satisfy reciprocity. The error is too big, even considering that the boundary could cause some numerical errors because the anomalous body is located at the left part of the survey line. As mentioned in the previous paragraph, the finite-difference and finite-element solutions to the DC resistivity problem mathematically ensure reciprocity, and the degree to which reciprocity is satisfied is closely related to the accuracy of the solutions. Another problem with their result is that negative apparent resistivity occurs when $n = 1$. For the given model, background resistivity is 1000 Ω.m and the depth to the topmost edge of the anomalous body is 5 m. Furthermore the dipole length is 5 m. For such survey parameters and model, we cannot expect negative apparent resistivity. Note that a two-layered model with resistivity, 1000 Ω.m, and first layer thickness, 5 m, yields apparent resistivity values greater than several hundred Ω.m when the dipole length is 5 m and $n=1$, even if the second layer is a perfect conductor. All these observations strongly imply that the modelling result by Jung et al. has a numerical accuracy problem.

Figure 3*b* shows our calculation results: the apparent resistivity pseudo-section obtained from the finite-element method for the same U-shaped model. There is no negative apparent resistivity in the section and apparent resistivity is symmetric about the centre of the U-shaped body. Also, apparent resistivity when $n=1$ is always greater than 300 Ω.m. We further calculated the response of a similar U-shaped model with a short strike-length using the integral equation method (not shown here) and obtained the same conclusion: the model does not produce any negative apparent resistivity data.

In this discussion, we have reinvestigated the problem of negative apparent resistivity through theoretical considerations and numerical testing. Theoretically analysing the behaviour of surface charge, electric field, potential, and current inside and outside an inhomogeneity, we confirm that the apparent resistivity must have a positive value if a 2D resistivity survey is performed along a straight line on the flat surface of an isotropic earth. We carefully re-examined the apparent resistivity pseudo-section presented by Jung et al. for a U-shaped model as a typical model producing negative apparent resistivity, and found the pseudo-section does not satisfy reciprocity in spite of the symmetry of the model and survey line; some numerical error, which cannot be ignored, might be involved in their calculations. Furthermore, we performed 3D finite-element modelling for the same U-shaped model, and the result does not produce any negative apparent resistivity over the whole apparent resistivity pseudo-section. Consequently, we are sure that negative apparent resistivity cannot be measured under the given conditions.

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