Embedded Distributivity

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Yoon-kyoung Joh. 2010. Embedded Distributivity. *Language and Information 14.2*, 17–32. Distributivity has been one of the central topics in formal semantics. However, no due attention has been paid to embedded distributivity that very frequently occurs in natural languages. In this paper, I propose a formal analysis for embedded distributivity. In analyzing embedded distributivity, I employ no complicated mechanisms but pluralization. Since distributivity is reduced to plurality as Landman (2000) argues, employing plural formation is not an ad hoc approach to embedded distributivity. That is, the plural variable inserted in the process of deriving embedded distributivity is motivated in a principled manner since the pluralization occurs inside a pluralization operator. Moreover, I point out that the plural variable made available is not restricted to entities. (**Hankuk University of Foreign Studies**)

1. Introduction

The sentence in (1.a) generates a distributive interpretation, evoking the $\forall\exists$-structure illustrated in (1.b). The formula in (1.b) describes three essential components involved with distributivity. In this paper, following Choe (1987), I will call the plural element that serves as the distributive antecedent such as *the students* a Sorting Key (henceforth, SrtKy) and the indefinite element like *one ball* a Distributed Share (henceforth, DstrShr). The relation-denoting expression such as *have* will simply be called a Relation.

(1) a. The students have one ball each.

b. $\forall x[x \leq \text{the students}(X) \rightarrow \exists y[\text{ball}(y) \ \& \ \text{have}(x,y)]]$

Yet, distributivity expressed in a sentence is not always as simple as the one shown in (1). More than one distributive relation can occur in a sentence and they can be embedded to one another. These are the cases that this paper will mainly be interested in. For example, there are two distributive relations in (2). First, in (2), *the baskets* is in a distributive relation with *two apples* and the relation is defined via the preposition *in*. Another distributivity involved in (2) resides at the sentential level. The entire phrase in the subject *two apples in each of the*

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baskets serves as the distributive antecedent in relation with the noun phrase *two leaves.* In the second case, the relation-denoting expression is the main verb *have.* Interestingly, the two distributive relations are not parallel to each other but are embedded to each other.

(2) Two apples in each of the baskets have two leaves each.

Embedded distributivity not only occurs in the nominal domain. It can also appear in association with the event. In (3), one distributive relation is found between *the boys* and *two ties* in the relation of buying but there is another distributivity that is associated with the plural event described in the main clause. The noun phrase *two stores* in the prepositional phrase is in a distributive relation with the plural event where each of the boys bought two ties. That is, each of the plural events where the boys bought two ties each took place in two stores in (3).

(3) The boys bought two ties in two stores each.

In this paper, I will provide a formal semantic analysis for sentences like (2) and (3). To do so, I organize this paper as follows. First, I discuss how distributivity is to be analyzed, introducing my previous work, Joh (2008b). In section 3, I analyze embedded distributivity on the basis of the account discussed in section 2 and briefly point out what implication my analysis has. Section 4 concludes this paper.

2. Distributivity

In this section, I will briefly introduce my previous work that analyzes distributivity in terms of a pluralization operator that has the ∀∃-structure inside it. My claim that pluralization essentially generates both forward and inverse distributive interpretations has been made on the basis of Landman (2000) who reduces distributivity to semantic plurality in terms of the definition in (4). The formula in (4) shows us how Landman (2000) defines distributivity in terms of pluralization. Under the condition that a predicate is a set of atoms which is abbreviated as AT in (4), we can get a distributive reading when the predicate is pluralized and applies to another plural. Landman (2000) argues that this is a linguistic fact and provides a proof as in (5).

(4) If P is a set of atoms then: $\alpha \in *P$ iff $\forall a \in \mathrm{AT}(\alpha) : a \in P$

(5) Assume that P is a set of atoms. Assume that every atomic part of α is in P. By definition of *P, if every atomic part of α is in P then their sum is in *P. Hence $\cup(\mathrm{AT}(\alpha)) \in *P$. Since D is an atomic part-of structure, $\alpha = \cup(\mathrm{AT}(\alpha))$. Hence $\alpha \in *P$. Assume $\alpha \in *P$. Since D is an atomic part-of structure, *P is itself also an atomic part-of sub-structure of D with set of atoms P. Hence If $\alpha \in *P$, every part of α is also in *P. If every part of $\alpha \in *P$, then every atomic part of α is in *P, and – since P is the set of atoms in *P – it follows that every atomic part of α is in P.

The essence of (4) and (5) is as follows. The grammar has only a single operation that forms semantically plural predicates out of semantically singular predicates: the *-operation leads to plural nouns in the nominal domain while the same operation creates the distributive interpretation in the verbal domain. That is, there are two modes of predication. Singular predication applies a predicate to singular entities while plural predication applies a plural predicate distributively to a sum of singular individuals.

The example with a distributive quantifier in (6) empirically attests Landman's (2000) claim that distributivity is, in fact, semantic plurality. In (6), the number of toys is multiplied by the number of girls and thus six toys were bought in total. The fact that we can get the total number of elements involved in a distributive interpretation through multiplying the number of the DstrShr by that of the SrtKy shows us that distributivity is plurality. After all, plurality amounts to multiplying.

(6) The two girls bought three toys each.

Based on Landman (2000), I have proposed a pluralization operator defined as in (7). In the denotation of the pluralization operator, there are three variables — Z, P and R. The variables mirror the three essential components of distributivity. Z is the distributive antecedent (SrtKy) and P is the Distributed Share (DstrShr). R expresses the relation between the SrtKy and the DstrShr. The pluralization operator in (7) is evoked not only by an overt distributive marker but also by a covert distributive particle.

(7) $[[*_{ij}]] = \lambda P_{<\alpha,t>}.\forall z[z_{<\alpha>} \in Z_{i<\alpha,t>} \rightarrow \exists x_{<\alpha>}[P(x) \,\&\, R_{j<e,<\alpha,t>>}(x)(z)]]$

In the denotation of the pluralization operator, the semantic type of the variables Z and P is $<\alpha, t>$. The former is a plural set while the latter is an atomic set. The relation-denoting variable R is of type $<e, <\alpha, t>>$. In all the variables, type $<\alpha>$ that can be either type $<e>$ or type $<v>$ expresses the parallel between the nominal domain and the verbal domain since not only the nominal but also the eventual adverbial can serve as the SrtKy and the DstrShr as shown in (8). In (8.a), John wears a (single) necktie each time when he goes to work while, in (8.b), each of the students who played did so loudly. To reflect this, I have defined the semantic type of Z and P as $<\alpha, t>$.

(8) a. John always wears neckties to go to work.

b. Haksayng-tul-i sikkurupkke-tul nol-ass-ta.
Student-IPM loudly-EPM play-Pst-Dec.
'The students played loudly.'

The reason why the relation variable R has α in its type is for its flexibility. The variable R incorporates intransitives, transitives, and ditransitives. Furthermore, in the denotation of the pluralization operator, the type $<\alpha>$ is assigned to the variables z and x. The lower-case of the variables z and x indicates that they are singular. The variable z is necessarily a pure atom since it constitutes the sub-parts of a set plural. However, the variable x is either a pure or an impure atom.

Even more, the pluralization operator I have proposed is designed to apply a group-forming operation when the DstrShr is plural.[1] The lower-case variable x ensures that the DstrShr must be atomic. When the predicate that comes to serve as the DstrShr is singular in itself, nothing happens and the DstrShr is a pure atom. However, when the predicate that comes to function as the DstrShr is semantically plural, a group-forming operation takes place and an impure atom is created out of the plural predicate. A pluralization operator can only apply to a set of atoms. Therefore, when the DstrShr is plural, a group-forming operation has to occur to shift the plural into an impure atom correlate. For example, the process can be explicated as in (9). When the pluralization operator is evoked, the plural DstrShr *three baskets* goes through group formation and becomes *a group of three baskets.* Then, the pluralization operator can apply to it and the group with plural members is pluralized as in *two groups of three baskets.*

(9) three baskets
→ a group of three baskets (group formation)
→ two groups of three baskets

Zabbal (2002) notes that the same process is evidenced in Arabic, as shown in (10). That is, in Arabic, a broken plural that corresponds to a group is systematically used to make a plural out of a plural. In fact, Lakoff (1970), Choe (1987), and Landman (2000) describe the group denotation of the plural DstrShr. The pluralization operator defined in (7) can explain why the group reading arises when the DstrShr is plural in a principled manner. Since the pluralization operator can apply only to sets of atoms as insured by the singular variable x inside the pluralization operator, group formation must take place when the DstrShr is apparently plural in its own. This additional process of group formation changes the plural into an impure atom.

(10)	bayt-un	singular	'a house/rent'
	buyuut-un	broken plural	'houses' → 'family'
	buyuut-aat-un	sound plural of broken plural	'families'

To get compositional facts correctly, I assume two indices for the variables that remain free and are ready to be lambda-abstracted in order to be filled with a proper value. The indices capture the dependency character of distributivity and make sure that each variable gets the right value. That is, in my system, the indices constrain the pluralization operator that is defined in a highly flexible manner. This is one of the crucial aspects I depart from Zimmermann (2002). In addition, for compositionality, I employ Bittner's (1994) λ-abstraction rule defined in (11).[2]

[1] Both sums and groups are plural terms. The former is composed of discrete entities while in the latter the entities are lumped into one set. Thus, the group individual is semantically plural but grammatically singular. For this reason, singular entities constitute pure atoms while group entities constitute impure atoms. Since the group has plural members but is regarded as a singular, it can be utilized to make a plural out of a plural.

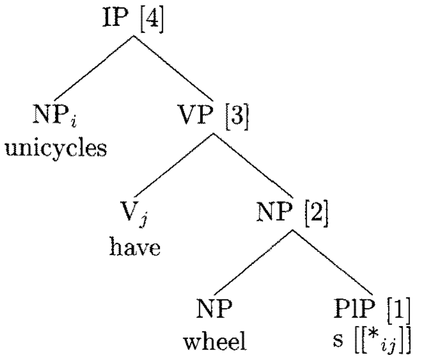
[2] Lambda abstraction defined in (11) is a rather liberated rule compared to Heim and Kratzer's (1998) Predicate Abstraction rule. In brief, I would like to note that Bittner's rule has wider

(11) Let α have a translation $[[\alpha]]$ and let the index 'i' be the index of either α or a sister of α, and let $[[\alpha]]$ contain a variable u with index 'i.' Then $\lambda u_i.[[\alpha]]$ is a translation of α.

With the tools introduced above, now we can analyze distributivity in a compositional fashion. First, I will show how the sentence with a dependent plural can be analyzed. The dependent plural is a distributive particle so that it first is translated into a pluralization operator in (7) that is placed under the PlP phrase.[3] Then, lambda abstraction defined in (11) occurs twice: one for the Relation variable and the other for the SrtKy variable. The last formula generated in (12.b) correctly captures the meaning of (12.a): unicycles each have a (singular) wheel.

(12) a. Unicycles have wheels.

b.



$[[1]] = \lambda P.\forall z[z \in Z_i \rightarrow \exists x[P(x) \; \& \; R_j(x)(z)]]$

$[[2]] = \forall z[z \in Z_i \rightarrow \exists x[\text{wheel}(x) \; \& \; R_j(x)(z)]]$
<Function Application>

$[[2]] = \lambda R_j.\forall z[z \in Z_i \rightarrow \exists x[\text{wheel}(x) \; \& \; R_j(x)(z)]]$
< λ-abstraction>

$[[3]] = \forall z[z \in Z_i \rightarrow \exists x[\text{wheel}(x) \; \& \; \text{have}(z, x)]]$
<Function Application>

$[[3]] = \lambda Z_i.\forall z[z \in Z_i \rightarrow \exists x[\text{wheel}(x) \; \& \; \text{have}(z, x)]]$
< λ-abstraction>

$[[4]] = \forall z[z \in [[\text{unicycles}]] \rightarrow \exists x[\text{wheel}(x) \; \& \; \text{have}(z, x)]]$
<Function Application>

To be more specific, I would like to explain the semantic derivation described in (12) step by step. The denotation in node [[1]] is the very extension of the

application than Heim and Kratzer's (1998) rule of the same kind. The former can account for various constructions such as possessed nominal phrases in West Greenlandic Inuit and the hanging topic construction in German that the latter cannot address, as discussed in Bittner (1994) and Zimmermann (2002). Since I believe that Bittner's rule is more desirable in that it does not have to postulate stipulative approaches such as covert movement in various circumstances, I employ the rule in (11), instead of relying on Heim and Kratzer's more standard form.

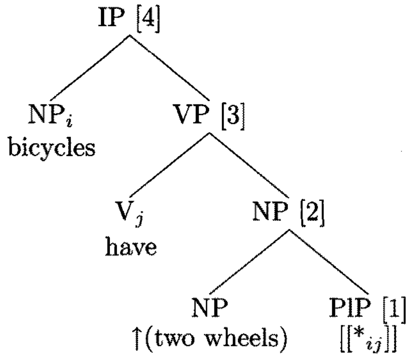
[3] The PlP is used as the abbreviation of the Plurality Phrase. In this paper, all kinds of distributive particles are being placed under the PlP, regardless of their overtness or covertness,

pluralization operator. This denotation directly applies to its syntactic sister to the effect that the value of the variable P is filled with *wheel.* Yet, the resulting proposition-denoting saturated expression cannot apply to the next constituent that it must be combined with. To open it up, lambda abstraction of the variable R applies over the index j. Now, the unsaturated expression can apply to its next syntactic sister *have* at node [[3]] via functional application. The same process of applying lambda abstraction that is followed by function application is repeated one more time. The proposition of type $< t >$ is opened up via lambda abstraction of the variable Z over the index i. Only after this remedy, the SrtKy *unicycles* can be factored in via function application.

The exact mechanism for (12.a) also explains the distributive sense of (13.a).[4] The anti-quantifier *each* translates into the * operator under the Plurality Phrase. The denotation of the pluralization operator first applies to the predicate it forms a constituent with. The value of the variable R is determined by the co-indexed relation-denoting main verb *have.* The distributive antecedent *bicycles* is factored in via lambda abstraction over the index i and then functional application takes place.

(13) a. Bicycles have two wheels each.

b.



$[[1]] = \lambda P.\forall z[z \in Z_i \rightarrow \exists x[P(x)\ \&\ R_j(x)(z)]]$

$[[2]] = \forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two wheels})(x)\ \&\ R_j(x)(z)]]$

<Function Application>

$[[2]] = \lambda R_j.\forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two wheels})(x)\ \&\ R_j(x)(z)]]$

< λ-abstraction>

$[[3]] = \forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two wheels})(x)\ \&\ \text{have}(z, x)]]$

[4] As the two semantic derivations in (12) and (13) show us, the dependent plural and the anti-quantifier are uniform fundamentally as I have claimed in Joh (2008b, 2010). In addition, the same interpretation can arise under the covert operation of the two kinds of distributivity markers. Yet, there are two minor differences among them. In contrast to anti-quantifiers, dependent plurals are, to large extent, affected by world knowledge and thus their interpretation can fluctuate between the distributive reading and the genuine plural reading. A difference between overt distributivity markers and covert distributivity markers is also found in the respect that the former generates distributivity obligatorily while the latter produces it optionally, as Choe (1998) notes. However, I would like to point out that the basic universal-existential structure that they generate is uniform, as long as distributivity is projected.

<Function Application>
$[[3]] = \lambda Z_i.\forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two wheels})(x)\ \&\ \text{have}(z,x)]]$
< λ-abstraction>
$[[4]] = \forall z[z \in [[\text{bicycles}]] \rightarrow \exists x[\uparrow(\text{two wheels})(x)\ \&\ \text{have}(z,x)]]$
<Function Application>

However, different from (12.a), the DstrShr NP *two wheels* in (13.a) is analyzed as a group since the DstrShr is a plural. The pluralization operator cannot apply to a plural directly since it is impossible to pluralize an element that is already plural. A plural can be pluralized only when it is shifted to an (impure) atom by group formation. The semantic derivation illustrated in (13.b) precisely yields the reading of (13.a): each of the bicycles has a group of two wheels.

There is one more point to be discussed about the system presented above. The three variables Z, P and R in the extension of the pluralization operator can be permuted to be the first semantic argument. That is, the first constituent that combines with the operator does not have to be the DstrShr but it can be the SrtKy or the Relation. The permutation is empirically attested by the three uses of a distributive quantifier – the anti-quantifier use in (14.a), the floated-quantifier use in (14.b) and the determiner-quantifier use in (14.c) – that select each variable as their first semantic argument.[5]

(14) a. The boys have two phones each.

b. The boys each have two phones.

c. Each of the boys have two phones.

3. Embedded Distributivity

In this section, I would like to provide a novel semantic analysis for embedded distributivity, based on the account for distributivity discussed in the previous section. In essence, in the analysis of embedded distributivity, a plural variable that is of <e,t> type will be made available through a plural forming operation in the process of semantic derivation and the operation that forms a plural will be represented by the ∈ relation. What is significant is that the plural variable introduced does not come from out of nowhere but is motivated by a linguistic fact in the grammar: distributivity is reduced to semantic plurality.

Let's first look at how the plural variable can be inserted in the formula evoked by the pluralization operator. In the denotation of the pluralization operator, repeated in (15), there are two singular variables: one is x and the other is z. I would like to argue that a plural variable can be introduced on the basis of the singular variables that are present in the denotation of the pluralization operator. To embed one distributive relation to another, a variable is needed to connect the two. Yet, the variable will not be inserted on no empirical grounds but be introduced by being mediated by the singular variables that already exist in the formula.

[5] For further details regarding the semantic derivations of the sentences in (14), refer to Joh (2009).

(15) $[[*_{ij}]] = \lambda P_{<\alpha,t>}.\forall z[z_{<\alpha>} \in Z_{i<\alpha,t>} \rightarrow \exists x_{<\alpha>}[P(x) \,\&\, R_{j<e,<\alpha,t>>}(x)(z)]]$

Illustration with examples seems to be in order. The sentence in (16) contains two distributive relations and one distributive relation embeds another distributive relation. The two distributive relations found in (16) can be described as in (17). First, one distributive relation is present inside the subject noun phrase. The preposition *in* that functions as the Relation makes it possible for the distributive reading to occur. At the sentential level, there is another distributive relation. The entire noun phrase that has expressed the first distributive relation functions as the SrtKy in the second distributive relation and the DstrShr is another plural noun phrase *two leaves.* In the second case, the main verb *have* denotes the Relation.

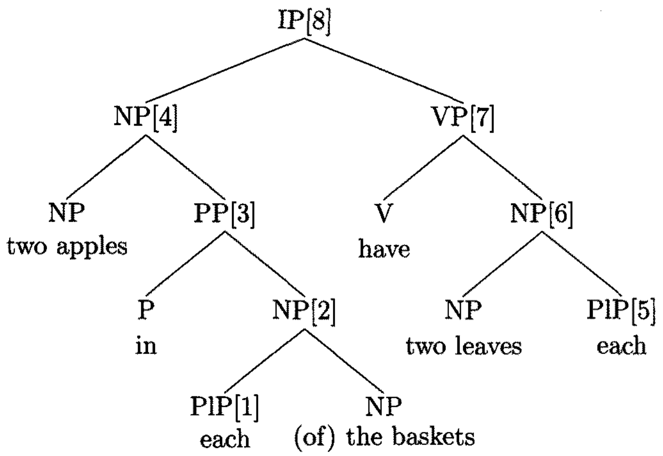
(16) Two apples in each of the baskets have two leaves each.

(17) a. Distributivity in *two apple in each of the baskets*
DstrShr: two apples
SrtKy: the baskets
Relation: in

b. Distributivity in *two apples in each of the baskets have two leaves each.*
DstrShr: two leaves
SrtKy: two apples in each of the baskets
Relation: have

To derive the correct interpretation of (16) compositionally, I propose its LF-structure as in (18). In (18), two pluralization operators are evoked. One is induced by a distributive particle in its determiner use that is placed inside the subject noun phrase and the other pluralization operator is introduced by *each* in its anti-quantifier use which is positioned in the direct object.

(18)



$[[1]] = \lambda Z.\forall z[z \in Z \rightarrow \exists x[P_i(x) \,\&\, R_j(x)(z)]]$
$[[2]] = \forall z[z \in [[\text{the baskets}]] \rightarrow \exists x[P_i(x) \,\&\, R_j(x)(z)]]$

$[[2]] = \lambda R_j.\forall z[z \in [[\text{the baskets}]] \rightarrow \exists x[P_i(x)\ \&\ R_j(x)(z)]]$
$[[3]] = \forall z[z \in [[\text{the baskets}]] \rightarrow \exists x[P_i(x)\ \&\ in(x,z)]]$
$[[3]] = \lambda P_i.\forall z[z \in [[\text{the baskets}]] \rightarrow \exists x[P_i(x)\ \&\ in(x,z)]]$
$[[4]] = \forall z[z \in [[\text{the baskets}]] \rightarrow \exists x[\uparrow(\text{two apples})(x)\ \&\ in(x,z)]]$
$[[4]] = \forall z[z \in [[\text{the baskets}]] \rightarrow \exists x \in X_m[\uparrow(\text{two apples})(x)$
$\&\ \text{in}(x,z)]]$ <Pluralization>
$[[4]] = \lambda X_m.\forall z[z \in [[\text{the baskets}]] \rightarrow \exists x \in X_m[\uparrow(\text{two apples})(x)$
$\&\ \text{in}(x,z)]]$ < λ-abstraction>
$[[5]] = \lambda P.\forall u[u \in U_k \rightarrow \exists y[P(y)\ \&\ R_h(y)(u)]]$
$[[6]] = \forall u[u \in U_k \rightarrow \exists y[\uparrow(\text{two leaves})(y)\ \&\ R_h(y)(u)]]$
$[[6]] = \lambda R_h.\forall u[u \in U_k \rightarrow \exists y[\uparrow(\text{two leaves})(y)\ \&\ R_h(y)(u)]]$
$[[7]] = \forall u[u \in U_k \rightarrow \exists y[\uparrow(\text{two leaves})(y)\ \&\ \text{have}(u,y)]]$
$[[7]] = \lambda U_k.\forall u[u \in U_k \rightarrow \exists y[\uparrow(\text{two leaves})(y)\ \&\ \text{have}(u,y)]]$
$[[8]] = \lambda X.\forall z[z \in [[\text{the baskets}]] \rightarrow \exists x \in X[\uparrow(\text{two apples})(x)$
$\&\ \text{in}(x,z)]]\ \&\ \forall u[u \in X \rightarrow \exists y[\uparrow(\text{two leaves})(y)$
$\&\ \text{have}(u,y)]]$ <Predicate Modification>
$[[8]] = \exists X.\forall z[z \in [[\text{the baskets}]] \rightarrow \exists x \in X[\uparrow(\text{two apples})(x)$
$\&\ \text{in}(x,z)]]\ \&\ \forall u[u \in X \rightarrow \exists y[\uparrow(\text{two leaves})(y)$
$\&\ \text{have}(u,y)]]$ <E-closure>

As illustrated above, inside the subject, an inverse distributive reading is generated. The pluralization operator first applies to the SrtKy *the baskets* and then the relation-denoting expression *in* is factored in via λ-abstraction that is followed by function application. For the DstrShr, another λ-abstraction must be applied and function application follows. Then, on the basis of the singular variable x, a plural variable X is introduced to the proposition via plural formation and gets λ-abstracted. The formula generated so far provides the value for the SrtKy of the distributive interpretation that is evoked by the anti-quantifier in the direct object. Since both node [4] and node [7] are of <<e,t>,t> type[6], they can be combined via predicate modification and E-closure ends the derivation. Then, the two distributive relations which are embedded to each other are well represented in the last formula. The last formula is interpreted as 'each of the baskets has a group of two apples and each of the apples has two leaves' and the two instances of distributivity are expressed as in (19).

(19) a. in(x,z): a group of two apples is in each of the baskets

b. has(u,y): each of the apples has two leaves

[6] In the semantic derivation above, not only the NP but also the VP are analyzed as <<e,t>,t> type. Concerning the VP that is defined as a generalized quantifier type, I would like to make a brief note. The <<e,t>,t> type simply tells us that the given expression takes a set as argument and maps it onto a truth-value. Thus, there is no principled reason why VPs should not denote such type. Zimmermann (2002) points out that even CPs can be defined as a generalized quantifier type with the example in (i).

(i) $[_{CP}$ Whoever comes in first$]$ will win.

It is well known that, in the nominal domain, even the expression like *John* can have <<e,t>t> type. In the verbal domain as well, the generalized quantifier type can be assigned to any expressions including the VP, the IP and the CP.

One thing to note is the type of the plural variable introduced. In the pluralization operator, the plural SrtKy is represented as a set by the variable Z and is of $<\alpha$,t> type. The immediate advantage of having the set-denoting variable Z is that any noun phrase can be shifted to a term of type <e,t>. In this way, various quantificational phrases of the basic type <<e,t>,t> such as *no* NPs, *most* NPs, *every* NPs, *few* NPs, *the* NPs can fill the value of the SrtKy Z.[7] Following this set denotation of the plural, I define the semantic type of the plural variable inserted in cases of embedded distributivity as $<\alpha$,t> as well.

Next, the example in (20)[8] shows us another type of embedded distributivity. The plural noun phrase *the boys* in (20) functions as the SrtKy with respect to *with two cars* and is also part of the SrtKy with respect to *have two tires*. In the first distributive relation inside the noun phrase in the subject, the relation is denoted by the preposition *with* while, in the second distributive relation, the main verb *have* behaves as a proper relation-denoting expression as shown in (21).

(20) The boys with two cars each have two tires each.

(21) a. Distributivity in *the boys with two cars each*
DstrShr: two cars
SrtKy: the boys
Relation: with

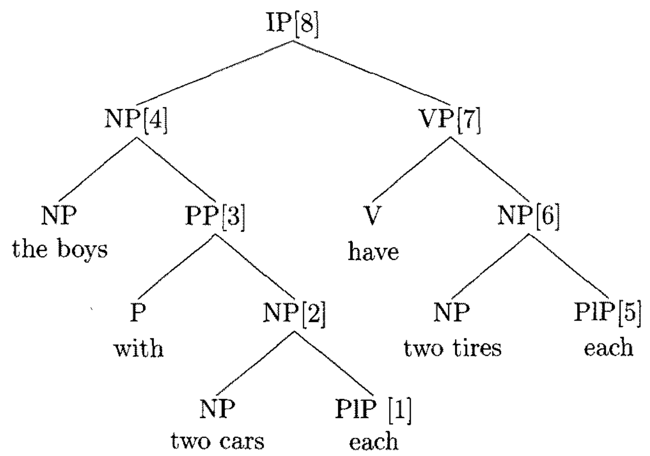
b. Distributivity *in the boys with two cars each have two tires each.*
DstrShr: two tires
SrtKy: the boys with two cars each
Relation: have

Embedded distributivity in (20) can be analyzed on the basis of the LF-structure described in (22). Just like the first case, there are two Plurality Phrases, each of which projects a pluralization operator. Both of the distributive relations are evoked by the pluralization operator in its anti-quantifier use that takes P as the first semantic argument.

[7] The operator *LOWER* discussed in Partee (1987) shifts an expression of <<e,t>,t> type to a denotation of <e> type but it is a partial function. Yet, we can easily get a denotation of type <e,t> from a denotation of type <<e,t>,t> by applying the function, $\lambda P \lambda x[P(\lambda y[y = x])]$, Montague's translation of English *be*. Also, the operator *ident* can always apply to shift <e> to <e,t>. Both *ident* and *be* are total functions and the least restrictions can be imposed on the SrtKy.

[8] A similar example has been discussed in my previous work, Joh (2008a), to illustrate a characteristic of inverse distributivity.

(22)



$[[1]] = \lambda P.\forall z[z \in Z_i \rightarrow \exists x[P(x) \ \& \ R_j(x)(z)]]$
$[[2]] = \forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two cars})(x) \ \& \ R_j(x)(z)]]$
$[[2]] = \lambda R_j.\forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two cars})(x) \ \& \ R_j(x)(z)]]$
$[[3]] = \forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two cars})(x) \ \& \ \text{with}(z,x)]]$
$[[3]] = \lambda Z_i.\forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two cars})(x) \ \& \ \text{with}(z,x)]]$
$[[4]] = \forall z[z \in [[\text{the boys}]] \rightarrow \exists x[\uparrow(\text{two cars})(x) \ \& \ \text{with}(z,x)]]$
$[[4]] = \forall z \in Q_h[z \in [[\text{the boys}]] \rightarrow \exists x[\uparrow(\text{two cars})(x)$
$\& \ \text{with}(z,x)]]$ <Pluralization>
$[[4]] = \lambda Q_h.\forall z \in Q_h[z \in [[\text{the boys}]] \rightarrow \exists x[\uparrow(\text{two cars})(x)$
$\& \ \text{with}(z,x)]]$ < λ-abstraction>
$[[5]] = \lambda P.\forall u[u \in U_k \rightarrow \exists y[P(y) \ \& \ R_h(y)(u)]]$
$[[6]] = \forall u[u \in U_k \rightarrow \exists y[\uparrow(\text{two tires})(y) \ \& \ R_h(y)(u)]]$
$[[6]] = \lambda R_h.\forall u[u \in U_k \rightarrow \exists y[\uparrow(\text{two tires})(y) \ \& \ R_h(y)(u)]]$
$[[7]] = \forall u[u \in U_k \rightarrow \exists y[\uparrow(\text{two tires})(y) \ \& \ \text{have}(u,y)]]$
$[[7]] = \lambda U_k.\forall u[u \in U_k \rightarrow \exists y[\uparrow(\text{two tires})(y) \ \& \ \text{have}(u,y)]]$
$[[8]] = \lambda Q.\forall z \in Q[z \in [[\text{the boys}]] \rightarrow \exists x[\uparrow(\text{two cars})(x)$
$\& \ \text{with}(z,x)]] \ \& \ \forall u[u \in Q \rightarrow \exists y[\uparrow(\text{two tires})(y)$
$\& \ \text{have}(u,y)]]$ <Predicate Modification>
$[[8]] = \exists Q.\forall z \in Q[z \in [[\text{the boys}]] \rightarrow \exists x[\uparrow(\text{two cars})(x)$
$\& \ \text{with}(z,x)]] \ \& \ \forall u[u \in Q \rightarrow \exists y[\uparrow(\text{two tires})(y)$
$\& \ \text{have}(u,y)]]$ <E-closure>

More specifically, the pluralization operator in node [1] directly applies to the DstrShr *two cars* and two occurrences of lambda-abstraction help to make the rest of the distributive relation be expressed. At node [4], a plural variable is formed on the basis of the singular variable z and the plural variable made available gets λ-abstracted. This formula combines with another distributivity projected by the distributive particle *each* in the direct object. The distributive sense inside the subject must be the value of the SrtKy of the distributive sense at the sentential level and this is done via predication modification. The two distributive relations are represented in the last formula via the two relations: $with(z,x)$ such that each of the boys is with two cars and $have(u, y)$ such that each of the boys with two cars has two tires.

So far, I have illustrated two cases of embedded distributivity. In the first case, the singular variable x in the extension of the pluralization operator got pluralized while, in the second case, I have shown that the atomic variable z can also be pluralized. It seems that we have seen all the possible cases of a singular variable being pluralized inside the pluralization operator since, after all, there are only two singular variables in the denotation of the pluralization operator. Yet, I would like to point out that there is a third case. The third singular variable can be introduced by a stage-level predicate.

In the example (23), the main verb *bought* is a stage-level predicate and, as commonly assumed, it carries an implicit event argument with it. When this atomic event variable is present, a third type of embedded distributivity can occur. In (23), the adjunct phrase *in two stores each* is a verbal modifier and the plural event denoted by the entire clause *the boys bought two ties* serves as the antecedent of the distributive reading projected by the pluralization operator *each*, as shown in (24).

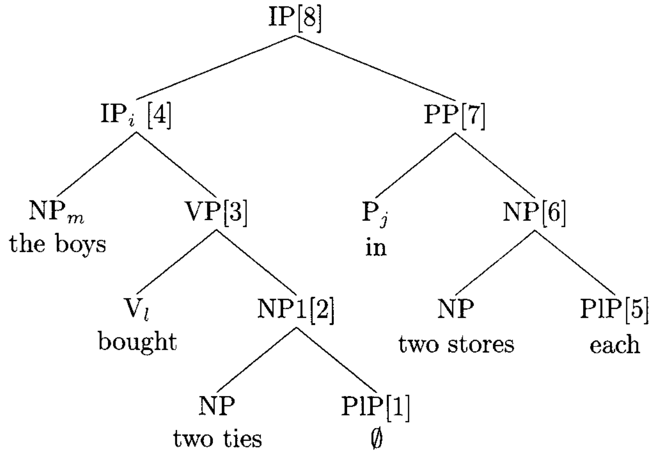
(23) The boys bought two ties in two stores each.

(24) a. Distributivity in *the boys bought two ties*
DstrShr: two ties
SrtKy: the boys
Relation: bought

b. Distributivity in *the boys bought two ties in two stores each*
DstrShr: two stores
SrtKy: the boys bought two ties (the plural event)
Relation: in

As schematically illustrated in (25), in this third type of embedded distributivity, there are also two projections of a pluralization operator. Yet, in this case, what provides a plural variable is the atomic event variable introduced by the stage-level predicate *bought*.

(25)



$$
\begin{aligned}
[[1]] &= \lambda P.\forall z[z \in Z_i \rightarrow \exists x[P(x) \;\&\; R_j(z,x)]]\\
[[2]] &= \forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two ties})(x) \;\&\; R_j(z,x)]]\\
[[2]] &= \lambda R_j.\forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two ties})(x) \;\&\; R_j(z,x)]]\\
[[3]] &= \forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two ties})(x)\\
&\quad \& \; \exists e[\text{bought}(z,x,e)]]]\\
[[3]] &= \lambda Z_i.\forall z[z \in Z_i \rightarrow \exists x[\uparrow(\text{two ties})(x)\\
&\quad \& \; \exists e[\text{bought}(z,x,e)]]]\\
[[4]] &= \forall z[z \in [[\text{the boys}]] \rightarrow \exists x[\uparrow(\text{two ties})(x)\\
&\quad \& \; \exists e[\text{bought}(z,x,e)]]]\\
[[4]] &= \forall z[z \in [[\text{the boys}]] \rightarrow \exists x[\uparrow(\text{two ties})(x)\\
&\quad \& \; \exists e \in E_k[\text{bought}(z,x,e)]]] \quad \text{<Pluralization>}\\
[[4]] &= \lambda E_k.\forall z[z \in [[\text{the boys}]] \rightarrow \exists x[\uparrow(\text{two ties})(x)\\
&\quad \& \; \exists e \in E_k[\text{bought}(z,x,e)]]] \quad \text{< }\lambda\text{-abstraction>}\\
[[5]] &= \lambda P.\forall u[u \in U_m \rightarrow y[P(y) \;\&\; R_l(u,y)]]\\
[[6]] &= \forall u[u \in U_m \rightarrow y[\uparrow(\text{two stores})(y) \;\&\; R_l(u,y)]]\\
[[6]] &= \lambda R_l.\forall u[u \in U_m \rightarrow y[\uparrow(\text{two stores})(y) \;\&\; R_l(u,y)]]\\
[[7]] &= \forall u[u \in U_m \rightarrow \exists y[\uparrow(\text{two stores})(y) \;\&\; \text{in}(u,y)]]\\
[[7]] &= \lambda U_m.\forall u[u \in U_m \rightarrow \exists y[\uparrow(\text{two stores})(y) \;\&\; \text{in}(u,y)]]\\
[[8]] &= \lambda E.\forall z[z \in [[\text{the boys}]] \rightarrow \exists x[\uparrow(\text{two ties})(x)\\
&\quad \& \; \exists e \in E[\text{bought}(z,x,e)]]] \;\&\; \exists u[u \in E\\
&\quad \rightarrow \exists y[\uparrow(\text{two stores})(y) \;\&\; \text{in}(u,y)]]\\
&\quad \text{<Predicate Modification>}\\
[[8]] &= \exists E.\forall z[z \in [[\text{the boys}]] \rightarrow \exists x[\uparrow(\text{two ties})(x)\\
&\quad \& \; \exists e \in E[\text{bought}(z,x,e)]]] \;\&\; \forall u[u \in E\\
&\quad \rightarrow \exists y[\uparrow(\text{two stores})(y) \;\&\; \text{in}(u,y)]]\\
&\quad \text{<E-closure>}
\end{aligned}
$$

In the above, first, a covert distributive particle evokes the ∀∃-structure in the main clause and, at node [4], a plural variable is made available based on the atomic event variable of the main verb and the plural variable inserted gets lambda-abstracted. This combines with the distributive relation projected by the overt distributive marker *each* in the adjunct phrase via predicate modification. The last formula derived states that each of the plural events where each of the boys bought two ties took place in two stores.

My analysis for the anti-quantifier occurring in the adjunct phrase and having the distributive relation with the main clause overcomes some limitations that Zimmermann's (2002) analysis faces. The semantic derivation in (26) is what Zimmermann (2002) puts forward for the anti-quantifier appearing in the PP. Zimmermann (2002) treats such a case simply as having one distributivity. In (26), the relation-denoting expression of the distributive sense is only the preposition, in contrast to my analysis where both the main verb and the preposition fill the value of the variable R in two different occurrences of distributivity. Therefore, in Zimmermann's (2002) system, complications arise in combining the PP of type <t> with the VP denoting a set of events. Lambda-abstraction over index j has already applied to combine with the preposition. Thus, Zimmermann (2002) applies lambda-abstraction over E that serves as the restriction for the existential quantifier over the event argument of the verb.

(26) [[in jeweils zwei Länden]]
$= \forall z[z \in Z_i \rightarrow \exists X[2\ \text{stores}(X)\ \&\ \exists e.\text{in}(z, X, e)]]$
$= \lambda E.\forall z[z \in Z_i \rightarrow \exists X[2\ \text{stores}(X)\ \&\ \exists e \in E[in(z, X, e)]]]$
[[t_i Rosen gekauft]]
$= \lambda E.\exists Y[\text{roses}(Y)\ \&\ \text{bought}(z_i, Y, E)]$
[[in jeweils zwei Länden Rosen gekauft]]
$= \lambda E.\exists Y[\text{roses}(Y)\ \&\ \text{bought}(z_i, Y, E)]$
$\&\ \forall z[z \in Z_i \rightarrow \exists X[2\ \text{stores}(X)\ \&\ \exists e \in E[\text{in}(z, X, e)]]]$

However, an immediate problem is that he neglects that the predicate *bought roses* also has a distributive relation with respect to each member denoted by the subject. That is, his analysis does not correctly capture the two distributive relations occurring in the sentence. Sentences like (23), in fact, have two distributive relations. The two distributive relations clearly emerge when we use a numeral expression in the direct object. Suppose that there are three students. Then, in (27), not only the number of stores involved is six but also the number of roses involved is six.

(27) The students bought two roses in two stores each.

Furthermore, his formula does not accurately capture the fact that the boys did not have to be physically present in two stores while the event took place in the stores. Suppose that the students bought two roses in e-shops on the internet. Then, the students themselves did not need to be physically in the stores. Only each of the plural events occurred in the stores.

Related to this point, he cannot address why the prepositional phrase can function as a verbal modifier only when the main verb has an event argument. In Zimmermann's (2002) analysis, the main clause is dependent on the event in the prepositional phrase. However, the reverse is true. The verbal modifier reading of the prepositional phrase depends on the presence or the absence of the event in the main clause. The PP can function as the verbal modifier with respect to the main clause because the verb in the main clause is a stage-level predicate and contains an event argument that can be pluralized. After all, the PP is a verbal modifier so that it must be the case that the PP depends on the event of the main clause, not the other way around.

The example in (28) clarifies this point. As Zimmermann (2002) notes, the sentence in (28) sounds odd when the stage-level predicate *bought* is replaced by the individual-level predicate *knew*. Zimmermann (2002) couldn't explain this but, in my analysis, the reason is obvious. There is no event variable that can be pluralized in the main clause.

(28) #The students knew two roses in two stores each.

Lastly, examining embedded distributivity both in the nominal domain and in the verbal domain as above, I would like to briefly point out that my current study is part of a bigger project that investigates parallelisms between the two domains. Abney (1987) and Szabolcsi (1989) study these parallelisms with reference to functional projections. Barker and Dowty (1993) claim that there are nominal proto-roles that are essentially parallel to verbal proto-roles. Krifka (1991) finds such parallels in the phenomenon of cumulativity. Placed within the context of this larger research program, my current study serves as another step towards a better understanding of the linguistic system.

4. Conclusion

What is needed in analyzing embedded distributivity in addition to the mechanisms that deal with distributivity is simply introducing a plural variable into the formula evoked by the pluralization operator and the multiple distributive relations generated are combined via predicate modification that is followed by existential closure. In other words, in my analysis presented in this paper, nothing other than plural formation has been employed to derive embedded distributivity which seems very complicated at first glance. Since distributivity is reduced to plurality as Landman (2000) claims, it is by no means an ad hoc approach. A plural variable is inserted on the basis of the singular variable inside the pluralization operator and this plural variable inserted mediates more than one distributive relations to be embedded to one another. In a sense, singular variables in the denotation of the pluralization operator are potential targets of additional pluralization and this is not restricted to the variables that already exist in the pluralization operator but can also apply to the event atomic variable that can be introduced by a stage-level relation-denoting expression.

It is noteworthy that both in distributivity and embedded distributivity the parallelism between the nominal domain and the verbal domain is compelling. Related to this point, an extremely intriguing question has been raised by an anonymous reviewer. In the current work, I have applied a group-forming operation when the DstrShr is plural. The question is whether this can hold true when the DstrShr denotes an event as in (29). In terms of the reasoning I have pursued in my current work, group formation seems to be in order for the adverbial *twice* in (29). However, it has never been thoroughly studied whether we have the group correlate for the eventive plural. If group formation occurs, function application will be employed to make the semantic derivation go through. If group formation does not take place, predicate modification must be used instead. At this point, I would like to leave it as a future research question. This will be an exciting future work that will lead us to investigate plural terms with more scrutiny.

(29) The three students came back twice each.

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