On the Design of Block Lengths for Irregular LDPC Codes Based on the Maximum Variable Degree

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ABSTRACT

This paper presents the design of block lengths for irregular low-density parity-check (LDPC) codes based on the maximum variable degree $d_{v,\max}$. To design a block length, the performance degradation of belief-propagation (BP) decoding performance from upper bounds on the maximum likelihood (ML) decoding performance is used as an important factor. Since for large block lengths, the performance of irregular LDPC codes is very close to the Shannon limit, we focus on moderate block lengths $(5\times10^2 \le N \le 4\times10^3)$. Given degree distributions, the purpose of our paper is to find proper block lengths based on the maximum variable degree $d_{v,\max}$. We also present some simulation results which show how a block length can be optimized.

Key Words : BP Decoding, Irregular LDPC Codes, Maximum Variable Degree, ML Decoding, Upper Bounds

Ⅰ. Introduction

Iterative decoding [1]-[5] has been a great advancement in communications theory because their performance is close to the Shannon limit. Especially, low-density parity-check (LDPC) codes [1]-[3] have several advantages over other codes, such as efficient parallel hardware implementation as well as their excellent performance. Numerous simulations and bounds have demonstrated their remarkable performance.

The belief propagation (BP) decoding performance of irregular LDPC codes of long block length $(N\ge 10^5)$ is near Shannon limit [3]. We conjecture that the BP decoding performance of irregular LDPC codes of long block length $(N\ge 10^5)$ is close to the maximum-likelihood (ML) decoding performance. For moderate block lengths $(5\times10^2 \le N\le 4\times10^3)$, however, it is generally believed that the BP decoding performance of irregular LDPC codes is far from the ML decoding performance. This motivates us to find short block lengths which show the good performance. This purpose is achieved based on the maximum variable degree $d_{v,\max}$.

The remainder of this paper is organized as follows. In Section II, we describe bounding technique used in this paper. In Section III, we present simulation results. In Section IV, we conclude the paper.

Ⅱ. Bounding Technique

Consider a linear binary (N,K) block code C, where N is the code word length and K the information frame length.

For a given code C, d is the Hamming weight of a code word, $d_{\min}$ is the minimum distance of the code and $Q(\cdot)$ is the complementary unit variance Gaussian distribution function. The simple bound [6] on the bit-error-rate (BER) with ML code word decoding is given by

$$P_b \le \sum_{d=d_{\min}}^{N-K+1} \min\{e^{-NE(c,d)}, e^{Ng(\delta)}Q(\sqrt{2cd})\}. \quad (1)$$

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In (1), the exponent $E(c,d)$ is defined as follows.

$$\text{If } c_0(\delta) < c < \frac{e^{2g(\delta)}-1}{2\delta(2-\delta)},$$

$$E(c,d) \equiv \frac{1}{2} ln[1-2c_0(\delta)f(c,\delta)] + \frac{cf(c,\delta)}{1+f(c,\delta)} \quad (2)$$

otherwise,

$$E(c,d) \equiv -g(\delta) + \delta c. \quad (3)$$

Parameters are defined as $\delta \equiv d/N$, $c \equiv r(E_b/N_0)$ with E_b being the energy per information bit and N_0 being the one-sided noise spectral density,

$$c_0(\delta) \equiv (1-e^{-2g(\delta)})\frac{1-\delta}{2\delta}, \quad (4)$$

$$f(c,\delta) \equiv \sqrt{\frac{c}{c_0(\delta)} + 2c + c^2} - c - 1, \quad (5)$$

and

$$g(\delta) \equiv \frac{1}{N} ln\left\{\sum_w \frac{w}{K} A_{w,d}\right\}. \quad (6)$$

where $A_{w,d}$ is the number of code words for an input sequence weight w and output codeword weight d. In order to calculate the bound to a particular code, the input-output weight distribution (IOWD) $A_{w,d}$ should be obtained for that particular code, which is usually very complicated. Therefore, an upper bound is obtained using an ML estimated IOWD [7], which is given by

$$\widehat{A_{w,d}} = \binom{K}{w} \frac{k}{N_s} \quad (7)$$

where k is the number of code words with the Hamming weight d for input sequences of the Hamming weight w among N_s generated sample code words. In order to calculate an ML estimated IOWD, a specific encoder is required. Using Gaussian elimination, it is possible to obtain LDPC encoders. Sample code words are generated randomly.

Ⅲ. Evaluation Method

In this paper, the bounding technique which is introduced in Section II is used to evaluate the performance of LDPC codes with particular maximum variable degrees $d_{v,\max}$. Given two LDPC codes with particular maximum variable degrees $d_{v,\max}$, first we calculate bounds and then the BER performance of BP decoding. Now the degradation of BP decoding performance from bounds is obtained. For each block length, the LDPC code with the maximum variable degree of smaller degradation is obviously recommended.

Ⅳ. Simulation Results

We consider irregular LDPC codes with the degree distribution pairs which have the maximum variable degree $d_{v,\max}$ of 5 or 9 [3]. The block lengths N are 500, 1000, 1500, 2000, and 4000. The rate for these irregular LDPC codes is 1/2. In each case, the maximum number of iterations is 100. A specific encoder is constructed so that BERs are given for systematic bits. The parity check matrices were not chosen entirely randomly. The degree-two nodes were made loop-free and all of them correspond to nonsystematic bits. Short cycles of length 4,6, and 8 were avoided. For an ML estimated IOWD, the number of randomly generated sample code words N_s is 10000 for each $w = 1, 2, \cdots, K$.

Fig. 1 or 3 show BP decoding performances for $d_{v,\max}$ of 9 or 5 respectively. Fig. 2 or 4 show bounds for $d_{v,\max}$ of 9 or 5 respectively. Fig. 5 shows that the performance degradation curves for $d_{v,\max}$ of 5 and 9 intersect at the block length N=2750. Given the block length of an LDPC code, the smaller the BP decoding performance degradation from an ML bound is, the better the LDPC code performs. This implies

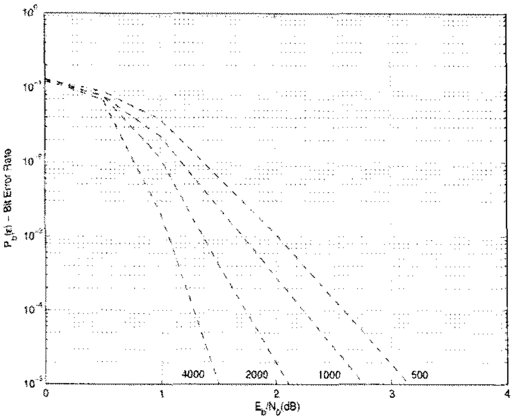

Fig. 1. BP decoding performance ($d_{v,\max}$=9, N = 4000, 2000, 1000, and 500) (Fig.1. is reprinted with permission of the original author).

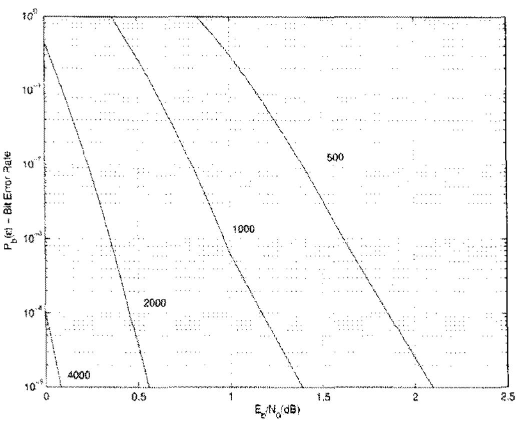

Fig. 2. Bounds on ML decoding performance ($d_{v,\max}$=9, N = 4000, 2000, 1000, and 500) (Fig.2. is reprinted with permission of the original author).

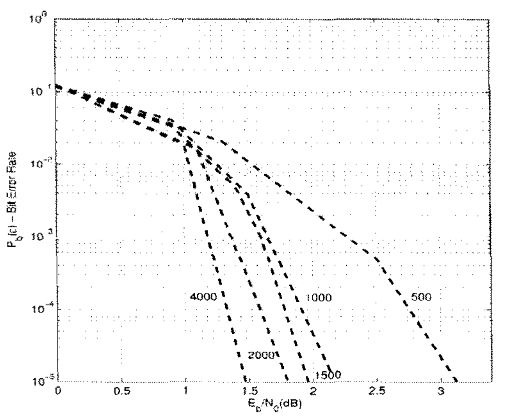

Fig. 3. BP decoding performance ($d_{v,\max}$=5, N = 4000, 2000, 1500, 1000, and 500).

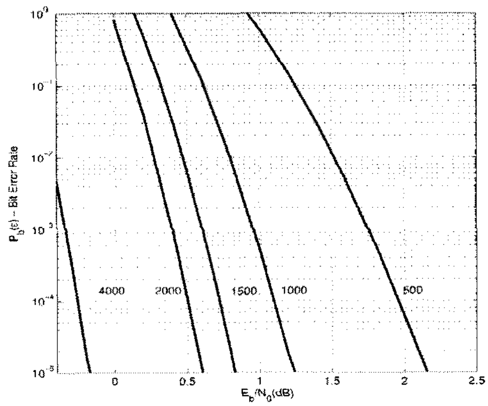

Fig. 4. Bounds on ML decoding performance ($d_{v,\max}$=5, N = 4000, 2000, 1500, 1000, and 500).

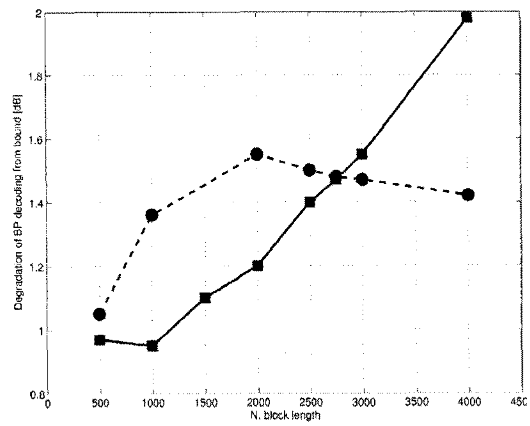

Fig. 5. At a BER of 10^{-5}, E_b/N_0 dB performance degradation of BP decoding from bounds (Solid line is for $d_{v,\max}$=5 and dashed line is for $d_{v,\max}$=9).

that it is desirable for irregular LDPC codes with $d_{v,\max}$ of 5 to choose block lengths belows N of 2750. By the same token, irregular LDPC codes with $d_{v,\max}$ of 9 should choose block lengths above N of 2750.

V. Conclusion

This paper presented the design of block lengths based on the maximum variable degree $d_{v,\max}$ for irregular LDPC codes. The performance degradation curves of BP decoding performance from bounds on the ML decoding performance were used to design block lengths. Since for large block lengths, the performance of irregular LDPC codes is very close to the Shannon limit, we tackled moderate block lengths $(5\times 10^2 \leq N \leq 4\times 10^3)$.

Given degree distributions, the purpose of our paper was to find reasonable block lengths based on the maximum variable degree $d_{v,\max}$. We also presented some simulation results which show how a block length can be optimized.

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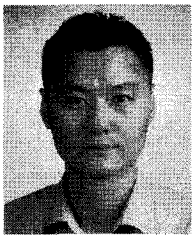

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