Performance Analysis of Multi-Hop Decode-and-Forward Relaying with Selection Combining

Vo Nguyen Quoc Bao and Hyung Yun Kong

Abstract: **In this paper, exact closed-form expressions for outage probability and bit error probability (BEP) are presented for multi-hop decode-and-forward (DF) relaying schemes in conjunction with cooperative diversity, in which selection combining technique is employed at each node. We have shown that the proposed protocol offers remarkable diversity advantage over direct transmission as well as the conventional DF relaying schemes with the same combining technique. We then investigate the system performance when different diversity schemes are employed. It has been observed that the system performance loss due to selection combining relative to maximal ratio combining is not significant. Simulations are performed to confirm our theoretical analysis.**

Index Terms: **Bit error probability (BEP), cooperative communication, decode-and-forward (DF), M-ary phase-shift keying (MPSK), multi-hop relaying, outage probability, rayleigh fading, selection combining (SC).**

I. INTRODUCTION

Recently, relaying dual-hop transmission has gained more attention under forms of cooperative communications and it is treated as one of the candidates to overcome the channel impairment like fading, shadowing and path loss [1], [2]. The main idea is that relay terminals in a multi-user network effectively form a virtual multiple-input multiple-output (MIMO) channel to assist the source-destination communication since the deployment of multiple antennas on mobile handsets is infeasible due to space limitation. This approach can obtain spatial diversity at the expense of reduced spectral efficiency.

A series of recent work concerning evaluating performance of the decode-and-forward (DF) relaying protocol with multi relays have been published (see, e.g., [3]–[14]). Specifically, in [3], [4], the performance of conventional DF relaying (CDFR) networks equipped with selection combining (SC) at the destination terminal in terms of outage probability and bit error probability (BEP) were provided when the statistics of the channels between the source, relays, and destination are assumed to be independent and identically distributed (i.i.d.) and independent but not identically distributed (i.n.d.). In [5]–[7], the outage probability and BEP of CDFR with maximal ratio combining (MRC) at the destination over dissimilar Rayleigh fading channels were examined. In [8]–[12], a class of multi-hop cooperative scheme employing DF relaying, called multi-hop DF relaying (MDFR) schemes, with MRC was proposed, and various performance metrics were also provided.

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V. N. Q. Bao is with the School of Telecommunications, Posts and Telecommunications Institute of Technology (PTIT), Ho Chi Minh City, Vietnam, email: baovnq@ptithcm.edu.vn.

H. Y. Kong is with the Department of Electrical Enginnering, University of Ulsan, Korea, email: hkong@mail.ulsan.ac.kr.

However, to the best of the authors' knowledge, there is no publication concerning the exact performance of MDFR with selection combining in both independent identically distributed and independent but not identically distributed Rayleigh fading channels.

In this paper, we derive exact closed-form expressions for outage probability and BEP of MDFR networks equipped with selection combining at each terminal. The concept of multi-hop cooperative diversity is applied to allow relay terminals to exploit all information they overhear from their previous terminals along the route to the destination to increase the chance of cooperation. At each relay terminal, receiving several replicas of the same signal from its predecessor terminals requires some kind of combining techniques in order to obtain a single representation of the desired symbol. To that effect, the receiver at each relay terminal can employ a variety of diversity combining techniques: SC, equal-gain combining (EGC), and MRC. Among them, SC gives the most inferior performance, MRC gives the best and the optimum performance, and EGC has a performance quality in between the others. SC and MRC are the two extremes of complexity quality tradeoff. Although optimum performance is highly desirable, practical systems often sacrifice some performance in order to reduce their complexity, i.e., instead of using MRC which requires exact knowledge of the channel state information, a system may use selection combining which is the simplest combining method. It only selects the best signal out of all replicas for further processing and neglects all the remaining ones. The benefit of using SC as opposed to MRC is reduced hardware complexity at each node in the network. In addition, it also reduces the computational costs and may even lead to a better performance than MRC, because channels with very low signal-to-noise ratio (SNR) can not accurately estimated and contribute much noise in reality.

The major contributions of this paper are as follows. We derive the exact closed-form expressions of two most important performance metrics, i.e., outage probability and BEP for M-ary phase-shift keying (MPSK) of MDFR schemes. In addition, the comparison between the performance of MDFR and that of CDFR [3], [4] is performed and it confirms that the proposed protocol outperforms CDFR in all range of operating SNRs.

The rest of this paper is organized as follows. In Section II, we introduce the model under study and describe the proposed protocol. Section III shows the formulas allowing for evaluation of outage probability and BEP of MDFR systems. In Section IV, we contrast the simulations and the results yielded by theory. Finally, the paper is closed in Section V.

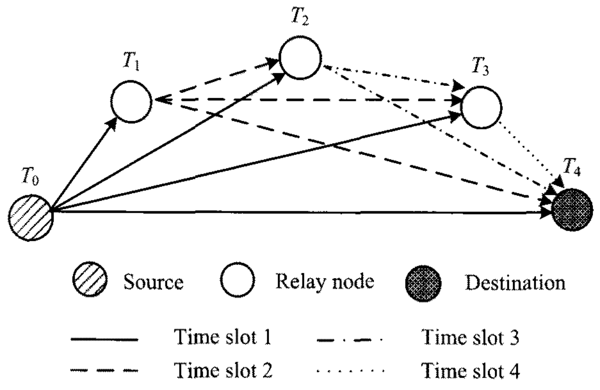

Fig. 1. A MDFR system with 3 relays ($K = 3$).

II. SYSTEM MODEL

We consider a wireless relay network consisting of one source, K relays, and one destination operating over slow, flat, Rayleigh fading channels as illustrated in Fig. 1. It is assumed that each node is equipped with a single antenna. The source terminal (T_0) communicates with the destination (T_{K+1}) via K relay nodes denoted as $T_1, \cdots, T_k, \cdots, T_K$. Due to Rayleigh fading, the channel powers, denoted by $\alpha_{T_i,T_j} = |h_{T_i,T_j}|^2$ are independent and exponential random variables where h_{T_i,T_j} is the fading coefficient from node T_i to node T_j with $i = 0, \cdots, K, j = 1, \cdots, K+1$ and $i < j$. We define λ_{T_i,T_j} as the expected value of α_{T_i,T_j}. The average transmit powers for the source and the relays are denoted by $\mathcal{P}_{T_i}$ with $i = 0, \cdots, K$, respectively. We further define $\gamma_{T_i,T_j} = \mathcal{P}_{T_i}\alpha_{T_i,T_j}$ as the instantaneous SNR per bit for the link $T_i \to T_j$ with $\bar{\gamma}_{T_i,T_j} = E(\gamma_{T_i,T_j}) = \mathcal{P}_{T_i}\lambda_{T_i,T_j}$.

For medium access, a time-division channel allocation scheme with $K+1$ time slots is used in order to realize orthogonal channelization, thus no inter-relay interference is considered in the signal model. To simplify the analytical approach, we assume perfect synchronization among nodes in the network.

Different from CDFR schemes [3], each relay node in MDFR schemes receives the signals transmitted by all its preceding terminals and then combines them by using selection combining. According to the selective DF relaying protocol [1], the relay node decides to cooperate or not with the source in its own time slot, based on the quality of its received signals. Since SC technique is used, the relay will choose the maximum instantaneous received SNR at the output of the selection combiner to demodulate and then check whether the received data are right or wrong. If they are right, that relay will cooperate with the source in its transmission time slot, otherwise, it will keep silent. It should be noted that neither relay selection technique nor feedback signals are used in CDFR or MDFR schemes, e.g., the relays as well as the destination need to receive all the signals transmitted from its decoding set as well as the source before choosing the best signal among them for decoding.

For the purpose of analysis, we define a decoding set $D(T_k)$ for node T_k, $k = 1, 2, \cdots, K+1$, whose members are its preceding relays which are able to decode correctly. It is obvious that $D(T_k)$ is a subset of $C(T_k) = \{T_1, \cdots, T_{k-1}\}$.

We assume that the receivers at the destination and relays have perfect channel state information but no transmitter channel state information is available at the source and relays.

III. PERFORMANCE ANALYSIS

To evaluate the outage probability and BEP of MDFR schemes, we shall need the conditional probability density function (PDF) of the instantaneous SNR of the selection combiner at each node conditioned on the decoding set of the destination, $D(T_{K+1})$.

Consider node T_k, with selection combining, the signal providing largest SNR is always selected from the signals sent from its decoding set $D(T_k)$ as well as from the source (T_0). Let us define $\{\gamma_i\}_{i=1}^{n_k}$ as the instantaneous SNR per bit of each path received by the node T_k from the set $D^*(T_k)$ with their expected values $\{\bar{\gamma}_i\}_{i=1}^{n_k}$, respectively, where $D^*(T_k) = D(T_k) \cup \{T_0\}$ and n_k is the cardinality of the set $D^*(T_k)$, i.e., $n_k = |D^*(T_k)|$.

Under the assumption that all links are subject to independent fading, the conditional cumulative distribution function (CDF) of $\beta_k = \max_{T_i \in D^*(T_k)} \mathcal{P}_{T_i}\alpha_{T_i,T_k} = \max_{i=1,\cdots,n_k} \gamma_i$ can be determined by [15]

$$F_{\beta_k}(\gamma) = \Pr[\gamma_1 < \gamma, \cdots, \gamma_i < \gamma, \cdots, \gamma_{n_k} < \gamma] = \prod_{i=1}^{n_k}\left(1 - e^{-\gamma/\bar{\gamma}_i}\right) \tag{1}$$

where $\bar{\gamma}_i = E(\gamma_i)$. Hence, the conditional PDF of β_k is given by differentiating (1) with respect to γ [15].

$$f_{\beta_k}(\gamma) = \frac{\partial}{\partial\gamma}F_{\beta_k}(\gamma) = \sum_{i=1}^{n_k}\frac{1}{\bar{\gamma}_i}e^{-\frac{\gamma}{\bar{\gamma}_i}}\prod_{\substack{j=1\\ j\neq i}}^{n_k}\left(1 - e^{-\frac{\gamma}{\bar{\gamma}_j}}\right) = \sum_{i=1}^{n_k}\left[(-1)^{i-1}\sum_{\substack{m_1,\cdots,m_i=1\\ m_1<\cdots<m_i}}^{n_k}\omega_i e^{-\omega_i\gamma}\right] \tag{2}$$

where $\omega_i = \sum_{l=1}^{i}\bar{\gamma}_{m_l}^{-1}$.

It is noted that the conditional PDF of β_k is expressed under a mathematically tractable form, which offers a convenient way to derive system performance metrics such as outage probability and BEP.

A. Outage Probability

We note that the system is in outage when the end-to-end mutual information defined as $I_{K+1} = \log_2(1 + \max_{T_k \in D^*(T_{K+1})}\gamma_{T_k,T_{K+1}})/(K+1)$ falls below the prespecified transmission rate R [bps/Hz] where $D^*(T_{K+1}) = D(T_{K+1}) \cup \{T_0\}$. The term $1/(K+1)$ is included to reflect that the data transmission takes place in $K+1$ time slots. Mathematically speaking, applying the theorem of total probability, the outage probability of the MDFR can be derived as a weighted sum of the outage probability at the destination, $\Pr[I_{K+1} < R|D(T_{K+1})]$, corresponding to each set of decoding relay $D(T_{K+1})$. Thus, the system outage probability can be written as

$$P_o = \Pr[I_{K+1} < R] = \sum_{D(T_{K+1}) \in 2^{C(T_{K+1})}} \Pr[D(T_{K+1})] \Pr[I_{K+1} < R | D(T_{K+1})] \quad (3)$$

where $2^{C(T_{K+1})}$ denotes the *power set* of $C(T_{K+1})$ that is the set of all subsets of $C(T_{K+1})$ and $\Pr[D(T_{K+1})]$ is the decoding set probability.

The mutual information between the source and node T_k can be expressed as $I_k = 1/(K+1)\log_2(1+\beta_k)$. Hence, from (2), it is straightforward to obtain the conditional probability that relay node T_k is involved in the cooperative transmission as

$$\Pr[T_k \in D(T_{K+1}) | D^*(T_k)] = \Pr[I_k \geq R | D^*(T_k)] = \int_{\gamma_{th}}^{\infty} f_{\beta_k}(\gamma) d\gamma = \sum_{i=1}^{n_k} \left[(-1)^{i-1} \sum_{\substack{m_1,\cdots,m_i=1 \\ m_1<\cdots<m_i}}^{n_k} e^{-\omega_i \gamma_{th}} \right] \quad (4)$$

where $\gamma_{th} = 2^{(K+1)R} - 1$.

By using the relation of joint probability of mass function (PMF) and sequence of conditional PMF [16], the decoding set probability can be written as follows

$$\begin{aligned} \Pr[D(T_{K+1})] = & \left(\prod_{T_p \in D(T_{K+1})} \Pr[T_p \in D(T_{K+1}) | D^*(T_p)] \right) \\ & \cdot \left[\prod_{T_q \in C(T_{K+1}) \backslash D(T_{K+1})} (\Pr[T_q \notin D(T_{K+1}) | D^*(T_p)]) \right] \\ = & \left(\prod_{T_p \in D(T_{K+1})} \Pr[T_p \in D(T_{K+1}) | D^*(T_p)] \right) \\ & \cdot \left[\prod_{T_q \in C(T_{K+1}) \backslash D(T_{K+1})} (1 - \Pr[T_q \in D(T_{K+1}) | D^*(T_p)]) \right]. \end{aligned} \quad (5)$$

The closed-form expression for the conditional outage probability for SC at the destination can be obtained as

$$\Pr[I_{K+1} < R | D(T_{K+1})] = \int_0^{\gamma_{th}} f_{\beta_{K+1}}(\gamma) d\gamma = \sum_{i=1}^{n_{K+1}} \left[(-1)^{i-1} \sum_{\substack{m_1,\cdots,m_i=1 \\ m_1<\cdots<m_i}}^{n_{K+1}} \left(1 - e^{-\omega_i \gamma_{th}}\right) \right]. \quad (6)$$

Substituting (5)–(6) into (3), we can obtain the exact closed-form expression for the outage probability of the system.

B. Bit Error Probability

Similar to outage probability, the end-to-end BEP for M-PSK of MDFR schemes can be written as

$$P_b = \sum_{D(T_{K+1}) \in 2^{C(T_{K+1})}} \Pr[D(T_{K+1})] B_D[D(T_{K+1})] \quad (7)$$

where $B_D[D(T_{K+1})]$ denotes the BEP for SC at the destination relative to the decoding set $D(T_{K+1})$.

In real scenarios, the decoding set is determined after receiving one frame from the source by employing cyclic-redundancy-check (CRC). However, for mathematical tractability of BEP calculation [4], [5], [14], we assumed that the decoding set can be determined on *symbol-by-symbol* basic. Therefore, the average conditional probability that relay T_k belongs to the decoding set of the destination is obtained as follows

$$\Pr[T_k \in D(T_{K+1}) | D^*(T_k)] = 1 - S_{T_k} \quad (8)$$

where S_{T_k} denotes the average symbol error probability (SEP) of M-PSK modulated symbols transmitted from the decoding set $D(T_k)$.

For the case of coherently detected M-PSK, to evaluate S_{T_k}, the moment-generating function (MGF)-based approach is used [17], namely

$$S_{T_k} = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_{\beta_k}\left(-\frac{g_{\mathsf{MPSK}}}{\sin^2\theta} \right) d\theta \quad (9)$$

where $g_{\mathsf{MPSK}} = \sin^2(\pi/M)$ and $M_{\beta_k}(s)$ is defined as follows [17]

$$M_{\beta_k}(s) = \int_0^{\infty} f_{\beta_k}(\gamma) e^{s\gamma} d\gamma = \sum_{i=1}^{n_k} \left\{ (-1)^{i-1} \sum_{\substack{m_1,\cdots,m_i=1 \\ m_1<\cdots<m_i}}^{n_k} \left[\left(1 - s\log_2(M)\omega_i{}^{-1}\right)^{-1} \right] \right\} \quad (10)$$

Having the MGF of the selection combiner output allows one to immediately evaluate S_{T_k}. Specifically, substituting (10) into (9) and after some manipulations [17, (5.79)] gives us the final desired result as (11) shown at the top of the next page. Next, by replacing (11) on (8) and then again employing the relation of joint probability of mass function and sequence of conditional PMF, we are able to show that the decoding set probability, $\Pr[D(T_{K+1}]$, has the same form of that in (5).

The average conditional BEP at the destination corresponding to each decoding set $D(T_{K+1})$ can be obtained by proceeding analogous to [18].

$$B_D[D(T_{K+1})] = \frac{1}{\log_2 M} \sum_{m=1}^{M} e_m \Pr\{\theta \in \Theta_m\} \quad (12)$$

where $\Theta_m = [\theta_L^m, \theta_U^m] = [(2m-3)\pi/M, (2m-1)\pi/M]$ for $m = 1, \cdots, M$ and e_m is the number of bit errors in the decision region Θ_m. Without loss of generality, it is assumed that the phase angle of the transmitted signal $\phi = 0$, the probability $\Pr\{\theta \in \Theta_m\}$ is

$$\begin{aligned} \Pr\{\theta \in \Theta_m\} &= \int_{\theta_L^m}^{\theta_U^m} \int_0^{\infty} f_\theta(\theta | \phi, \gamma) f_{\beta_{K+1}}(\gamma) d\gamma d\theta \\ &= \sum_{i=1}^{n_{K+1}} \left[(-1)^{i-1} \sum_{\substack{m_1,\cdots,m_i=1 \\ m_1<\cdots<m_i}}^{n_{K+1}} \int_{\theta_L^m}^{\theta_U^m} \int_0^{\infty} \begin{bmatrix} f_\theta(\theta | \phi, \gamma) \\ \cdot \omega_i e^{-\omega_i \gamma} \end{bmatrix} d\gamma d\theta \right] \\ &= \sum_{i=1}^{n_{K+1}} \left[(-1)^{i-1} \sum_{\substack{m_1,\cdots,m_i=1 \\ m_1<\cdots<m_i}}^{n_{K+1}} I\left(\theta_U^m, \theta_L^m; \omega_i^{-1}\right) \right] \end{aligned} \quad (13)$$

$$S_{T_k} = \sum_{i=1}^{n_k}\left[(-1)^{i-1}\sum_{\substack{m_1,\cdots,m_i=1\\ m_1<\cdots<m_i}}^{n_k}\frac{1}{\pi}\int_0^{(M-1)\pi/M}\frac{\sin^2\theta}{\sin^2\theta + g_{\mathrm{MPSK}}{\omega_i}^{-1}\log_2 M}d\theta\right]$$
$$= \sum_{i=1}^{n_k}\left\{(-1)^{i-1}\sum_{\substack{m_1,\cdots,m_i=1\\ m_1<\cdots<m_i}}^{n_k}\left[\left(\frac{M-1}{M}\right)\left[1-\sqrt{\frac{g_{\mathrm{MPSK}}{\omega_i}^{-1}\log_2 M}{1+g_{\mathrm{MPSK}}{\omega_i}^{-1}\log_2 M}}\left(\frac{M}{(M-1)\pi}\right)\right.\right.\right.$$
$$\left.\left.\left.\cdot\left[\frac{\pi}{2}+\tan^{-1}\left(\sqrt{\frac{g_{\mathrm{MPSK}}{\omega_i}^{-1}\log_2 M}{1+g_{\mathrm{MPSK}}{\omega_i}^{-1}\log_2 M}}\cot\frac{\pi}{M}\right)\right]\right]\right]\right\} \tag{11}$$

where $f_\theta(\theta|\phi,\gamma)$ is defined by [18, (9b)]. Furthermore, using the analysis in [18], $I(\theta_U^m, \theta_L^m; \omega_i^{-1})$ can be derived as follows

$$I\left(\theta_U^m, \theta_L^m; {\omega_i}^{-1}\right) = \frac{\theta_U^m - \theta_L^m}{2\pi} + \frac{1}{2}\begin{bmatrix} \psi_U^m\left(\frac{1}{2}+\frac{\tan^{-1}(\alpha_U^m)}{\pi}\right) \\ -\psi_L^m\left(\frac{1}{2}+\frac{\tan^{-1}(\alpha_L^m)}{\pi}\right)\end{bmatrix} \tag{14}$$

with

$$\mu_U^m = \sqrt{\log_2(M)\omega_i^{-1}}\sin(\theta_U^m) \tag{15a}$$
$$\mu_L^m = \sqrt{\log_2(M)\omega_i^{-1}}\sin(\theta_L^m) \tag{15b}$$
$$\alpha_U^m = \frac{\sqrt{\log_2(M)\omega_i^{-1}}\cos(\theta_U^m)}{\sqrt{(\mu_U^m)^2+1}} \tag{15c}$$
$$\alpha_L^m = \frac{\sqrt{\log_2(M)\omega_i^{-1}}\cos(\theta_L^m)}{\sqrt{(\mu_L^m)^2+1}} \tag{15d}$$
$$\psi_U^m = \frac{\mu_U^m}{\sqrt{(\mu_U^m)^2+1}} \tag{15e}$$
$$\psi_L^m = \frac{\mu_L^m}{\sqrt{(\mu_L^m)^2+1}} \tag{15f}$$

Finally, inserting (5) and (12) into (7), we can obtain the exact closed-form expression for BEP of the MDFR system.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we provide some simulation results of the proposed protocol and verify these results with our derived formulas. We consider a linear network consisting of multiple nodes as shown in Fig. 2 where the relays are located across the straight line connecting the source and the destination. This scheme is investigated due to its applicable characteristics to multihop ad-hoc networks with minimal impact to existing serial relaying networks where the broadcast nature of wireless channels is fully exploited [11]. Furthermore, beside cooperative gain offered by cooperative transmission, the network topology benefits from path loss reduction since the communication is achieved by relaying the information from the source to the destination via intermediate nodes in between.

The average channel power due to transmission between node T_i and node T_j is modeled as $\lambda_{T_i,T_j} = \kappa_0 d_{T_i,T_j}^{-\eta}$ where d_{T_i,T_j} is the distance from node T_i to node T_j, η is the path loss exponent and κ_0 captures the effects due to antenna gain, shadowing, etc [19]. However, for a fair of comparison to direct transmission, the overall distance of all hops is normalized to be one, i.e., $\sum_{k=0}^{K} d_{T_k,T_{k+1}} = 1$, and the uniform power allocation is employed in order to keep the total power constraint. Without loss of generality, we assume $\kappa_0 = 1$, $\eta = 3$ and the source, relays, and destination are equidistant from each other, i.e., $d_{T_i,T_j} = (j-i)/(K+1)$ with $i = 0,\cdots,K, j = 1,\cdots,K+1$ and $i < j$ for all results except those in Figs. 8 and 9.

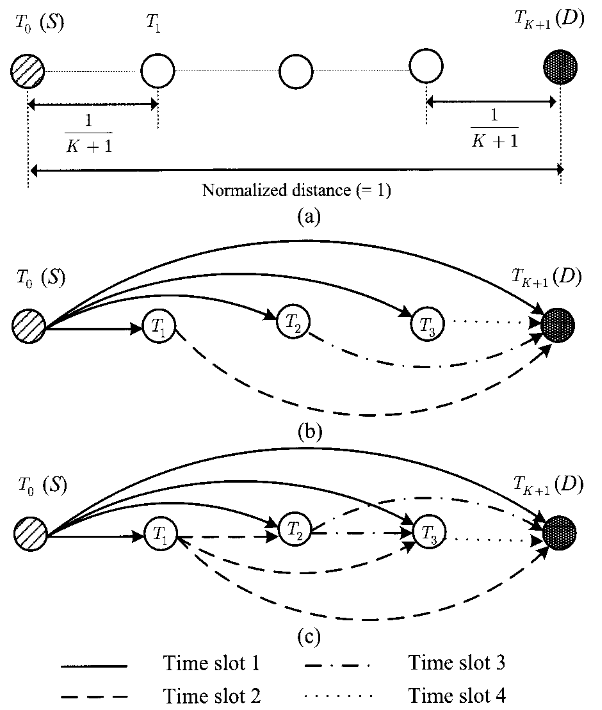

Fig. 2. (a) A linear network with K relays, (b) CDFR network, and (c) MDFR network.

We first investigate the participation probabilities of each relay for CDFR and MDFR schemes with 3 relays in Figs. 3 and 4, defined as the average probability that relay involves in the cooperative transmission. Mathematically, the participation probability of node k, $\Pr[T_k \in D(T_{K+1})]$, denotes probability that its total instantaneous received SNR is greater than the SNR threshold, γ_{th} (or T_k correctly decodes the symbol transmitted by the source), namely

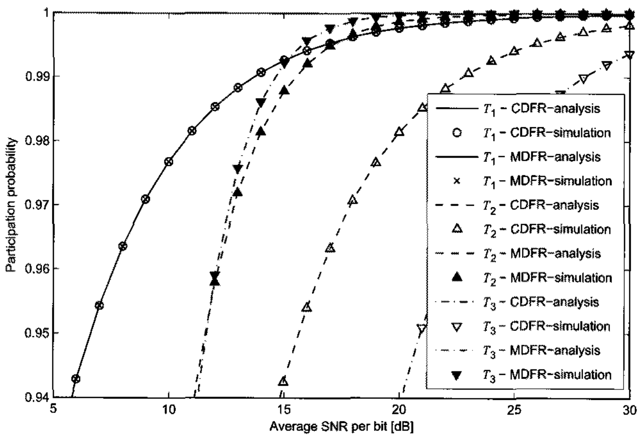

Fig. 3. Participation probabilities for each relay in MDFR and CDFR network in sense of outage probability ($R = 1$ bps/Hz).

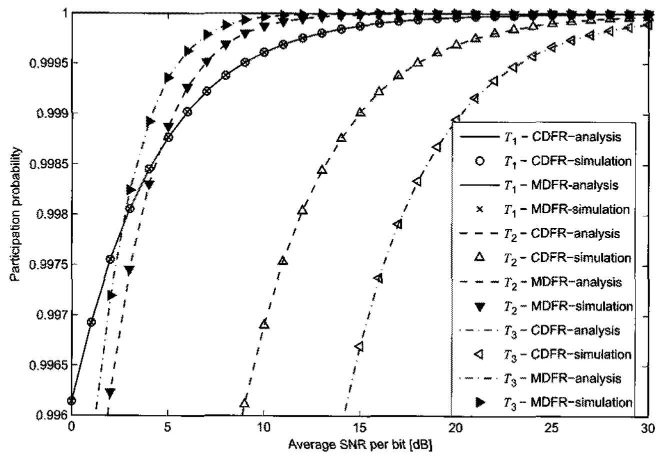

Fig. 4. Participation probabilities for each relay in MDFR and CDFR network in sense of SEP (BPSK).

- For CDFR [20]
  - In sense of outage probability,

$$\Pr[T_k \in D(T_{K+1})] = \Pr(I_k \geq R) = \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{T_0,T_k}}\right). \quad (16)$$

  - In sense of SEP,

$$\Pr[T_k \in D(T_{K+1})] = 1 - S_{T_k}. \quad (17)$$

In this case, the relays receive only the signal transmitted from the source. Therefore, S_{T_k} can be obtained by using (11) with $n_k = 1$ and $\omega_i^{-1} = \bar{\gamma}_{T_0,T_k}$.

- For MDFR
  - In sense of outage probability,

$$\begin{aligned}\Pr[T_k \in D(T_{K+1})] &= \Pr[I_k \geq R] \quad (18)\\ &= \sum_{D(T_k)\in 2^{C(T_k)}} \Pr[D(T_k)] \Pr[I_k \geq R | D(T_k)]\end{aligned}$$

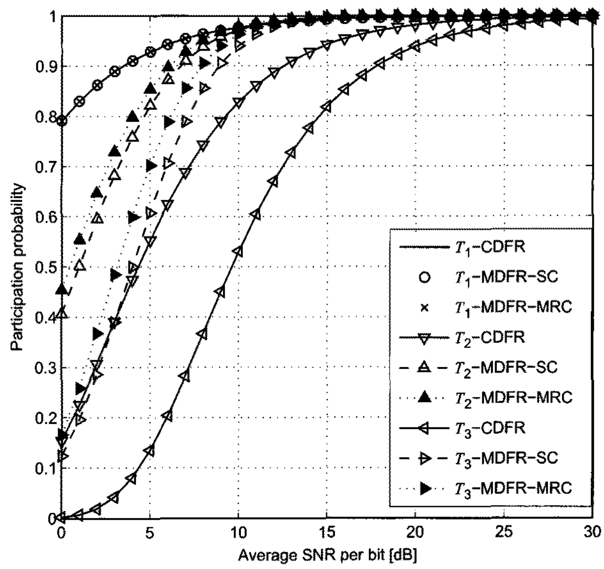

Fig. 5. Effect of combining technique on participation probability in sense of outage probability ($R = 1$ bps/Hz).

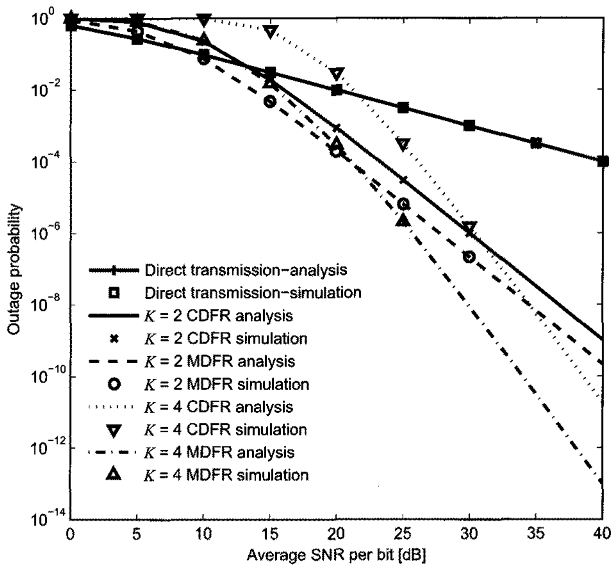

Fig. 6. Outage probability for MDFR and CDFR ($R =$ 1bps/Hz).

where $\Pr[D(T_k)]$ can be obtained by using the same approach as for (5). Furthermore, employing (2) again yields

$$\begin{aligned}\Pr[I_k \geq R | D(T_k)] &= \int_{\gamma_{th}}^{\infty} f_{\beta_k}(\gamma) d\gamma \quad (19)\\ &= \sum_{i=1}^{n_k} (-1)^{i-1} \sum_{\substack{m_1,\cdots,m_i=1\\ m_1<\cdots<m_i}}^{n_k} e^{-\omega_i \gamma_{th}}.\end{aligned}$$

  - In sense of SEP,

$$\begin{aligned}&\Pr[T_k \in D(T_{K+1})]\\ &= \sum_{D(T_k)\in 2^{C(T_k)}} \Pr[D(T_k)] \Pr[T_k \in D(T_{K+1}) | D(T_k)] \quad (20)\end{aligned}$$

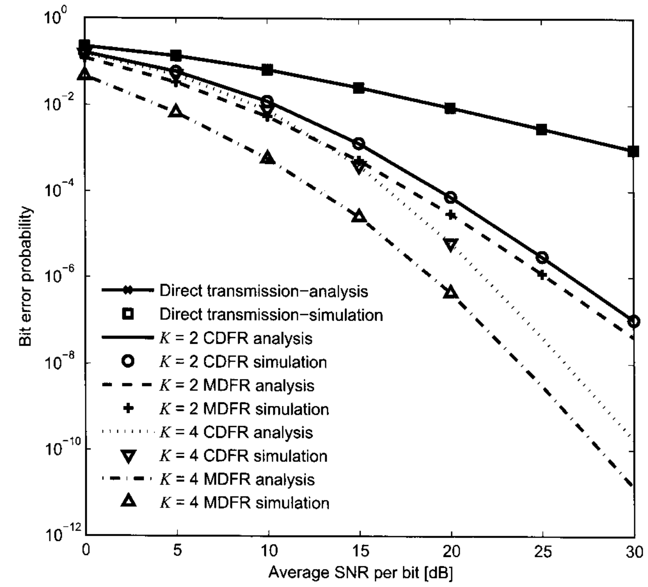

Fig. 7. BEP for MDFR and CDFR (16-PSK).

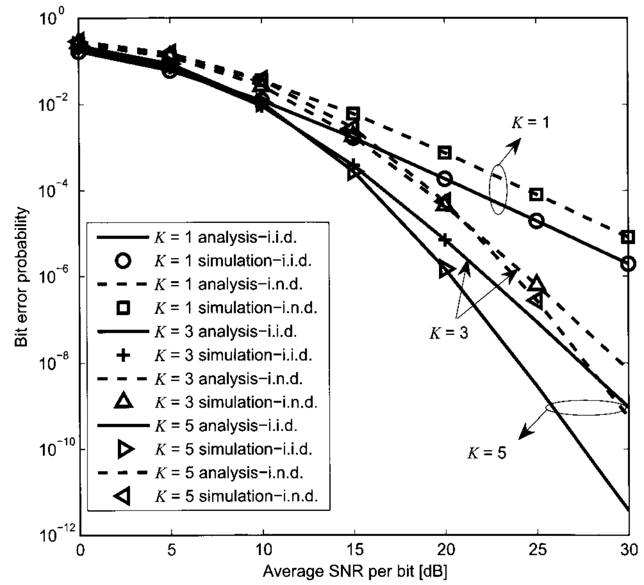

Fig. 9. BEP of MDFR schemes over i.i.d. and i.n.d. channels (QPSK).

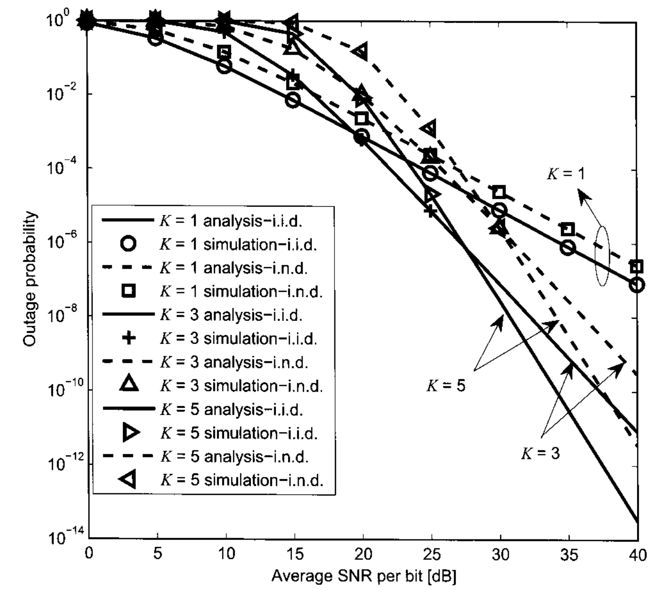

Fig. 8. Outage probability of MDFR schemes over i.i.d. and i.n.d. channels ($R = 0.5$ bps/Hz).

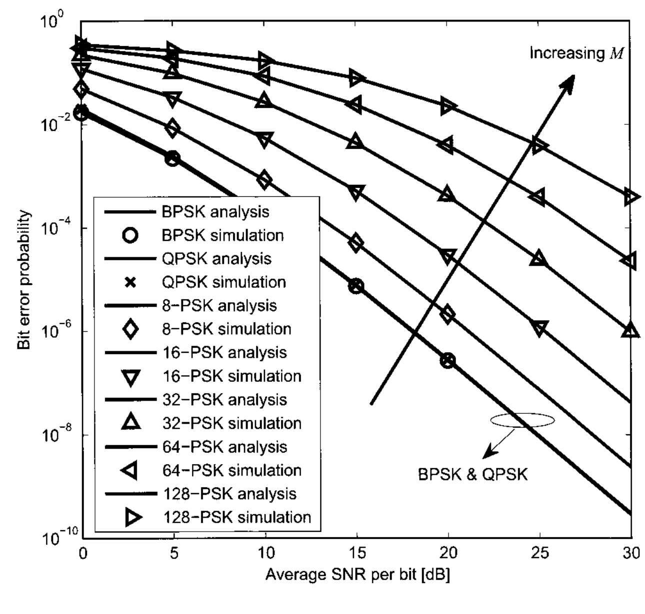

Fig. 10. Effect of modulation levels on the average BEP of MDFR schemes ($K = 2$).

where $\Pr[T_k \in D(T_{K+1}) | D(T_k)] = 1 - S_{T_k}$.

As can be observed from these figures, in the medium SNR regime, although both schemes (MDFR and CDFR) cannot make the relays perfectly involve in the cooperative transmission, MDFR schemes significantly increase the participation probability of relays as compared to CDFR schemes except for the closest relay (T_1). Furthermore, it is interesting to note that the farther the relays from the source are, the bigger participation probability gap between the curves of MDFR and CDFR is and the less reliable the signals received by those relays are forwarded to the destination. On the other hand, we can see that at high SNR, the gap between the curves caused by MDFR and CDFR gets negligible. In summary, from Fig. 5 and other numerical experiments not included here for brevity, we notice that, as expected, the advantage of the proposed schemes is also hold for other combining techniques, namely EGC and MRC.

Figs. 6 and 7 show the outage probability and the average BEP of the MDFR scheme with different number of cooperative nodes. In addition, the performance of CDFR and MDFR schemes are also compared and illustrated. For comparison, the performance of direct transmission is also plotted as a reference. As can be clearly seen, at high SNR regime, MDFR schemes always outperform direct transmission as well as CDFR schemes and the improvement of the outage and error probability will be proportional to the number of relays in

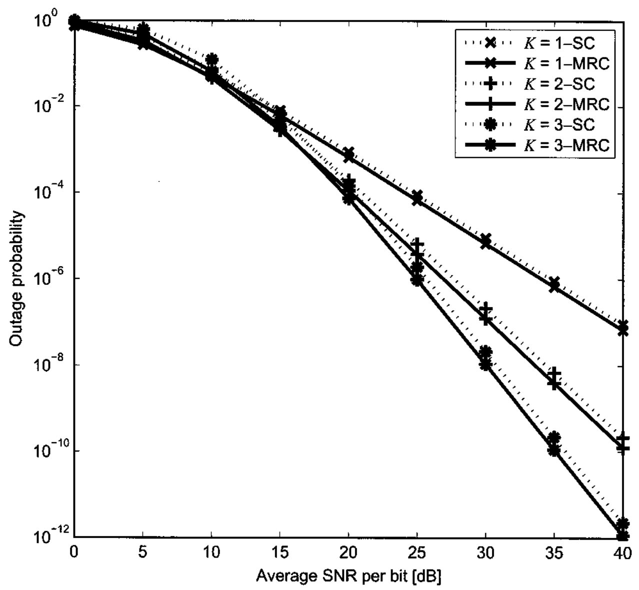

Fig. 11. Effect of combining technique on the outage probability of MDFR schemes ($R = 1$ bps/Hz).

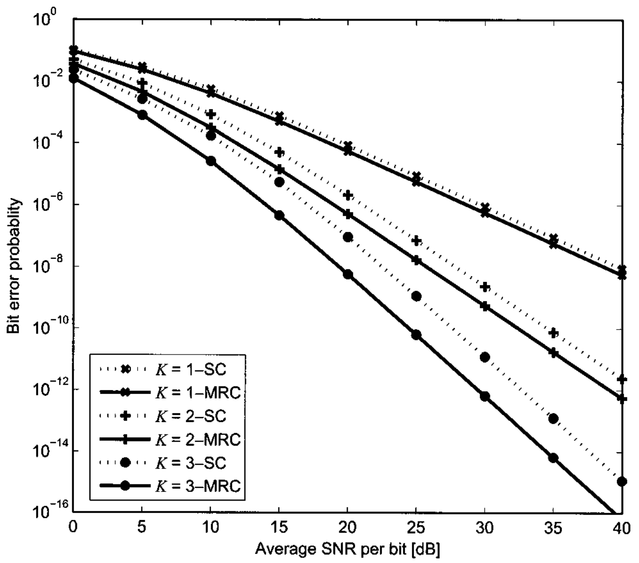

Fig. 12. Effect of combining technique on the average BEP of MDFR schemes (8-PSK).

high SNR regime. However, with low SNR regime, using more relays could make the system performance worsen. For example, at 10 dB, we can see that the best performance is obtained with 2 relays not with 4 relays. It means that under constraint of fixed transmit power; the optimal number of relays in MDFR significantly depends on the operating SNRs. Furthermore, as illustrated in Figs. 6 and 7, the advantage of MDFR over CDFR is linearly increasing according number of relays.

In Figs. 8 and 9, the performance of MDFR schemes in both i.i.d. and i.n.d. channels were examined. The results are based on the assumption that λ_{T_i,T_j} is set to be 1 and to be uniformly distributed between 0 and 1 for the i.i.d. and i.n.d. case, respectively. It can be seen that our analytical results and the simulation results are in excellent agreement. At high SNR regime, the curves obtained for i.n.d. cases have the same form with those for i.i.d. cases and seem to be shifted from those for i.i.d. case to the right, indicating that MDFR schemes can achieve same diversity gain under both i.i.d. and i.n.d. channels. Furthermore, under the same conditions, the gap between two curves of i.i.d. and i.n.d. case tends to linearly increase according number of relays. This is also consistent with the results reported in Figs. 6 and 7 where the similar observation can be achieved.

In Fig. 10, we study the average BEP performance for different levels of M-PSK. We can see that as gray coding is used for bit-symbol mapping, average BER of BPSK is same with that of QPSK. In addition, as expected, analytical curves match very well with the ones obtained from Monte Carlo simulations.

Figs. 11 and 12 show the effect of combining technique, i.e., SC and MRC, on the outage and BEP of MDFR cooperative networks. It is observed from these figures that the system with MRC outperforms that with SC, as expected; however, their outage probability and BEP curves have the same slope. Furthermore, the performance loss between SC and MRC systems tends to increase to be proportional to the number of relays. For example, in Fig. 12, the performance loss for the cases $K = 1, 2, 3$ are 1, 2, and 3 dB, respectively. We can conclude that the performance loss due to using a less complex combiner is not substantial.

V. CONCLUSION

We have presented the exact closed-form expressions for the outage and BEP of MDFR over Rayleigh fading channels. Its validity was demonstrated by a variety of Monte-Carlo simulations. The expression is general and offers a convenient way to evaluate MDFR system which exploits SC technique with any network topologies. In addition, the results were shown that employing the MDFR significantly enhances the system performance compared to that of CDFR.

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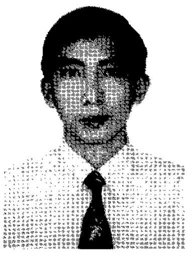
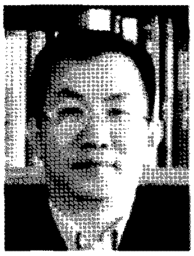


Vo Nguyen Quoc Bao received the B.E. and the M.S. degrees in Electronics and Telecommunications Engineering from Ho Chi Minh City University of Technology, Vietnam, in 2002 and 2005, respectively. He received Ph.D. degree from University of Ulsan, Korea in Feb 2010. Currently, he is an Assistant Professor at School of Telecommunications, Posts and Telecommunications Institute of Technology (PTIT), Ho Chi Minh City, Vietnam. His major research interests are modulation and coding techniques, MIMO system, combining technique, cooperative communications. He is a Member of KICS, IEICE, and IEEE.



Hyung Yun Kong received the Ph.D. and M.E. degrees in Electrical Engineering from Polytechnic University, Brooklyn, New York, USA, in 1996 and 1991, respectively. He received B.E. in Electrical Engineering from New York Institute of Technology, New York in 1989. Since 1996, he was with LG electronics Co., Ltd. in multimedia research lab developing PCS mobile phone systems and LG chair man's office planning future satellite communication systems from 1997. Currently, he is a Professor in Electrical Engineering at University of Ulsan, Ulsan, Korea. He performs several government projects supported by ITRC (Information Technology Research Center), KOSEF (Korean Science and Engineering Foundation), etc. His research area includes high data rate modulation, channel coding, detection and estimation, cooperative communications, and sensor network. He is Member of IEEK, KICS, KIPS, IEICE, and IEEE.