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Reliability analysis of a complex system, attended by two repairmen with vacation under marked process with the application of copula

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Abstract. This paper deals with the reliability analysis of a complex system, which consists of two subsystems A and B connected in series. Subsystem A has only one unit and B has two units B_1 and B_2. Marked process has been applied to model the complex system. Present reliability model incorporated two repairmen: supervisor and novice to repair the failed units. Supervisor is always there and the novice remains in vacation and is called for repair as per demand. The repair rates for supervisor and novice follow general and exponential distributions respectively and the failure time for both the subsystems follows exponential distribution. The model is analyzed under "Head of line repair discipline". By employing supplementary variable technique, Laplace transformation and Gumbel-Hougaard family of copula various transition state probabilities, reliability, availability and cost analysis have been obtained along with the steady state behaviour of the system. At the end some special cases of the system have been taken.

Key Words: *Reliability, M.T.T.F, marked process, head of line repair policy, Gumbel-Hougaard family of copula, vacation*

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1. INTRODUCTION

Many modern repairable complex systems are multistate, that is they and their elements are capable of assuming a whole range of performance levels, varying from perfect functioning, partial functioning to complete failure. In these systems not only the combination of failure events matter, but so does the sequence ordering of failures. Therefore, it may be the case that the data taken from any repairable complex system possesses more information than the time of failure. For instance, in place of just the time of failure, the system can have some additional information such as the type of failure, identity of failed component, type of repair etc. Hence, it is reasonable to model the system under marked process where the marks label the types of events. For example, these marks may be of two types, viz. the down state of the system is due to preventive or corrective maintenance. Here in this paper we have used marked process to model the system.

Reliability works performed by Kovalenko and Smolich (2001) and Ram et al. (2008) for complex repairable systems concentrated on one repairman without giving any attention to two repairmen. Though Kovalenko and Smolich (2001) and Goel and Gupta (1985) while analyzing the complex system incorporated the concepts of two repairmen but without giving any thought for vacations of repairman. Traditionally it is expected that the repairman is always available but practically it has been observed that the repairman may not be a full time employee of the organization. He is busy somewhere else doing some other part time job and called for repair as per requirements. Hu and Li (2009) analyzed a three unit system with vacation and priority but they did not take into account the case of two repairmen. Repairman is one of the essential parts of any repairable system and can affect the economy of the system directly or indirectly on the basis of his skills, efficiency and availability. His availability also plays key role in determining the reliability measures of a system.

Keeping above facts in view unlike many other reliability models, in the present work special attention has been paid to reliability modelling of a complex system incorporating two repairmen with different skills and availability. Here complex system considered consists of two subsystems A and B arranged in series. Subsystem A has only one unit while B has two identical units B_1 and B_2 in which B_2 is in hot standby with B_1. B can fail in two ways viz. partially and catastrophically. System A has only two states: good and failed while B can be in either of the three states namely good, degraded and failed. Partial failure brings subsystem B to the state of reduced efficiency and hence the whole system. On the other hand in case of catastrophic failure the subsystem B fails completely which leads to the complete breakdown of the system. Further, the system fails completely if subsystem A fails or both the units of subsystem B fail. In the present study it has been assumed that the entire system is attended by two repairmen to avoid the undesirable delays in repair. First repairman (supervisor) remains with the system and the second repairman (novice) always in vacation who is called for repair as per requirements. It is further assumed that the supervisor is skilled to repair both subsystems A and B whereas novice is good in repairing subsystem B only. Also, whenever there is a failure in a unit of subsystem B supervisor starts repairing it but if at the same time there occurs any failure in other unit of B then novice is called for repairing. The system is under "Head of line repair discipline" i.e. first come first get policy. However since the system is under marked process the possibility of more than one type of repairs cannot be ruled out and when this possibility exists the reliability of the system can be analyzed with the help of copula (Nelson, 2006). The copula approach allows us to incorporate two different distributions in repair simultaneously hence overcome some of the well known limitations of traditional methods. Here in the present system the repair from state S_1 to S_0 follows two types, i.e. exponential and general. Also the joint probability distribution of repairs follows Gumbel-Hougaard copula. Failure rates are assumed to be constant whereas the repairs follow general and exponential distribution respectively. The following characteristics of the system have been analyzed:

(1) Transition state probabilities of the system.
(2) Asymptotic behaviour of the system.
(3) Various measures such as reliability, availability, M.T.T.F. analysis and cost effectiveness of the system.

At last, we also present a numerical example to demonstrate the applications of the complex system under study.

2. ASSUMPTIONS

(1) Initially the system is in good state.
(2) Subsystems A and B are connected in series.
(3) Subsystem A has constant failure rate and two states good and failed.
(4) Each unit of the subsystem B has constant failure rate and three states good, degraded and failed.
(5) System is under marked process.
(6) Repairs done by the supervisor follow general time distribution and repairs done by novice follow exponential time distribution. There exist two types of repair from S_1 to S_0 state one is general and other is exponential.
(7) Subsystem A can be repaired by supervisor only.
(8) System is under head of line repair discipline (first come first served) it means that if at any instant the subsystem B is under repair and at the same time there occur any failure in subsystem A then repair of subsystem B is continued and repair of subsystem A is undertaken only when the repair of B has completed.
(9) After repair the system is as good as new.
(10) Joint probability distribution of repair rates, where repair is done by supervisor and novice follows Gumbel-Hougaard family of copula.

3. STATE SPECIFICATION

G = Good state, F = Failed state, D = Degraded state, F_R = Failed under repair,
D_R = Degraded under repair,
F_{Rv} = Failed, one unit is under repair and novice is in vacation

States	State of subsystem A	Subsystem B number of good units	System state
S_0	G	2	G
S_1	F	2	F_R
S_2	G	0	F_R
S_3	G	1	D_R
S_4	F	1	F_R
S_5	G	0	F_{Rv}
S_6	G	0	F_R

Figure 3.1. State specification chart

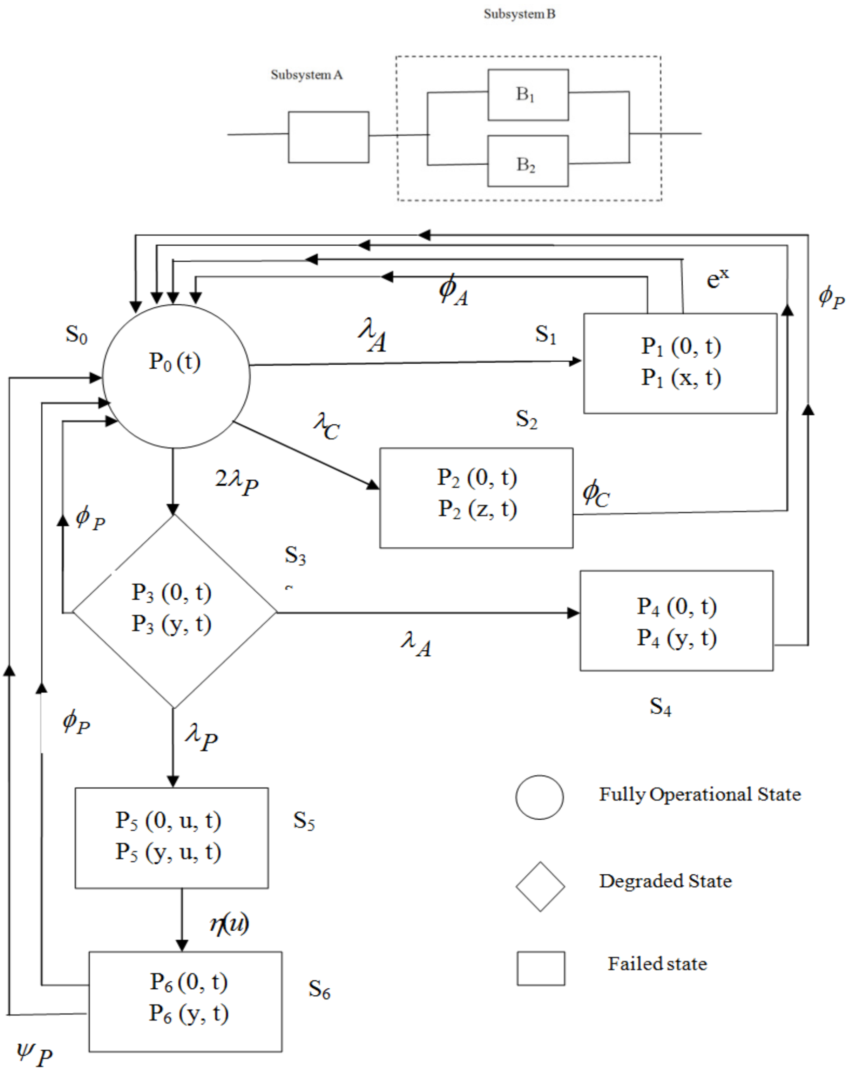

Figure 3.2. State transition diagram

4. NOTATIONS

λ_A	Failure rate of subsystem A.
λ_P, λ_C	Failure rates of subsystem B for partial and catastrophic failures for both the units.
$\phi_i(r)$	Repair rate of supervisor.
$\psi_i(r)$	Repair rate of novice. If i = A then r = x, if i = B then r = y and if i = C then r = z.
x	Elapsed repair time for the subsystem A.
y, z	Elapsed repair times for the partial and catastrophic failures respectively in subsystem B.
η, u	Vacation rate and variable for vacation.
$P_i(t)$	Probability that the system is in S_i state at instant t for i = 1 to i = 6.
$\overline{P_i}(s)$	Laplace transform of $P_i(t)$.
$P_4(y, t)$	Probability density function that at time t the system is in failed state S_4 and the system is under repair, elapsed repair time is y.
$E_p(t)$	Expected profit during the interval (0, t].
K_1, K_2	Revenue per unit time and service cost per unit time respectively.

If $u_1 = \phi_P(y)$, $u_2 = \psi_P(y)$ then the expression for the joint probability according to Gumbel-Hougaard family of copula is given as

$$C_\theta(u_1,u_2) = \exp[-\{(-\log\phi_P(y))^\theta + (-\log\psi_P(y))^\theta\}^{1/\theta}]$$
$$S_\eta(s) = \int_0^\eta \eta(x)\exp[-sx - \int_0^x \eta(x)dx]dx \tag{4.1}$$

5. FORMULATION OF MATHEMATICAL MODEL

By probability considerations and continuity arguments, we obtain the following set of integro-differential equations governing the behaviour of the system.

$$\begin{aligned}\left[\frac{d}{dt} + \lambda_A + \lambda_C + 2\lambda_P\right]P_0(t) = & \int_0^\infty \exp[-\{(-x)^\theta + (-\log\phi_A(x))^\theta\}^{1/\theta}]P_1(x,t)dx \\ & + \int_0^\infty \phi_C(z)P_2(z,t)dz + \int_0^\infty \phi_P(y)P_3(y,t)dy \\ & + \int_0^\infty \exp[-\{(-\log\psi_P(y))^\theta + (-\log\phi_P(y))^\theta\}^{1/\theta}]P_6(y,t)dy \\ & + \int_0^\infty \phi_P(y)P_4(y,t)dy\end{aligned} \tag{5.1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]\right]P_1(x,t)=0 \quad (5.2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\phi_C(z)\right]P_2(z,t)=0 \quad (5.3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_A+\lambda_P+\phi_P(y)\right]P_3(y,t)=0 \quad (5.4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_P(y)\right]P_4(y,t)=\lambda_A P_3(y,t) \quad (5.5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\eta(u)\right]P_5(y,u,t)=0 \quad (5.6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\exp[-\{(-\log\psi_P(y))^{\theta}+(-\log\phi_P(y))^{\theta}\}^{1/\theta}]\right]P_6(y,t)=0 \quad (5.7)$$

Boundary conditions

$$P_1(0,t)=\lambda_A P_0(t) \quad (5.8)$$

$$P_2(0,t)=\lambda_C P_0(t) \quad (5.9)$$

$$P_3(0,t)=2\lambda_P P_0(t) \quad (5.10)$$

$$P_4(0,t)=0 \quad (5.11)$$

$$P_5(0,u,t)=\lambda_p P_3(y,t) \quad (5.12)$$

$$P_6(0,t)=\eta(u)P_5(y,u,t) \quad (5.13)$$

Initial condition

$P_0(0)=0$, and other state probabilities are zero at t =0 (5.14)

6. SOLUTION OF THE MODEL

Taking Laplace transformation of equations (5.1-5.13) and using (5.14), we get

$$\left[s+\lambda_A+\lambda_C+2\lambda_P\right]\overline{P_0}(s)=1+\int_0^{\infty}\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]\overline{P_1}(x,s)dx$$
$$\int_0^{\infty}\phi_C(z)\overline{P_2}(z,s)dz+\int_0^{\infty}\phi_P(y)\overline{P_3}(y,s)dy$$

$$+\int_0^\infty \exp[-\{(-\log\psi_P(y))^\theta + (-\log\phi_P(y))^\theta\}^{1/\theta}]\overline{P_6}(y,s)dy$$
$$+\int_0^\infty \phi_P(y)\overline{P_4}(y,s)dy$$

(6.1)

$$\left[s+\frac{\partial}{\partial x}+\exp[-\{(-x)^\theta+(-\log\phi_A(x))^\theta\}^{1/\theta}]\right]\overline{P_1}(x,s)=0$$
(6.2)

$$\left[s+\frac{\partial}{\partial z}+\phi_C(z)\right]\overline{P_2}(z,s)=0$$
(6.3)

$$\left[s+\frac{\partial}{\partial y}+\lambda_A+\lambda_P+\phi_P(y)\right]\overline{P_3}(y,s)=0$$
(6.4)

$$\left[s+\frac{\partial}{\partial y}+\phi_P(y)\right]\overline{P_4}(y,s)=\lambda_\mathrm{A}\,\overline{\mathrm{P}_3}(\mathrm{y,s})$$
(6.5)

$$\left[s+\frac{\partial}{\partial y}+\eta(u)\right]\overline{P_5}(y,u,s)=0$$
(6.6)

$$\left[s+\frac{\partial}{\partial y}+\exp[-\{(-\log\psi_P(y))^\theta+(-\log\phi_P(y))^\theta\}^{1/\theta}]\right]\overline{P_6}(y,s)=0$$
(6.7)

Boundary conditions

$$\overline{P_1}(0,s)=\lambda_A\overline{P_0}(s) \tag{6.8}$$
$$\overline{P_2}(0,s)=\lambda_C\overline{P_0}(s) \tag{6.9}$$
$$\overline{P_3}(0,s)=2\lambda_P\overline{P_0}(s) \tag{6.10}$$
$$\overline{P_4}(0,s)=0 \tag{6.11}$$
$$\overline{P_5}(0,u,s)=\lambda_P\overline{P_3}(y,s) \tag{6.12}$$
$$\overline{P_6}(0,s)=\eta(u)\overline{P_5}(y,u,s) \tag{6.13}$$

Solving equations (6.1-6.7) with the help of equations (6.8-6.13) one can obtain the following transition state probabilities:

$$\overline{P_0}(s)=\frac{\lambda_A+\lambda_P}{D(s)} \tag{6.14}$$

$$\overline{P_1}(s) = \lambda_A \left[\frac{1-\overline{S_{\phi}}_x(s)}{s} \right] \overline{P_0}(s) \tag{6.15}$$

$$\overline{P_2}(s) = \lambda_C \left[\frac{1-\overline{S_{\phi}}_C(s)}{s} \right] \overline{P_0}(s) \tag{6.16}$$

$$\overline{P_3}(s) = 2\lambda_P \left[\frac{1-\overline{S_{\phi}}_P(s+\lambda_A+\lambda_P)}{s+\lambda_A+\lambda_P} \right] \overline{P_0}(s) \tag{6.17}$$

$$\overline{P_4}(s) = \frac{2\lambda_A\lambda_P}{\lambda_P+\lambda_A} \left[\frac{1-\overline{S_{\phi}}_P(s)}{s} \right] \overline{P_0}(s) - \frac{2\lambda_A\lambda_P}{\lambda_P+\lambda_A} \left[\frac{1-\overline{S_{\phi}}_P(s+\lambda_A+\lambda_P)}{s+\lambda_A+\lambda_P} \right] \overline{P_0}(s) \tag{6.18}$$

$$\overline{P_5}(s) = 2\lambda_P{}^2 \left[\frac{1-\overline{S_{\phi}}_P(2s+\lambda_A+\lambda_P+\eta(u))}{2s+\lambda_A+\lambda_P+\eta(u)} \right] \overline{P_0}(s) \tag{6.19}$$

$$\overline{P_6}(s) = 2\lambda_P{}^2\eta(u) \left[\frac{1-\overline{S_{\psi}}_y(3s+\lambda_A+\lambda_P+\eta(u))}{3s+\lambda_A+\lambda_P+\eta(u)} \right] \overline{P_0}(s) \tag{6.20}$$

$$\begin{aligned} D(s) = {} & s + \lambda_A + \lambda_C + 2\lambda_P - \lambda_A \overline{S_{\phi}}_x(s) - \lambda_C \overline{S_{\phi}}_C(s) \\ & - 2\lambda_P \overline{S_{\phi}}_P(s+\lambda_A+\lambda_P) \\ & - \frac{2\lambda_A\lambda_P}{\lambda_A+\lambda_P} \left\{ \overline{S_{\phi}}_P(s) - \overline{S_{\phi}}_P(s+\lambda_A+\lambda_P) \right\} \\ & - 2\lambda_P{}^2\eta(u) \int_0^{\infty} \psi_P \exp\{-(3s+\lambda_A+\lambda_P+\eta(u))y - \int_0^{y} \psi_y dy\} \end{aligned} \tag{6.21}$$

$$\phi_x = \exp[-\{(-x)^{\theta} + (-\log\phi_A(x))^{\theta}\}^{1/\theta}] \tag{6.22}$$

$$\psi_y = \phi_P(y) + \exp[-\{(-\log\psi_P(y))^{\theta} + (-\log\phi_P(y))^{\theta}\}^{1/\theta}], \psi_P = \psi_y - \phi_P(y) \tag{6.23}$$

$$\begin{aligned} \overline{P}up &= \overline{P_0}(s) + \overline{P_3}(s) \\ &= \frac{(\lambda_A+\lambda_P)}{D(s)} \left[1 + \frac{2\lambda_P}{s+\lambda_P+\lambda_A} \{1-\overline{S}\phi_P(s+\lambda_A+\lambda_P)\} \right] \end{aligned} \tag{6.24}$$

$$\overline{P}down = \overline{P_1}(s) + \overline{P_2}(s) + \overline{P_4}(s) + \overline{P_5}(s) + \overline{P_6}(s)$$

$$= \left[\lambda_{\mathrm{A}} \left\{ \frac{1-\overline{S_{\phi}}_{x}(s)}{s} \right\} + \lambda_{\mathrm{C}} \left\{ \frac{1-\overline{S_{\phi}}_{C}(s)}{s} \right\} + \frac{2\lambda_A\lambda_P}{\lambda_A+\lambda_P} \left\{ \frac{1-\overline{S_{\phi}}_{P}(s)}{s} \right\} \right.$$

$$-\frac{2\lambda_A\lambda_P}{\lambda_A+\lambda_P} \left\{ \frac{1-\overline{S_{\phi}}_{P}(s+\lambda_A+\lambda_P)}{s+\lambda_A+\lambda_P} \right\} + 2{\lambda_P}^2 \left\{ \frac{1-\overline{S_{\phi}}_{P}(2s+\lambda_A+\lambda_P+\eta(u))}{2s+\lambda_A+\lambda_P+\eta(u)} \right\}$$

$$\left. +2{\lambda_P}^2\eta(u) \left\{ \frac{1-\overline{S_{\psi}}_{y}(3s+\lambda_A+\lambda_P+\eta(u))}{3s+\lambda_A+\lambda_P+\eta(u)} \right\} \right] \frac{\lambda_{\mathrm{A}}+\lambda_P}{\mathrm{D(s)}} \tag{6.25}$$

Also it is noticeable that

$$\overline{P}up + \overline{P}down = 1/s \tag{6.26}$$

7. ASYMPTOTIC BEHAVIOUR OF THE SYSTEM

Using Able's lemma

$$\lim_{s\to 0} \{s\overline{F}(s)\} = \lim_{t\to\infty} F(t) = F(say)$$

in equations (6.26) and (6.27) we get the following time independent probabilities

$$\overline{P}up = \frac{(\lambda_A+\lambda_P)}{D(0)} \left[1 + \frac{2\lambda_P}{\lambda_A+\lambda_P} \{1-\overline{S_{\phi}}_{P}(\lambda_A+\lambda_P)\} \right] \tag{7.1}$$

$$\overline{P}down = \frac{(\lambda_A+\lambda_P)}{D(0)} \left[\lambda_A \overline{M_{\phi}}_{x} + \lambda_C \overline{M_{\phi}}_{C} + \frac{2\lambda_A\lambda_P}{\lambda_A+\lambda_P} \overline{M_{\phi}}_{P} \right.$$

$$\left. -\frac{2\lambda_A\lambda_P}{\lambda_A+\lambda_P} \left(\frac{1-\overline{S_{\phi}}_{P}(\lambda_A+\lambda_P)}{\lambda_A+\lambda_P} \right) \right] + 2{\lambda_{\mathrm{P}}}^2 \left(\frac{1-\overline{S_{\phi}}_{P}(\lambda_A+\lambda_P+\eta(u))}{\lambda_A+\lambda_P+\eta(u)} \right)$$

$$+2{\lambda_{\mathrm{P}}}^2\eta(u) \left(\frac{1-\overline{S_{\psi}}(\lambda_A+\lambda_P+\eta(u))}{\lambda_A+\lambda_P+\eta(u)} \right) \tag{7.2}$$

where

$$D(0) = \lim_{s\to 0} D(s)$$

$$\overline{M_{\phi\ P}} = \lim_{s\to 0}\left\{\frac{1-\overline{S_{\phi\ P}}(s)}{s}\right\}$$

8. PARTICULAR CASES

(1) When catastrophic failure does not occur in the subsystem B

In this case the result can be derived by putting $\lambda_C = 0$ in equations (6.14-6.23), Laplace transformation of various state probabilities are as follows:

$$\overline{P_0}(s) = \frac{\lambda_A + \lambda_P}{D_1(s)}$$

$$\overline{P_1}(s) = \frac{\lambda_A(\lambda_A + \lambda_P)}{D_1(s)}\left[\frac{1-\overline{S_{\phi}}_{x}(s)}{s}\right]$$

$$\overline{P_2}(s) = 0$$

$$\overline{P_3}(s) = \frac{2\lambda_P(\lambda_A + \lambda_P)}{D_1(s)}\left[\frac{1-\overline{S_{\phi}}_{P}(s+\lambda_A+\lambda_P)}{s+\lambda_A+\lambda_P}\right]$$

$$\overline{P_4}(s) = \frac{2\lambda_A\lambda_P}{D_1(s)}\left[\frac{1-\overline{S_{\phi}}_{P}(s)}{s}\right] - \frac{2\lambda_A\lambda_P}{D_1(s)}\left[\frac{1-\overline{S_{\phi}}_{P}(s+\lambda_A+\lambda_P)}{s+\lambda_A+\lambda_P}\right]$$

$$\overline{P_5}(s) = \frac{2\lambda_P{}^2(\lambda_A+\lambda_P)}{D_1(s)}\left[\frac{1-\overline{S_{\phi}}_{P}(2s+\lambda_A+\lambda_P+\eta(u))}{2s+\lambda_A+\lambda_P+\eta(u)}\right]$$

$$\overline{P_6}(s) = \frac{2\lambda_P{}^2\eta(u)(\lambda_A+\lambda_P)}{D_1(s)}\left[\frac{1-\overline{S_{\psi}}_{y}(3s+\lambda_A+\lambda_P+\eta(u))}{3s+\lambda_A+\lambda_P+\eta(u)}\right]$$

$$D_1(s) = s + \lambda_A + 2\lambda_P - \lambda_A\overline{S_{\phi}}_{x}(s) - 2\lambda_P\overline{S_{\phi}}_{P}(s+\lambda_A+\lambda_P)$$
$$-\frac{2\lambda_A\lambda_P}{\lambda_A+\lambda_P}\left\{\overline{S_{\phi}}_{P}(s) - \overline{S_{\phi}}_{P}(s+\lambda_A+\lambda_P)\right\} - 2\lambda_P{}^2\eta(u)$$
$$\int_0^\infty \psi_y \exp\{-(3s+\lambda_A+\lambda_P+\eta(u))y - \int_0^y \psi_y dy\}$$

$$\phi_x = \exp[-\{(-x)^\theta + (-\log\phi_A(x))^\theta\}^{1/\theta}]$$

$$\psi_y = \phi_P(y) + \exp[-\{(-\log\psi_P(y))^\theta + (-\log\phi_P(y))^\theta\}^{1/\theta}]$$

(2) When repair follows exponential distribution

In this case the result can be derived by putting

$$\overline{S_{\phi_x}}(s)=\frac{\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]}{s+\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]},\ \overline{S_{\phi_P}}(s)=\frac{\phi_P(y)}{s+\phi_P(y)}$$

$$\overline{S_{\psi_y}}(s)=\frac{\phi_P+\exp[-\{(-\log\psi_P(y))^{\theta}+(-\log\phi_P(y))^{\theta}\}^{1/\theta}]}{s+\phi_P+\exp[-\{(-\log\psi_P(y))^{\theta}+(-\log\phi_P(y))^{\theta}\}^{1/\theta}]}$$

in equations (6.14-6.23), we get

$$\overline{P_0}(s)=\frac{\lambda_A+\lambda_P}{D_2(s)}\quad \overline{P_1}(s)=\frac{\lambda_A(\lambda_A+\lambda_P)}{D_2(s)}\left[\frac{1}{s+\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]}\right]$$

$$\overline{P_2}(s)=\frac{\lambda_C(\lambda_A+\lambda_P)}{D_2(s)}\left[\frac{1}{s+\phi_C(z)}\right],\ \overline{P_3}(s)=\frac{2\lambda_P(\lambda_A+\lambda_P)}{D_2(s)}\left[\frac{1}{s+\lambda_A+\lambda_P+\phi_P(y)}\right]$$

$$\overline{P_4}(s)=\frac{2\lambda_A\lambda_P}{D_2(s)}\left[\frac{1}{s+\phi_P(y)}\right]-\frac{2\lambda_A\lambda_P}{D_2(s)}\left[\frac{1}{s+\lambda_A+\lambda_P+\phi_P(y)}\right]$$

$$\overline{P_5}(s)=\frac{2\lambda_P{}^2(\lambda_A+\lambda_P)}{D_2(s)}\left[\frac{1}{2s+\lambda_A+\lambda_P+\eta(u)+\phi_P(y)}\right]$$

$$\overline{P_6}(s)=\frac{2\lambda_P{}^2\eta(u)(\lambda_A+\lambda_P)}{D_2(s)}\left[\frac{1}{3s+\lambda_A+\lambda_P+\eta(u)+\psi_y}\right]$$

$$\begin{aligned}D_2(s)={}&s+\lambda_A+2\lambda_P+\lambda_C-\frac{\lambda_A\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]}{s+\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]}\\&-\frac{\lambda_C\phi_C(z)}{s+\phi_C(z)}-\frac{2\lambda_P\phi_P(y)}{s+\lambda_A+\lambda_P+\phi_P(y)}\\&-\frac{2\lambda_A\lambda_P}{\lambda_A+\lambda_P}\left\{\frac{\phi_P(y)}{s+\phi_P(y)}-\frac{\phi_P(y)}{s+\lambda_A+\lambda_P+\phi_P(y)}\right\}-2\lambda_P{}^2\eta(u)\\&\int_0^{\infty}\psi_y\exp\{-(3s+\lambda_A+\lambda_P+\eta(u))y-\int_0^{y}\psi_y dy\}\end{aligned}$$

$$\phi(x)=\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]$$

$$\psi_y=\phi_P(y)+\exp[-\{(-\log\psi_P(y))^{\theta}+(-\log\phi_P(y))^{\theta}\}^{1/\theta}]$$

9. NUMERICAL COMPUTATION

(1) Availability analysis

Let the vacation rate be η(u) = 0.20, failure rates of subsystem A and B for partial and catastrophic failures be λA = 0.20, λP = 0.15, λC = 0.05, repair rates be ΦP = ΦC =

ΦA = ψP =1, θ = 1 and x = y = z = 1. Also let the repair follows exponential distribution i.e.

$$\overline{S_{\phi_x}}(s)=\frac{\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]}{s+\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]},\overline{S_{\phi_P}}(s)=\frac{\phi_P(y)}{s+\phi_P(y)},$$

$$\overline{S_{\phi_C}}(s)=\frac{\phi_C(y)}{s+\phi_C(y)}$$

Putting all these values in equation (6.14) and taking inverse Laplace transformation, we get

$$P_{up}= 0.09696483693\ e^{(-3.015211563\ t)}-0.1478308979\ e^{(-1.511354028\ t)}+.1831230754e^{(-1.162033724\ t)} +0.0004142060614\ e^{(-0.8841029318\ t)}+0.8673287795\ e^{(-0.02063108694t)} \quad (9.1)$$

Now in equation (9.1) setting t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, one can obtain Table 9.1 and correspondingly Figure 9.1 which shows the variation of availability with respect to time.

(2) Reliability analysis

Let failure rates of subsystem A and B for partial and catastrophic failures be λ_A = 0.25, λ_P = 0.10, λ_C = 0.05, repair rates be $\Phi_P = \Phi_C = \Phi_A = \psi_P$ = 0, θ = 1 and x = y = z = 1. Also let the repair follows exponential distribution i. e.

$$\overline{S_{\phi_x}}(s)=\frac{\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]}{s+\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]},\overline{S_{\phi_P}}(s)=\frac{\phi_P(y)}{s+\phi_P(y)},$$

$$\overline{S_{\phi_C}}(s)=\frac{\phi_C(y)}{s+\phi_C(y)}$$

Putting all these values in equation (6.14) and taking inverse Laplace transform and varying time, one can obtain Table 9.2. Also Figure 9.2 shows how reliability varies with respect to time in this case.

(3) M.T.T.F. analysis

Suppose that the repair follows exponential distribution, we have

$$\overline{S_{\phi_x}}(s)=\frac{\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]}{s+\exp[-\{(-x)^{\theta}+(-\log\phi_A(x))^{\theta}\}^{1/\theta}]},\overline{S_{\phi_P}}(s)=\frac{\phi_P(y)}{s+\phi_P(y)},$$

$$\overline{S_{\phi_C}}(s)=\frac{\phi_C(y)}{s+\phi_C(y)}$$

$$\text{M.T.T.F.} = \lim_{s\to 0} \overline{P}up(s)$$

(a) Setting ΦA = ΦC = ΦP = ψA = ψP = 0, λP = 0.10, λC =0.05, x = y = z =1, θ = 1 and varying λA as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, one can obtain Table 9.3 which demonstrates variation of M.T.T.F. with respect to λA.
(b) Setting ΦA = ΦC = ΦP = ψA = ψP = 0, λA = 0.25, λC = 0.05, x = y = z =1, θ = 1 and varying λP as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, one can obtain Table 9.4 which demonstrates variation of M.T.T.F. with respect to λP.
(c) Setting ΦA = ΦC = ΦP = ψA = ψP = 0, λA = 0.25, λP = 0.10, x = y = z =1, θ = 1 and varying λC as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, one can obtain Table 9.5 which demonstrates variation of M.T.T.F. with respect to λC.
In cases (a), (b) and (c), one can obtain Figure 9.3, 9.4 and 9.5 respectively which represent the variation of M.T.T.F with respect to λA, λP and λC respectively.

(4) Cost analysis

Let the vacation rate be η(u) = 0.20, failure rates of subsystem A and B for partial and catastrophic failures be λA = 0.25, λP = 0.15, λC = 0.05, repair rates be ΦP = ΦC = ΦA = ψP =1 and x = y = z = 1. Also if the repair follows exponential distribution then from equation (6.14), on putting all these values and taking inverse Laplace transform one can obtain equation (9.1). Let the service facility be always available, then expected profit during the interval (0, t] is given by

$$E_P(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$$

where K_1 and K_2 are the revenue per unit time and service cost per unit time respectively, we get

$$E_P(t) = K_1[-0.03215855170\, e^{(-3.015211563\, t)} + 0.09781354677\, e^{(-1.511354028\, t)} - 1.575884345\, e^{(-1.162033724\, t)} - 0.004685043410\, e^{(-0.8841029318\, t)} - 42.03989746\, e^{(-0.020631086941)} + 42.13229940] - K_2 t \quad (9.2)$$

Keeping K1 = 1 and varying K2 at 0.1, 0.2, 0.3, 0.4, 0.5 in equation (9.2), one can obtain Table 9.6 which is depicted by Figure 9.6.

Table 9.1. Time vs. Availability

Time	P_{up}
0	1.000000000
1	.8792207761
2	.8433013232
3	.8193350997
4	.8000434892
5	.7827956429
6	.7665010503
7	.7507466059
8	.7353831278
9	.7203560351
10	.7056431166

Table 9.2. Time vs. Reliability

Time	P_{up}
0	1.000000000
1	0.7374072327
2	0.5394872580
3	0.3922069455
4	0.2836841908
5	0.2043369251
6	0.1466795483
7	0.1049923209
8	0.07497487060
9	0.05343317031
10	0.03801719556

Table 9.3. Time vs. M.T.T.F.

λ_A	MTTF
.10	5.714285714
.20	3.703703704
.30	2.727272727
.40	2.153846154
.50	1.777777777
.60	1.512605042
.70	1.315789474
.80	1.164021164
.90	1.043478261

Table 9.4. Time vs. M.T.T.F.

λ_P	MTTF
.10	3.142857142
.20	2.698412699
.30	2.323232323
.40	2.027972028
.50	1.794871795
.60	1.607843137
.70	1.455108359
.80	1.328320802
.90	1.221532091

Table 9.5. Time vs. M.T.T.F.

λ_C	MTTF
.10	2.857142856
.20	2.417582417
.30	2.095238095
.40	1.848739495
.50	1.654135338
.60	1.496598693
.70	1.366459627
.80	1.257142857
.90	1.164021164

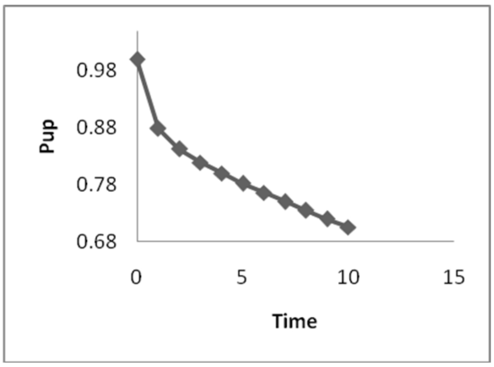

Figure 9.1. Time vs. Availability

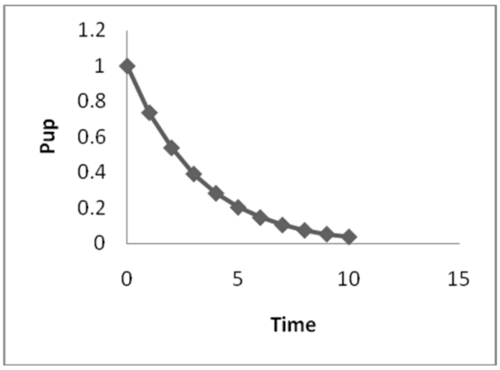

Figure 9.2. Time vs. Reliability

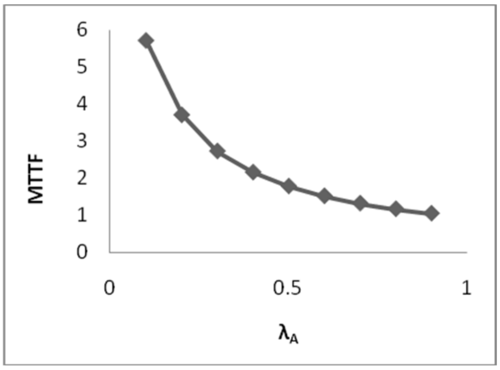

Figure 9.3. λ_A vs. M.T.T.F.

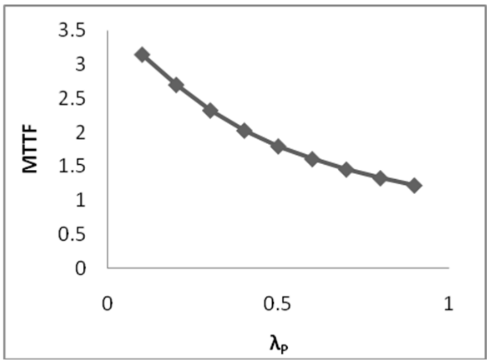

Figure 9.4. λ_P vs. M.T.T.F.

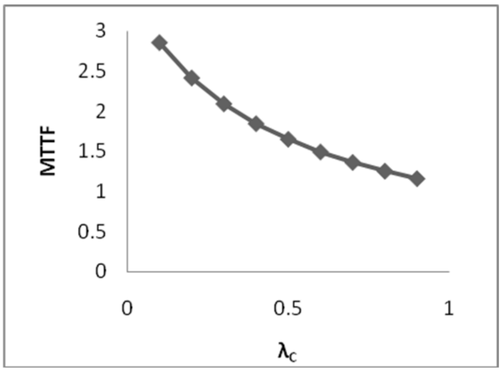

Figure 9.5. λ_C vs. M.T.T.F.

Table 9.6. Time vs. expected profit

Time	$E_P(t)$				
	K_2=0.1	K_2=0.2	K_2=0.3	K_2=0.4	K_2=0.5
0	0	0	0	0	0
1	0.82135184	0.72135184	0.62135184	0.52135184	0.42135184
2	1.58093814	1.38093814	1.18093814	0.98093814	0.78093814
3	2.31168949	2.01168949	1.71168949	1.41168949	1.11168949
4	3.02113173	2.62113173	2.22113173	1.82113173	1.42113173
5	3.71244158	3.21244158	2.71244158	2.21244158	1.71244158
6	4.38703384	3.78703384	3.18703384	2.58703384	1.98703384
7	5.04562110	4.34562110	3.64562110	2.94562110	2.24562110
8	5.68865644	4.88865644	4.08865644	3.28865644	2.48865644
9	6.31649917	5.41649917	4.51649917	3.61649917	2.71649917
10	6.92947313	5.92947313	4.92947313	3.92947313	2.92947313

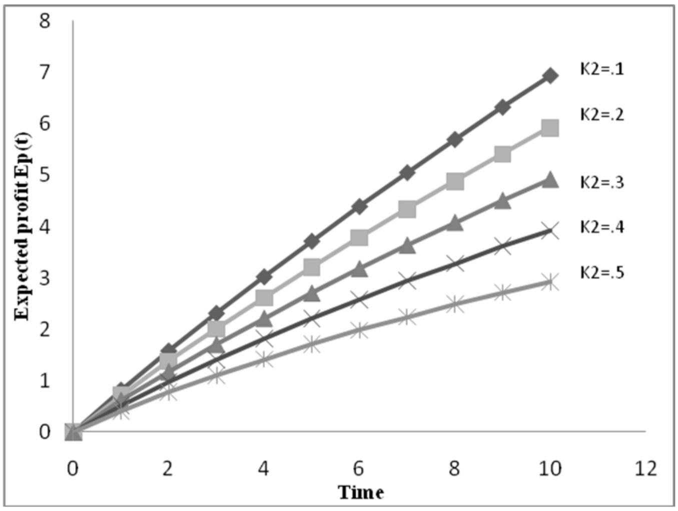

Figure 9.6. Time vs. expected profit

10. RESULTS AND DISCUSSION

It can be easily concluded from Figure9.1 that when $\eta(u) = 0.20$, $\lambda_A = 0.25$, $\lambda_P = 0.15$ and $\lambda_C = 0.05$, then availability of the system initially decreases rapidly with respect to time and later on stabilizes at value 0.7. Also examination of Figure 9.2 reveals that reliability of the system decreases with the increment in time.

When revenue cost per unit time K_1 kept at value 1 and service cost K_2 varied at values 0.1, 0.2, 0.3, 0.4, 0.5, Figure9.6 is obtained. This graph draws an important conclusion that increasing service cost leads decrement in expected profit. The highest and lowest values of expected profit are obtained to be 6.92 and 0.421 respectively. Expected profit decreases with the increment in K_2.

Further Figure 9.3, 9.4 and 9.5 represents how M.T.T.F. of the system changes with respect to λ_A, λ_P and λ_C respectively when other parameters have been kept constant. M.T.T.F. varies from 5.71 to 1.04, from 3.14 to 1.22 and from 2.80 to 1.1 with respect to time in the cases of λ_A, λ_P and λ_C respectively. By observing these graphs one can easily conclude that M.T.T.F. of the complex system decreases as the value of λ_A, λ_P and λ_C increases. One remarkable observation is that M.T.T.F depends more on λ_A in comparison to λ_P and λ_C.

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