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PERIODIC SOLUTION TO DELAYED HIGH-ORDER COHEN-GROSSBERG NEURAL NETWORKS WITH REACTION-DIFFUSION TERMS

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ABSTRACT. In this paper, we study delayed high-order Cohen-Grossberg neural networks with reaction-diffusion terms and Neumann boundary conditions. By using inequality techniques and constructing Lyapunov functional method, some sufficient conditions are given to ensure the existence and convergence of the periodic oscillatory solution. Finally, an example is given to verify the theoretical analysis.

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1. Introduction

Consider the following delayed high-order Cohen-Grossberg neural networks with reaction-diffusion terms:

$$
\begin{aligned}
\frac{\partial u_i(x,t)}{\partial t} &= \sum_{k=1}^{m} \frac{\partial}{\partial x_k}\Big(D_{ik}\frac{\partial u_i(x,t)}{\partial x_k}\Big) \\
&\quad -a_i(u_i(x,t))[b_i(u_i(x,t)) - \sum_{j=1}^{n} c_{ij} f_j(u_j(x,t)) \\
&\quad -\sum_{j=1}^{n} \omega_{ij} f_j(u_j(x,t-t_{ij})) - \sum_{j,k=1}^{n} d_{ijk} f_j(u_j(x,t)) f_k(u_k(x,t)) \\
&\quad -\sum_{j,k=1}^{n} e_{ijk} f_j(u_j(x,t-t_{ij})) f_k(u_k(x,t-t_{ik})) + I_i(t)], \\
u_i(x,t) &= \varphi_i(x,t), \quad -\tau \le t \le 0,\ x \in \Omega, \qquad (1)
\end{aligned}
$$

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$$\frac{\partial u_i(x,t)}{\partial \nu} = 0, \quad x \in \partial\Omega.$$

Where $i = 1, \cdots, n$. n is the number of neurons in the networks. $u(x,t) = (u_1(x,t), \cdots, u_n(x,t))^T$, $u_i(x,t)$ denotes the state of the ith neural unit at time t and in space $x \in \Omega$. Ω is a bounded open domain in $\mathbb{R}^m$ with smooth boundary $\partial\Omega$ and $mes\Omega > 0$ denotes the measure of Ω. ν is the outer normal direction of $\partial\Omega$. $D_{ik} \geq 0$ corresponds to the transmission diffusion coefficient along the ith unit. $a_i(\cdot)$ and $b_i(\cdot)$ represent an amplification function and an appropriately behaved function, respectively. c_{ij} represents the strength of the jth neuron connecting on the ith neuron at time t and in space x, and ω_{ij} represents the strength of the jth neuron connecting on the ith neuron at time $t - t_{ij}$ and in space x. d_{ijk} and e_{ijk} represent the second-order strength of the neuron interconnections within the network without delays and with delays, respectively. t_{ij} corresponds to the transmission delays along the axon of the jth neuron from the ith neuron and satisfies $0 \leq t_{ij} \leq \tau$, where $\tau > 0$ is a given constant. f_i shows how the ith neuron reacts to the input. $I(t) = (I_1(t), \cdots, I_n(t))^T$, I_i is the input from outside the system.

Since Cohen-Grossberg neural networks(CGNNs)which include the traditional neural networks as special cases were first introduced in 1983 by Cohen and Grossberg in [1], CGNNs have been extensively investigated and successfully applied to parallel computation, associative memory and optimization problems, etc[2−17]. Because these applications heavily depend on the dynamical behaviors of the networks, the analysis of the dynamical behaviors is the necessary step to design of neural networks.

However, both in biological and artificial neural networks, diffusion effect cannot be avoided when electrons are moving in asymmetric electromagnetic field, thus we must consider that the activations vary in space as well as in time. Refs [13,16,18-23] have considered the stability of neural networks with diffusion terms, which are expressed by partial differential equations. But the results in [13,16,18-23] are about the stability of equilibrium point of high-order CGNNs or about the periodicity of other neural networks.

Studies on the dynamical behaviors of the neural networks not only involve a discussion of stability properties, but also involve many dynamic behaviors such as periodic oscillatory behavior, bifurcation and chaos[24−25]. In many applications, the properties of periodic oscillatory solutions to the high-order CGNNs are of great interest, for example, the human brain is in periodic oscillatory of chaos, hence it is of prime importance to study periodic oscillatory and chaos phenomena of the high-order neural networks.

Moreover, because of the finite processing speed of information, it is sometimes necessary to take account of time delays in the modeling of the biological or artificial neural networks. Time delays may led to bifurcation, oscillation, divergence or instability which may be harmful to a system, thus, the study of neural dynamics with consideration of the delayed problem becomes extremely important to manufacture high quality neural networks. Refs [2,4-9,12-23] have studied the stability of delayed neural networks. The periodicity of high-order CGNNs has not been studied.

To the best of our knowledge, few authors have considered periodicity of delayed reaction-diffusion high-order CGNNs with Neumann boundary conditions. But it is important in theories and applications, and also is a very challenging problem. In this paper, we will investigate the existence and global exponential stability of the periodic solution to delayed reaction-diffusion high-order CGNNs with Neumann boundary conditions.

This paper is organized as follows. In section 2, some preliminaries and the main result are given. In section 3, by employing inequality techniques and constructing suitable Lyapunov functional, some sufficient conditions are obtained to ensure the existence and global exponential stability of the periodic oscillatory solution. In section 4, one example is given to verify the theoretical analysis. Section 5 is the conclusions of the paper.

2. Preliminaries and main result

Let $X = C(\mathbb{R}^m \times [-\tau, 0], \mathbb{R}^n)$ be the Banach space of continuous functions which map $\mathbb{R}^m \times [-\tau, 0]$ into $\mathbb{R}^n$ with the topology of uniform convergence. $L^2(\Omega)$ is the space of real functions on Ω which are L^2 for the Lebesgue measure. It is a Banach space for the following norm:

$$\|u(\cdot,t)\|_2 = (\sum_{i=1}^{n} \int_\Omega |u_i(x,t)|^2 dx)^{\frac{1}{2}},$$

where $u(\cdot,t) = (u_1(\cdot,t), \cdots, u_n(\cdot,t))^T$.

For any $\varphi(x,t) \in C(\mathbb{R}^m \times [-\tau,0], \mathbb{R}^n)$, we define $\|\varphi\|_2 = (\sum_{i=1}^n \|\varphi_i\|_2^2)^{\frac{1}{2}}$, where $\varphi = (\varphi_1, \cdots, \varphi_n)^T$,

$$\|\varphi_i\|_2 = (\int_\Omega |\varphi_i|_\tau^2 dx)^{\frac{1}{2}}, \quad |\varphi_i|_\tau = \sup_{-\tau \le s \le 0} |\varphi_i(x,s)|.$$

Definition 1. The solution $\tilde{u} = \tilde{u}(x,t;\phi) \in \mathbb{R}^n$ of system (1) with the initial value ϕ is said to be global exponential stability, if there exist positive constants $\gamma > 0$ and $\varepsilon > 0$ such that

$$\|u(\cdot,t;\varphi) - \tilde{u}(\cdot,t;\phi)\|_2 \le \gamma \|\varphi - \phi\|_2 \exp^{-\varepsilon t}, \quad \forall t \ge 0,$$

where $u = u(x,t;\varphi) \in \mathbb{R}^n$ is any solution to system (1) with the initial value φ.

Definition 2. Let $V : \mathbb{R} \to \mathbb{R}$ be a continuous function, the upper right Dini-derivative D^+V is defined as follows:

$$D^+V = \limsup_{h \to +0} \frac{V(t+h) - V(t)}{h}.$$

Throughout the paper, we always assume that system (1) has a smooth solution $u(x,t)$ with the following norm $\|u(\cdot,t)\|_2 = \sqrt{\sum_{i=1}^n \int_\Omega |u_i(x,t)|^2 dx}$, $\forall t \in [0, +\infty)$.

In this paper, we always assume that:

$(H1)$ There exist constants m_i, M_i such that $0 < m_i \leq a_i(u_i) \leq M_i$, $i = 1, \cdots, n$. Moreover, $D_{ik} \geq 0, k = 1, \cdots, m$, $a_i(\cdot)$ is differentiable and there exists positive constant Γ_i such that $0 < a_i'(\cdot) \leq \Gamma_i$ for all $i = 1, \cdots, n$.

$(H2)$ $I_i(t)$ is $\omega-$periodic function satisfying $I_i(t+\omega) = I_i(t)$ with the upper bounded value $B_i \geq 0$, $(i = 1, \cdots, n)$. $b_i(\cdot) \geq 0$ is differentiable, $b_i(0) = 0$, $i = 1, \cdots, n$. Moreover, $\alpha_i = \inf_{x\in\mathbb{R}}\{\dot{b}_i(x)\} > 0$, where $\dot{b}_i(\cdot)$ denotes the derivative of $b_i(\cdot)$, $i = 1, \cdots, n$.

$(H3)$ There exist positive constants Λ_i, β_i such that

$$\|f_i\| = \sup_u |f_i(u)| \leq \Lambda_i, i = 1, \cdots, n. \tag{2}$$

$$|f_i(x_i) - f_i(y_i)| \leq \beta_i |x_i - y_i|, \ \forall x_i, y_i \in \mathbb{R}, \quad i = 1, \cdots, n. \tag{3}$$

Remark 1. $(H3)$ can be got rid of for the first-order Cohen-Grossberg neural networks which can be obtained by $d_{ijk} = e_{ijk} \equiv 0$ in (1), but it is indispensable for the high-order Cohen-Grossberg neural networks.

Remark 2. By $(H3)$, we know that the functions $f_i (i = 1, \cdots, n)$ also satisfy

$$|f_i(x_i)f_j(x_j) - f_i(y_i)f_j(y_j)| \leq \gamma_{ij}|x_i - y_i| + \gamma_{ji}|x_j - y_j|, \ \forall \ x_i, x_j, y_i, y_j \in \mathbb{R},$$

where $\gamma_{ij} = \beta_i \Lambda_j$, and β_i, $\Lambda_i (i = 1, \cdots, n)$ are defined in $(H3)$.

$(H4)$ There exist constants p_{ij}, q_{ij}, m_{ijk}, n_{ijk}, r_{ijk}, s_{ijk}, p^*_{ij}, q^*_{ij}, m^*_{ijk}, n^*_{ijk}, r^*_{ijk}, s^*_{ijk} such that for $i = 1, \cdots, n$ it holds

$$\begin{aligned}
2\alpha_i m_i > \ & + \sum_{j=1}^n \Gamma_i\big((|c_{ij}| + |\omega_{ij}|)\Lambda_j + B_i\big) \\
& + \sum_{j=1}^n M_i\Big(|c_{ij}|^{2p_{ij}}\beta_j^{2q_{ij}} + |\omega_{ij}|^{2p^*_{ij}}\beta_j^{2q^*_{ij}}\Big) \\
& + \sum_{j=1}^n M_j\Big(|c_{ji}|^{2-2p_{ji}}\beta_i^{2-2q_{ji}} + |\omega_{ji}|^{2-2p^*_{ji}}\beta_i^{2-2q^*_{ji}}\Big) \\
& + \sum_{j,k=1}^n (M_i|d_{ijk}|^{2m_{ijk}}\gamma_{jk}^{2n_{ijk}} + M_j|d_{jik}|^{2-2m_{jik}}\gamma_{ik}^{2-2n_{jik}}) \\
& + \sum_{j,k=1}^n (M_i|d_{ijk}|^{2r_{ijk}}\gamma_{kj}^{2s_{ijk}} + M_k|d_{kji}|^{2-2r_{kji}}\gamma_{ij}^{2-2s_{kji}}) \\
& + \sum_{j,k=1}^n (M_i|e_{ijk}|^{2m^*_{ijk}}\gamma_{jk}^{2n^*_{ijk}} + M_j|e_{jik}|^{2-2m^*_{jik}}\gamma_{ik}^{2-2n^*_{jik}}) \\
& + \sum_{j,k=1}^n (M_i|e_{ijk}|^{2r^*_{ijk}}\gamma_{kj}^{2s^*_{ijk}} + M_k|e_{kji}|^{2-2r^*_{kji}}\gamma_{ij}^{2-2s^*_{kji}}).
\end{aligned}$$

Remark 3. $(H4)$ is important for the asymptotical behavior of system (1), it assures that the neuron reaction can be controlled by the amplification function and the behavior function.

The main result of this paper is the following theorem:

Theorem 1. *If $(H1)-(H4)$ hold, then system (1) has a unique $\omega-$periodic solution and all other solutions converge exponentially to it as $t \to +\infty$.*

Remark 4. If $p_{ij} = q_{ij} = m_{ijk} = n_{ijk} = r_{ijk} = s_{ijk} = p_{ij}^* = q_{ij}^* = m_{ijk}^* = n_{ijk}^* = r_{ijk}^* = s_{ijk}^* = 1$ or $p_{ij} = q_{ij} = m_{ijk} = n_{ijk} = r_{ijk} = s_{ijk} = p_{ij}^* = q_{ij}^* = m_{ijk}^* = n_{ijk}^* = r_{ijk}^* = s_{ijk}^* = \frac{1}{2}$, then $(H4)$ can be changed into the following:

$$\begin{aligned}(H5) : 2\alpha_i m_i > \; & + \sum_{j=1}^{n} \Gamma_i\Big(\big(|c_{ij}| + |\omega_{ij}|\big)\Lambda_j + B_i\Big) \\ & + \sum_{j=1}^{n} \Big[M_i\Big(|c_{ij}|^2\beta_j^2 + |\omega_{ij}|^2\beta_j^2\Big) + 2M_j\Big] \\ & + \sum_{j,k=1}^{n} (M_i|d_{ijk}|^2\gamma_{jk}^2 + M_j) + \sum_{j,k=1}^{n} (M_i|d_{ijk}|^2\mu_{jk}^2 + M_k) \\ & + \sum_{j,k=1}^{n} (M_i|e_{ijk}|^2\gamma_{jk}^2 + M_j) + \sum_{j,k=1}^{n} (M_i|e_{ijk}|^2\mu_{jk}^2 + M_k)\end{aligned}$$

or

$$\begin{aligned}(H6) : 2\alpha_i m_i > \; & + \sum_{j=1}^{n} \Gamma_i\Big(\big(|c_{ij}| + |\omega_{ij}|\big)\Lambda_j + B_i\Big) \\ & + \sum_{j=1}^{n} \Big[M_i\Big(|c_{ij}|\beta_j + |\omega_{ij}|\beta_j\Big) + M_j\Big(|c_{ji}|\beta_i + |\omega_{ji}|\beta_i\Big)\Big] \\ & + \sum_{j,k=1}^{n} (M_i|d_{ijk}|\gamma_{jk} + M_j|d_{jik}|\gamma_{ik} + M_i|d_{ijk}|\gamma_{kj} + M_k|d_{kji}|\gamma_{ij}) \\ & + \sum_{j,k=1}^{n} (M_i|e_{ijk}|\gamma_{jk} + M_j|e_{jik}|\gamma_{ik} + M_i|e_{ijk}|\gamma_{kj} + M_k|e_{kji}|\gamma_{ij}).\end{aligned}$$

Then we have the following corollary:

Corollary 1. *If $(H1)$-$(H3)$ hold, furthermore, $(H5)$ or $(H6)$ holds, then system (1) has a unique $\omega-$periodic solution and all other solutions converge exponentially to it as $t \to +\infty$.*

Remark 5. When $D_{ik} \equiv 0$, system (1) becomes the system which expressed by ordinary differential equations. Furthermore, while $d_{ijk} \equiv 0$ and $e_{ijk} \equiv 0$, system (1) becomes the first-order CGNNs. Therefore our results contain those in [7],[19] and [22].

3. The Proof of Theorem 1

In this section, we will begin to discuss the periodicity of the solution to system (1). The following is the proof of Theorem 1:

Proof. For any smooth vector functions φ and ψ, let $u(x,t;\varphi)=(u_1(x,t;\varphi),\cdots,u_n(x,t;\varphi))^T$ and $u(x,t;\psi)=(u_1(x,t;\psi),\cdots,u_n(x,t;\psi))^T$ denote the solutions to system (1) which satisfy the assumptions $(H1)$-$(H4)$ through $(\varphi,0)$ and $(\psi,0)$, respectively.

Let $v_i(x,t)=u_i(x,t;\varphi)-u_i(x,t;\psi)$, $i=1,\cdots n$, then from (1) we know that $v_i(x,t)$ satisfies the following

$$
\begin{aligned}
\frac{\partial v_i(x,t)}{\partial t} &= \sum_{k=1}^{m}\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial v_i(x,t)}{\partial x_k}) - a_i(u_i(x,t;\varphi))\Big[I_i(t)+b_i(u_i(x,t;\varphi))\\
&- \sum_{j=1}^{n} c_{ij}f_j(u_j(x,t;\varphi)) - \sum_{j=1}^{n}\omega_{ij}f_j(u_j(x,t-t_{ij};\varphi))\\
&- \sum_{j,k=1}^{n} d_{ijk}f_j(u_j(x,t;\varphi))f_k(u_k(x,t;\varphi))\\
&- \sum_{j,k=1}^{n} e_{ijk}f_j(u_j(x,t-t_{ij};\varphi))f_k(u_k(x,t-t_{ik};\varphi))\Big]\\
&+ a_i(u_i(x,t;\psi))\Big[b_i(u_i(x,t;\psi)) - \sum_{j=1}^{n} c_{ij}f_j(u_j(x,t;\psi))\\
&- \sum_{j=1}^{n}\omega_{ij}f_j(u_j(x,t-t_{ij};\psi))\\
&- \sum_{j,k=1}^{n} d_{ijk}f_j(u_j(x,t;\psi))f_k(u_k(x,t;\psi)) \qquad (4)\\
&- \sum_{j,k=1}^{n} e_{ijk}f_j(u_j(x,t-t_{ij});\psi)f_k(u_k(x,t-t_{ik};\psi)) + I_i(t)\Big]\\
v_i(x,t) &= \varphi_i(x,t)-\psi_i(x,t), \quad -\tau\le t\le 0,\ x\in\Omega,\\
\frac{\partial v_i(x,t)}{\partial \nu} &= 0, \quad x\in\partial\Omega.
\end{aligned}
$$

By simple calculation, we see that $v_i(x,t)$ also satisfies the following equation:

$$
\frac{\partial v_i(x,t)}{\partial t} = \sum_{k=1}^{m}\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial v_i(x,t)}{\partial x_k}) - \Big(a_i(u_i(x,t;\varphi)) - a_i(u_i(x,t;\psi))\Big)
$$

$$
\begin{aligned}
&\cdot \Big[b_i(u_i(x,t;\psi)) - \sum_{j=1}^{n} c_{ij} f_j(u_j(x,t;\psi)) - \sum_{j=1}^{n} \omega_{ij} f_j(u_j(x,t-t_{ij};\psi)) \\
&- \sum_{j,k=1}^{n} d_{ijk} f_j(u_j(x,t;\psi)) f_k(u_k(x,t;\psi)) \\
&- \sum_{j,k=1}^{n} e_{ijk} f_j(u_j(x,t-t_{ij});\psi) f_k(u_k(x,t-t_{ik};\psi)) + I_i(t)\Big] \\
&- a_i(u_i(x,t;\varphi))\Big[\Big(b_i(u_i(x,t;\varphi)) - b_i(u_i(x,t;\psi))\Big) \\
&- \sum_{j=1}^{n} \omega_{ij}\Big(f_j(u_j(x,t-t_{ij};\varphi)) - f_j(u_j(x,t-t_{ij};\psi))\Big) \\
&- \sum_{j=1}^{n} c_{ij}\Big(f_j(u_j(x,t;\varphi)) - f_j(u_j(x,t;\psi))\Big) - \sum_{j,k=1}^{n} d_{ijk} \cdot \\
&\quad \Big(f_j(u_j(x,t;\varphi)) f_k(u_k(x,t;\varphi)) - f_j(u_j(x,t;\psi)) f_k(u_k(x,t;\psi))\Big) \\
&- \sum_{j,k=1}^{n} e_{ijk}\Big(f_j(u_j(x,t-t_{ij});\varphi) f_k(u_k(x,t-t_{ik};\varphi)) \\
&\quad - f_j(u_j(x,t-t_{ij});\psi) f_k(u_k(x,t-t_{ik};\psi))\Big)\Big].
\end{aligned}
$$

By $(H4)$, there exists a small positive constant $0 < \lambda < \min\{\frac{1}{2}, \min_i\{m_i\alpha_i\}\}$ such that

$$
\begin{aligned}
W_i &= \lambda - \alpha_i m_i + \frac{1}{2}\Big[\sum_{j=1}^{n} \Gamma_i\big((|c_{ij}| + |\omega_{ij}|)\Lambda_j + B_i\big) \\
&+ \sum_{j=1}^{n} (M_i |c_{ij}|^{2p_{ij}} \beta_j^{2q_{ij}} + M_j |c_{ji}|^{2-2p_{ji}} \beta_i^{2-2q_{ji}}) \\
&+ \sum_{j=1}^{n} (M_i |\omega_{ij}|^{2p^*_{ij}} \beta_j^{2q^*_{ij}} + M_j |\omega_{ji}|^{2-2p^*_{ji}} \beta_i^{2-2q^*_{ji}} e^{2\lambda\tau}) \\
&+ \sum_{j,k=1}^{n} (M_i |d_{ijk}|^{2m_{ijk}} \gamma_{jk}^{2n_{ijk}} + M_j |d_{jik}|^{2-2m_{jik}} \gamma_{ik}^{2-2n_{jik}}) \qquad (5) \\
&+ \sum_{j,k=1}^{n} (M_i |d_{ijk}|^{2r_{ijk}} \gamma_{kj}^{2s_{ijk}} + M_k |d_{kji}|^{2-2r_{kji}} \gamma_{ij}^{2-2s_{kji}}) \\
&+ \sum_{j,k=1}^{n} (M_i |e_{ijk}|^{2m^*_{ijk}} \gamma_{jk}^{2n^*_{ijk}} + M_j |e_{jik}|^{2-2m^*_{jik}} \gamma_{ik}^{2-2n^*_{jik}} e^{2\lambda\tau})
\end{aligned}
$$

$$+ \sum_{j,k=1}^{n}(M_i|e_{ijk}|^{2r^*_{ijk}}\gamma_{kj}^{2s^*_{ijk}} + M_k|e_{kji}|^{2-2r^*_{kji}}\gamma_{ij}^{2-2s^*_{kji}}e^{2\lambda\tau})\Big] \leq 0.$$

Taking Laypunov functional as follows:

$$\begin{aligned} V(t) &= \sum_{i=1}^{n}\int_{\Omega}\Big[|v_i(x,t)|^2e^{2\lambda t} \\ &+ \sum_{j=1}^{n}M_i|\omega_{ij}|^{2-2p^*_{ij}}\beta_j^{2-2q^*_{ij}}\int_{t-t_{ij}}^{t}|v_j(x,s)|^2e^{2\lambda(s+t_{ij})}ds \\ &+ \sum_{j,k=1}^{n}M_i|e_{ijk}|^{2-2m^*_{ijk}}\gamma_{jk}^{2-2n^*_{ijk}}\int_{t-t_{ij}}^{t}|v_j(x,s)|^2e^{2\lambda(s+t_{ij})}ds \\ &+ \sum_{j,k=1}^{n}M_i|e_{ijk}|^{2-2r^*_{ijk}}\gamma_{kj}^{2-2s^*_{ijk}}\int_{t-t_{ik}}^{t}|v_k(x,s)|^2e^{2\lambda(s+t_{ik})}ds\Big]dx. \end{aligned}$$

Calculating $D^+V(t)$ along system (4), we have

$$\begin{aligned} &D^+V(t) \\ &= \sum_{i=1}^{n}\int_{\Omega}[2\lambda|v_i(x,t)|^2e^{2\lambda t} + 2v_i(x,t)e^{2\lambda t}\frac{\partial v_i(x,t)}{\partial t} \qquad (6) \\ &+ \sum_{j=1}^{n}M_i|\omega_{ij}|^{2-2p^*_{ij}}\beta_j^{2-2q^*_{ij}}(|v_j(x,t)|^2e^{2\lambda(t+t_{ij})} - |v_j(x,t-t_{ij})|^2e^{2\lambda t}) \\ &+ \sum_{j,k=1}^{n}M_i(|e_{ijk}|^{2-2m^*_{ijk}}\gamma_{jk}^{2-2n^*_{ijk}}(|v_j(x,t)|^2e^{2\lambda(t+t_{ij})} - |v_j(x,t-t_{ij})|^2e^{2\lambda t}) \\ &+ \sum_{j,k=1}^{n}M_i|e_{ijk}|^{2-2r^*_{ijk}}\gamma_{kj}^{2-2s^*_{ijk}})(|v_k(x,t)|^2e^{2\lambda(t+t_{ik})} - |v_k(x,t-t_{ik})|^2e^{2\lambda t})]dx. \end{aligned}$$

Noting $(H1)$, by using the Green formula and the boundary condition, it has

$$\begin{aligned} &\int_{\Omega}\sum_{k=1}^{m}\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial v_i(x,t)}{\partial x_k})v_i(x,t) \\ &= \int_{\Omega}\sum_{k=1}^{m}\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial v_i(x,t)}{\partial x_k}v_i(x,t))dx - \int_{\Omega}\sum_{k=1}^{m}\frac{\partial v_i(x,t)}{\partial x_k}D_{ik}\frac{\partial v_i(x,t)}{\partial x_k}dx \\ &= -\sum_{k=1}^{m}\int_{\Omega}D_{ik}(\frac{\partial v_i(x,t)}{\partial x_k})^2dx \leq 0. \end{aligned}$$

Hence, it follows from (6) to get

$$D^+V(t)$$

$$
\begin{aligned}
\leq\ & e^{2\lambda t}\sum_{i=1}^{n}\int_{\Omega}[2\lambda|v_i(x,t)|^2-2m_i\alpha_i|v_i(x,t)|^2\\
&+\sum_{j=1}^{n}\Gamma_i\big(\big(|c_{ij}|+|\omega_{ij}|\big)\Lambda_j+B_i\big)|v_i(x,t)|^2\\
&+\sum_{j=1}^{n}M_i(|c_{ij}|\beta_j|v_i(x,t)||v_j(x,t)|+|\omega_{ij}|\beta_j|v_i(x,t)||v_j(x,t-t_{ij})|)\\
&+\sum_{j,k=1}^{n}M_i|d_{ijk}|(\gamma_{jk}|v_i(x,t)||v_j(x,t)|+\gamma_{kj}|v_i(x,t)||v_k(x,t)|)\\
&+\sum_{j,k=1}^{n}M_i|e_{ijk}|(\gamma_{jk}|v_i(x,t)||v_j(x,t-t_{ij})|+\gamma_{kj}|v_i(x,t)||v_k(x,t-t_{ik})|)\\
&+\sum_{j=1}^{n}M_i|\omega_{ij}|^{2-2p_{ij}^*}\beta_j^{2-2q_{ij}^*}(|v_j(x,t)|^2e^{2\lambda t_{ij}}-|v_j(x,t-t_{ij})|^2)\\
&+\sum_{j,k=1}^{n}M_i|e_{ijk}|^{2-2m_{ijk}^*}\gamma_{jk}^{2-2n_{ijk}^*}(|v_j(x,t)|^2e^{2\lambda t_{ij}}-|v_j(x,t-t_{ij})|^2)\\
&+\sum_{j,k=1}^{n}M_i|e_{ijk}|^{2-2r_{ijk}^*}\gamma_{kj}^{2-2s_{ijk}^*}(|v_k(x,t)|^2e^{2\lambda t_{ik}}-|v_k(x,t-t_{ik})|^2)]dx\\
\leq\ & e^{2\lambda t}\sum_{i=1}^{n}\int_{\Omega}\Big\{|v_i(x,t)|^2\Big(2\lambda-2m_i\alpha_i+\sum_{j=1}^{n}\Gamma_i\big(\big(|c_{ij}|+|\omega_{ij}|\big)\Lambda_j+B_i\big)\Big)\\
&+\sum_{j=1}^{n}(M_i|c_{ij}|^{2p_{ij}}\beta_j^{2q_{ij}}+M_j|c_{ji}|^{2-2p_{ji}}\beta_i^{2-2q_{ji}}+M_i|\omega_{ij}|^{2p_{ij}^*}\beta_j^{2q_{ij}^*})|v_i(x,t)|^2\\
&+\sum_{j,k=1}^{n}(M_i|d_{ijk}|^{2m_{ijk}}\gamma_{jk}^{2n_{ijk}}+M_j|d_{jik}|^{2-2m_{jik}}\gamma_{ik}^{2-2n_{jik}}\\
&\quad+M_i|d_{ijk}|^{2r_{ijk}}\gamma_{kj}^{2s_{ijk}}+M_k|d_{kji}|^{2-2r_{kji}}\gamma_{ij}^{2-2s_{kji}})|v_i(x,t)|^2\\
&+\sum_{j,k=1}^{n}(M_i|e_{ijk}|^{2m_{ijk}^*}\gamma_{jk}^{2n_{ijk}^*}+M_i|e_{ijk}|^{2r_{ijk}^*}\gamma_{kj}^{2s_{ijk}^*})|v_i(x,t)|^2\\
&+\sum_{j,k=1}^{n}(M_i|e_{ijk}|^{2-2m_{ijk}^*}\gamma_{jk}^{2-2n_{ijk}^*}|v_j(x,t-t_{ij})|^2\\
&\quad+M_i|e_{ijk}|^{2-2r_{ijk}^*}\gamma_{kj}^{2-2s_{ijk}^*}|v_k(x,t-t_k)|^2)\\
&+\sum_{j=1}^{n}M_i|\omega_{ij}|^{2-2p_{ij}^*}\beta_j^{2-2q_{ij}^*}|v_j(x,t-t_{ij})|^2
\end{aligned}
$$

$$+\sum_{j=1}^{n} M_i|\omega_{ij}|^{2-2p^*_{ij}}\beta_j^{2-2q^*_{ij}}(|v_j(x,t)|^2e^{2\lambda t_{ij}}-|v_j(x,t-t_{ij})|^2)$$
$$+\sum_{j,k=1}^{n} M_i|e_{ijk}|^{2-2m^*_{ijk}}\gamma_{jk}^{2-2n^*_{ijk}}(|v_j(x,t)|^2e^{2\lambda t_{ij}}-|v_j(x,t-t_{ij})|^2)$$
$$+\sum_{j,k=1}^{n} M_i|e_{ijk}|^{2-2r^*_{ijk}}\gamma_{kj}^{2-2s^*_{ijk}}(|v_k(x,t)|^2e^{2\lambda t_{ik}}-|y_k(x,t-t_k)|^2)\Big\}dx.$$

Then we also have the following

$$\begin{aligned}
&D^+V(t)\\
\le\ & e^{2\lambda t}\sum_{i=1}^{n}\int_\Omega\Big[|v_i(x,t)|^2\Big(2\lambda-2m_i\alpha_i+\sum_{j=1}^{n}\Gamma_i\big((|c_{ij}|+|\omega_{ij}|)\Lambda_j+B_i\big)\\
&+\sum_{j=1}^{n}(M_i|c_{ij}|^{2p_{ij}}\beta_j^{2q_{ij}}+M_j|c_{ji}|^{2-2p_{ji}}\beta_i^{2-2q_{ji}}+M_i|\omega_{ij}|^{2p^*_{ij}}\beta_j^{2q^*_{ij}})\\
&+\sum_{j,k=1}^{n}(M_i|d_{ijk}|^{2m_{ijk}}\gamma_{jk}^{2n_{ijk}}+M_j|d_{jik}|^{2-2m_{jik}}\gamma_{ik}^{2-2n_{jik}}\\
&\quad+M_i|d_{ijk}|^{2r_{ijk}}\gamma_{kj}^{2s_{ijk}}+M_k|d_{kji}|^{2-2r_{kji}}\gamma_{ij}^{2-2s_{kji}})\Big)\\
&+\sum_{j,k=1}^{n}(M_i|e_{ijk}|^{2m^*_{ijk}}\gamma_{jk}^{2n^*_{ijk}}+M_i|e_{ijk}|^{2r^*_{ijk}}\gamma_{kj}^{2s^*_{ijk}})|v_i(x,t)|^2\\
&+\sum_{j=1}^{n}M_i|\omega_{ij}|^{2-2p^*_{ij}}\beta_j^{2-2q^*_{ij}}e^{2\lambda\tau}|v_j(x,t)|^2\\
&+\sum_{j,k=1}^{n}M_i|e_{ijk}|^{2-2m^*_{ijk}}\gamma_{jk}^{2-2n^*_{ijk}}e^{2\lambda\tau}|v_j(x,t)|^2\\
&+\sum_{j,k=1}^{n}M_i|e_{ijk}|^{2-2r^*_{ijk}}\gamma_{kj}^{2-2s^*_{ijk}}e^{2\lambda\tau}|v_k(x,t)|^2\Big]dx\\
=\ & 2e^{2\lambda t}\sum_{i=1}^{n}W_i\int_\Omega|v_i(x,t)|^2dx\le 0,
\end{aligned}$$

where W_i is defined by (5). It means that

$$V(t)\le V(0)=\sum_{i=1}^{n}\int_\Omega\Big[|\varphi_i(x,0)-\psi_i(x,0)|^2$$
$$+\quad\sum_{j=1}^{n}M_i|\omega_{ij}|^{2-2p^*_{ij}}\beta_j^{2-2q^*_{ij}}\int_0^{t_{ij}}|\varphi_j(x,s-t_{ij})-\psi_j(x,s-t_{ij})|^2e^{2\lambda s}ds$$

$$+\sum_{j,k=1}^{n} M_i|e_{ijk}|^{2-2m^*_{ijk}}\gamma_{jk}^{2-2n^*_{ijk}}\int_0^{t_{ji}}|\varphi_j(x,s-t_{ji})-\psi_j(x,s-t_{ji})|^2e^{2\lambda s}ds$$

$$+\sum_{j,k=1}^{n} M_i|e_{ikj}|^{2-2r^*_{ikj}}\gamma_{jk}^{2-2s^*_{ikj}}\int_0^{t_{ik}}|\varphi_j(x,s-t_{ij})-\psi_j(x,s-t_{ij})|^2e^{2\lambda s}ds\Big]dx.$$

Let

$$\begin{aligned}\Upsilon &= \max_i\Big\{1+(e^{2\lambda\tau}-1)\sum_{j,k=1}^{n}\frac{M_i}{2\lambda}\Big(|\omega_{ij}|^{2-2p^*_{ij}}\beta_j^{2-2q^*_{ij}}\\ &\quad +|e_{ijk}|^{2-2m^*_{ijk}}\gamma_{jk}^{2-2n^*_{ijk}}+|e_{ikj}|^{2-2r^*_{ikj}}\gamma_{jk}^{2-2s^*_{ikj}}\Big)\Big\}>1.\end{aligned}$$

By the definition of $V(t)$ we know that $V(t)\geq\|v(\cdot,t)\|_2^2e^{2\lambda t}$(where the norm $\|\cdot\|_2$ is defined in page 3), so it holds:

$$\|v(\cdot,t)\|_2^2\leq\Upsilon\|\varphi-\psi\|_2^2e^{-2\lambda t},$$

according to the definition of function $v(x,t)$ in page 6, that is

$$\|u(\cdot,t;\varphi)-u(\cdot,t;\psi)\|_2\leq\sqrt{\Upsilon}\|\varphi-\psi\|_2e^{-\lambda t}. \tag{7}$$

For $\tau>0$, we can choose a positive integer N and a positive constant κ such that

$$\sqrt{\Upsilon}e^{-\lambda(N\omega-\tau)}\leq\kappa<1. \tag{8}$$

If we define a poincaré mapping $P:\ C\to C$ (here C denotes the continuous function space) by $P(\Phi)=u(x,\omega+\theta;\Phi)$, $(\theta\in[-\tau,0])$, then $P^N(\Phi)=u(x,N\omega+\theta;\Phi)$, $\theta\in[-\tau,0]$. Let $t=N\omega$, by (7)-(8), we get

$$\begin{aligned}\|P^N(\varphi)-P^N(\psi)\|_2 &= \|u(x,N\omega+\theta;\varphi)-u(x,N\omega+\theta;\psi)\|_2\\ &\leq \sqrt{\Upsilon}\|\varphi-\psi\|_2e^{-\lambda(N\omega+\theta)}\leq\sqrt{\Upsilon}\|\varphi-\psi\|_2e^{-\lambda(N\omega-\tau)}\\ &\leq \kappa\|\varphi-\psi\|_2.\end{aligned}$$

This implies that P^N is a contraction mapping because of $0<\kappa<1$. According to the Banach fixed point theorem, there exists a unique point $\varphi^*\in C$ such that $P^N(\varphi^*)=\varphi^*$. Since $P^N(P(\varphi^*))=P(P^N(\varphi^*))=P(\varphi^*)$, then $P(\varphi^*)$ is also a fixed point of P^N. By the uniqueness of the fixed point of P^N, we have

$$P(\varphi^*)=\varphi^* \text{ that is } u(x,\omega+\theta;\varphi^*)=\varphi^*.$$

Let $u(x,\omega+\theta;\varphi^*)$ be the solution of CGNNs (1) through $(\varphi^*,0)$, then $u(x,t+\omega+\theta;\varphi^*)$ is also the solution of CGNNs (1) through $(u(x,\omega+\theta;\varphi^*),0)$. obviously, for all $t\geq0$, it has

$$u(x,t+\omega+\theta;\varphi^*)=u(x,t+\theta;u(x,\omega+\theta;\varphi^*))=u(x,t+\theta;\varphi^*),\forall\theta\in[-\tau,0].$$

Hence, for all $t\geq0$, it has

$$u(x,t+\omega;\varphi^*)=u(x,t;\varphi^*).$$

This shows that $u(x,t;\varphi^*)$ is exactly one ω-periodic solution of CGNNs (1), and according to (7) it is easy to see that all other solutions of CGNNs (1) converge exponentially to it as $t\to+\infty$.

4. Example

In order to illustrate the feasibility of our criteria established above, we consider the following high-order Cohen-Grossberg neural networks with delays and reaction-diffusion terms

$$\begin{aligned}
\frac{\partial u_1(x,t)}{\partial t} &= \frac{\partial}{\partial x}(D_1\frac{\partial u_1(x,t)}{\partial x}) - a_1(u_1(x,t))[b_1(u_1(x,t)) - \sum_{j=1}^{2} c_{1j}f_j(u_j(x,t)) \\
&\quad -\sum_{j=1}^{2}\omega_{1j}f_j(u_j(x,t-t_j)) - \sum_{j,k=1}^{2} d_{1jk}f_j(u_j(x,t))f_k(u_k(x,t)) \\
&\quad - \sum_{j,k=1}^{2} e_{1jk}f_j(u_j(x,t-t_j))f_k(u_k(x,t-t_k) + I_1(t)], \\
\frac{\partial u_2(x,t)}{\partial t} &= \frac{\partial}{\partial x}(D_2\frac{\partial u_2(x,t)}{\partial x}) - a_2(u_2(x,t))[b_2(u_2(x,t)) - \sum_{j=1}^{2} c_{2j}f_j(u_j(x,t)) \\
&\quad -\sum_{j=1}^{2}\omega_{2j}f_j(u_j(x,t-t_j)) - \sum_{j,k=1}^{2} d_{2jk}f_j(u_j(x,t))f_k(u_k(x,t)) \\
&\quad - \sum_{j,k=1}^{2} e_{2jk}f_j(u_j(x,t-t_j))f_k(u_k(x,t-t_k) + I_2(t)], \\
u_i(x,t) &= \varphi_i(x,t), \quad -\tau\le t\le 0,\ x\in\Omega,\ i=1,2 \qquad (9)\\
\frac{\partial u_i(x,t)}{\partial \nu} &= 0, \quad x\in\partial\Omega,\ i=1,2.
\end{aligned}$$

Let $D_1>0, D_2>0$. Let $a_1 = 6+\sin u_1(x,t)$ and $a_2 = 3-\sin u_2(x,t)$ which satisfy (H1) with $m_1=5, M_1=7, m_2=2, M_2=4, \Gamma_1=1, \Gamma_2=1$. $b_1 = 5u_1(x,t)+\frac{1}{2}\sin u_1(x,t)$ and $b_2 = 6u_2(x,t)+\frac{1}{2}\sin u_2(x,t)$ satisfy (H2) with $\alpha_1=\frac{9}{2}, \alpha_2=\frac{11}{2}$. $f_1=\tanh u_1(x,t)$ and $f_2=\tanh u_2(x,t)$ satisfy (H3) with $\beta_1=1, \beta_2=1, \gamma_{ij}=1, i,j=1,2$ and $\Lambda_1=1$, $\Lambda_2=1$. Furthermore, let $c_{ij}=d_{ijk}=0, i,j,k=1,2$ and $\omega_{11}=\frac{1}{20}, \omega_{12}=\frac{1}{60}, \omega_{21}=\frac{1}{50}, \omega_{22}=\frac{1}{10}. e_{111}=\frac{1}{20}, e_{112}=e_{121}=-\frac{1}{64}, e_{122}=\frac{1}{12}, e_{211}=\frac{1}{40}, e_{212}=e_{221}=-\frac{1}{32}, e_{222}=-\frac{1}{30}$. $I_1=2+\sin t, I_2=1-3\cos t$ with $B_1=3$ and $B_2=4$. By simple calculation, for all $i,j=1,2$ we can choose $p_{ij}=q_{ij}=m_{ijk}=n_{ijk}=r_{ijk}=s_{ijk}=\frac{1}{2}$ and $p^*_{ij}=q^*_{ij}=m^*_{ijk}=n^*_{ijk}=r^*_{ijk}=s^*_{ijk}=\frac{1}{2}$, such that

$$-2\alpha_1m_1+\sum_{j=1}^{2}\Gamma_1\big((|c_{1j}|+|\omega_{1j}|)\Lambda_j+B_1\big)+\sum_{j=1}^{2}(M_1|\omega_{1j}|\beta_j+M_j|\omega_{j1}|\beta_1)$$

$$
\begin{aligned}
& + \sum_{j,k=1}^{2} (M_1|e_{1jk}|\gamma_{jk} + M_j|e_{j1k}|\gamma_{1k}) + \sum_{j,k=1}^{2} (M_1|e_{1jk}|\mu_{jk} + M_k|e_{kj1}|\mu_{j1}) \\
= & -2 \cdot \frac{9}{2} \cdot 5 + 1 \cdot [(\frac{1}{20} + \frac{1}{60}) \cdot 1 + 3] + [7 \cdot (\frac{1}{20} + \frac{1}{60}) + 7 \cdot \frac{1}{20} + 4 \cdot \frac{1}{50}] \\
& +2 \cdot [7 \cdot (\frac{1}{20} + \frac{1}{64} + \frac{1}{64} + \frac{1}{12}) + 7 \cdot (\frac{1}{20} + \frac{1}{64}) + 4 \cdot (\frac{1}{40} + \frac{1}{32})] \\
= & -45 + 7\frac{509}{800} = -37\frac{291}{800} < 0.
\end{aligned}
$$

Similarly, we also have

$$
\begin{aligned}
& -2\alpha_2 m_2 + \sum_{j=1}^{2} \Gamma_2\big((|c_{2j}| + |\omega_{2j}|)\Lambda_j + B_2\big) + \sum_{j=1}^{2} (M_2|\omega_{2j}|\beta_j + M_j|\omega_{j2}|\beta_2) \\
+ & \sum_{j,k=1}^{2} (M_2|e_{2jk}|\gamma_{jk} + M_j|e_{j2k}|\gamma_{2k}) + \sum_{j,k=1}^{2} (M_2|e_{2jk}|\mu_{jk} + M_k|e_{kj2}|\mu_{j2}) < 0.
\end{aligned}
$$

That means (H6) holds. It follows from Corollary 1 that system (9) has unique $\omega-$periodic solution and all other solutions converge exponentially to it as $t \to +\infty$.

Although the selection of the coefficients and functions in the above example is somewhat artificial, the possible application of our theoretical results should have been clearly expressed.

5. Conclusions

In this paper, the periodic behaviors of high-order Cohen-Grossberg neural networks model with delays and reaction-diffusion terms are studied. By employing inequality techniques and constructing Lyapunov functional, some sufficient conditions have been given to ensure the existence of the periodic solution. These conditions are useful in the design and applications of high-order Cohen-Grossberg neural networks model with delays and reaction-diffusion terms and the method in this paper may be extended for more complex networks.

On the other hand, our criteria for the high-order CGNNs require that the behavior function should be bounded which is not needed in first-order neural networks. In the future work, whether this condition can be got rid of for high-order neural networks or not should be studied.

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REFERENCES

1. M. Cohen, S. Grossberg, *Absolute stability and global pattern formation and parallel memory storage by competitive neural networks*, IEEE Trans. Syst. Man Cybernet **SMC-13**(1983), 15-23.
2. J. Cao, L. Liang, *Boundedness and stability for Coheb-Grossberg neural network with time-varying delays*, J.Math. Anal.Application **296**(2004), 665-685.
3. J. Cao, X. Li, *Stability in delayed Cohen-Grossberg neural networks: LMI optimization approach*, Physica D **212**(2005), 54-65.
4. Z. Chen, H. Zhao & J. Ruan, *Dynamic analysis of high-order Cohen-Grossberg neural networks with time delay*, Chaos Solitons & Fractals **24**(2006), 421-430.
5. H. Jiang, J. Cao, *Dynamics of Cohen-Grossberg neural networks with time-varying delays*, Phys. Lett.A **354**(2006), 414-422.
6. T. Chen, L. Rong, *Robust global exponential stability of Cohen-Grossberg neural networks with time delays*, IEEE trans. Neur. Network **15**(2004), 203-206.
7. Y. Li, *Existence and stability of periodic solutions for Cohen-Grossberg neural networks with multiple delays*, Chaos Solitons & Fractals **20**(2004), 459-466.
8. C. Hwang, C. Cheng & T. Li, *Globally exponential stability of generalized Cohen-Grossberg neural networks with time delay*, Phys. Lett. A **319**(2003), 157-166.
9. H. Wan, H. Qiao & J. Peng, *Delay-independent criteria for exponential stability generalized Cohen-Grossberg neural networks with dicrete delay*, Phys. Lett.A **353**(2006), 151-157.
10. X. Liao, C. Li & K. Wong, *Criteria for exponential stability of Cohen-Grossberg neural networks*, Neural Networks **17**(2004), 1401-1414.
11. W. Xiong, D. Xu, *Global exponential stability of discrete-time Cohen-Grossberg neural networks*, Neurocomputing **64**(2005), 433-446.
12. J. Xu, Z. Liu,& X. Liao, *Global asymptotic stability of high-order Hopfield type neural networks with time delays*, Comp.Math. Application **45**(2003), 1729-1737.
13. P. Yan, T. Lv, *Stability of delayed reaction diffusion high-order Cohen-Grossberg neural networks with variable coefficient*, Proccedings of Second International Symposium on Intelligent Information Technology Application, IEEE CS Press **1**(2008), 741-745.
14. K. Yuan, J. Cao, *Global exponential stability of Cohen-Grossberg neural networks with multiple time-varying delays*, LNCS **3173**(2004), 745-753.
15. J. Zhang, Y. Suda, H. Komine, *Global exponential Stability of Cohen-Grossberg neural networks with variable delays*, Phys. Lett.A **338**(2005), 44-50.
16. H. Zhao, K. Wang, *Dynamical behaviors of Cohen-Grossberg neural networks with delays and reaction-diffusion terms*, Neurocomputing **70**(2006), 536-543.
17. J. Cao, L. Liang & J. Lamb,*Exponential Stability of high-order bidirectional associative memory neural networks with time delays*, Physica D **199**(2004), 425-436.
18. J. Liang, J. Cao, *Global exponential Stability of reaction-diffusion recurrent neural networks with time-varying delays*, Phys. Lett. A **314**(2003), 434-442.
19. Q. Song, J. Cao, *Global exponential Stability and existence of periodic solutions in BAM neural networks with distributed delays and reaction-diffusion terms*, Chaos Solitons & Fractals **23**(2005), 421-430.
20. Q. Song, Z. Zhao, Y. Li, *Global exponential Stability of BAM neural networks with distributed delays and reaction-diffusion terms*, Phys. Lett. A **335** (2005), 213-225.
21. Q. Song,J. Cao, Z. Zhao, *Periodic solutions and its exponential stability of reaction-diffusion recurrent neural netwokes with continuously distributed delays*, Nonlinear Anal: Real World Application **7**(2006), 421-430.
22. H. Zhao, G. Wang, *Existence of periodic oscillatory solution of reaction-diffusion neural networks with delays*, Phys. Lett. A **343**(2005), 372-383.

23. G. Lu, *Global exponential Stability and periodicity of reaction-diffusion delayed recurrent neural networks with Dirichlet Boundary conditons*, Chaos Solitons & Fractals **35**(2008), 116-125.
24. S. Schiff, K. Jerger, D. Duong, etc, *Controlling chaos in the brain*, Nature **370**(1994), 615-620.
25. X. Liao, K. Wong, C. Leung, etc, *Hopf bifurcation and chaos in a single delayed neuron equation with non monotonic activation function*, Chaos Solitons & Fractals **12** (2001), 1535-1547.

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