J. Appl. Math. & Informatics Vol. 28(2010), No. 1 - 2, pp. 217 - 231
Website: http://www.kcam.biz

EXISTENCE OF LARGE SOLUTIONS FOR A QUASILINEAR ELLIPTIC PROBLEM†

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ABSTRACT. We consider a class of elliptic problems of a logistic type

$$-\mathrm{div}(|\nabla u|^{m-2}\nabla u) = w(x)u^q - (a(x))^{\frac{m}{2}} f(u)$$

in a bounded domain of $\mathbf{R}^N$ with boundary $\partial\Omega$ of class C^2, $u|_{\partial\Omega} = +\infty$, $\omega \in L^\infty(\Omega)$, $0 < q < 1$ and $a \in C^\alpha(\overline{\Omega})$, $\mathbf{R}^+$ is non-negative for some $\alpha \in (0,1)$, where $\mathbf{R}^+ = [0,\infty)$. Under suitable growth assumptions on a, b and f, we show the exact blow-up rate and uniqueness of the large solutions. Our proof is based on the method of sub-supersolution.

AMS Mathematics Subject Classification : 35J60, 35J40.
Key word and phrases : Large solutions, blow-up rate, uniqueness, sub-supersolutions.

1. Introduction

The purpose of this paper is to investigate the blow-up rate and uniqueness of solutions to the problem

$$-\mathrm{div}(|\nabla u|^{m-2}\nabla u) = w(x)u^q - (a(x))^{\frac{m}{2}} f(u) \;\; x \in \Omega, u|_{\partial\Omega} = +\infty \tag{1.1}$$

where the boundary condition means $u(x) \to +\infty$ as $d(x) = \mathrm{dist}(x, \partial\Omega) \to 0$, Ω is a bounded domain in $\mathbf{R}^N (N \geq 1)$ with smooth boundary of class $C^2, m > 2, q \in (0,1), w \in L^\infty(\Omega)$ and $a \in C^\alpha(\overline{\Omega}, \mathbf{R}^+)$ is non-negative for some $\alpha \in (0,1)$ where $\mathbf{R}^+ = [0,\infty)$. The solutions to the above problems are called large solutions or explosive solutions.

This problem appears in the study of non-Newton fluids [1-3] and non-Newtonian filtration [4]. The quantity p is a characteristic of the medium. Media with $p > 2$ are called dilatant fluids and those with $p < 2$ are called pseudo-plastics. If $p = 2$, they are Newtonian fluids. Such problems arise in the study of the

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Received June 18, 2009. Accepted October 14, 2009. *Corresponding author.
†Project Supported by the National Natural Science Foundation of China(Grant No.10871060). Project Supported by the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (Grant No.08KJB110005).

sub-sonic motion of a gas [16], the electric potential in some bodies [17] and Riemanmian geometry [18].

Explosive solutions of the problem

$$\triangle u(x) = f(u(x)), x \in \Omega \tag{1.2}$$

$$u|_{\partial\Omega} = +\infty \tag{1.3}$$

where Ω is a bounded domain in $\mathbf{R}^N (N \geq 1)$ have been extensively studied, see [5-15]. A problem with $f(u) = -e^u$ and $N = 2$ was first considered by Bieberbach [10] in 1916.Bieberbach showed that if Ω is a bounded domain in R^2 such that $\partial\Omega$ is a C^2 sub-manifold of R^2, then there exists a unique $u \in C^2(\Omega)$ such that $\triangle u = -e^u$ in Ω and $|u(x) - (\ln(d(x))^{-2}|$ is bounded on Ω. Here $d(x)$ denotes the distance from a point $x$to $\partial\Omega$. Rademacher [14], using the idea of Bieberbach, extended the above result to a smooth bounded domain in R^3. In this case the problem plays an important role, when $N = 2$, in the theory of Riemanmian surfaces of constant negative curvature and in the theory of automorphic functions, and when $N = 3$, according to [14], in the study of the electric potential in a glowing hollow metal body. Lazer and McKenna [8] extended the results for a bounded domain Ω in $\mathbf{R}^N (N \geq 1)$ satisfying a uniform external sphere condition and the non-linearity $f = f(x,u) = p(x)(e^u)$ where $p(x)$ is continuous and strictly negative on $\overline{\Omega}$. The existence, but not uniqueness, of solutions of Eqs. (1.2) and (1.3) with f monotone was studied by Keller [15]. Problem

$$\triangle u = k(x)g(u), x \in \Omega, u|_{\partial\Omega} = +\infty \tag{1.4}$$

arises from many branches of mathematics and applied mathematics, and has been discussed by many authors and in many contexts, see e.g.[19-21]. Moreover, by analyzing the corresponding ordinary differential equations, combing with the maximum principle, Bandle and Marcus [20] obtain further conclusion through a series of assumptionson the function g.

In the paper, we extend the results of [22] to problem (1.1), nonlinearity f satisfying the Keller-Osserman condition, and the weight function $a(x)$ vanishes in some region of Ω, as well as on some piece of the boundary $\partial\Omega$. The basic structural assumptions of this paper are as the following

(H1) The open set $\Omega_+ = \{x \in \Omega : a(x) > 0\}$ is connected with boundary $\partial\,\Omega_+$ of class C^2, and the open $\Omega_0 = \Omega\backslash\overline{\Omega}_+$ satisfies $\overline{\Omega}_0 \subset \Omega$.

(H2) $f \in C^1(R_+, R_+)$ satisfies $f(0) = 0, f(s) > 0$ for each $s > 0$, $s \mapsto s^{-1}f(s)$ is nondecreasing in $(0, +\infty)$, and there exist $p > 1$ and $K_0 > 0$ such that $\lim\limits_{s\to+\infty} s^{-p}f(s) = K_0$.

(H3) There exists a positive nondecreasing function $b \in C([0,\delta])$, such that $a(x) = b(d(x))$ in Ω_δ, where

$$\Omega_\delta = \{x \in \Omega; d(x) = \text{dist}(x, \partial\Omega) < \delta\}$$

with

$$\delta \leq \frac{1}{2}\text{dist}(\Omega_0, \partial\Omega).$$

Moreover, $b(r)$ satisfies

$$\frac{1}{b(r)}\int_0^r b(s)ds, \quad \frac{1}{b(r)}\int_r^\delta b(s)ds \in C^1([0,\delta])$$

and

$$\lim_{r\to 0}\frac{1}{b(r)}\int_0^r b(s)ds = 0.$$

We modify the method developed in [22], and give the following Theorem

Theorem 1. *Under the assumptions (H1), (H2) and (H3), any positive solution u to the problem (1.1) satisfies*

$$\lim_{d(x)\to 0}\frac{u(x)}{(\int_0^{d(x)} A(r)dr)^{-\alpha}} = M, \tag{1.5}$$

where

$$M = \Big(\frac{\alpha^{m-1}(\alpha+1)(m-1)(A_0)^{\frac{m}{2}} - (m-1)\alpha^{m-1}(A_0)^{\frac{m}{2}-1}}{K_0}\Big)^{\frac{2}{m}\alpha},$$

$$\alpha = \frac{1}{p-1} A(r) = \int_0^r b(s)ds, \quad A_0 = \lim_{r\to 0}\frac{(\int_0^r b(s)ds)^2}{b(r)\int_0^r A(s)ds} \geq 1.$$

Furthermore, if $\omega(x) \geq 0$, then (1.1) admits a unique positive solution.

The distribution of this paper is as follows. In Section 2 we collect some preliminary results of a technical nature that are going to be used later. In Section 3 we give the asymptotic behavior of the solutions of two auxiliary problems that will be used in the next section. In Section 4 we prove the main Theorem.

2. Some preliminary results

In this section we collect some useful preliminary results to be used in Section 3. Let

$$A(r) = \int_0^r b(s)ds, \quad A^*(r) = \int_0^r A(s)ds \tag{2.1}$$

and

$$B(r) = \int_r^\delta b(s)ds, \quad B^*(r) = \int_r^\delta B(s)ds. \tag{2.2}$$

We give two propositions as follows

Proposition 1. *For the small $\delta > 0$, consider the following singular value problem*

$$\begin{cases} -div(|\nabla u|^{m-2}\nabla u) = \omega(x)u^q - a(x)^{\frac{m}{2}} f(u), & in\ \Omega_\delta \\ u = \infty, & on\ \partial\Omega \\ u = 0, & on\ \partial\Omega_\delta\backslash\partial\Omega \end{cases} \tag{2.3}$$

Suppose that $\underline{u}, \overline{u}$ satisfy

$$-div(|\nabla\underline{u}|^{m-2}\nabla\underline{u}) \leq \omega(x)\underline{u}^q - a(x)^{\frac{m}{2}}f(\underline{u}), \ in \ \Omega_\delta$$
$$-div(|\nabla\overline{u}|^{m-2}\nabla\overline{u}) \geq \omega(x)\overline{u}^q - a(x)^{\frac{m}{2}}f(\overline{u}), in \ \Omega_\delta$$
$$\lim_{d(x)\to 0}\underline{u} = \lim_{d(x)\to 0}\overline{u} = \infty, \quad \lim_{d(x)\to\delta}\underline{u} \leq 0 \leq \lim_{d(x)\to\delta}\overline{u}$$

and

$$\underline{u} \leq \overline{u} \ \ in \ \Omega_\delta.$$

Then the problem (2.3) possesses a solution u satisfying $\underline{u} \leq u \leq \overline{u}$.

Proposition 2. *Consider the following singular value problem*

$$\begin{cases} -div(|\nabla u|^{m-2}\nabla u) = \omega(x)u^q - a(x)^{\frac{m}{2}}f(u), & in \ \Omega_\delta \\ u = \infty, \ \ on \ \partial\Omega_\delta \end{cases} \tag{2.4}$$

Suppose that $\underline{u}$, $\overline{u}$ satisfy

$$-div(|\nabla\underline{u}|^{m-2}\nabla\underline{u}) \leq \omega(x)\underline{u}^q - a(x)^{\frac{m}{2}}f(\underline{u}), \ \ in \ \Omega_\delta$$
$$-div(|\nabla\overline{u}|^{m-2}\nabla\overline{u}) \geq \omega(x)\overline{u}^q - a(x)^{\frac{m}{2}}f(\overline{u}), \ \ in \ \Omega_\delta$$
$$\lim_{d(x)\to 0}\underline{u} = \lim_{d(x)\to 0}\overline{u} = \infty, \quad \lim_{d(x)\to\delta}\underline{u} = \lim_{d(x)\to\delta}\overline{u} = \infty,$$

and $\underline{u} \leq \overline{u}$ in Ω_δ. Then the problem (2.4) possesses a solution u satisfying $\underline{u} \leq u \leq \overline{u}$.

To prove Theorem 1, we also need the following two lemmas (see [22]).

Lemma 1. *Let $b(r) \in C([0,\delta],[0,+\infty))$. If $\frac{A(r)}{b(r)}$ is a differentiable in $[0,\delta]$, $\lim_{r\to 0}\frac{A(r)}{b(r)} = 0$, and $\lim_{r\to 0}(\frac{A(r)}{b(r)})' \geq 0$, then we have*

$$\lim_{r\to 0}\frac{A^\mu(r)}{b(r)} = 0 \ \forall\mu \geq 1, \quad \lim_{r\to 0}\frac{A^*(r)}{b(r)} = 0$$

and

$$\lim_{r\to 0}\frac{A^2(r)}{b(r)A^*(r)} = A_0 \geq 1$$

where $A(r)$, $A^*(r)$ defined by (2.1).

Lemma 2. *Let $b(r) \in C([0,\delta],[0,+\infty))$. If $\frac{B(r)}{b(r)}$ is a differentiable in $[0,\delta]$ and $\lim_{r\to\delta}(\frac{B(r)}{b(r)})' \leq 0$, then we have*

$$\lim_{r\to\delta}\frac{B^\mu(r)}{b(r)} = 0 \ \forall\mu \geq 1, \quad \lim_{r\to\delta}\frac{B^*(r)}{b(r)} = 0$$

and

$$\lim_{r\to\delta}\frac{B^2(r)}{b(r)B^*(r)} = B_0 \geq 1$$

where $B(r)$, $B^*(r)$ defined by (2.2).

3. Two auxiliary problem

Let

$$\lambda = \inf_{\Omega} \omega \text{ and } \Lambda = \sup_{\Omega} \omega$$

we prove the following two theorems which will be crucial in proving Theorem 1.

Theorem 2. *Suppose (H2) and (H3) hold. Then, for each $\epsilon > 0$, the problem*

$$\begin{cases} -div(|\nabla u|^{m-2}\nabla u) = \lambda u^q - b(d(x))^{\frac{m}{2}} f(u), & in\ \Omega_\delta \\ u = \infty, & on\ \partial\Omega \\ u = 0, & on\ \partial\Omega_\delta \backslash \partial\Omega \end{cases} \tag{3.1}$$

possesses a positive solution Φ_ϵ such that

$$1 - \epsilon \leq \lim \inf_{d(x)\to 0} \frac{\Phi_\epsilon(x)}{M(A^*(d(x)))^{-\alpha}} \leq \lim \sup_{d(x)\to 0} \frac{\Phi_\epsilon(x)}{M(A^*(d(x)))^{-\alpha}} \leq 1 + \epsilon,$$

where

$$M = \Big(\frac{\alpha^{m-1}(\alpha+1)(m-1)(A_0)^{\frac{m}{2}} - (m-1)\alpha^{m-1}(A_0)^{\frac{m}{2}-1}}{K_0}\Big)^{\frac{2}{m}\alpha},$$

$$\alpha = \frac{1}{p-1}$$

Proof. First we claim that, for each $\epsilon > 0$ sufficiently small, there exists a constant $A_\epsilon > 0$ such that for $A \geq A_\epsilon$,

$$\overline{\Phi}_\epsilon(x) = A + B_+(A^*(d(x)))^{-\alpha}$$

is a positive supersolution of (3.1) if

$$B_+ = (1+\epsilon)\Big(\frac{\alpha^{m-1}(\alpha+1)(m-1)(A_0)^{\frac{m}{2}} - (m-1)\alpha^{m-1}(A_0)^{\frac{m}{2}-1}}{K_0}\Big)^{\frac{2}{m}\alpha},$$

$$\alpha = \frac{1}{p-1} \tag{3.2}$$

Indeed, $\lim_{d(x)\to 0} \overline{\Phi}_\epsilon(x) = \infty$ since $\alpha > 0$. Thus, $\overline{\Phi}_\epsilon(x)$ is a supersolution of (3.1′) if and only if

$$\begin{aligned} &-(B_+\alpha)^{m-1}(-\alpha-1)(m-1)(A^*(d(x)))^{-\alpha m+\alpha-m}(A(d(x)))^m|\nabla d(x)|^m \\ &-(B_+\alpha)^{m-1}(m-1)(A^*(d(x)))^{(-\alpha-1)(m-1)}(A(d(x)))^{m-2}b(d(x))|\nabla d(x))|^m \\ &-(B_+\alpha)^{m-1}(A^*(d(x)))^{(-\alpha-1)(m-1)}(A(d(x)))^{m-1}\triangle_m(d(x)) \\ &\leq -\lambda(A + B_+(A^*(d(x)))^{-\alpha})^q - b(d(x))\frac{f(\overline{\Phi}_\epsilon(x))}{\overline{\Phi}_\epsilon^{\frac{m}{2}+\frac{mp}{2}-1}} \\ &\quad (A + B_+(A^*(d(x)))^{-\alpha})^{\frac{m}{2}+\frac{mp}{2}-1} \end{aligned} \tag{3.3}$$

Thus, multiplying this inequality by

$$\frac{(A(d(x)))^{2-m}(A^*(d(x)))^{\alpha m+m-\alpha p}}{b(d(x))}$$

and taking into account $|\nabla d(x)| = 1$ and $\alpha + 1 - \alpha p = 0$, we find that $\overline{\Phi}_\epsilon(x)$ is a supersolution of (3.1) if and only if

$$-(B_+\alpha)^{m-1}(-\alpha-1)(m-1)\frac{A^2(d(x))}{A^*(d(x))b(d(x))} - (B_+\alpha)^{m-1}(m-1) - (B_+\alpha)^{m-1}$$
$$\frac{A(d(x))}{b(d(x))}\triangle_m(d(x))$$
$$\leq (A(A^*(d(x)))^\alpha + B_+)^q(\frac{A^*(d(x))}{A(d(x))})^{m-2}\frac{(A^*(d(x)))^{-\alpha q+\alpha m-\alpha p+2}}{b(d(x))}$$
$$+\frac{f(\overline{\Phi_\epsilon}(x))}{\overline{\Phi}_\epsilon^{\frac{m}{2}+\frac{mp}{2}-1}}(\frac{b(d(x))A^*(d(x))}{A^2(d(x))})^{\frac{m}{2}-1}(A(A^*(d(x)))^\alpha + (B_+)^{\frac{m}{2}+\frac{mp}{2}-1}) \quad (3.4)$$

At the value $d(x) = 0$, we see by $-\alpha q + \alpha m - \alpha p + 2 > 1$ and Lemma 1 that the inequality (3.4) becomes

$$(B_+\alpha)^{m-1}(\alpha+1)(m-1)A_0 - (m-1)(B_+\alpha)^{m-1}$$
$$\leq (\frac{1}{A_0})^{\frac{m}{2}-1}(B_+)^{\frac{m}{2}+\frac{mp}{2}-1}K_0$$

which is satisfied if and only if

$$(B_+)^{\frac{m}{2}(p-1)} \geq (A_0)^{\frac{m}{2}-1}\frac{\alpha^{m-1}(\alpha+1)(m-1)(A_0) - (m-1)\alpha^{m-1}}{K_0}.$$

By our choice of (3.2) and the continuity, we see that the inequality (3.3) is satisfied in Ω_σ for some $\sigma = \sigma(\epsilon) \in (0, \delta)$. Finally, by choosing a sufficiently large, it is clear that the inequality (3.3) is satisfied in Ω_δ, since $p > 1 > q$ and $b(r)$ is bounded away from zero in $\Omega_\delta\Omega_\sigma$.

Next we construct a subsolution of (3.1) with the same blow-up rate. In fact, for each sufficiently small $\epsilon > 0$, there exists $C < 0$ for which the function

$$\underline{\Phi}_\epsilon(x) = \max\{0, C + B_-(A^*(d(x)))^{-\alpha}\},$$

provides us with a non-negative subsolution of (3.1) if

$$B_- = (1-\epsilon)M$$
$$= (1-\epsilon)(\frac{\alpha^{m-1}(\alpha+1)(m-1)(A_0)^{\frac{m}{2}} - (m-1)\alpha^{m-1}(A_0)^{\frac{m}{2}-1}}{K_0})^{\frac{2}{m}\alpha} \quad (3.5)$$

Indeed, it is easy to see that $\underline{\Phi}_\epsilon(x)$ is a subsolution if in the region where

$$C + B_-(A^*(d(x)))^{-\alpha} \geq 0$$

the following inequality is satisfied

$$-(B_-\alpha)^{m-1}(-\alpha-1)(m-1)(A^*(d(x)))^{-\alpha m+\alpha-m}(A(d(x)))^m|\nabla d(x)|^m$$
$$-(B_-\alpha)^{m-1}(m-1)(A^*(d(x)))^{(-\alpha-1)(m-1)}(A(d(x)))^{m-2}b(d(x))|\nabla d(x))|^m$$
$$-(B_-\alpha)^{m-1}(A^*(d(x)))^{(-\alpha-1)(m-1)}(A(d(x)))^{m-1}\triangle_m(d(x))$$
$$\geq -\lambda(A + B_-(A^*(d(x)))^{-\alpha})^q$$

$$- b(d(x))\frac{f(\overline{\Phi}_\epsilon(x))}{\overline{\Phi}_\epsilon^{\frac{m}{2}+\frac{mp}{2}-1}(A+B_-(A^*(d(x)))^{-\alpha})^{\frac{m}{2}+\frac{mp}{2}-1}} \tag{3.6}$$

At $d(x)=0$, we see by the same proceeding above that the inequality (3.6) is equivalent to

$$(B_+\alpha)^{m-1}(\alpha+1)(m-1)A_0-(m-1)(B_+\alpha)^{m-1}$$
$$\geq (\frac{1}{A_0})^{\frac{m}{2}-1}(B_+)^{\frac{m}{2}+\frac{mp}{2}-1}K_0$$

By our choice of (3.5) and the continuity, there exists $\sigma=\sigma(\epsilon)>0$ for which (3.6) is satisfied in Ω_σ. It is easy to see that

$$\lim_{r\to 0} B_-(A^*(r)^{-\alpha})+C=\infty, \lim_{r\to\delta} B_-(A^*(r)^{-\alpha})+C<0$$

if $C<0$ with $|C|$ being large enough, and $(B_-(A^*(r)^{-\alpha})+C)'<0$ in $[0,\delta]$, where represents the derivative with respect to r. Thus, for each $C<0$ with $|C|$ large enough, there exists a constant $Z(C)\in[0,\delta)$ such that

$$B_-(A^*(d(x)))^{-\alpha}+C\leq 0 \text{ if } d(x)\in[Z(C),\delta],$$

while

$$B_-(A^*(d(x)))^{-\alpha})+C>0 \text{ if } d(x)\in[0,Z(C)).$$

Then by choosing C such that $Z(C)=\sigma$, it follows that $\underline{\Phi}_\epsilon(x)$ provides us a subsolution of (3.1).

It follows from Proposition 1 that there exists a solution of (3.1), denoted by Φ_ϵ, satisfying

$$1-\epsilon=\lim_{d(x)\to 0}\frac{\underline{\Phi}_\epsilon(x)}{M(A^*(d(x)))^{-\alpha}}\leq \lim\inf_{d(x)\to 0}\frac{\Phi_\epsilon(x)}{M(A^*(d(x)))^{-\alpha}}$$
$$\leq \lim\sup_{d(x)\to 0}\frac{\Phi_\epsilon(x)}{M(A^*(d(x)))^{-\alpha}}\leq \lim\sup_{d(x)\to 0}\frac{\overline{\Phi}_\epsilon(x)}{M(A^*(d(x)))^{-\alpha}}=1+\epsilon.$$

The proof is completed.

Theorem 3. *Suppose (H2) and (H3) hold. Then, for each $\epsilon>0$, the problem*

$$\begin{cases} -div(|\nabla u|^{m-2}\nabla u)=\Lambda u^q-b(d(x))^{\frac{m}{2}}f(u), & in\ \Omega_\delta \\ u=\infty, & on\ \partial\Omega_\delta \end{cases} \tag{3.7}$$

possesses a positive solution Ψ_ϵ satisfying

$$1-\epsilon\leq \lim\inf_{d(x)\to 0}\frac{\Psi_\epsilon(x)}{M[A^*(d(x))]^{-\alpha}}\leq \lim\sup_{d(x)\to 0}\frac{\Psi_\epsilon(x)}{M[A^*(d(x))]^{-\alpha}}\leq 1+\epsilon,$$

$$1-\epsilon\leq \lim\inf_{d(x)\to \delta}\frac{\Psi_\epsilon(x)}{N[A^*(d(x))]^{-\alpha}}\leq \lim\sup_{d(x)\to \delta}\frac{\Psi_\epsilon(x)}{N[A^*(d(x))]^{-\alpha}}\leq 1+\epsilon,$$

where

$$M=(\frac{\alpha^{m-1}(\alpha+1)(m-1)(A_0)^{\frac{m}{2}}-(m-1)\alpha^{m-1}(A_0)^{\frac{m}{2}-1}}{K_0})^{\frac{2}{m}\alpha},$$

$$N = (\frac{\alpha^{m-1}(\alpha+1)(m-1)(B_0)^{\frac{m}{2}} - (m-1)\alpha^{m-1}(B_0)^{\frac{m}{2}-1}}{K_0})^{\frac{2}{m}\alpha},$$
$$\alpha = \frac{1}{p-1}.$$

Proof. First we claim that, for each $\epsilon > 0$ sufficiently small, there exists a constant $A_\epsilon > 0$ such that for $A \geq A_\epsilon$,

$$\overline{\Psi}_\epsilon(x) = A + B_+(A^{*(d(x))})^{-\alpha} + C_+(B^{*(d(x))})^{-\alpha}$$

is a positive supersolution of (3.7) if

$$B_+ = (1+\epsilon)(\frac{\alpha^{m-1}(\alpha+1)(m-1)(A_0)^{\frac{m}{2}} - (m-1)\alpha^{m-1}(A_0)^{\frac{m}{2}-1}}{K_0})^{\frac{2}{m}\alpha}$$
$$C_+ = (1+\epsilon)(\frac{\alpha^{m-1}(\alpha+1)(m-1)(B_0)^{\frac{m}{2}} - (m-1)\alpha^{m-1}(B_0)^{\frac{m}{2}-1}}{K_0})^{\frac{2}{m}\alpha}$$

and

$$B_+(A^*(d(x)))^{-\alpha-1}A(d(x)) - C_+(B^*(d(x)))^{-\alpha-1}B(d(x)) > 0. \tag{3.8}$$

Indeed,

$$\lim_{d(x))\to 0} \overline{\Psi}_\epsilon(x) = \lim_{d(x))\to \delta} \overline{\Psi}_\epsilon(x) = \infty$$

since $\alpha > 0$. Thus, $\overline{\Psi}_\epsilon$ is a supersolution of $(3.7)'$ if and only if

$$\begin{aligned}
-\alpha^{m-1}&\Delta_m(d(x))(B_+(A^*(d(x)))^{-\alpha-1}A(d(x)) - C_+(B^*(d(x)))^{-\alpha-1}B(d(x))^{m-1}\\
&- \alpha^{m-1}(m-1)(B_+(A^*(d(x)))^{-\alpha-1}A(d(x)) - C_+(B^*(d(x)))^{-\alpha-1}\\
&\quad B(d(x))^{m-2}(-\alpha-1)(B_+(A^*(d(x)))^{-\alpha-2}A^2(d(x))\\
&+ C_+(B^*(d(x)))^{-\alpha-2}B^2(d(x))|\nabla d(x))|^m\\
&- \alpha^{m-1}(m-1)(B_+(A^*(d(x)))^{-\alpha-1}\\
&\quad A(d(x)) - C_+(B^*(d(x)))^{-\alpha-1}B(d(x)))^{m-2}(B_+(A^*(d(x)))^{-\alpha-1}\\
&+ C_+(B^*(d(x)))^{-\alpha-1}b(d(x)))|\nabla d(x))|^m\\
&\leq -\Lambda(A + B_+(A^*(d(x)))^{-\alpha} + C_+(B^*(d(x)))^{-\alpha})^q\\
&+ b(d(x))\frac{f(\overline{\Psi}_\epsilon(x))}{(\overline{\Psi}_\epsilon(x))^{\frac{m}{2}+\frac{mp}{2}-1}}(A + B_+(A^*(d(x)))^{-\alpha}\\
&+ C_+(B^*(d(x)))^{-\alpha})^{\frac{m}{2}+\frac{mp}{2}-1}.
\end{aligned} \tag{3.9}$$

Multiplying on both sides of the above inequality by

$$\frac{(A(d(x)))^{2-m}(A^*(d(x)))^{-(-\alpha-1)(m-1)}}{b(d(x))}$$

and taking into account $|\nabla d| = 1$ and $\alpha + 1 - \alpha p = 0$, we find that $\overline{\Psi}_\epsilon(x)$ is a supersolution of (3.7) if and only if

$$-\alpha^{m-1}\Delta_m(d(x))\frac{A(d(x))}{b(d(x))}(B_+ - C_+(\frac{A^*(d(x))}{B^*(d(x))})^{\alpha+1}\frac{B(d(x))}{A(d(x))})^{m-1}$$

$$- \alpha^{m-1}(m-1)(-\alpha-1)(B_+ - C_+(\frac{A^*(d(x))}{B^*(d(x))})^{\alpha+1}\frac{B(d(x))}{A(d(x))})^{m-2}$$
$$(B_+\frac{A^2(d(x))}{A^*(d(x))b(d(x))} + C_+(\frac{A^*(d(x))}{B^*(d(x))})^{\alpha+1}\frac{B^2(d(x))}{B^*(d(x))b(d(x))}$$
$$- \alpha^{m-1}(m-1)(B_+ - C_+(\frac{A^*(d(x))}{B^*(d(x))})^{\alpha+1}\frac{B(d(x))}{A(d(x))})^{m-2}$$
$$(B_+ + C_+(\frac{A^*(d(x))}{B^*(d(x))})^{\alpha+1}$$
$$\leq -\Lambda(A(A^*(d(x)))^{\alpha} + B_+ + C_+(\frac{A^*(d(x))}{B^*(d(x))})^{\alpha})^q\frac{A(d(x))}{b(d(x))}$$
$$\frac{(A^*(d(x)))^{\alpha m+m-1-\alpha-\alpha q}}{(A(d(x)))^{m-1}} + \frac{f(\overline{\Psi}_\epsilon(x))}{(\overline{\Psi}_\epsilon(x))^{\frac{m}{2}+\frac{mp}{2}-1}}(\frac{b(d(x))A^*(d(x))}{A^2(d(x))})^{\frac{m}{2}-1}$$
$$(A(A^*(d(x)))^{\alpha} + B_+ + C^+\frac{A^*(d(x))}{(B^*(d(x)))^{\frac{m}{2}+\frac{mp}{2}-1}}. \tag{3.10}$$

At the value $d(x) = 0$, by Lemma 1, the inequality (3.10) becomes

$$\alpha^{m-1}(m-1)(\alpha+1)B_+^{m-1}A_0 - \alpha^{m-1}(m-1)B_+^{m-1}$$
$$\leq K_0(\frac{1}{A_0})^{\frac{m}{2}-1}B_+^{\frac{mp}{2}-\frac{m}{2}}.$$

since $B^*(r)$ and $B(r)$ being bounded above near $r = 0$.

Multiplying on both sides of the inequality (3.9) by

$$\frac{(B(d(x)))^{2-m}(B^*(d(x)))^{-(-\alpha-1)(m-1)}}{b(d(x))}$$

and taking into account $|\nabla d| = 1$ and $\alpha + 1 - \alpha p = 0$, we find that $\overline{\Psi}_\epsilon(x)$ is a supersolution of (3.7) if and only if

$$-\alpha^{m-1}\triangle_m(d(x))\frac{B(d(x))}{b(d(x))}(B_+(\frac{B^*(d(x))}{A^*(d(x))})^{\alpha+1}\frac{A(d(x))}{B(d(x))} - C_+)^{m-1}$$
$$+ \alpha^{m-1}(m-1)(\alpha+1)(B_+(\frac{B^*(d(x))}{A^*(d(x))})^{\alpha+1}\frac{A(d(x))}{B(d(x))} - C_+)^{m-2}$$
$$(B_+\frac{A^2(d(x))}{A^*(d(x))b(d(x))}(\frac{B^*(d(x))}{A^*(d(x))})^{\alpha+1} + C_+\frac{B^2(d(x))}{B^*(d(x))b(d(x))})$$
$$- \alpha^{m-1}(m-1)(B_+(\frac{B^*(d(x))}{A^*(d(x))})^{\alpha+1}\frac{A(d(x))}{B(d(x))} - C_+)^{m-2}$$
$$(B_+(\frac{B^*(d(x))}{A^*(d(x))})^{\alpha+1} + C_+)$$
$$\leq -\Lambda(A(B^*(d(x)))^{\alpha} + B_+(\frac{B^*(d(x))}{A^*(d(x))})^{\alpha} + C_+)^q$$

$$\frac{(B^*(d(x)))^{\alpha m+m-1-\alpha-\alpha q}}{(B(d(x)))^{m-1}}$$

$$\frac{B(d(x))}{b(d(x))}+\frac{f(\overline{\Psi}_\epsilon(x))}{(\Psi_\epsilon(x))^{\frac{m}{2}+\frac{mp}{2}-1}}(\frac{b(d(x))B^*(d(x))}{B^2(d(x))})^{\frac{m}{2}-1}(A(B^*(d(x)))^\alpha$$

$$+B_+(\frac{B^*(d(x))}{A^*(d(x))}+C_+)^{\frac{m}{2}+\frac{mp}{2}-1}. \tag{3.11}$$

At the value $d(x)=\delta$, choose suitable α, m, B_0 s.t.

$$\alpha^{m-1}(m-1)(-1)^{m-2}(\alpha+1)B_0>1$$

by Lemma 2, (3.11) becomes

$$\alpha^{m-1}(m-1)(\alpha+1)(-1)^{m-2}(C_+)^{m-1}B_0-\alpha^{m-1}(m-1)(-1)^{m-2}(C_+)^{m-1}$$

$$\le K_0(C_+)^{\frac{m}{2}+\frac{mp}{2}-1}(\frac{1}{B_0})^{\frac{m}{2}-1}$$

since $A^*(r)$ and $A(r)$ being bounded above near $r=\delta$.

Therefore, by the choice of (3.8) and the continuity,we see that the inequality (3.9) is satisfied if $d(x)\in[0,\sigma)\cup(\delta-\sigma,\delta]$ for some $\sigma=\sigma(\epsilon)>0$. Finally, by choosing A as sufficiently large, it is clear that the inequality is satisfied in Ω_δ, since $p>q$ and $b(r)$ is bounded away from zero.

Next we construct a subsolution with the same blow-up rate. In fact, for each sufficiently small $\epsilon>0$, there exists $C<0$ for which the function

$$\underline{\Psi}_\epsilon(x)=\max\{0, C+B_-(A^*(d(x)))^{-\alpha}+C_-(B^*(d(x)))^{-\alpha}\}$$

provides us with a non-negative subsolution of (3.7) if

$$B_-=(1-\epsilon)(\frac{\alpha^{m-1}(\alpha+1)(m-1)(A_0)^{\frac{m}{2}}-(m-1)\alpha^{m-1}(A_0)^{\frac{m}{2}-1}}{K_0})^{\frac{2}{m}\alpha}$$

$$C_-=(1-\epsilon)(\frac{\alpha^{m-1}(\alpha+1)(m-1)(B_0)^{\frac{m}{2}}-(m-1)\alpha^{m-1}(B_0)^{\frac{m}{2}-1}}{K_0})^{\frac{2}{m}\alpha}$$

and

$$B_-(A^*(d(x)))^{-\alpha-1}A(d(x))-C_-(B^*(d(x)))^{-\alpha-1}B(d(x))>0 \tag{3.12}$$

Indeed, it is easy to see that $\underline{\Psi}_\epsilon(x)$ is a subsolution if in the region where

$$C+B_-(A^*(d(x)))^{-\alpha}+C_-(B^*(d(x)))^{-\alpha}\ge 0,$$

the following inequality is satisfied

$$-\alpha^{m-1}\Delta_m(d(x))(B_-(A^*(d(x)))^{-\alpha-1}A(d(x))-C_-(B^*(d(x)))^{-\alpha-1}$$
$$B(d(x))^{m-1}-\alpha^{m-1}(m-1)(B_-((A^*(d(x)))^{-\alpha-1}A(d(x))$$
$$-C_-(B^*(d(x)))^{-\alpha-1}B(d(x))^{m-2}(-\alpha-1)(B_-(A^*(d(x)))^{-\alpha-2}A^2(d(x))$$
$$+C_-(B^*(d(x)))^{-\alpha-2}B^2(d(x))|\nabla d(x)|^m-\alpha^{m-1}(m-1)(B_-(A^*(d(x)))^{-\alpha-1}$$
$$A(d(x))-C_-(B^*(d(x))^{-\alpha-1}B(d(x)))^{m-2}(B_-(A^*(d(x)))^{-\alpha-1}$$
$$+C_-(B^*(d(x)))^{-\alpha-1}b(d(x))|\nabla d(x)|^m-C_-(B^*(d(x)))^{-\alpha-1}(B(d(x))^{m-1}$$

$$\geq -\Lambda(C + B_-(A^*(d(x)))^{-\alpha} + C_-(B^*(d(x)))^{-\alpha})^q + b(d(x))$$
$$\frac{f(\overline{\Psi}_\epsilon(x))}{(\overline{\Psi}_\epsilon(x))^{\frac{m}{2}+\frac{mp}{2}-1}}(C + B_-(A^*(d(x)))^{-\alpha} + C_-(B^*(d(x)))^{-\alpha}))^{\frac{m}{2}+\frac{mp}{2}-1} \quad (3.13)$$

Similarly, at the value $d(x) = 0$, the inequality (3.13) is equivalent to

$$\alpha^{m-1}(m-1)(\alpha+1)B_-^{m-1}A_0 - \alpha^{m-1}(m-1)B_-^{m-1} \geq K_0(\frac{1}{A_0})^{\frac{m}{2}-1}B_-^{\frac{mp}{2}-\frac{m}{2}}.$$

At the value $d(x) = \delta$, the inequality (3.13) is equivalent to

$$\alpha^{m-1}(m-1)(\alpha+1)(-1)^{m-2}(C_-)^{m-1}B_0 - \alpha^{m-1}(m-1)(-1)^{m-2}(C_-)^{m-1}$$
$$\leq K_0(C_-)^{\frac{m}{2}+\frac{mp}{2}-1}(\frac{1}{B_0})^{\frac{m}{2}-1}.$$

Thus, there exists $\sigma = \sigma(\epsilon) > 0$, for which (3.13) is satisfied if $d(x) \in [0, \sigma) \cup (\delta - \sigma, \delta]$. We see by the proof of Theorem 2 that for each $C < 0$ with $|C|$ large enough, there exists a constant $Z(C) \in (0, \delta)$ such that

$$C + B_-(A^*(d(x)))^{-\alpha} + C_-(B^*(d(x)))^{-\alpha} \leq 0 \quad \text{if} \quad d(x) \in [Z(C), \delta - Z(C)],$$

while

$$C + B_-(A^*(d(x)))^{-\alpha} + C_-(B^*(d(x)))^{-\alpha} > 0$$

if

$$d(x) \in [0, Z(C)) \cup (\delta - Z(C), \delta].$$

Then by choosing $C < 0$ such that $Z(C) = \sigma$, it follows that $\underline{\Psi}_\epsilon(x)$ provides us a subsolution of (3.7).

It follows from Proposition 2 that there exists a solution of (3.7), denote by $\Psi_\epsilon(x)$, satisfying

$$1 - \epsilon = \lim_{d(x)\to 0} \frac{\underline{\Psi}_\epsilon(x)}{M[A^*(d(x))]^{-\alpha}} \leq \lim \inf_{d(x)\to 0} \frac{\Psi_\epsilon(x)}{M[A^*(d(x))]^{-\alpha}}$$
$$\leq \lim \sup_{d(x)\to 0} \frac{\Psi_\epsilon(x)}{M[A^*(d(x))]^{-\alpha}} \leq \lim \sup_{d(x)\to 0} \frac{\overline{\Psi}_\epsilon(x)}{M[A^*(d(x))]^{-\alpha}} = 1 + \epsilon,$$

$$1 - \epsilon = \lim_{d(x)\to \delta} \frac{\underline{\Psi}_\epsilon(x)}{N[B^*(d(x))]^{-\alpha}} \leq \lim \inf_{d(x)\to \delta} \frac{\Psi_\epsilon(x)}{N[B^*(d(x))]^{-\alpha}}$$
$$\leq \lim \sup_{d(x)\to \delta} \frac{\Psi_\epsilon(x)}{N[B^*(d(x))]^{-\alpha}} \leq \lim \sup_{d(x)\to \delta} \frac{\overline{\Psi}_\epsilon(x)}{N[B^*(d(x))]^{-\alpha}} = 1 + \epsilon.$$

The proof is completed.

4. Proof of Theorem 1

Let u be any positive solution of (1.2). Since Ω is of class C^2, there exists $0 < \mu_0 < \frac{\delta}{2}$ such that

$$\Omega_{\delta,\mu} = \{x \in \Omega; \mu < d(x) < \delta\} \subset \Omega_\delta \text{ for each } \mu \in (0, \mu_0).$$

Since $b(r)$ is nondecreasing on $(0,\delta)$, then $b(d(x)-\mu)\le b(d(x))$ in $\Omega_{\frac{\delta}{2}+\mu,\mu}$ and

$$u_{\frac{\delta}{2}+\mu,\mu}=u|_{\Omega_{\frac{\delta}{2}+\mu,\mu}}$$

provide us a positive subsolution of

$$\begin{cases} -\mathrm{div}(|\nabla u|^{m-2}\nabla u)=\Lambda u^q-b(d(x)-\mu)^{\frac{m}{2}}f(u), & \text{in } \Omega_{\frac{\delta}{2}+\mu,\mu}, \\ u=\infty, \quad \text{on } \partial\Omega_{\frac{\delta}{2}+\mu,\mu} \end{cases} \tag{4.1}$$

where

$$\Lambda=\sup_{\Omega}\omega.$$

Thus, any positive solution ζ_μ of (4.1) is a supersolution of (1.2) that u verifies in $\Omega_{\frac{\delta}{2}+\mu,\mu}$. So, thanks to the uniqueness, we see from the strong maximum principle that

$$u_{\frac{\delta}{2}+\mu,\mu}\le\zeta_\mu \ \text{ in } \ \Omega_{\frac{\delta}{2}+\mu,\mu} \tag{4.2}.$$

We see by Theorem 3 that for each $\epsilon>0$ sufficiently small, any positive solution Ψ_ϵ of

$$\begin{cases} -\mathrm{div}(|\nabla u|^{m-2}\nabla u)=\Lambda u^q-b(d(x))^{\frac{m}{2}}f(u), & \text{in } \Omega_{\frac{\delta}{2}}, \\ u=\infty, \quad \text{on } \partial\Omega_{\frac{\delta}{2}} \end{cases}$$

satisfies

$$\lim\sup_{d(x)\to0}\frac{\Psi_\epsilon(x)}{M[A^*(d(x))]^{-\alpha}}\le1+\epsilon \tag{4.3}$$

here M,α defined in (1.4).

Fixed $\epsilon>0$ and $\mu\in(0,\mu_0)$, considering the function ζ_μ defined by

$$\zeta_\mu(x)=\Psi_\epsilon(x-\mu\overrightarrow{n_x}),x\in\Omega_{\frac{\delta}{2}+\mu,\mu}$$

where $\overrightarrow{n}(x)$ stands for the inward unit normal at $x_0=\overline{B}_{\mathrm{dist}(x,\partial\Omega)}(x)\cap\partial\Omega$.

We have that for each sufficiently small $\mu>0$, ζ_μ provides us a large supersolution of (1.2) in $\Omega_{\frac{\delta}{2}+\mu,\mu}$, and hence, (4.2)implies that

$$u(x)\le\Psi_\epsilon(x-\mu\overrightarrow{n_x}) \ \text{ for each } \ x\in\Omega_{\frac{\delta}{2}+\mu,\mu} \ \text{ and } \mu\in(0,\mu_0).$$

Letting $\mu\to0$, we have

$$u\le\Psi_\epsilon \ \text{ in } \ \Omega_{\frac{\delta}{2}},$$

we see by (4.3) that

$$\lim\sup_{d(x)\to0}\frac{u(x)}{M[A^*(d(x))]^{-\alpha}}\le1+\epsilon \tag{4.4}$$

To complete the proof of Theorem 1, we have to show that

$$\lim\inf_{d(x)\to0}\frac{u(x)}{M[A^*(d(x))]^{-\alpha}}\ge1-\epsilon. \tag{4.5}$$

For each sufficiently small $\mu\in(0,\delta)$, set

$$C_\mu=\{x\notin\Omega;\mathrm{dist}(x,\partial\Omega)<\mu\},$$

and

$$D_{\delta,\mu} = \Omega_\delta \cup C_\mu.$$

Let $d(x) = \operatorname{dist}(x, \partial\Omega)$ for $x \in \Omega$, and $d(x) = -\operatorname{dist}(x, \partial\Omega)$ for $x \notin \Omega$.

By Theorem 2, for each sufficiently small $\epsilon > 0$, (3.1) possesses a positive solution Φ_ϵ such that

$$\lim_{d(x)\to 0}\inf \frac{\Phi_\varepsilon(x)}{M[A^*(d(x))]^{-\alpha}} \geq 1 - \epsilon \tag{4.6}$$

where M, α defined in (1.4). For each sufficiently small $\mu > 0$, set

$$\xi_\mu(x) = \Phi_\epsilon(x + \mu\overrightarrow{n_x}), x \in D_{\delta-\mu,\mu}.$$

Then ξ_μ provides us with a large positive solution of

$$\begin{cases} -\operatorname{div}(|\nabla u|^{m-2}\nabla u) = \lambda u^q - b(d(x)+\mu)^{\frac{m}{2}} f(u), & \text{in } D_{\delta-\mu,\mu} \\ u = \infty, \quad \text{on } \partial C_\mu \backslash \partial\Omega \\ u = 0, \quad \text{on } \partial\Omega_{\delta-\mu}\backslash\partial\Omega \end{cases} \tag{4.7}$$

where

$$\lambda = \inf_\Omega \omega.$$

Thanks to the uniqueness, $\xi_\mu|\Omega_{\delta-\mu}$ is unique and provides us a positive subsolution of (1.2) in $\Omega_{\delta-\mu}$, we have that

$$\Phi_\epsilon(x + \mu\overrightarrow{n}_x) = \xi_\mu(x) \leq u_{\delta-\mu}(x) = u|_{\Omega_{\delta-\mu}} \text{ for each } x \in \Omega_{\delta-\mu}.$$

Letting $\mu \to 0$, we obtain

$$\Phi_\epsilon \leq u \text{ in } \Omega_\delta,$$

and hence, we see by (4.6) that

$$\lim_{d(x)\to 0}\inf \frac{u(x)}{M[A^*(d(x))]^{-\alpha}} \geq 1 - \epsilon,$$

combining with (4.4) and letting $\epsilon \to 0$, we have

$$\lim_{d(x)\to 0} \frac{u(x)}{M(\int_0^{d(x)} A(r)dr)^{-\alpha}} = 1.$$

We now show the uniqueness. Assume that u_1 and u_2 are two positive solution of (1.2). Then (1.3) holds for u_1 and u_2. For any $\epsilon > 0$ there exists $\sigma = \sigma(\epsilon) \in (0, \delta)$ such that

$$(1-\epsilon)u_1 \leq u_2 \leq (1+\epsilon)u_1 \text{ in } \Omega_\sigma.$$

Now, consider the problem

$$\begin{cases} -\operatorname{div}(|\nabla u|^{m-2}\nabla u) = \omega(x)u^q - a(x)f(u), & \text{in } \Omega^\sigma = \Omega\backslash\overline{\Omega}_\sigma, \\ u = u_2, \quad \text{on } \partial\Omega^\sigma \end{cases} \tag{4.8}$$

By the uniqueness theorem, (4.8) possesses a uniqueness positive solution since $\omega(x) \geq 0$, necessarily, u_2. It is easy to see that the pair $(1-\epsilon)u_1, (1+\epsilon)u_1$ provides us an ordered sub-solution pair of (4.8). Therefore,

$$(1-\epsilon)u_1 \leq u_2 \leq (1+\epsilon)u_1 \text{ in } \Omega^\sigma,$$

and, consequently,

$$(1-\epsilon)u_1 \leq u_2 \leq (1+\epsilon)u_1 \text{ in } \Omega.$$

As this is true for any $\epsilon > 0$, we obtain that $u_1 = u_2$. This concludes the proof of Theorem 1.

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