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Numerical Study Of H_2O-Cu Nanofluid Using Lattice-Boltzmann Method

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Abstract : In the present study, a laminar natural convection flow of H2O-Cu nanofluid in a two dimensional enclosure has been investigated using a thermal lattice Boltzmann approach with the Bhatnagar-Gross-Krook (BGK) model. The effect of suspended nanoparticles on the fluid flow and heat transfer process have been studied for different controlling parameters such as particle volume fraction (Φ), Rayleigh number (Ra). For this investigation the Rayleigh number changes from 104 to 106 and volume fraction varied from 0 to 10% with three different particle diameters (dp), say 10 nm, 20 nm and 40 nm. It is shown that increasing the Rayleigh number (Ra) and the volume fraction of nanofluid causes an increase of the effective heat transfer rate in terms of average Nusselt number (Nu) as well as the thermal conductivity of nanofluid. On the other hand, increasing the particle diameter causes the decrease of the heat transfer rate and thermal conductivity. The result of the analysis are compared with experimental and numerical data both for pure and nanofluids and it is seen a relatively good agreement.

Key words : Nanofluid, Lattice-Boltzmann Method, Rayleigh number, Volume fraction, Particle diameter.

1. Introduction

In the recent years, the micro and nano systems have become of great interest due to their important and promising applications in various fields. Nanofluids are envisioned to describe a solid-liquid mixer which consists of nano-sized solid particles and a base liquid. It has potential applications in the micro electro mechanical systems (MEMS) and electronics cooling industries. The performance of heat transfer of nanofluid depends on more factors such as shape, dimension and the volume fractions of particle in the suspensions, and the thermal properties of particle materials [1-2]. However, significant amounts of experimental and theoretical work have been performed on buoyancy induced flow in conventional fluid [3,4,5]. From the microscopic point of view, classical mechanics has no insight into the microstructure of the substance. However, statistical mechanics can calculate the properties of state on the basis of molecular motions in a space, and on the basis of the intermolecular interactions. The Lattice Boltzmann Method (LBM) is one of the methods available to deal with such problems. LBM has been developed to simulate flows containing multiple

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phases and components [8-9]. Using the two-component LBM, Shan [10] simulated the Rayleigh-Benard convection in two and three dimensions. However, Buick and Gretaed [11] introduced a body force into LBM by modifying the collision function. To overcome the limitations of multi-component flows in the existing traditional computational methods, many researchers interested to use multi-component LBM [12-13]. In the present work, multi-component thermal Lattice-Boltzmann method (TLBM) is used for simulating natural convection H_2O-Cu nanofluid with Boussinesq approximation in a square cavity. As far we know, there is no work on nanofluid heat transfer under natural convection using TLBM. The results of the analysis are compared with experimental and numerical data both for pure and nanofluids, and shows a relatively good agreement.

2. Formulation of the problem

To ensure the model satisfies the N-S equations for a fluid under the influence of body force, multi-component Lattice-Boltzmann equation (LBE) can be written as [10]

$$\begin{aligned} f_i^\sigma(x+e_i\Delta t, t+\Delta t) - f_i^\sigma(x,t) \\ = -\frac{1}{\tau_m^\sigma}(f_i^\sigma(x,t) - f_i^{\sigma,eq}(x,t) \\ + \frac{2\tau_m^\sigma - 1}{\tau_m^\sigma}\frac{D}{A_i c^2}\vec{e}_i \cdot \vec{F}^\sigma \end{aligned} \tag{1}$$

Here, $\omega^\sigma = 1/\tau_m^\sigma$ is the relaxation parameter of $\sigma = 1$, 2 components with lattice velocity vectors e_i, A_i is the adjustable coefficient, D is the dimension and F^σ is applied force. For two dimensional D2Q9 (two dimensional lattice with nine velocities) model, the equilibrium distribution function, $f_i^\sigma(x,t)$, can be defined as

$$\begin{aligned} f_i^{\sigma,eq} = \rho^\sigma w_i [1 + \frac{3}{c^2}\vec{e}_i \cdot \vec{u}^{\sigma,eq} \\ + \frac{9}{2c^4}(\vec{e}_i \cdot \vec{u}^{\sigma,eq})^2 - \frac{3}{2c^2}\vec{u}^{\sigma,eq^2}] \end{aligned} \tag{2}$$

where w_i is the lattice weighting factors. Therefore the equilibrium velocity becomes

$$\vec{u}^{eq} = \frac{\sum_\sigma \rho^\sigma \vec{u}^\sigma / \tau_m^\sigma}{\sum_\sigma \rho^\sigma / \tau_m^\sigma} + \frac{\vec{F}^\sigma}{2\rho^\sigma} \tag{3}$$

Simultaneously, the lattice Boltzmann energy equation without viscous dissipation for nanofluid defined as [12]

$$\begin{aligned} g_i^\sigma(x+e_i\Delta t, t+\Delta t) - g_i^\sigma(x,t) \\ = -\frac{1}{\tau_\theta^\sigma}(g_i^\sigma(x,t) - g_i^{\sigma,eq}(x,t) \end{aligned} \tag{4}$$

The energy distribution function can be written as

$$\begin{aligned} g_i^{\sigma,eq} = \varepsilon^\sigma w_i [1 + \frac{3}{c^2}\vec{e}_i \cdot \vec{u}^{\sigma,eq} \\ + \frac{9}{2c^4}(\vec{e}_i \cdot \vec{u}^{\sigma,eq})^2 - \frac{3}{2c^2}\vec{u}^{\sigma,eq^2}] \end{aligned} \tag{5}$$

$\varepsilon^\sigma(x,t) = \sum_i g_i^\sigma(x,t)$ is the internal energy variable. The mean velocity, temperature, viscosity and thermal diffusivity of the nanofluid can be written as

$$\vec{u} = \frac{\sum_\sigma m^\sigma \sum_i f_i^\sigma(x,t)\vec{e}_i}{\sum_\sigma \rho^\sigma}, \quad T = \frac{\sum_\sigma \varepsilon^\sigma(x,t)}{\sum_\sigma \rho^\sigma C_p^\sigma},$$

$$\nu = (\sum_\sigma c_m^\sigma \tau_m^\sigma - \frac{1}{2}) Cs^2 \Delta t, \text{ and}$$

$$\alpha = (\sum_\sigma c_\theta^\sigma \tau_\theta^\sigma - \frac{1}{2}) Cs^2 \Delta t$$

Where c_m^σ and c_θ^σ are the concentration of viscosity and diffusivity of each component respectively. Solving the equations (1) and (5) with other approximations, we get all information that we interested in our study.

3. Results and discussion

Consider a two dimensional square cavity as shown in Figure 1. The horizontal walls are assumed to be insulated whereas the left vertical walls is high temperature T_h(hot) and right vertical temperature is cold temperature T_c(cold). The non-dimensionalized velocity and temperature fields are defined as

$$Y=y/H,\ X=x/H,\ U=\frac{uH}{\sqrt{g\beta\triangle TH^3}},$$
$$V=\frac{vH}{\sqrt{g\beta\triangle TH^3}},\quad \theta=\frac{T-T_c}{T_h-T_c}$$

The characteristic speed $\sqrt{g\beta\triangle TH}$ must be chosen carefully so that the low Mach number approximation holds in order to insure the problem is in the incompressible regime. For better approximation, the value $\sqrt{g\beta\triangle TH}$ should be less than about 0.1. Throughout the calculation, the lattice units are considered.

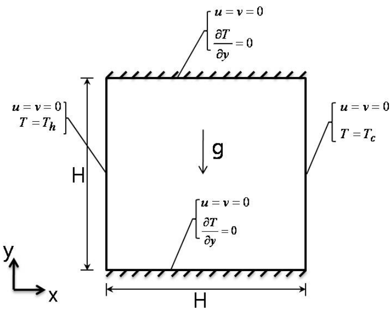


Figure 1: The schematic diagram of the computational configuration and coordinate system

For natural convection, the momentum and energy equations are coupled and the flow is driven by temperature or mass gradient, Under Boussinesq approximation, the force term per unit mass can be written as

$$\vec{F}(\vec{x},t)=\rho(\vec{x},t)g\beta(T(\vec{x},t)-T_{ref}) \quad (6)$$

The ratio of the buoyancy force to the product of viscous force and heat diffusion rates defined by the Rayleigh number, $Ra=\Pr\times Gr=\frac{g\beta\triangle TH^3}{\nu\alpha}$, where Gr is the Grashof number, Pr is the Prandtl number. In order to assess the accuracy of this method, the result of LBM is verified with the conventional benchmark and with experimental results as shown in Figue 2 and 3

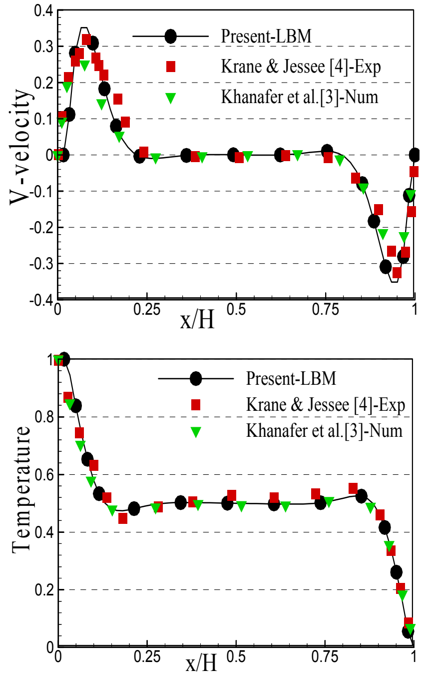


Figure 2: Comparison of temperature and velocity profiles for pure fluid (Pr=0.71,Ra=1.89×10^5).

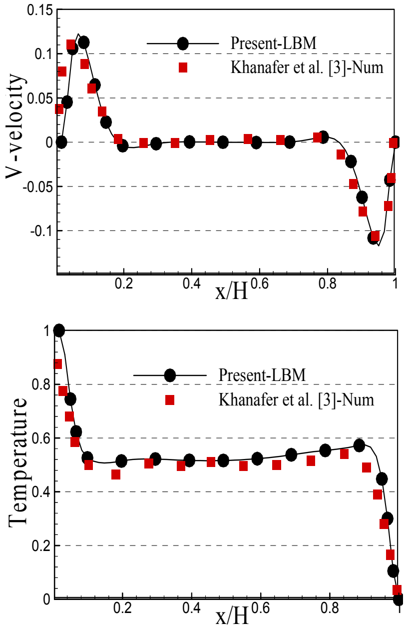


Figure 3: Comparison of temperature and velocity profiles for nanofluid ($Pr=6.2$, $Gr=10^5$,$\Phi=10\%$).

It can be seen from Figure 2 that our present numerical simulation has a good agreement with both numerical results of and experimental solutions of reference 〔4〕 for pure fluid (air, Pr=0.71). However, agreement is obtained between the present solution and bench mark solution of Khanafer 〔6〕 as illustrated in Figure 3 for H_2O-Cu nanofluids, the diameter of Cu nanoparticle is taken as 10 nm. This confirms that our method is correct both for pure and nanofluids.

It is seen from Figure 4 and 5 that the non dimensional temperature of nanofluid increases remarkably in the core region with increasing the Rayleigh numbers and the particle volume fractions. The temperature drops very gradually from the value at the wall to the value at the center. For natural convection in a cavity, the rate of cooling is expected to be higher near the heated and unheated walls due to the fluid motion and hydrodynamics effects. This phenomenon is attributed to buoyancy-induced cellular flows in the boundary layer adjoining the heated and cold walls. On the other hand, Figure 6, it is observed that increasing the particle diameters causes the decreases of temperature distribution on the core regions. In addition, from the above figures, it is clear that the temperature profiles of nanofluid are similar to pure fluid. This suggested that nanofluid behave like the carrier fluid not as a solid fluid mixer.

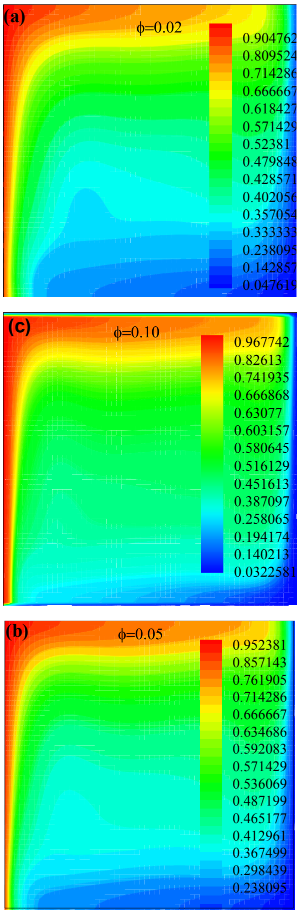


Figure 4: Temperature distribution with d=10nm and Ra=105 for different particle volume fractions (Φ)

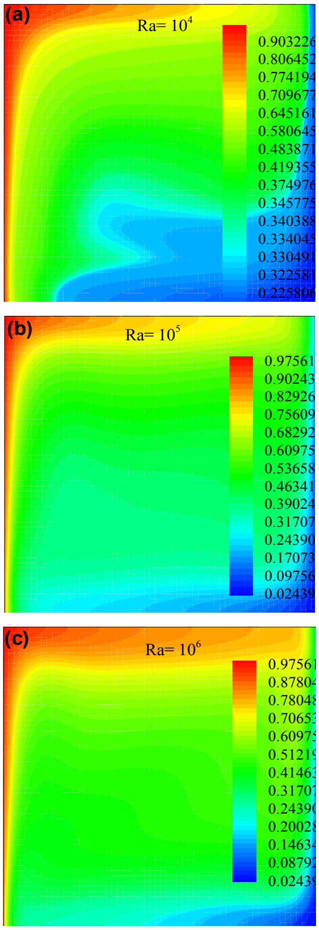
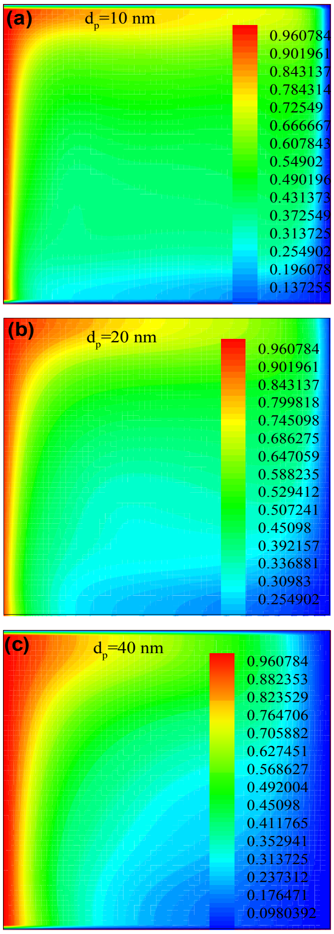


Figure 5: Temperature contours for different Rayleigh numbers (Ra) with d=10nm and Φ=0.05



Figuer 6: Temperature contours for different particle diameters (d) with Φ = 0.05 and Ra=10^5

To investigate the heat transfer performance in terms of effective thermal conductivity of the nanofluid, the local Nusselt number Nu_L and the averaged Nusselt number Nu are respectively defined in dimensionless form as follows:

$$Nu_L(Y) = -\frac{k_{nf}}{k_f}\left(\frac{\partial\theta}{\partial X}\right)_{wall} \quad \text{and}$$

$$Nu = \int_0^1 Nu_L(Y)dY$$

where k_{nf} and k_f are the thermal conductivity of the nanofluid and base fluid respectively.

It is seen from Figure 7(a), the rate of heat transfer linearly increases with increasing the particle volume fractions. As the volume fraction increases, irregular and random movement of nanoparticles increases energy exchange rates in the fluid. However, it is not observed for large particle diameter in natural convection. It is noted that, the surface area of nanoparticles enhances the heat conduction of nanofluid, since heat transfer takes place at the surface of particles. Figure 7(b) has shown that, if the Rayleigh number increased, the heat transfer rate also increased significantly due to higher convection heat transfer. It is obvious from above figures that nanofluids exhibit superior heat transfer characteristics to the convectional heat transfer fluids.

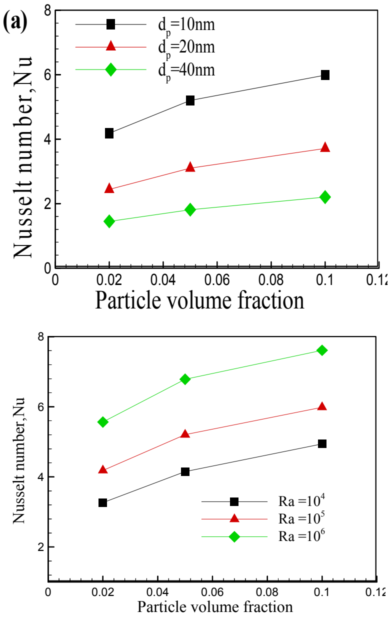


Figure 7: Nusselt number (Nu) as a function of volume fraction for different (a) particle diameter, (b) Rayleigh number (Ra)

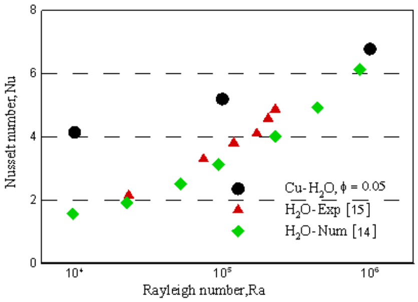
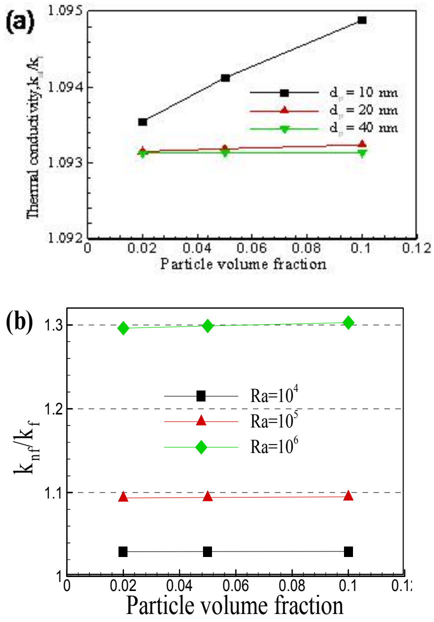


Figure 8: Comparisons of Cu-H_2O nanofluid with pure fluid (H_2O) for different Rayleigh numbers with dp=10nm and Φ=5%

Figure 8 represents the variation of the average Nusselt number against Rayleigh number, product of Grashof number(Gr) and Prandtl number(Pr), for dp = 10nm with particle volume fraction Φ=0.05. For natural convection of base fluid, water, the numerical and experimental results are obtained by Leong et al.[14] and Wen and Ding [15] respectively. It is shown that the Nusselt number increases with Rayleigh number in the entire range of $10^4 \le Ra \le 10^6$. Comparing with base fluid, it is observed that the enhancement of heat transfer in nanofluid significantly increases at low Rayleigh number. The enhancement rates of nanofluid heat transfer are 158% for Ra $=10^4$ and 63% for Ra$=10^5$ respectively. However, for Ra$=10^6$, the enhancement rate is very small, approximately 3%, comparing with numerical works[14].



Figuer 9: Conductivity ratio versus volume fraction for different (a) particle diameter, and (b) Rayleigh number (Ra)

On the basis of the definition, the effective thermal conductivity for a two-component mixture (H_2O-Cu) is defined as $k_{nf}=\alpha_{nf}(\rho C_p)_{nf}$. Figure 9 shows the enhancement of thermal conductivity of nanofluid within the enclosure at different Rayleigh numbers, volume fractions as well as the particle diameters due to its practical applications.

It can be seen from Figure 9(a) that thermal conductivity is increased with increased of volume fraction. This increased is significantly changed for small particle diameter. The particle size is important factor in this case. Because, in natural convection flow, only buoyancy force is a dominant force, therefore particle movement is very important. The main problem for large size of particle is the rapid settling in the fluid. These problems are highly undesirable for many practical cooling industries. However, it is seen from Figure 9 (b) that the thermal conductivity of nanofluids can be enhanced with a factor 1.02, 1.09 and 1.29 for $Ra=10^4,10^5,10^6$ respectively under the same particle volume fraction, Φ = 2%. When the Rayleigh number (Ra) increases, two tends were observed for nanofluid: enhanced mixing of the hot and cold fluid, and increases the temperature gradient near the hot and cold walls. As a result the irregular and random movement of nanoparticles increases energy exchange rates in the fluid and consequently enhanced the thermal conductivity in the nanofluid.

4. Conclusions

In the present paper, the thermal Lattice-Boltzmann Method (TLBM) is successfully applied to study buoyancy-driven heat transfer characteristics and flow performance of H_2O-Cu nanofluid in a square cavity. The suspended nanoparticles remarkably enhance heat transfer process and the nanofluid has larger heat transfer coefficient than that of the original base liquid under the same Raleigh number.

The present results indicate that the heat transfer rate of nanofluid increases with the increase of nanoparticles volume fraction as well as Rayleigh numbers. However, it is significantly decreases, if the particle diameter increases. The enhancement rates of nanofluid heat transfer comparing with base fluid are 158% for $Ra=10^4$ and 63% for $Ra=10^5$ respectively. However, for $Ra=10^6$, the enhancement rate is very small, approximately 3%, comparing with numerical works. In addition, in the same range of buoyancy parameter, the thermal conductivity enhanced 26% with the same particle volume fraction. The results from our analysis shows that LBM can be a good tool to investigate nanofluids and heat trasfer enhancement plays a significant role in engineering applications such as in the electronic cooling industries or MEMS devices.

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