Communications of the Korean Statistical Society
2010, Vol. 17, No. 1, 99–106

# Prediction Intervals for Proportional Hazard Rate Models Based on Progressively Type II Censored Samples

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**Abstract**

In this paper, we present two methods for obtaining prediction intervals for the times to failure of units censored in multiple stages in a progressively censored sample from proportional hazard rate models. A numerical example and a Monte Carlo simulation study are presented to illustrate the prediction methods.

Keywords: Progressive Type-II censoring, proportional hazard rate model, prediction interval, highest conditional density, Monte Carlo simulation.

## 1. Introduction

Recently, the Type-II progressively censoring scheme has received considerable interest among the statisticians. It can be described as follows. $n$ units are placed on a life-testing experiment and only $m(< n)$ units are completely observed until failure. The censoring occurs progressively in $m$ stages. These $m$ stages offer failure times of the $m$ completely observed units. At the time of the first failure (the first stage), $R_1$ of the $n-1$ surviving units are randomly withdrawn (censored intentionally) from the experiment, $R_2$ of the $n-2-R_1$ surviving units are withdrawn at the time of the second failure (the second stage), and so on. Finally, at the time of the $m^{th}$ failure (the $m^{th}$ stage), all the remaining $R_m = n-m-R_1-\cdots-R_{m-1}$ surviving units are withdrawn. We will refer to this as progressive Type-II right censoring scheme $(R_1, R_2, \ldots, R_m)$. It is clear that this scheme includes the conventional Type-II right censoring scheme (when $R_1 = R_2 = \cdots = R_{m-1} = 0$ and $R_m = n-m$) and complete sampling scheme (when $n = m$ and $R_1 = R_2 = \cdots = R_m = 0$). The ordered lifetime data which arise from such a progressive Type-II right censoring scheme are called progressively Type-II right censored order statistics. For further details on progressively censoring, inferences and their applications, one may refer to Balakrishnan (2007).

Censoring occurs when exact survival times are known only for a portion of the individuals or items under study. The complete survival times may not have been observed by the experimenter either intentionally or unintentionally, and therefore prediction of unobserved or censored observations is an interesting topic, especially in the viewpoint of acturial, medical and engineering sciences. In this article, we consider the problem of predicting times to failure of units censored in multiple stages in a progressively censored sample from proportional hazard rate models.

Viveros and Balakrishnan (1994) used the conditional method of inference to develop conditional prediction interval for an observation from an independent future sample based on an observed progressively Type-II right censored sample. Balakrishnan and Lin (2002) discussed exact prediction

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intervals for last censored failure times in a progressively Type-II right censored sample from an exponential distribution, based on the best linear unbiased estimator(BLUE). Basak *et al.* (2006) presented a detailed discussion on the point prediction of censored failure times in a progressively Type-II right censored sample. They obtained the different point predictors and discussed their properties such as unbiasedness, consistency and efficiency, and then compared these predictors in terms of mean squared prediction error.

Let us consider the continuous random variable $X$ with the cumulative distribution function(cdf) $F(x;\theta)$. In many situations $F(x;\theta)$ can be written as

$$F(x;\theta) = 1 - \left[\bar{F}_0(x)\right]^{\theta}, \quad -\infty \le c < x < d \le \infty, \ \theta > 0, \tag{1.1}$$

where $\bar{F}_0(.) = 1 - F_0(.)$ and $F_0(.)$ is a baseline *cdf* with $F_0(c) = 0$ and $F_0(d) = 1$. This family of distributions is well-known in the life time experiments as proportional hazard rate(PHR) models (see Marshall and Olkin, 2007), which includes several well-known lifetime distributions such as exponential, Pareto, Lomax, Burr type XII and so on. The PHR model has gained widespread attention from both applied and theoretical statisticians to model failure time data. This model is flexible enough to accommodate both monotonic as well as non-monotonic failure rates even though the baseline failure rate models are frequently encountered in modeling failure data. Recently, Ahmadi *et al.* (2009a, 2009b) have studied the problems of estimation and prediction for the PHR models based $k$-record data.

From the model (1.1), the probability density function(pdf) is given, by

$$f(x;\theta) = \theta f_0(x)\left[\bar{F}_0(x)\right]^{\theta-1}, \quad -\infty \le c < x < d \le \infty, \ \theta > 0, \tag{1.2}$$

where $f_0(.) = F_0'(.)$ is the corresponding *pdf*.

Let $X_{1:m:n}, \ldots, X_{m:m:n}$ denote a progressively Type-II censored sample from the PHR model (1.1) obtained from a sample of size $n$ with the censoring scheme $(R_1, \ldots, R_m)$. To simplify the notation, we will use $x_i$ in place of $x_{i:m:n}$. The aim of this paper is to discuss the prediction interval of the life-lengths $Y = X_{s:R_i}(s = 1, 2, \ldots, R_i;\ i = 1, 2, \ldots, m)$ of all censored units in all $m$ stages of censoring based on the observed data $\mathbf{X} = (X_1, \ldots, X_m)$. Here $Y = X_{s:R_i}$ denotes the $s^{th}$ order statistic out of $R_i$ removed units at $i$ $(i = 1, 2, \ldots, m)$. In Section 2, we present two methods for obtaining prediction intervals(PIs) for the times to failure of units. In Section 3, we present a numerical example and a Monte Carlo simulation study to illustrate two prediction methods discussed in this paper.

## 2. PIs for the Times to Failure

Let $X_1, X_2, \ldots, X_m$ denote a progressively Type-II right censored sample from model (1.1), with $(R_1, R_2, \ldots, R_m)$ being the progressive censoring scheme. In this section, we consider two methods to obtain PIs for $Y = X_{s:R_i}$ $(s = 1, 2, \ldots, R_i;\ i = 1, 2, \ldots, m)$ in all $m$ stages of censoring based on the observed progressively Type-II right censored sample $\mathbf{x} = (x_1, \ldots, x_m)$.

The joint density function of $\mathbf{X} = (X_1, X_2, \ldots, X_m)$ is given (see Balakrishnan (2007)) by

$$f(\mathbf{x};\theta) = A\prod_{i=1}^{m}\left[f(x_i;\theta)\{1 - F(x_i;\theta)\}^{R_i}\right], \tag{2.1}$$

where $A = n(n-1-R_1)(n-2-R_1-R_2)\cdots(n-m+1-R_1-\cdots-R_{m-1})$. For model (1.1), (2.1) reduces to

$$f(\mathbf{x};\theta) = A\left[\prod_{i=1}^{m}\frac{f_0(x_i)}{\bar{F}_0(x_i)}\right]\theta^m e^{-\theta T(\mathbf{x})}, \tag{2.2}$$

where $T(\mathbf{x}) = -\sum_{i=1}^{m}(R_i + 1)\ln \bar{F}_0(x_i)$. From (2.2), the maximum likelihood estimator(MLE) of $\theta$ is derived to be

$$\widehat{\theta}_{ML} = \frac{m}{T(\mathbf{x})}.$$

It is well-known that the conditional distribution of $X_{s:R_i}$ given $\mathbf{X}$ is just the distribution of $X_{s:R_i}$ given $X_i = x_i$ (due to the Markovian property of progressively Type-II right censored-order statistics). This implies that the density of $Y_{s:R_i}$ given $\mathbf{X} = \mathbf{x}$ is the same as the density of the $s^{th}$ order statistic out of $R_i$ units from the population with density $f(y)/(1 - F(x_i)), y \geq x_i$ (left truncated density at $x_i$). Therefore, the conditional density of $Y = X_{s:R_i}$ given $X_i = x_i$, for $y \geq x_i$, is given by

$$f(y|x_i;\theta) = s\binom{R_i}{s} f(y;\theta)\,[F(y;\theta) - F(x_i;\theta)]^{s-1}\,[1 - F(y;\theta)]^{R_i-s}\,[1 - F(x_i;\theta)]^{-R_i}. \tag{2.3}$$

For model (1.1), (2.3) reduces to

$$f(y|x_i;\theta) = s\binom{R_i}{s}\theta\frac{f_0(y)}{\bar{F}_0(y)}\left[\left\{\bar{F}_0(x_i)\right\}^{\theta} - \left\{\bar{F}_0(y)\right\}^{\theta}\right]^{s-1}\left[\left\{\bar{F}_0(y)\right\}^{\theta}\right]^{R_i-s+1}\times\left[\left\{\bar{F}_0(x_i)\right\}^{\theta}\right]^{-R_i}, \quad y \geq x_i. \tag{2.4}$$

## 2.1. Pivot method

Let us now define the random variable $U$ as

$$U = \left(\frac{\bar{F}_0(Y)}{\bar{F}_0(X_i)}\right)^{\theta}. \tag{2.5}$$

From (2.4), the conditional distribution of $U$ given $X_i = x_i$ can be shown to be

$$g(u|x_i) = s\binom{R_i}{s}u^{R_i-s}(1-u)^{s-1}, \quad 0 < u < 1.$$

So, when the parameters $\theta$ is known and $X_i = x_i$, $U$ is a pivot statistic and it has the Beta$(R_i - s + 1, s)$ distribution. Hence, a $100(1-\gamma)$ PI for $Y$ is $(L_1(x_i), U_1(x_i))$ where

$$L_1(x_i) = \bar{F}_0^{-1}\left(\bar{F}_0(x_i)\left(b_{1-\frac{\gamma}{2}}\right)^{\frac{1}{\theta}}\right) \quad \text{and} \quad U_1(x_i) = \bar{F}_0^{-1}\left(\bar{F}_0(x_i)\left(b_{\frac{\gamma}{2}}\right)^{\frac{1}{\theta}}\right). \tag{2.6}$$

Here $b_\gamma$ denotes $100\gamma^{th}$ percentile of Beta$(R_i - s + 1, s)$.

When $\theta$ is unknown, this parameters in (2.6) has to be estimated. By replacing $\theta$ by the its MLE, $\widehat{\theta}_{ML} = m/T(\mathbf{x})$, we obtain the prediction limits for $Y$ as follows:

$$L_1(x_i) = \bar{F}_0^{-1}\left(\bar{F}_0(x_i)\left(b_{1-\frac{\gamma}{2}}\right)^{\frac{T(\mathbf{x})}{m}}\right) \quad \text{and} \quad U_1(x_i) = \bar{F}_0^{-1}\left(\bar{F}_0(x_i)\left(b_{\frac{\gamma}{2}}\right)^{\frac{T(\mathbf{x})}{m}}\right).$$

***Example 1.*** (i) (Exponential distribution): Taking $\overline{F}_0(x) = e^{-x}$, $x > 0$, in (1.1), X has exponential distribution, and we obtain the prediction limits as

$$L_1(x_i) = -\ln\left(e^{-x_i}\left(b_{1-\frac{\gamma}{2}}\right)^{\frac{T_0(\mathbf{x})}{m}}\right) = x_i - \frac{T_0(\mathbf{x})}{m}\ln\left(b_{1-\frac{\gamma}{2}}\right)$$

and

$$U_1(x_i) = -\ln\left(e^{-x_i}\left(b_{\frac{\gamma}{2}}\right)^{\frac{T_0(\mathbf{x})}{m}}\right) = x_i - \frac{T_0(\mathbf{x})}{m}\ln\left(b_{\frac{\gamma}{2}}\right), \tag{2.7}$$

where

$$T_0(\mathbf{x}) = \sum_{j=1}^{m}(R_j + 1)x_j. \tag{2.8}$$

(ii) (Burr type XII distribution): Taking $\overline{F}_0(x) = (1 + x^c)^{-1}$, $x > 0$, $c > 0$, with known $c$, in (1.1), X has Burr type XII distribution, and we obtain the prediction limits as

$$L_1(x_i) = \left(\frac{1 + x_i^c}{\left(b_{1-\frac{\gamma}{2}}\right)^{\frac{T_0(\mathbf{x})}{m}}} - 1\right)^{\frac{1}{c}} \quad \text{and} \quad U_1(x_i) = \left(\frac{1 + x_i^c}{\left(b_{\frac{\gamma}{2}}\right)^{\frac{T_0(\mathbf{x})}{m}}} - 1\right)^{\frac{1}{c}}. \tag{2.9}$$

## 2.2. HCD method

Now let us consider another prediction interval for $Y = X_{s:R_i}$. An HCD prediction interval is such that the conditional pdf of, $Y$ given $\mathbf{X} = (X_1, \ldots, X_m)$ for every point inside the interval, is greater than that for every point outside of it. For more details about the HCD method, see Raqab and Nagaraja (1995) and Awad and Raqab (2000). By substituting the MLE of $\theta$ in (2.4), we can obtain the approximate density of $Y$ given $X_i = x_i$ as

$$\hat{f}_{Y|X_i}(y|x_i) = s\binom{R_i}{s}\widehat{\theta}\frac{f_0(y)}{\bar{F}_0(y)}\left[\left(\bar{F}_0(y)\right)^{\widehat{\theta}}\right]^{R_i-s+1}\left[\left(\bar{F}_0(x_i)\right)^{\widehat{\theta}} - \left(\bar{F}_0(y)\right)^{\widehat{\theta}}\right]^{s-1} \times \left[\left(\bar{F}_0(x_i)\right)^{\widehat{\theta}}\right]^{-R_i}, \quad y \geq x_i. \tag{2.10}$$

We can easily observe that this conditional density is a unimodal function of

$$\hat{U} = \left(\frac{\bar{F}_0(Y)}{\bar{F}_0(X_i)}\right)^{\widehat{\theta}},$$

for $s > 1$ and $R_i > s$ ($i.e$; $s = 2, \ldots, R_i - 1$). Hence, the $(1-\gamma)100\%$ highest conditional density(HCD) PI for $Y$ is $(L_2(x_i), U_2(x_i))$ where

$$L_2(x_i) = \bar{F}_0^{-1}\left(\bar{F}_0(x_i)(w_2)^{\frac{T_0(\mathbf{x})}{m}}\right) \quad \text{and} \quad U_2(x_i) = \bar{F}_0^{-1}\left(\bar{F}_0(x_i)(w_1)^{\frac{T_0(\mathbf{x})}{m}}\right),$$

where $w_1$ and $w_2$ are the simultaneous solution of the following equations:

$$1 - \gamma = \int_{w_1}^{w_2} \widehat{g}(u|x_i)du \tag{2.11}$$

and

$$\widehat{g}(w_1|x_i) = \widehat{g}(w_2|x_i). \tag{2.12}$$

On using (2.9), we simplify equations (2.10) and (2.11) as

$$B_{w_2}(R_i - s + 1, s) - B_{w_1}(R_i - s + 1, s) = 1 - \gamma \tag{2.13}$$

and

$$\left(\frac{1 - w_2}{1 - w_1}\right)^{s-1} = \left(\frac{w_1}{w_2}\right)^{R_i - s} \tag{2.14}$$

where

$$B_t(a,b) = \frac{1}{B(a,b)} \int_0^t x^{a-1}(1-x)^{b-1}\,dx$$

is the incomplete beta function.

***Example 2.*** (i) (Exponential distribution): Taking $\overline{F}_0(x) = e^{-x}$, for the case of exponential distribution, we obtain the HCD prediction limits for $Y$ as

$$L_2(x_i) = -\ln\left(e^{-x_i}(w_2)^{\frac{T_0(\mathbf{x})}{m}}\right) = x_i - \frac{T_0(\mathbf{x})}{m}\ln(w_2)$$

and

$$U_2(x_i) = -\ln\left(e^{-x_i}(w_1)^{\frac{T_0(\mathbf{x})}{m}}\right) = x_i - \frac{T_0(\mathbf{x})}{m}\ln(w_1), \tag{2.15}$$

where $w_1$ and $w_2$ are the simultaneous solution of the Equations (2.13) and (2.14) and $T_0(\mathbf{x})$ is as given in (2.8).

(ii) (Burr type XII distribution): Taking $\overline{F}_0(x) = (1+x^c)^{-1}$, $x > 0$, $c > 0$, with known $c$, in (1.1), X has Burr type XII distribution, and we obtain the HCD prediction limits as

$$L_2(x_i) = \left[\frac{1+x_i^c}{(\omega_2)^{\frac{T_0(\mathbf{x})}{m}}} - 1\right]^{\frac{1}{c}} \quad \text{and} \quad U_2(x_i) = \left[\frac{1+x_i^c}{(\omega_1)^{\frac{T_0(\mathbf{x})}{m}}} - 1\right]^{\frac{1}{c}}. \tag{2.16}$$

## 3. Numerical Illustration

In this section, we present a numerical example and a Mont Carlo simulation study to illustrate all the prediction methods discussed in this paper. We consider exponential distribution with *pdf*

$$F(x;\theta) = 1 - e^{-\theta x}, \quad x > 0,\ \theta > 0$$

as a special case from the model (1.1).

### 3.1. Numerical example

We consider the following set of data reported in Nelson (1982, Table 6.1). Nelson presents the results of a life-test experiment in which specimens of a type of electrical insulating fluid were subject to a constant voltage stress(34 KV/minutes). The 19 times to breakdown are:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.19 | 0.78 | 0.96 | 1.31 | 2.78 | 3.16 | 4.15 | 4.67 | 4.85 | 6.50 |
| 7.35 | 8.01 | 8.27 | 12.06 | 31.75 | 32.52 | 33.91 | 36.71 | 72.89 | |

For this example, the two following censored schemes are considered:

(i) progressive Type-II censored sample generated by Viveros and Balakrishnan (1994). In that sample, the vector of observed failure times and the progressive censoring scheme are

$$\mathbf{x} = (0.19, 0.78, 0.96, 1.31, 2.78, 4.85, 6.50, 7.35)$$

and

$$\mathbf{R} = (0,0,3,0,3,0,0,5).$$

Table 1: 95% PIs for the times to failure

| | | Pivot Method | HCD Method |
|---|---|---|---|
| Case 1 | $X_{1:R_3}$ | ( 1.037, 12.133) | — |
| | $X_{2:R_3}$ | ( 1.860, 22.415) | ( 1.860, 22.415) |
| | $X_{3:R_3}$ | ( 4.103, 44.384) | — |
| | $X_{1:R_5}$ | ( 2.857, 13.953) | — |
| | $X_{2:R_5}$ | ( 3.680, 24.235) | ( 3.680, 24.235) |
| | $X_{3:R_5}$ | ( 5.923, 46.204) | — |
| | $X_{1:R_8}$ | ( 7.396, 14.054) | — |
| | $X_{2:R_8}$ | ( 7.842, 18.801) | ( 7.889, 17.429) |
| | $X_{3:R_8}$ | ( 8.791, 24.794) | ( 8.790, 23.796) |
| | $X_{4:R_8}$ | (10.380, 34.084) | (12.135, 33.512) |
| | $X_{5:R_8}$ | (13.260, 2.400) | — |
| Case 2 | $X_{1:R'_2}$ | ( 0.807, 4.769) | — |
| | $X_{2:R'_2}$ | ( 1.054, 7.1124) | ( 1.155, 6.228) |
| | $X_{3:R'_2}$ | ( 1.518, 9.455) | ( 1.673, 8.705) |
| | $X_{4:R'_2}$ | ( 2.156, 11.972) | ( 2.646, 11.327) |
| | $X_{5:R'_2}$ | ( 2.960, 14.791) | ( 2.834, 14.401) |
| | $X_{6:R'_2}$ | ( 3.947, 18.067) | ( 3.947, 18.066) |
| | $X_{7:R'_2}$ | ( 5.157, 22.034) | ( 5.336, 22.083) |
| | $X_{8:R'_2}$ | ( 6.664, 27.155) | ( 7.098, 28.981) |
| | $X_{9:R'_2}$ | ( 8.609, 34.202) | ( 9.419, 28.920) |
| | $X_{10:R'_2}$ | (11.305, 45.738) | (12.687, 43.055) |
| | $X_{11:R'_2}$ | (15.715, 73.045) | — |

For this scheme, we have $n = 19$, $m = 8$.

(ii) another progressive Type-II censored sample with $n = 19$, $m = 8$ generated by Ng *et al.* (2004) using the optimal censoring plan. The vector of observed failure times and the progressive censoring scheme are

$$\mathbf{x} = (0.19, 0.78, 1.31, 3.16, 4.67, 8.01, 31.75, 36.71)$$

and

$$\mathbf{R} = (0, 11, 0, 0, 0, 0, 0, 0).$$

We obtain the 95% PI's for $X_{s:R_i}$ $(s = 1, 2, \ldots, R_i;\ i = 3, 5, 8)$ using the two methods discussed in Section 2. The results are displayed in Table 1. It is observed from Table 1 that the PIs obtained using the HCD method are shorter than the PIs obtained from the pivot method.

## 3.2. Simulation study

A Monte Carlo simulation study is used to evaluate the means and standard errors of prediction limits. We randomly generated 2000 progressively censored sample from exponential distribution with $\theta = 0.75, 1$ and 2. We also used the censoring scheme $\mathbf{R} = (0, 0, 3, 0, 3, 0, 0, 5)$ used by Viveros and Balakrishnan (1994). We computed 95% PI's for $X_{s:R_i}$ $(s = 1, 2, \ldots, R_i;\ i = 3, 5, 8)$ by using (2.7) and (2.14) and the corresponding means and standard errors of prediction limits. Table 2 presents the means and standard errors of prediction limits and means and standard errors of the lengths of PI's, respectively.

From Table 2, we observe that the means and standard errors of prediction limits obtained by pivot method are almost higher than those of prediction limits obtained by the HCD method in the most of considered cases. From Table 2, we note that the HCD method performs well based on PI length

Table 2: Means and standard errors of prediction limits and the lengths of PIs.

| | | $\theta = 0.75$ | | $\theta = 1$ | | $\theta = 2$ | |
|---|---|---|---|---|---|---|---|
| | | Pivot | HCD | Pivot | HCD | Pivot | HCD |
| $X_{1:R_3}$ | lower (SE) | 0.235(0.131) | — | 0.176(0.100) | — | 0.088(0.050) | — |
| | upper (SE) | 1.839(0.660) | — | 1.401(0.499) | — | 0.701(0.252) | — |
| | length(SE) | 1.604(0.522) | — | 1.325(0.384) | — | 0.013(0.241) | — |
| $X_{2:R_3}$ | lower (SE) | 0.352(0.163) | 0.352(0.163) | 0.271(0.123) | 0.271(0.123) | 0.134(0.059) | 0.134(0.059) |
| | upper (SE) | 3.349(1.205) | 3.349(1.205) | 2.562(0.905) | 2.562(0.905) | 1.268(0.436) | 1.268(0.436) |
| | length(SE) | 2.997(1.037) | 2.997(1.037) | 2.291(0.760) | 2.291(0.760) | 1.134(0.304) | 1.134(0.304) |
| $X_{3:R_3}$ | lower (SE) | 0.683(0.256) | — | 0.510(0.201) | — | 0.260(0.101) | — |
| | upper (SE) | 6.603(2.287) | — | 4.935(1.818) | — | 2.503(0.879) | — |
| | length(SE) | 5.902(0.539) | — | 4.425(1.594) | — | 2.343(0.754) | — |
| $X_{1:R_5}$ | lower (SE) | 0.449(0.196) | — | 0.338(0.148) | — | 0.166(0.074) | — |
| | upper (SE) | 2.077(0.729) | — | 1.564(0.548) | — | 0.773(0.275) | — |
| | length(SE) | 1.628(0.524) | — | 1.326(0.385) | — | 0.607(0.184) | — |
| $X_{2:R_5}$ | lower (SE) | 0.566(0.235) | 0.566(0.235) | 0.429(0175) | 0.429(0175) | 0.215(0.089) | 0.215(0.089) |
| | upper (SE) | 3.539(1.269) | 3.539(1.269) | 2.685(0.941) | 2.685(0.941) | 1.356(0.479) | 1.356(0.479) |
| | length(SE) | 2.973(1.021) | 2.973(1.021) | 2.256(0.752) | 2.256(0.752) | 1.141(0.346) | 1.141(0.346) |
| $X_{3:R_5}$ | lower (SE) | 0.900(0.334) | — | 0.670(0.254) | — | 0.342(0.130) | — |
| | upper (SE) | 6.815(2.338) | — | 5.079(1.789) | — | 2.578(0.901) | — |
| | length(SE) | 5.915(2.012) | — | 4.391(1.526) | — | 2.236(0.761) | — |
| $X_{1:R_8}$ | lower (SE) | 1.023(0.387) | — | 0.769(0.289) | — | 0.381(0.147) | — |
| | upper (SE) | 2.003(0.715) | — | 1.506(0.533) | — | 0.744(0.271) | — |
| | length(SE) | 0.980(0.321) | — | 0.737(0.254) | — | 0.363(0.124) | — |
| $X_{2:R_8}$ | lower (SE) | 1.074(0.408) | 1.037(0.396) | 0.820(0.319) | 0.792(0.309) | 0.406(0.154) | 0.392(0.150) |
| | upper (SE) | 2.667(0.959) | 2.468(0.889) | 2.040(0.750) | 1.887(0.695) | 1.005(0.358) | 0.930(0.332) |
| | length(SE) | 1.593(0.537) | 1.431(0.493) | 1.220(0.430) | 1.095(0.371) | 0.599(0.197) | 0.538(0.146) |
| $X_{3:R_8}$ | lower (SE) | 1.227(0.468) | 1.227(0.468) | 0.911(0.335) | 0.911(0.335) | 0.463(0.175) | 0.463(0.175) |
| | upper (SE) | 3.570(1.284) | 3.570(1.284) | 2.654(0.927) | 2.654(0.927) | 1.374(0.476) | 1.374(0.476) |
| | length(SE) | 2.343(0.826) | 2.343(0.836) | 1.743(0.583) | 1.743(0.583) | 0.911(0.285) | 0.911(0.285) |
| $X_{4:R_8}$ | lower (SE) | 1.459(0.533) | 1.716(0.621) | 1.088(0.394) | 1.280(0.459) | 0.548(0.208) | 0.644(0.242) |
| | upper (SE) | 4.932(1.755) | 4.873(1.589) | 3.682(1.292) | 3.385(1.038) | 1.846(0.671) | 1.698(0.598) |
| | length(SE) | 3.473(1.213) | 3.157(1.089) | 2.594(0.904) | 2.105(0.539) | 1.298(0.451) | 1.054(0.324) |
| $X_{5:R_8}$ | lower (SE) | 1.868(0.691) | — | 1.406(0.500) | — | 0.710(0.257) | — |
| | upper (SE) | 7.996(2.858) | — | 6.019(2.087) | — | 3.046(1.080) | — |
| | length(SE) | 6.128(2.167) | — | 5.613(1.591) | — | 2.336(0.765) | — |

as an optimality criterion. We note that also the means and standard errors of prediction limits and means and standard errors of the lengths of PI's decrease with increasing $\theta$. In the sense of PI length as an optimality criterion, it is clear that the HCD method provides the best results for predicting $Y = X_{s:R_i}$ $(s = 1, 2, \ldots, R_i;\ i = 1, 2, \ldots, m)$ when $s > 1$ and $s < R_i$. So, we recommend to use the HCD method for $s > 1$ and $s < R_i$ .

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*Received August 2009; Accepted November 2009*