Power Randomization Schemes for Random Beamforming Based MIMO Systems

Bang Chul Jung, Kil-Young Sung*, *Member, KIMICS*

Abstract— **In this paper, we propose two power randomization schemes for the random beamforming (RBF) based MIMO systems in cellular downlink. In the proposed system, a BS randomizes not only the pre-coding matrix but also the power allocation matrix, while the conventional RBF system allocates an equal power to each transmit stream. The proposed water-filling based power randomization scheme (Scheme-I) is proper in the low SNR values and the proposed random-power based randomization scheme (Scheme-II) is proper in the high SNR values. The proposed system with the power randomization outperforms the conventional RBF system which allocates the same power for each data stream..**

Index Terms— **Random beamforming, power allocation, MIMO, cellular system, scheduling**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless communications can increase throughput in multiuser cellular systems by using multiple antennas in exchanging multiple users' data simultaneously [1]–[4]. The sum capacity of the MIMO broadcast channel (MIMO-BC) can be achieved by a dirty paper coding (DPC) scheme [1], [5]. However, the DPC scheme requires complete channel state information and high complexity at the transmitter, which is difficult to implement in practice.

One approach to reduce feedback requirements is to use a random beamforming (RBF) scheme [6]. In the RBF scheme, the base station randomly selects a beam for transmission and uses it to send training sequences.

Users feedback their signal-to-noise ratio (SNR) values according to this beam pattern and the base station schedules a user with the highest SNR value (or uses another scheduling rule) for transmission. The performance of this RBF scheme is close to that the optimal beamforming strategy for a large number of users [6]. Since, the RBF scheme sends only one data stream, we do not take full advantage of MIMO-BC capacity gain. A possible approach is to use opportunistic space division multiple access (OSDMA) where the base station sends orthogonal beams [7]. In this case, each user reports the best beam and the signal-to-interference-plus-noise ratio (SINR) values to the base station. The base station then schedules transmissions to multiple users based on the received SINR values. In the conventional RBF techniques described above, random beams consist of orthonormal beams implemented by a unitary matrix. However, we can randomize not only beam patterns but also the allocated power levels of the beams.

Most previous results have considered a single receive antenna case at each mobile station. However, if a mobile station has multiple antennas for data reception, it can receive multiple data streams. The RBF scheme can also be applied in this case. In [8], an RBF technique for MIMO systems was proposed to simultaneously obtain downlink multiuser diversity gain, spatial multiplexing gain, and array gain by feeding only effective SINR values back to the BS. However, the RBF scheme in [8] is operated in a time-division multiplexing (TDM) manner, which selects only one user for data stream and all spatial resources in downlink are assigned to the selected user. When we use a singular value decomposition (SVD) technique at the base station, we can allocate the spatial resources to different users at the same time.

In this paper, we propose two power randomization schemes for RBF based MIMO systems and we extend the power randomization schemes to a multi-user RBF system in which different beams can be allocated to different users. We use an SVD-based RBF scheme as a more generalized RBF scheme. The rest of paper is organized as follows. In Section II, we introduce a system model to analyze both the conventional RBF system and the proposed RBF system. In Section III, the performance of both systems is evaluated buy simulation. Finally, summary and conclusions are presented in Section IV.

II. PROPOSED POWER RANDOMIZATION SCHEME

Fig. 1 shows the block diagram of a conventional SVD-based random beamforming system. In this system, each stream has an equal power. A BS generates a unitary matrix V_b in a pseudo-random manner and MSs estimate the wireless channel [8]. Each MS measures the effective signal-to-noise ratio (ESNR) of each stream and feedbacks the ESNR to the BS.

Manuscript received October 19, 2010; revised November 7, 2010; accepted November 15, 2010

* Corresponding Author

Bang Chul Jung and Kil-Young Sung are with the Dept. of Information and Communication Engineering and the Institute of Marine Industry, Collage of Marine Science, Gyeongsang National University, Tongyeong, Gyeongnam, 650-160, Korea (Email: bcjung@gnu.ac.kr, kysung3836@hanmail.net)

Based on this feedback information, the BS selects one MS to transmit data in the next transmission time. If the BS has *M* antennas and each MS has *N* antennas, the total number of transmit streams is equal to min{*M, N*}. The BS may select one MS that has the highest summation of the total ESNRs or the highest effective capacity estimated from the ESNR.

In this case, the received signal of the *k*-th MS can be written as:

$$\begin{aligned} y_k &= \mathbb{H}_k \mathbb{V}_b x + z_k \\ &= \mathbb{U}_k \Sigma_k \mathbb{V}_k \mathbb{V}_b x + z_k , \end{aligned} \tag{1}$$

where $\mathbb{H}_k$, x, and z_k denotes the wireless channel gain of the *k*-th MS, the transmitted symbol vector at the BS, the complex gaussian noise vector with a covariance matrix of (N_0 · I) at the *k*-th MS, respectively. After the channel estimation at the receiver and post-processing of SVD operation, the received vector y_k can be written as:

$$\begin{aligned} \hat{y}_k &= \mathbb{U}_k^H \mathbb{U}_k \Sigma_k \mathbb{V}_k \mathbb{V}_b x + \mathbb{U}_k^H z_k \\ &= \Sigma_k \mathbb{V}_k \mathbb{V}_b x + \hat{z}_k , \end{aligned} \tag{2}$$

where $\hat{z}_k$ has an identical statistical property with the given z_k. We assume that the channel estimation is perfect.

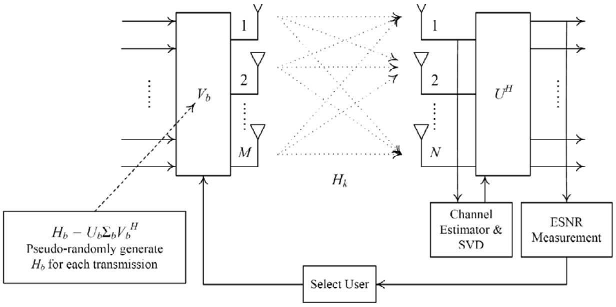

Fig. 1. Block diagram of a conventional random beamforming system with equal-power allocation

Let $\Psi^k = \mathbb{V}_k \mathbb{K}_b$. Then, the ESNR of the *i*-th stream at the *k*-th MS is expressed as:

$$\Gamma_i^k = \frac{\lambda_{k,i}^2 \Psi_{ii}^k}{\lambda_{k,i}^2 \cdot \sum_{j \neq i} \Psi_{k,i}^k + N_0 \min\{M, N\} / E_s} \tag{3}$$

where, $\lambda_{k,i}^2$ and E_s represent the *i*-th singular value of $\mathbb{H}_k$ and the allocated energy for transmitted symbol vector x, respectively. Therefore, the ESNR is affected by not only the thermal noise at the receiver but also the allocated power to the other streams. The effective capacity is obtained as:

$$R_k = \sum_{i=1}^{\min\{M,N\}} \log_2 (1 + \Gamma_i^k). \tag{4}$$

The BS selects the user which has the maximum *effective capacity* computed by the ESNR.

At the low SNR values, Eq. (3) can be approximated as:

$$\begin{aligned} \Gamma_i^k &= \frac{\lambda_{k,i}^2 \Psi_{ii}^k}{\lambda_{k,i}^2 \cdot \sum_{j \neq i} \Psi_{k,i}^k + N_0 \min\{M, N\} / E_s} \\ &\simeq \frac{\lambda_{k,i}^2 \Psi_{ii}^k}{N_0 \min\{M, N\} / E_s} . \end{aligned} \tag{5}$$

In this case, the ESNR reflects $\lambda_{k,i}^2$ exactly and is mainly affected by the thermal noise. Hence, at low SNR values the wireless channel $\mathbb{H}_k$ can be regarded as a *parallel gaussian channel* since the allocated power to each stream does not affect the other streams. At the high SNR values, Eq. (3) can be approximated as:

$$\begin{aligned} \Gamma_i^k &= \frac{\lambda_{k,i}^2 \Psi_{ii}^k}{\lambda_{k,i}^2 \cdot \sum_{j \neq i} \Psi_{k,i}^k + N_0 \min\{M, N\} / E_s} \\ &\simeq \frac{\lambda_{k,i}^2 \Psi_{ii}^k}{\lambda_{k,i}^2 \cdot \sum_{j \neq i} \Psi_{k,i}^k} . \end{aligned} \tag{6}$$

In this case, the ESNR does not reflect the $\lambda_{k,i}^2$ values anymore and is affected by the other signature and the allocated power to the other streams. Hence, at the high SNR values, the wireless channel $\mathbb{H}_k$ cannot be regarded as a *parallel gaussian channel* since the allocated power to each stream does affect the other streams.

Thus far, we have reviewed the conventional RBF system which has been considered as an effective MIMO downlink technique. However, this system did not consider the fact that we can randomize the allocated power allocation of each transmitted stream. We propose a power randomization scheme for the performance improvement of the conventional RBF schemes. Fig. 2 illustrates the proposed RBF system with ***power randomization scheme***. In the proposed system, the BS generates not only a pseudo-random unitary matrix ($\mathbb{V}_b$) but also a pseudo-random diagonal matrix (power randomization matrix, $\mathbb{P}_b$). If we use the proposed system at the transmitter, then the Eq. (3) can be rewritten as:

$$\Gamma_i^k = \frac{P_i \cdot \lambda_{k,i}^2 \Psi_{ii}^k}{P_i \cdot \lambda_{k,i}^2 \cdot \sum_{j \neq i} \Psi_{k,i}^k + N_0 \min\{M, N\} / E_s}, \tag{7}$$

where P_i represents the *i*-th diagonal element of the power randomization matrix $\mathbb{P}_b$.

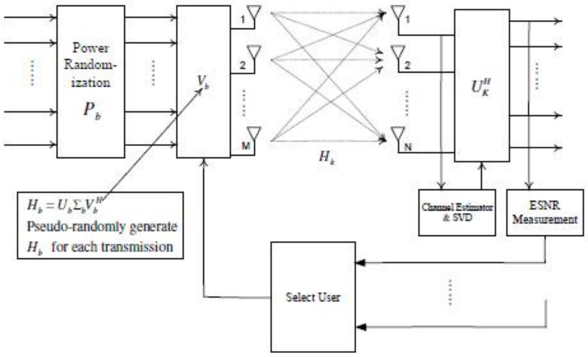

Fig. 2. Block diagram of the proposed random beamforming system with ***power randomization scheme***

We propose two power randomization schemes. We first propose a water-filling based power randomization scheme which jointly generates both a power randomization matrix and a unitary matrix generated from the original random matrix ($\mathbb{H}_b$). This scheme is called ***Scheme-I***. The allocated power for each stream is computed based on the matrix ($\sum_b$).

Therefore, this allocated power is optimal to the MS in which has the wireless channel of $\mathbb{H}_b$. This scheme is appropriate for the case of low SNR values since the power is allocated, assuming that the channel is parallel-gaussian.

When the SNR values are high, scheme-I is not optimal because the ESNR is dominated by the interference from other streams instead of the thermal noise. The channel cannot be assumed to be parallel gaussian in this case. Hence, we propose another power randomization scheme in which the power allocation matrix ($\mathbb{P}_b$) is *independently* generated to the unitary matrix ($\mathbb{V}_b$) for high SNR values. This scheme is called ***Scheme-II***. Through the proposed power randomization schemes, the variance of the resultant ESINR becomes large and the multiuser diversity gain also becomes increased.

The proposed systems can be easily extended to multi-user SVD technique in which each transmit stream is allocated to the different user. A BS randomly generates both a power allocation matrix and a unitary matrix, which is the same operation as the single user case.

However, a BS selects the MS that feeds back the highest ESINR for each stream, while one user with the highest effective capacity is selected in the single-user case. The multi-user SVD technique has more degrees of freedom and the multi-user SVD scheme is a generalized version of the single-user SVD technique.

III. SIMULATION RESULTS

Fig. 3 shows the capacity of the proposed RBF system with power randomization schemes and the conventional RBF system when the number of antennas at both transmitter and receiver is four. We assume that there exist 100 users in a cell. As a whole, the proposed systems outperform the conventional RBF system which allocates the equal power to all streams. As noted in Section II, the scheme-I yields excellent performance at low SNR values. Since the scheme-I allocates almost equal power at high SNR values, the scheme-I and the conventional equal power scheme yield the nearly identical performance. On the other hand, the scheme-II yields the highest capacity at high SNR values. A performance crossover point in the single-user SVD technique occurs when the SNR is equal to 18dB. If the SNR value is lower than 18dB, the thermal noise at the receiver is more dominant than the interference (*noise-limited*) and, thus, the proposed scheme-I yields better performance than the proposed scheme-II. In addition, if the SNR is higher than 18dB, the interference is more dominant than the thermal noise (*interference-limited*) and, thus, the proposed scheme-II yields better performance than the proposed scheme-I. Hence, we can obtain the improved capacity if we switch the proposed scheme-I and the proposed scheme-II according to the SNR value. The performance of the multi-user SVD case shows similar tendency and is improved due to the increased degrees of freedom in scheduling policy, compared to the single-user SVD case.

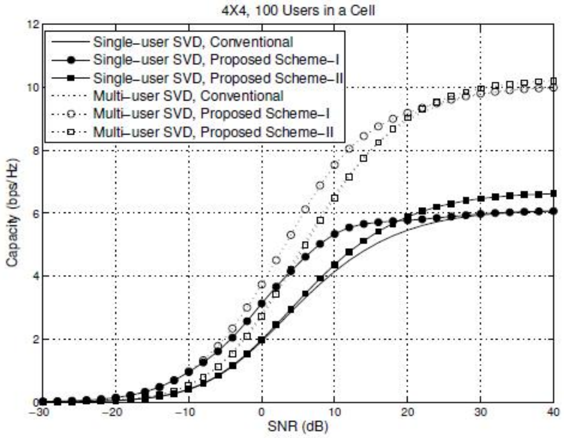

Fig. 3. Capacity comparison between the proposed power randomization system and the conventional system for varying SNR values in the case of 4 X 4, 100 users in a cell

Fig. 4 shows another example of the capacity of the proposed system and the conventional BF scheme when the number of antennas at both transmitter and receiver is three. The overall characteristics of the performance is similar to Fig. 3. Note that, the performance cross-over point between the proposed schemes becomes higher because the noise-limited situation more frequently occur as the number of antennas at transmitter decreases.

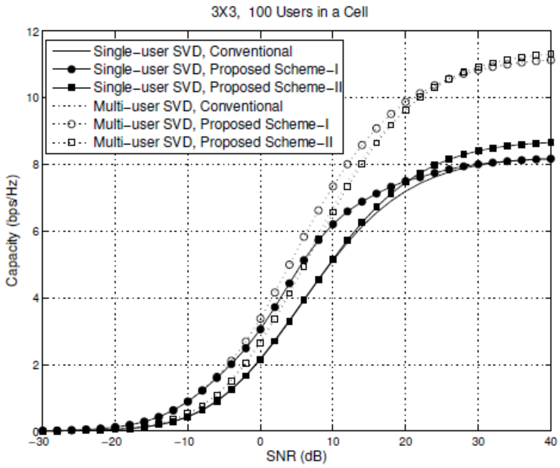

Fig. 4. Capacity comparison between the proposed power randomization system and the conventional system for varying SNR values in the case of 3 X 3, 100 users in a cell

IV. CONCLUSIONS

We proposed two power randomization schemes for the random beamforming based MIMO systems in downlink. The proposed water-filling based power randomization scheme (Scheme-I) is proper in low SNR values, while the proposed random-power based randomization scheme (Scheme-II) is proper in high SNR values. The proposed schemes can be switched according to the given SNR values. However, the multi-user SVD scheme with the proposed power randomization schemes yields the larger capacity than the single-user SVD scheme.

REFERENCES

[1] G. Caire and S. Shamai, “On the achievable throughput of a multiantenna Gaussian broadcast channel”, *IEEE Trans. on Information Theory*, Vol. 49, No. 7, pp. 1691-1706, July 2003.

[2] P. Viswanath and D. N. C. Tse, “Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality”, *IEEE Trans. on Information Theory*, Vol. 49, No. 8, pp. 1912-1921, Aug. 2003.

[3] S. Vishwanath, N. Jindal, and A. Goldsmith, “Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels”, *IEEE Trans. on Information Theory*, Vol. 49, No. 10, pp. 2658-2668, Oct. 2003.

[4] K. -K. Wong, R. D. Murch, and K. B. Letaief, “Performance enhancement of multiuser MIMO wireless communication systems”, *IEEE Trans. on Communications*, Vol. 50, No. 12, pp. 1960-1970, Dec. 2002.

[5] M. H. M. Costa, “Writing on dirty paper”, *IEEE Trans. on Information Theory*, Vol. 29, No. 3, pp. 439-441, May 1983.

[6] P. Viswanath, D. N. C. Tse, and R. Laroia, “Opportunistic beamforming using dumb antennas”, *IEEE Trans .on Information Theory*, Vol. 48, No. 6, pp. 1277-1294, June 2002.

[7] M. Sharif and B. Hassibi, “On the capacity of MIMO broadcast channels with partial side information”, *IEEE Trans. on Information Theory*, Vol. 51, No. 2, pp. 506-522, Feb. 2005.

[8] J. Chung, C. -S. Hwang, K. Kim, and Y. K. Kim, “A random beamforming technique in MIMO systems exploiting multiuser diversity”, *IEEE Journal of Sel. Areas in Communi.*, Vol. 21, No. 5, pp. 848-855, June 2003.

Bang Chul Jung received the B.S. degree in Electronics Engineering from Ajou University, Suwon, Korea, in 2002 and the M.S. and Ph.D degrees in Electrical & Computer Engineering from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 2004 and 2008, respectively. He was a research professor with KAIST Institute for Information Technology Convergence, Daejeon, Korea, until Feb. 2010. He is an assistant professor of department of Information and Communication Engineering, Gyeonsang National University, Korea. He received Bronze Prize in Intel Student Paper Contest in 2005. His research interests include orthogonal resource hopping systems for packet data transmission, OFDM systems, radio resource management, wireless scheduling algorithm, information theory, link & system-level simulations for 4G wireless communication systems, adaptive modulation & coding, and MIMO systems.

Kil-Young Sung received the B.S. degree in Electronics Engineering from Kyeongbuk National University, Daegu, Korea, in 1980 and the M.S. degree in Electronics Engineering from Kunkuk University, Seoul, Korea, in 1985, respectively. He received the Ph. D degree in Electronics Engineering from Pukyong National University, Pusan, Korea, in 2000. He is a professor of department of Information and Communication Engineering, Gyeongsang National University, Korea. His research interests include VLSI array, computer architecture, and image compression.