품질손실함수를 이용한 규격치 결정방법의 성능평가

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Comparative Performance of the Size Determination Method Using Quality Loss Function

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Abstract

This paper deals with the performance evaluation of determining production size specifications. A customer who does not find the size specification he or she wants may purchase rather a larger or smaller one, but the purchasing desire decreases as the difference between the required and the prepared sizes increases. Introducing a generalized quality loss function which reflects how much the purchasing desire changes according to the difference, Park and Kim(1992) formulated a mathematical model for determining the size specifications so as to minimize the expected loss. Afterward the model has been applied to the determination of sizing system for mail-order clothing and brassiere (Lee and Choi, 1996; Chun, et.al., 1996).

The performance of the size determination method proposed by Park and Kim is evaluated in this paper. Usually the intervals between two successive size specifications are determined to be equal, but the size determination method compares favorably with the equidistance case, and more favorably if the population distribution is more skewed.

1. Introduction

In a modern multi-item small quantity production environment, a trade-off between diversification to meet customers´ various needs and simplification to improve production efficiency is unavoidable. To compromise the two contradictory objectives, the optimal choice of specifications is a matter of interest. The question which production specifications should be manufactured or stocked has been called as the assortment problem and treated for several decades. In the assortment problem[1, 3], the next larger or the next stronger is used if a required size or strength is not prepared, and minimizing the loss due to trim wastage or extra strength is the objective function. Tryfos[7] dealt with the problem of determining the measurements of a given number of sizes of apparel. He also assumed that a customer who doesn't find one's size considers the next larger measurement.

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However, in many cases such as clothing and footwear, a customer who dose not find one's size may purchase rather a larger or a smaller one, but the purchasing desire decreases as the difference between the required and the prepared sizes incre ases. When buying a pair of comfort shoes, a customer might prefer rather larger than smaller ones. Likewise, a customer might prefer rather smaller ones when buying a pair of running shoes if the difference between the required and the prepared sizes is acceptable. Introducing a generalized quality loss function, Park and Kim[5] formulated the problem so as to include these situations. Afterw ard the method has applied to the determination of sizing system for mail-order clothing and brassiere[2, 4].

It is examined that how the performance of the proposed method by Park and Kim changes according to the shape of the distribution.

2. Mathematical Model for Determining Size Specifications[5]

2.1 Quality Loss Function

In determining size specifications to be manufactured, loss can be defined as the lost-sales due to the difference between the customer's required size and the prepared size. As the difference increases, customers tend to give up purchases more and more, and thus the opportunity loss increases. A manufacturer can reduce the loss by making many kinds of sizes, but the production and stocking costs constraint making too many kinds. Thus, it is of interest that how to determine the size specifications to minimize the loss for a limited number of sizes.

Taguchi's quality loss function is a measure of such a loss due to the deviation from a target value[6]. Since the amount of the loss is determined by the size of the deviation, the quality loss function is given by the following expression:

$$L(u) = f(u-x) \tag{1}$$

where

$L(u)$ = the amount of loss
x = the target value (i.e., the required size)
u = the provided value

When the required and the provided sizes are equal, there is no loss due to size matters (that is, $L(u)$ = 0). Furthermore, the first derivative of L(u) at u=x equals to zero because the loss function has the minimum value at the point. Hence the Taylor series expansion of the loss function approximates to

$$L(u) \approx C(x-u)^2 \tag{2}$$

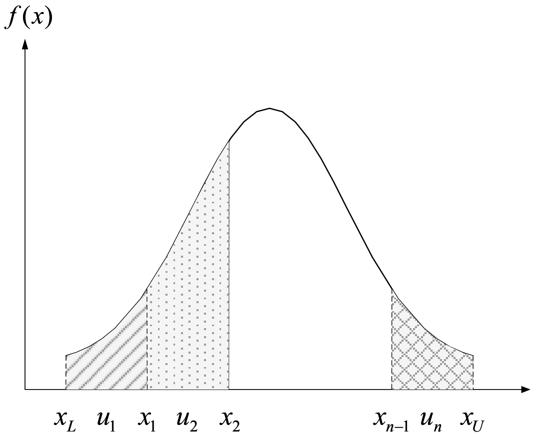
In the quality loss function, the amount of loss is determined by the absolute difference between the required and the provided sizes, $|x-u|$. However, as in the cases of clothing and footwear, a customer who does not find one's size might prefer a larger or a smaller one to a certain extent. Thus, we can generalize the quality loss function as follows:

$$L(u) = \begin{cases} C_1(u-x)^2, & \text{if } x \le u \\ C_2(x-u)^2, & \text{if } x > u \end{cases} \tag{3}$$

If $C_1 = C_2$, equation (3) coincides with the conventional quality loss function; and if $C_2 = \infty$, the problem reduces to the previous studies in which the next larger or stronger ones are used when the required specifications are not made[5].

2.2 Model Formulation

Suppose that the required sizes of target customers are distributed in the interval (x_L, x_U) and a manufacturer supplies n kinds of size specifications $u_i (; x_L \le u_1 < u_2 \cdots\cdots < u_n \le x_U)$. If a required size x is between the two specifications u_i and u_{i+1}, the customer would select u_i or u_{i+1} if he or she decides to buy. In this case, we can imagine an intermediate value x_i which is the boundary between selecting a larger one and a smaller one. The customer considers a larger size u_{i+1} if the required size is larger than the boundary value x_i, and considers a smaller size u_i if the required one is smaller than that. Figure 1 shows these relationships, where $f(x)$ denotes the probability density function of the required sizes.



<Figure 1>. Schematic representation of the size specifications problem

The expected loss $E(L)$ is expressed as

$$E(L) = \sum_{i=1}^{n} \int_{x_{i-1}}^{u_i} C_1 (u_i - x)^2 f(x) dx + \sum_{i=1}^{n} \int_{u_i}^{x_i} C_2 (x - u_i)^2 f(x) dx \quad (4)$$

where

$x_L = x_0$ and $x_U = x_n$

To find the optimal values of x_i and u_i minimizing the expected loss $E(L)$, setting the partial derivatives of equation (4) with respect to x_i and u_i equal to zero yield respectively

$$C_1 (u_{i+1} - x_i)^2 = C_2 (x_i - u_i)^2 \quad (5)$$

and

$$C_1 \int_{x_{i-1}}^{u_i} (u_i - x) f(x) dx = C_2 \int_{u_i}^{x_i} (x - u_i) f(x) dx \quad (6)$$

It is notable in equation (5) that the boundary value x_i balances the two losses resulting from choosing the larger size u_{i+1} and the smaller size u_i. Equations (5) and (6) can be rewritten as

$$u_{i+1} = (1 + \sqrt{C_2/C_1}) x_i - \sqrt{C_2/C_1} u_i \quad (7)$$

and

$$\int_{x_{i-1}}^{u_i} (u_i - x) f(x) dx = (C_2/C_1) \int_{u_i}^{x_i} (x - u_i) f(x) dx \quad (8)$$

The set of sizes $u_i (; x_L \le u_1 < u_2 \cdots\cdots < u_n \le x_U)$ satisfying equations (7) and (8) minimizes the expected loss[5].

2.3 Iterative Computation Procedure

The optimal values of x_i and u_i minimizing the expected loss $E(L)$ can be obtained by the following iterative procedure[5].

Step 1. Put $i = 1$, and compute x_i with an arbitrary value for u_i in (8).

Step 2. Compute u_{i+1} by substituting the values of x_i and u_i in (7).

Step 3. Compute x_{i+1} by substituting the values of x_i and u_{i+1} in (8). (If $i = n-1$ go to Step 4. Otherwise go to Step 2 after replacing i with $i+1$.)

Step 4. If $x_n = x_U$, then the optimal sizes $u_1 (; i = 1, 2, \cdots, n)$ are obtained. Otherwise using a method in numerical analysis such as bisection method, adjust the value for u_1 and repeat the iterative procedure.

3. Performance Evaluation

Normal population distributions are often assumed in statistical models, but the distributions of anthropometric data are generally asymmetric, ske wed to the right as the lognormal distribution. To evaluate the effect of skewness, the beta distribution is assumed in this section.

The probability density function of the beta distribution is expressed as

$$f(x;a,b) = \frac{1}{B(a,b)}[x^{a-1}(1-x)^{b-1}]$$

$$(;\ 0 \le x \le 1,\ a > 0,\ b > 0) \quad (9)$$

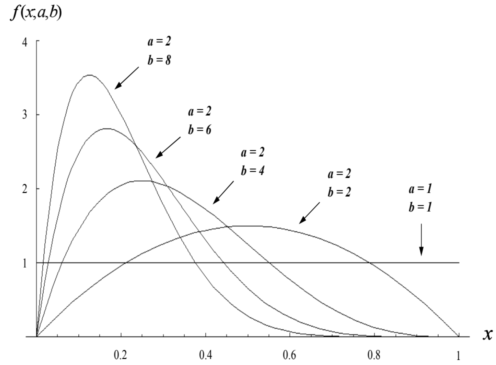
where,

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}dx \quad (10)$$

The beta distribution is useful for the purpose because many different shapes are available depending on the values of the shape parameters (a,b). Figure 2 is the plot of the beta distribution for various values of the parameters.

The shape of the beta distribution is symmetric when the values of a and b are equal. If a=b=1, the distribution becomes the uniform distribution. The greater the difference between a and b, the greater the asymmetry in Figure 2.

To illustrate the characteristics of the optimal size specifications, let us consider the following cases.



<Figure 2>. The shape of the beta distribution

- Distribution of the required sizes: The beta distributions shown in Figure 2.
- Coefficient of the quality loss function:

  (i) C_2/C_1= 0.25 (; a smaller one is somewhat preferred to a larger one.)

<Table 1>. The optimal size specifications and the expected loss

Shape parameters (a,b)	C2/C1	Optimal size specifications u_1	u_2	u_3	u_4	u_5	Expected loss $E(L)$
(1,1)	0.25	0.067	0.267	0.467	0.667	0.867	0.593
	1	0.100	0.300	0.500	0.700	0.900	0.333
	4	0.133	0.333	0.533	0.733	0.933	0.593
(2,2)	0.25	0.122	0.301	0.463	0.625	0.799	0.482
	1	0.163	0.339	0.500	0.661	0.837	0.269
	4	0.201	0.375	0.536	0.699	0.878	0.482
(2,4)	0.25	0.082	0.211	0.336	0.472	0.638	0.330
	1	0.111	0.240	0.368	0.508	0.683	0.191
	4	0.140	0.271	0.403	0.548	0.735	0.357
(2,6)	0.25	0.062	0.162	0.264	0.378	0.527	0.225
	1	0.085	0.187	0.291	0.411	0.570	0.133
	4	0.107	0.212	0.321	0.449	0.624	0.253
(2,8)	0.25	0.050	0.132	0.217	0.315	0.448	0.161
	1	0.068	0.152	0.241	0.345	0.488	0.096
	4	0.087	0.174	0.267	0.379	0.539	0.184

(ii) $C_2/C_1 = 1$ (; closest to the required size is always preferred.)

(iii) $C_2/C_1 = 4$ (; a larger one is somewhat preferred to a smaller one.)

- The number of size specifications: $n = 5$

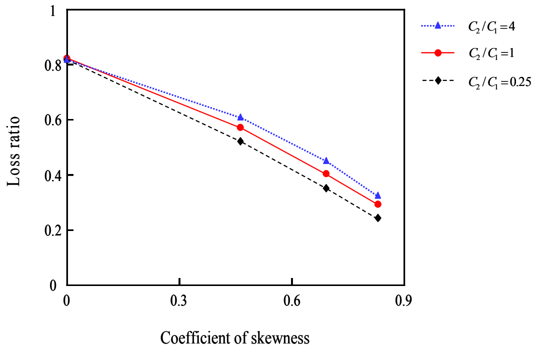
The optimal standard size specifications and the expected losses obtained using the iterative optimization procedure are summarized in Table 1.

It should be noted that the optimal size intervals are not equidistant unless the population distribution is uniform. The proposed model shows that the optimal size intervals are narrower where probability densities are higher. The optimal size intervals are not symmetric if the population distribution is not symmetric. Even in the case of symmetric distribution, the optimal intervals are not symmetric if the values of C_1 and C_2 are different.

To examine how the efficiency of the proposed model changes according to the degree of skewness, let us compare the expected losses of the proposed model with those of the equidistance model. The degree of skewness is measured by the coefficient of skewness. The coefficient of the beta distribution is

$$g = \frac{2(b-a)}{(a+b+2)} \sqrt{\frac{a+b+1}{ab}} \qquad (11)$$

The loss ratios of the proposed model to the equidistance case are summarized in Table 2. The set of size specifications {0.1, 0.3, 0.5, 0.7, 0.9} is used for the equidistance model. The loss ratios show that the proposed model compares favorably with the equidistance model, and more favorably if the population distribution is skewed. Figure 3 shows the comparative advantage with respect to the degree of skewness.



<Figure 3>. The loss ratios with respect to the coefficient of skewness

<Table 2>. The comparison of the expected losses between the proposed and the equidistance models

Shape Parameters (a,b)	Coefficient of Skewness	C2/C1	Expected loss		Loss ratio $E(L)/E_e(L)$
			Equidistance model $E_e(L)$	Proposed model $E(L)$	
(2,2)	0	0.25	0.592	0.482	0.814
		1	0.330	0.269	0.815
		4	0.592	0.482	0.814
(2,4)	0.47	0.25	0.599	0.330	0.551
		1	0.328	0.191	0.582
		4	0.585	0.357	0.610
(2,6)	0.69	0.25	0.605	0.225	0.372
		1	0.320	0.133	0.416
		4	0.572	0.253	0.442
(2,8)	0.83	0.25	0.617	0.161	0.261
		1	0.312	0.096	0.308
		4	0.554	0.184	0.332

4. Concluding Remarks

Quality loss function has been widely accepted and used in quality engineering as the financial measure of the user dissatisfaction with a product's performance when it deviates from a target value. In this paper, a generalized quality loss function is used to include the situation that the losses due to positive deviation and negative deviation from the required size are different.

Conventionally, the differences between two successive size specifications have been determined to be equal. However, the model shows that making size intervals narrower where the probability densities of the required sizes are higher and making them wider where the probability densities are lower reduce the expected loss. If the distribution of the required sizes is more skewed, the comparative advantage of the proposed model gets larger.

References

[1] Bongers, C.(1982), "Optimal size selection in standardization: a case study," *Journal of Operational Research Society,* Vol.33, pp.793-799.

[2] Chun, J.S., Park, K.H., and Park, Y.T.(1996), "A study on determining optimal sizes for mail-order clothing," *Journal of the Ergonomics Society of Korea,* Vol.15, pp.113-124.

[3] Jorjani, S., Scott, C.H., and Woodruff, D.L.(1999), "Selection of an optimal subset of sizes," *International Journal of Production Research,* Vol.37, pp. 3697-3710.

[4] Lee, K.H. and Choi, H.S.(1996), "A study on establishment of brassiere sizing system for elderly woman applying loss function," *Journal of the Ergonomics Society of Korea,* Vol.15, pp.1-13.

[5] Park, Y.T. and Kim, S.D.(1992), "On the determination of optimal sizes," *Journal of the Human Engineering Society of Korea,* Vol.11, pp.39-47.

[6] Phadke, M.S.(1989), *Quality Engineering Using Robust Design*. Prentice Hall: Englewood Cliffs, NJ.

[7] Tryfos, P.(1985), "On the optimal choice of sizes," *Operations Research,* Vol.33, pp.678-684.

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