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Statistical process control of dye solution stream using spectrophotometer

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Abstract

The need for statistical process control to check the performance of a process is becoming more important in chemical and pharmaceutical industries. This study illustrates the method to determine whether a process is "in control" and how to produce and interpret control charts. In the experiment, a stream of green dyed water and a stream of pure water were continuously mixed in the process. The concentration of the dye solution was measured before and after the mixer via a spectrophotometer. The in-line mixer provided benefits to the dye and water mixture but not for the stock dye solution. The control charts were analyzed, and the pre-mixer process was in control for both the stock and mixed solutions. The R and X-bar charts showed virtually all of the points within control limits, and there were no patterns in the X-bar charts to suggest nonrandom data. However, the post-mixer process was shown to be out of control. While the R charts showed variability within the control limits, the X-bar charts were out of control and showed a steady increase in values, suggesting that the data was nonrandom. This steady increase in dye concentration was due to discontinuous, non-steady state flow. To improve the experiment in the future, a mixer could be inserted into the stock dye tank. The mixer would ensure that the dye concentration of the stock solution is more uniform prior to entering the pre-mixer flow cell. Overall, this would create a better standard to judge the water and dye mixture data against as well.

Keywords: Histogram, R-charts, statistical process control, X-bar charts.

1. Introduction

The purpose of the experiment was to learn how construct control charts and determine if a certain process is "in control" or not. Both $\overline{X}$ and R charts can be analyzed to determine whether measurement variations are random fluctuations, or indicative of a problem in the process (Cho, 2010). Control charts are widely used in manufacturing processes to determine if equipment failures or other problems are affecting the quality of the product (Ko and Cho, 2008). The concentrations before and after an in-line mixer were measured for a stream of green dye and for a mixed stream of green dye and pure water. From this data, control charts were made and process capability calculated.

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2. Methods

The experiment followed the procedure outlined in the guide (Willson, 2010). Figure 2.1 below shows the process flow diagram. Appendix C discusses safety considerations. First, a 0.15 wt% solution was prepared from twenty liters of water and a concentrated dye solution. Next, the rotameter was calibrated by taking measurements at the rotameter levels of 50, 70, 90, 110 and 130. Using a stopwatch, the experimenters measured the time required to fill a graduated cylinder to the 100 mL mark. Linear regression then gave a relationship between the rotameter level and the volumetric flowrate. Next, the flowrate of the peristaltic pump was determined by measuring the time required to fill the graduated cylinder up to the 200 mL mark. Three measurements were taken, and were averaged to determine the flowrate of the pump.

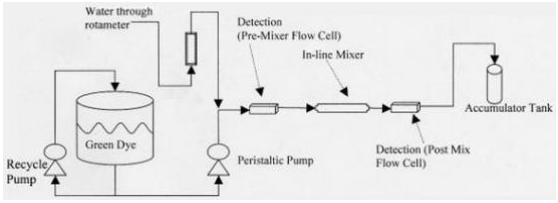

Figure 2.1 Process flow diagram

After calibrating the spectrophotometer, the absorbance of the dye solution was measured. This could be used to calculate the concentration of the dye using Beer's Law:

$$c = \frac{A}{\varepsilon l}. \tag{2.1}$$

However, since the average concentrations before and after the mixer should be the same for the dye solution, a correction factor was applied to the path length of the pre-mixer spectrometer cell. Next, in order to produce a 0.11 wt% dye solution, the rotameter level was set to 120, and the concentration of the dye and water mixture was determined, both before and after the in-line mixer. The path length correction factor used for the dye solution was also applied for the mixture, in order to keep all of the measurements on the same scale. The computer program which recorded the absorbances showed two different absorption peaks for the green dye. The reason for the difference could be because the color green is made from yellow and blue . Additionally, there is detector error present.

Using the data collected, control charts could be constructed in JMP to determine if the process was in control or not. JMP grouped the data into subgroups of five measurements, and calculated the range and mean concentration for each subgroup. For the $\overline{X}$ charts, the center line is simply the average of the means of all the subgroups (or $\overline{\overline{X}}$). The upper and lower limits for the $\overline{X}$ charts could be calculated from the standard deviation, or estimated using the range. Regardless, the upper and lower limits should be approximately ± 3 standard deviations from $\overline{\overline{X}}$. This ensures that the control limits will contain 99.7% of all data points, assuming they belong to a normal and random distribution. For the R charts, the centerline is the mean of the ranges of all of the subgroups ($\overline{R}$), and the upper and lower control limits are determined by constants that are based on the subgroup size.

The R charts help show the variability of the process. If all of the values lie within the control limits, this is a good indication that the process might be in control. However, to be sure, the $\overline{X}$ charts also need to be analyzed. As well as being within the control limits, the points on the $\overline{X}$ chart need to be randomly distributed above and below the center line. If any patterns or trends appear in the $\overline{X}$ chart, then the process is not in control.

Lastly, the process capability index (C_p) can be determined. Process capability is the ratio of the specification requirements to the statistical variability of the process. This is an indication of quality of the material that can be produced, given the random fluctuations that the process undergoes. To be considered capable, C_p should be at least 1.3 (Willson, 2010). A high process capability is desired, because it represents a process where almost all the measurements fall inside the specification limits. Higher C_p values represent lower variability. To determine the process capability, the natural variability of a stable process was compared with the process specification limits.

$$C_p = \frac{USL - LSL}{6s} = \frac{USL - LSL}{UCL - LCL}. \tag{2.2}$$

3. Results

Using the raw absorbance data collected for the dye-stream, Beer's law was applied to calculate the path length of the pre-mixer flow cell, 0.530 cm. The histogram of the concentrations of the stock dye solution in the pre-mixer flow cell, Figure 3.1, has gaps at regular intervals in the distribution. The gaps are most likely due to the imprecision of the spectrophotometer. In other words, the spectrophotometer is rounding these values up or down, which produces the gaps in the histogram. Nevertheless, the pre-mixer flow cell process shows a relatively normal distribution. Likewise, Figure 3.2 for the post-mixer flow cell of the stock dye solution shows a relatively normal distribution.

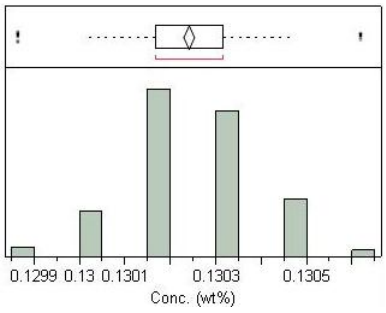

Figure 3.1 Histogram of stock dye pre-mixer concentrations

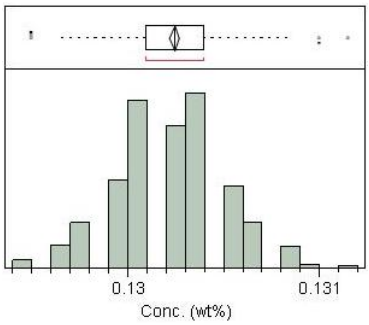

Figure 3.2 Histogram of stock dye post-mixer concentrations

Shown in Figure 3.3 and Figure 3.4 below are the $\overline{X}$ and R control charts of the stock dye solution in the pre-mixer, calculated using the estimation method. In Figure 3.4, there is one point that lies outside of the control limits. However, a process in statistical control can have 0.3% of the points outside of the control limits. Although 0.5% of the points in Figure 3.4 are outside the control limits, that value is so close to 0.3% that we will still consider the stock dye concentration in the pre-mixer to be in control. The concentration of the stock dye solution was supposed to be 0.15% in the bulk tank. According to both Figures 3.3 and 3.5, the concentration was at an average value of 0.13%.

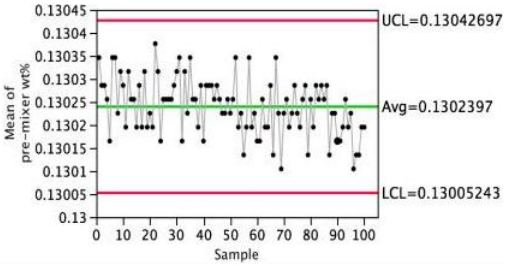

Figure 3.3 Stock dye pre-mixer $\overline{X}$ chart, estimation method

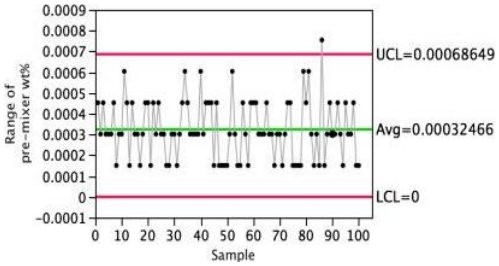

Figure 3.4 Stock dye pre-mixer R chart

There is variation in the bulk solution stream concentrations. Even though the solution is supposed to be uniform, the green dye, which was added to the tank of water, was prepared without thorough mixing. Based on the R chart (Figure 3.6), the post-mixer appears to be in control. However, the $\overline{X}$ chart, Figure 3.5, shows a trend in the form of a positive slope, meaning the data is not random. A system in control must be both normal and random. Therefore, the post-mixer flow cell of the stock dye solution is not in control.

The non-random fluctuations could be due to the fact that the system had not yet reached steady-state. The flow was not continuous, so the flow of dye in the stream changed. The fluid flowed because of a peristaltic pump, which caused the fluid to travel in pulses. As the fluid flowed through the system and the mixer, it slowed down because of friction. The concentration of dye could then build up, resulting in the increase in concentration over time which was observed.

Also, the period of time over which the experiment was performed was relatively short. Over a longer period of time, the concentration in the post-mixer would level off. In theory, the concentration and variability should not be changing because the only stream flowing through is of the stock dye solution. For the stock dye solution, $\overline{\overline{X}}$ should be and was the same in the pre- and post-mixer. The $\overline{R}$ value increased from 0.000325 in the pre-mixer to 0.000598 in the post-mixer. $\overline{R}$ should be greater in the pre-mixer. However, the concentration, which sloped upwards with time in the post-mixer, introduced greater variability into the post-mixer stream.

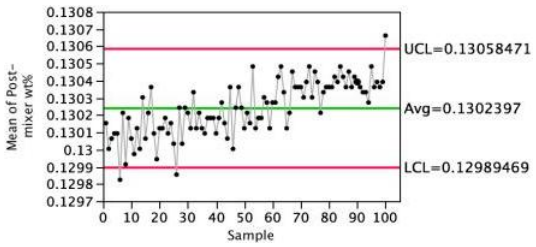

Figure 3.5 Stock dye post-mixer $\overline{X}$ chart, estimation method

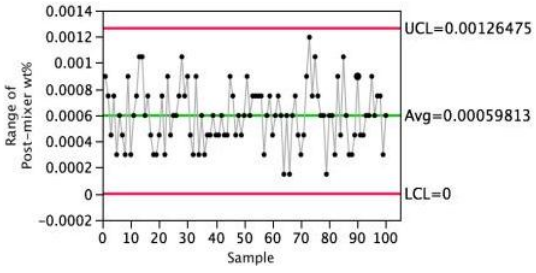

Figure 3.6 Stock dye post-mixer R chart

In the mixed solution of dye and water, the pre-mixer flow cell shows a normal distribution (Figure 3.7). For the post-mixer flow cell, the distribution is skewed to the right (Figure

3.8). Because the concentration distribution is not normal, the control charts will not be a good indication of whether the process is actually in control or not.

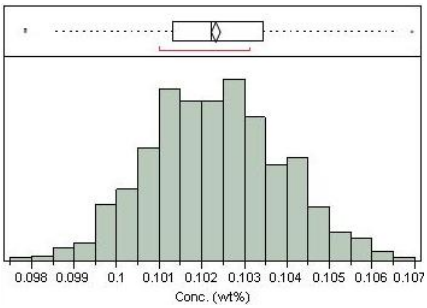

Figure 3.7 Histogram of dye and water pre-mixer concentrations

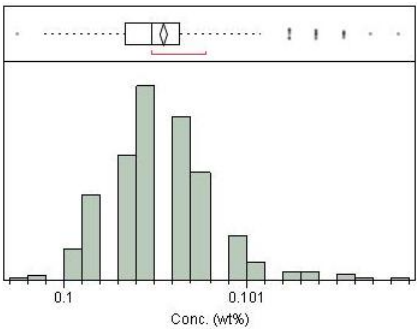

Figure 3.8 Histogram of dye and water post-mixer concentrations

$\overline{X}$ and R control charts (Figures 3.9 and 3.10 below, respectively) of the dye and water in the pre-mixer show that the process was in control. However, the control charts using the estimation method, Figure 3.11 and Figure 3.12, show that the process was out of control for the dye and water in the post-mixer. The same trend that appeared in the post-mixer of the stock dye solution appeared in the post-mixer of the dye and water solution. The positive slope is still present. For the same reasons, as stated before, the dye probably was not flowing at steady-state.

$\overline{\overline{X}}$ in the pre- and post-mixer for the dye and water stream should be the same. The target final concentration was 0.11%. According to Figure 3.9 and Figure 3.11, in the pre-mixer and post-mixer, $\overline{\overline{X}}$ values were 0.102% and 0.101%, respectively. The value of $\overline{R}$ should be greater before the mixer. The mixer creates less variability in concentrations, so the value of $\overline{R}$ decreases by an order of magnitude, from 0.00368 in the pre-mixer to 0.000534 in the post-mixer. The primary sources of variability were the precision of the spectrophotometer as well as changes in the local concentration of the dye.

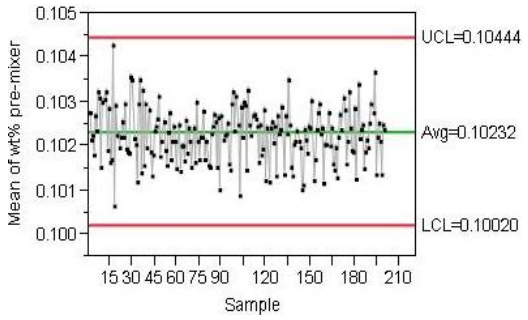

Figure 3.9 Dye and water pre-mixer $\overline{X}$ chart using estimation method

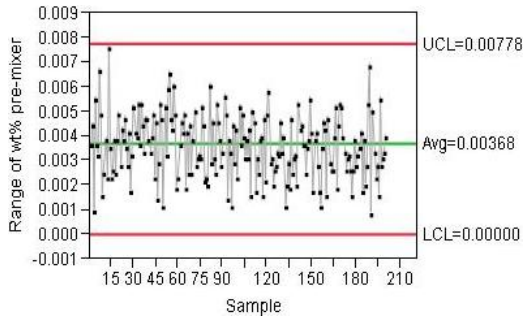

Figure 3.10 Dye and water pre-mixer R chart

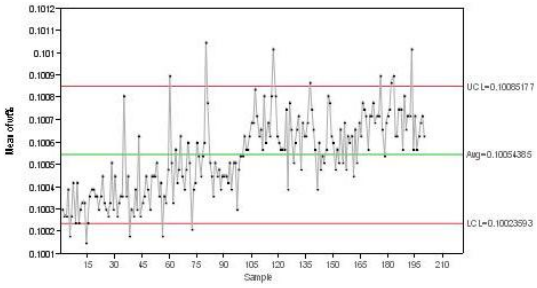

Figure 3.11 Dye and water post-mixer $\overline{X}$ chart using estimation method

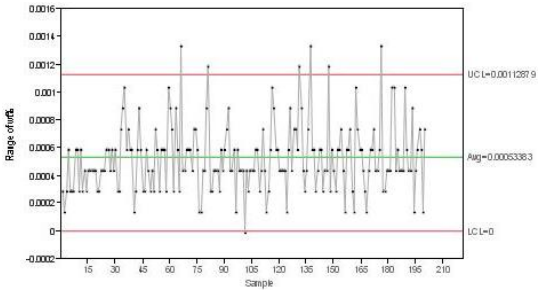

Figure 3.12 Dye and water post-mixer R chart

In Figure 3.13 and Figure 3.14 below, the control limits were calculated using the standard deviation method rather than the estimation method. The standard deviation method yielded wider control limits for $\overline{X}$ than the estimation method did. Comparing the two methods, the percent differences for the control limits for the pre- and post-mixer flow cells were 1.40% and 1.48%, respectively. This means that the two methods performed more or less the same for this experiment. However, when there is a limited amount of data available, the estimation method is typically used, since a reliable value of s can be hard to find, and $\overline{R}$ is relatively easy to calculate. In other words, the standard deviation method requires more data for accuracy.

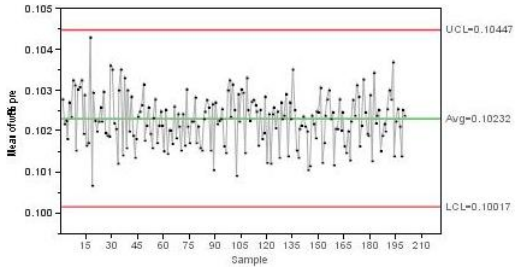

Figure 3.13 Dye and water pre-mixer $\overline{X}$ chart using standard deviation method

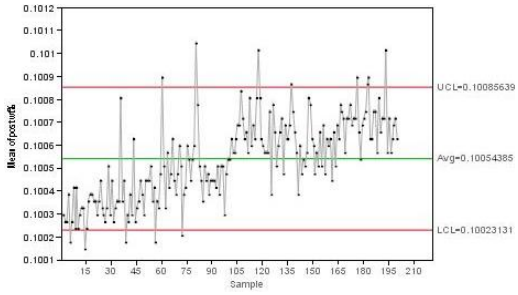

Figure 3.14 Dye and water post-mixer $\overline{X}$ chart using standard deviation method

The raw concentration data vs. time graph, Figure 3.15 below, shows that the mixer creates much more uniform concentrations. In the pre-mixer, the data points are scattered. In the post-mixer, the concentrations have much less variability. The mixer lowers the variability and fluctuations of the concentrations.

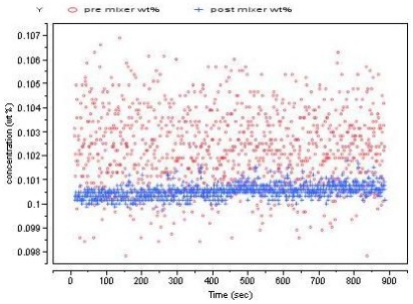

Figure 3.15 Pre- and post- mixer raw concentration data

$\overline{R}$ should be greater in the pre-mixer as compared to the post-mixer flow cell. This was true for the mixed dye and water solution but not for the stock dye solution. This could be attributed to experimental error. Given the precision of the spectrophotometer, the difference in $\overline{R}$ values of 2.73 $\times 10^{-4}$ can be neglected. However, in all cases, a mixer should decrease variability because that is its purpose.

The spectrophotometer was a useful way to measure the concentration since it allowed measurements to be taken continuously. With other measurement methods, it might be necessary to take manual samples to determine the concentration of the solution, which means less data would be collected in the same amount of time. However, the spectrophotometer also creates some heat. This could potentially create measurement error due changes in the physical properties of the solutions with temperature.

The post-mixer data is more sensitive to small nonrandom fluctuations since the variability decreases due to the in-line mixer. This is a good thing since this leads to tighter control limits, which means the overall product quality is better.

For this process, the concentrations needed to be within ± 0.002 wt% of the mean. The process capability index values, in Table 3.1 below, show that the stock dye solution process is very capable both before and after the in-line mixer. As was expected, C_p values dropped from the pre-mixer to the post-mixer. The variability was lower for the pre-mixer, so the C_p value was higher, as expected. The process capability index for the mixed dye and water solution increased from the pre-mixer to the post-mixer flow cell, which was expected. The mixer lowered variability, resulting in a process more capable of producing a product according to specifications.

The average velocities for the stock and mixed solutions were 16.1 and 21.9 cm/s, respectively. From this, the Reynolds numbers for the stock and mixed solutions were found to be 1020 and 1390, respectively, meaning the flow was laminar in both cases. Since turbulent flow would create better mixing, the in-line mixer should raise the Reynolds number of the stream by introducing turbulence.

For the stock dye solution, the in-line mixer shows no benefit, since the C_p actually

Table 3.1 Process capability index value.

	C_p	
Stock Dye Solution	pre-mixer	10.7
	post-mixer	5.80
Dye+water solution	pre-mixer	0.94
	post-mixer	6.50

decreases because of the mixer. This makes sense, since there is nothing for the mixer to mix, as the stock solution should be relatively uniform already. However, for the dye and water mixture, the Cp value increases significantly, meaning the mixer is worthwhile.

4. Conclusions and recommendations

In conclusion, the in-line mixer provided benefits to the dye and water mixture, but not for the stock dye solution. For the stock solution, the post-mixer $\overline{R}$ actually increased relative to the pre-mixer value, from 3.25×10^{-4} to 5.98×10^{-4}. This would suggest that the mixer actually increases the variability. As a result, the stock dye post-mixer C_p value (5.80) was lower than the pre-mixer value (10.7). For the mixed solution, the post-mixer $\overline{R}$ was 5.34×10^{-4}, compared to the pre-mixer value of 3.68×10^{-3}. This indicates a decrease in variability of the concentrations of the solution. This decrease in variability leads to a higher post-mixer C_p value (6.50), compared to the pre-mixer (0.94).

After analyzing the control charts, the pre-mixer process was in control for both the stock and mixed solutions. The R and $\overline{X}$ charts showed virtually all of the points within control limits, and there were no patterns in the $\overline{X}$ charts to suggest nonrandom data. However, the post-mixer process was shown to be out of control. While the R charts showed the variability staying within limits, the $\overline{X}$ charts were out of control, and showed a steady increase in values, suggesting that the data is nonrandom (Lim and Cho, 2008). This steady increase in dye concentration was due to the peristaltic pump, which caused the fluid to travel in pulses. As the fluid flowed through the system and the mixer, it slowed down because of friction. The concentration of dye could then build up, resulting in the increase in concentration over time that was observed.

To improve the experiment in the future, the tank holding the twenty liters of dye solution could have a mixer in it. This would ensure that the dye concentration of the stock solution is more uniform heading into the pre-mixer flow cell. This would create a better standard to judge the water and dye mixture data against as well.

Appendix A: Raw data

Linear regression data for rotameter calibration

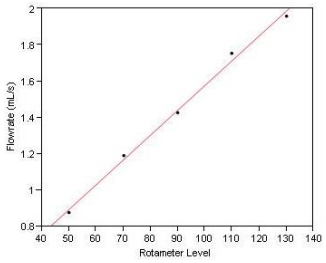

Figure A.1 Rotameter calibration curve

$$\text{Flowrate}(mL/s) = 0.2132906 + 0.0136555 \times \text{Rotameter Level}$$

Summary of Fit	
RSquare	0.995613
RSquare Adj	0.994151
Root Mean Square Error	0.033099
Mean of Response	1.442282
Observations (or Sum Wgts)	5

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	0.74588659	0.745887	680.8252
Error	3	0.00328669	0.001096	Prob > F
C. Total	4	0.74917328	0.0001*	

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> $\|t\|$
Intercept	0.2132906	0.049372	4.32	0.0229*
Rotameter Level	0.0136555	0.000523	26.09	0.0001*

Table A.1 Rotameter calibration measurements

Rotameter Level	Volume (mL)	Time (s)	Flowrate (mL/s)
50	100	114	0.877
70	100	84	1.190
90	100	70	1.429
11	100	57	1.754
130	100	51	1.961

Table A.2 Peristaltic pump flow measurements

Volume (mL)	Time (s)	Flowrate (ml/s)
200	40	5
200	39	5.128205128
200	39	5.128205128
	Average flowrate (mL/s)	5.085470085

Table A.3 Sample of bulk dye solution spectrophotometer data

Time (sec)	Post-mixer	Post-mixer concentration wt%	Pre-mixer	Pre-mixer concentration wt%	length pre-mixer	wt% (average l)
10.216	0.869	0.12994	0.864	0.12920	0.53810	0.13047
11.273	0.869	0.12994	0.862	0.12890	0.53934	0.13017
12.224	0.874	0.13069	0.862	0.12890	0.54245	0.13017
13.172	0.868	0.12979	0.865	0.12935	0.53686	0.13062
14.121	0.872	0.13039	0.863	0.12905	0.54058	0.13032
15.174	0.87	0.13009	0.864	0.12920	0.53872	0.13047
16.124	0.867	0.12964	0.863	0.12905	0.53748	0.13032
17.072	0.872	0.13039	0.863	0.12905	0.54058	0.13032
17.591	0.87	0.13009	0.862	0.12890	0.53997	0.13017
18.54	0.868	0.12979	0.862	0.12890	0.53872	0.13017
19.488	0.87	0.13009	0.863	0.12905	0.53934	0.13032
20.435	0.87	0.13009	0.863	0.12905	0.53934	0.13032
21.382	0.868	0.12979	0.863	0.12905	0.53810	0.13032
22.375	0.87	0.13009	0.864	0.12920	0.53872	0.13047
23.4	0.871	0.13024	0.861	0.12875	0.54121	0.13001
24.348	0.871	0.13024	0.864	0.12920	0.53933	0.13047
25.424	0.87	0.13009	0.862	0.12890	0.53997	0.13017
26.375	0.87	0.13009	0.862	0.12890	0.53997	0.13017
27.326	0.867	0.12964	0.863	0.12905	0.53748	0.13032
28.275	0.872	0.13039	0.862	0.12890	0.54121	0.13017
29.223	0.869	0.12994	0.862	0.12890	0.53934	0.13017
30.16	0.871	0.13024	0.863	0.12905	0.53996	0.13032
31.109	0.87	0.13009	0.862	0.12890	0.53997	0.13017
32.058	0.87	0.13009	0.862	0.12890	0.53997	0.13017
33.004	0.87	0.13009	0.861	0.12875	0.54059	0.13001
33.948	0.869	0.12994	0.864	0.12920	0.53810	0.13047
34.895	0.871	0.13024	0.864	0.12920	0.53933	0.13047
35.843	0.867	0.12964	0.863	0.12905	0.53748	0.13032
36.395	0.867	0.12964	0.862	0.12890	0.53810	0.13017
37.333	0.867	0.12964	0.863	0.12905	0.53748	0.13032
38.282	0.871	0.13024	0.863	0.12905	0.53996	0.13032
39.333	0.871	0.13024	0.863	0.12905	0.53996	0.13032
40.281	0.872	0.13039	0.865	0.12935	0.53933	0.13062
41.231	0.871	0.13024	0.862	0.12890	0.54059	0.13017
42.18	0.869	0.12994	0.863	0.12905	0.53872	0.13032
43.126	0.869	0.12994	0.862	0.12890	0.53934	0.13017
44.116	0.869	0.12994	0.863	0.12905	0.53872	0.13032
45.066	0.868	0.12979	0.863	0.12905	0.53810	0.13032
45.584	0.87	0.13009	0.862	0.12890	0.53997	0.13017
46.533	0.868	0.12979	0.862	0.12890	0.53872	0.13017

Table A.4 Sample of mixture dye solution spectrophotometer data

Time (sec)	Channel A	Channel C	pre-mixer concentration	post-mixer concentration
10.432	0.681	0.671	0.102833	0.100336
11.383	0.683	0.67	0.103135	0.100187
12.333	0.676	0.672	0.102078	0.100486
13.284	0.67	0.671	0.101172	0.100336
13.844	0.695	0.67	0.104947	0.100187
14.794	0.674	0.67	0.101776	0.100187

Table A.4 (Continued.)

Time (sec)	Channel A	Channel C	pre-mixer concentration	post-mixer concentration
15.742	0.661	0.67	0.099813	0.100187
16.689	0.686	0.671	0.103588	0.100336
17.636	0.691	0.671	0.104343	0.100336
18.585	0.673	0.671	0.101625	0.100336
19.533	0.674	0.67	0.101776	0.100187
20.483	0.68	0.671	0.102682	0.100336
21.433	0.681	0.672	0.102833	0.100486
22.383	0.676	0.67	0.102078	0.100187
23.334	0.677	0.67	0.102229	0.100187
24.28	0.677	0.671	0.102229	0.100336
25.228	0.653	0.674	0.098605	0.100785
26.178	0.667	0.67	0.100719	0.100187
27.129	0.686	0.672	0.103588	0.100486
28.081	0.69	0.67	0.104192	0.100187
29.076	0.676	0.67	0.102078	0.100187
30.025	0.683	0.67	0.103135	0.100187
30.545	0.67	0.669	0.101172	0.100037
31.482	0.678	0.67	0.10238	0.100187
32.431	0.695	0.671	0.104947	0.100336
33.376	0.678	0.67	0.10238	0.100187
34.323	0.67	0.67	0.101172	0.100187
35.271	0.692	0.67	0.104494	0.100187
36.219	0.672	0.672	0.101474	0.100486
37.211	0.679	0.671	0.102531	0.100336
38.164	0.652	0.671	0.098454	0.100336
39.114	0.68	0.673	0.102682	0.100636
40.064	0.696	0.671	0.105098	0.100336
41.071	0.697	0.672	0.105249	0.100486
42.065	0.695	0.671	0.104947	0.100336
43.082	0.689	0.668	0.104041	0.099888
43.601	0.673	0.669	0.101625	0.100037
44.598	0.668	0.672	0.10087	0.100486
45.549	0.701	0.671	0.105853	0.100336
46.498	0.685	0.672	0.103437	0.100486

Appendix B: Sample Calulations

Calculating peristaltic pump flow rate
On average, 39.33 s required to fill 200mL

$$\text{Flow rate} = V/t = 200/39.33 = 5.085 mL/s.$$

Calculating required rotameter setting for 0.11 wt% dye solution
Step 1: Material balance on dye:

$$w_{bulk} V_{pump} = w_{prod}(V_{pump} + V_{water}).$$

Then

$$V_{water} = \frac{V_{pump}(w_{bulk} - w_{prod})}{w_{prod}} = \frac{5.085(0.0015 - 0.0011)}{0.0011} = 1.849 mL/s.$$

Step 2: Plug into rotameter calibration equation to get required rotameter level:

$$\widehat{y} = ax + b,$$

therefore,

$$x = \frac{\widehat{y} - b}{a} = \frac{1.849 - 0.2133}{0.0137} = 119.8,$$

where

w_{bulk} =weight percent of bulk dye solution

w_{prod} =desired product weight percent

V_{pump} =volumetric flow rate of bulk dye solution(mL/s)

V_{water} =volumetric flow rate of water to be mixed with dye solution(mL/s).

Calculation of pre-mixer path length:

$$l = A/\epsilon/c_{post} = 0.86249/0.13024/12.5 = 0.52979 \approx 0.530cm,$$

where

l = length of pre-mixer path (cm)

c_{post} =average post-mixer wt%

A =absorbance = unitless

ϵ =extinction coefficient = 12.5 cm (calculated from Lab Manual "Calibration of Green Dye" Graph)

Calculation of upper and lower control limits for $\overline{X}$ and R charts
For the post-mixer dye + water mixture:
$\overline{X}$-bar = 0.100544wt%
R-bar = 0.000534wt%
$UCL_x = \mu + 3\sigma_x = \mu + A_2R - bar = 0.100544 + 0.58 \times 0.000534 = 0.10085wt\%$
$LCL_x = \mu + 3\sigma_x = \mu + A_2R - bar = 0.100544 - 0.58 * 0.000534 = 0.10023wt\%$
$UCL_R = D_4 \times R - bar = 2.11 \times 0.000534 = 0.00113wt\%$
$LCL_R = D_3 \times R - bar = 0 \times 0.000534 = 0.$
The values of A_2, D_3 and D_4 were obtained in the reference in the experimental handout (Willson, 2010).

Calculation of process capability:
For the pre-mixer, stock dye:

$$c_p = \frac{USL - LSL}{6s} = \frac{USL - LSL}{UCL - LCL} = \frac{0.004}{0.13042697 - 0.13005243} = 10.7,$$

where

USL = upper specification limit ($wt\%$) = $X \pm 0.002$

LSL = lower specification limit ($wt\%$) = $X \pm 0.002$

UCL = upper control limit ($wt\%$)

LCL = lower control limit ($wt\%$)

Calculation of percent difference of control limits of estimation vs. standard deviation method
Pre-mixer UCL-LCL for estimation method = $0.10444 - 0.10020 = 0.00424wt\%$
Pre-mixer UCL-LCL for standard deviation method = $0.10447 - 0.10017 = 0.00430wt\%$
Percent difference = $\frac{(0.00430-0.00424)}{0.00430} = .0140 = 1.40\%$ difference

Calculating the Reynolds number (water + dye solution pre-mixer)

Step 1: Calculate average velocity

$$v_{avg} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4 \times 6.934}{\pi \times 0.635^2} = \frac{21.894cm}{s}$$

Step 2: Calculate Reynold's number (assuming the properties of water)

$$Re = \frac{\rho \nu D}{\mu} = \frac{1 \times 21.894 \times 0.635}{0.01} = 1390,$$

Where

v_{avg} = average velocity (cm/s)

Q = volumetric flowrate (cm^3/s)

D = diameter of piping (cm)

ρ = density of water (g/cm^3)

μ = viscosity of water (poise).

Appendix C: Safety considerations

Safety goggles, long pants, and closed-toed shoes must be worn during the experiment. The spectrophotometer can get hot because it is a light source. The experimenters should avoid touching the spectrophotometer. The water and dye are safe substances. However, they do present a slipping hazard. Additionally, the experiment uses both electricity and water, so there is an electrical hazard.

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