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Approximation Method for QoS Analysis of Wireless Cellular Networks with Impatient Calls

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Abstract. Simple-closed expressions for approximate calculation of quality of service (QoS) metrics of isolated cell of wireless networks with either finite or infinite queues of both new and handover calls are developed. It is assumed that both kinds of calls might leave the system without receiving service if their waiting times exceed some threshold value. For the models with infinite queues of heterogeneous calls easily checkable ergodicity conditions are proposed. The high accuracy of the developed approximation formulas is shown. Results of numerical experiments are given.

Keywords: Cellular Wireless Networks, Impatient Calls, QoS Analysis, Approximation Method

1. INTRODUCTION

Apart from new (or original) call (o-calls) flows, additional classes of calls that require special approaches also exist in cellular wireless networks (CWNs). These calls are known as handover calls (h-calls), and they appear only on wireless cellular networks. This phenol-menon is characterized by moving mobile subscribers (MSs) with established network connections crossing boundaries between cells and being served by a new cell. From a new cell's point of view this is an h-call, and because the connection with the MS has already established, the transfer of the MS to new cell must be transparent for the user. In other words, a call may occupy channels from different cells several times throughout its duration, which means that channel occupation period is not the same as call duration.

It is known that h-calls re more susceptible to possible losses and delays than new calls. For this reason a number of different schemes for the prioritization of h-calls are suggested in various works, mostly employing guard channels for h-calls and/or creating a queue of h-calls in a base station (BS). Joint use of these schemes improves the QoS metrics for h-calls. Arranging a queue of h-calls can be achieved in networks where micro-cells are covered by a certain macro-cell, i.e., where there exists a certain zone (handover zone-h-zone), within which mobile users can be handled by any of the neighboring cells. The time when the user crosses the h-zone is called the degradation interval. When a user enters the h-zone, the availability of a free channel in a new cell is checked. If a free channel exists, then the h-call immediately occupies the channel and the h-procedure is successfully completed for the given stage; otherwise, the given h-call continues using the old channel of the previous cell while queuing for a channel of a new cell to be empty. If a free channel does not appear in the new cell before the end of the degradation interval, then a forced call interruption of

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the h-call occurs.

The mathematical basis of such networks can be found in (Akimaru and Kawashima, 1993; Yue and Matsumoto, 2002; Chen *et al*., 2004), and the recent results in the field were described in review (Das Bit and Mitra, 2003). Note that the primary topic of investigation is modeling single cells in CWNs with queues for h-calls (see (Hong and Rapoport, 1986; Yoon and Un, 1993; Kim *et al*., 2007) and references therein). The possibility of o-calls arising requires a queue (finite or infinite), whereas it is necessary to keep high the chances for h-calls to access the system via a reservation of channels. This scheme improves the total throughput of the cell. The models in which queues of o-calls are allowed are investigated in (Guerin, 1988; Kim *et al*., 2008) (see references therein, also).

Here, we consider models of isolated cells of CWNs with guard channels for h-calls and queues for both types of impatient calls (Chang *et al*., 1994). In (Chang *et al*., 1994), models with finite queues are considered only, and an approach based on Mason's formula for calculating QoS metrics is proposed. However, the proposed approach in (Chang *et al*., 1994) becomes inefficient even for a cell with a moderate size of buffers for heterogeneous calls. Therefore, this paper proposes an efficient and refined approximation method for calculating QoS metrics. Our approach is based on state space merging of two-dimensional Markov chains (2-D MC) (Melikov and Babaev, 2006). As a result, the desired QoS metrics can be obtained without any computational problems for the models in any dimensions (including infinite queues) because of the simple-closed expressions developed for their calculation.

This paper is organized as follows. In Section 2, we describe the model and develop simple, explicit formulae to calculate the QoS metrics of models with both finite and infinite buffers. Numerical results are given in Section 3. Finally, in Section 4, we provide some concluding remarks.

2. SYSTEM MODEL

We begin with a brief description of the model following (Chang *et al*., 1994). A cell contains $N>1$ radio channels, which are used by Poisson flows of new calls (with intensity λ_o) and handover calls (with intensity λ_h). When a user enters the h-zone, the availability of free channels in a new cell is checked. If a free channel exists, then the h-call immediately occupies the free channel, and the h-procedure is successfully completed for the given stage; otherwise, the given h-call continues using the old channel of the previous cell and either joins the queue for a free channel on a new cell or else is blocked due to buffer overflow (if the buffer size is finite). If the free channel does not appear in the new cell before the end of the degradation interval, then a forced call interruption of the h-call occurs. The system provides a buffer of size R_h for h-calls in the h-zone, and the degradation interval is assumed to be an exponential distribution with a mean of τ_h^{-1}.

Other types of calls are handled by a scheme of guard channels, i.e., an o-call is received only when the number of free channels is greater than g; otherwise, the o-calls will be either put in a buffer with size R_o or else blocked due to buffer overflow (if the buffer size is finite). At the moment of a channel becoming clear, the choice of a call from the queue is carried out as follows. If, at this moment, the number of free channels is greater than g, one o-call is selected from the queue (if there are any available) for service; otherwise, the released channel stands idle even if there is a queue of o-calls. A channel does not stand idle if there is a queue of h-calls. The queued o-call departs from the buffer unless it can be successfully served within a given amount of time, which has an exponential distribution with mean τ_o^{-1}.

Note that a channel's occupancy time considers both components of occupancy time: the time of calls duration and their mobility. Distribution functions of channel occupancy time of both types of calls are assumed to be exponential with the same mean μ^{-1}. If during call handling the h-procedure is initiated, the remaining handling time of this call in a new cell (as an h-call) is also exponentially distributed with the same mean due to the memoryless property of exponential distribution.

Our goal is to develop an approximation method for calculating the QoS metrics of the described models including the blocking (dropping) probability of heterogeneous calls and their average queue length, and their mean waiting time.

3. THE METHOD

First, consider a model with finite queues of heterogeneous calls. The state of the system at any time is described by a two-dimensional vector $\boldsymbol{k}=(k_1, k_2)$, where k_1 is the total number of busy channels and h-calls in the buffer, and k_2 is the number of o-calls in the buffer. Then, the state space of appropriate 2-D MCs is given by (for graph of the state space, see (Chang *et al*., 1994)):

$$S=\bigcup_{i=0}^{R_o} S_i\,, \tag{1}$$

where

$S_0 := \{\boldsymbol{k} : k_1 = 0, 1, \cdots, N+R_h;\ k_2 = 0\}$;

$S_i := \{\boldsymbol{k} : k_1 = N-g, N-g+1, \cdots, N+R_h; k_2 = i\},\ i \geq 1.$

Elements of the generating matrix of this MC, $\boldsymbol{k}$, $\boldsymbol{k}'\in S$, are determined from following relations:

$$q(\boldsymbol{k},\boldsymbol{k}')=\begin{cases}\lambda_o+\lambda_h, & \text{if } k_1\le N-g-1,\ k_2=0,\ \boldsymbol{k}'=\boldsymbol{k}+\boldsymbol{e}_1\\ \lambda_o, & \text{if } k_1\ge N-g,\ \boldsymbol{k}'=\boldsymbol{k}+\boldsymbol{e}_2\\ \lambda_h, & \text{if } k_1\ge N-g,\ \boldsymbol{k}'=\boldsymbol{k}+\boldsymbol{e}_1\\ f(k_1)\mu+(k_1-N)^+\tau_h, & \text{if } \boldsymbol{k}'=\boldsymbol{k}-\boldsymbol{e}_1\\ (N-g)\mu\delta(k_1,N-g)+k_2\tau_o, & \text{if } \boldsymbol{k}'=\boldsymbol{k}-\boldsymbol{e}_2\\ 0 & \text{in other cases}\end{cases} \quad (2)$$

where $\boldsymbol{e}_1=(1,0)$, $\boldsymbol{e}_2=(0,1)$, $f(x)=min(x,N)$, $x^+=\max(0,x)$ and $\delta(i,j)$ are Kronecker's symbols.

The average number of o-calls (L_o) and h-calls (L_h) in the queue, and the average number of busy channels (N_{av}), are determined following marginal distributions of the initial 2-D MC:

$$L_o=\sum_{k_2=1}^{R_o}k_2\sum_{k_1=N-g}^{N+R_h}p(k_1,k_2), \quad (3)$$

$$L_h=\sum_{k_1=N+1}^{N+R_h}(k_1-N)\sum_{k_2=0}^{R_o}p(k_1,k_2), \quad (4)$$

here $p(\boldsymbol{k})$-stationary probability of state $\boldsymbol{k}\in S$.

To calculate the blocking probability of a o-call (P_o), the following approach can be used. As mentioned above, o-calls may be blocked in the following cases: (i) at the moment the o-call arrives, the buffer is full; (ii) the waiting time of an o-call in buffer exceeds a given threshold τ_o^{-1}. Therefore, the desired QoS metric is defined as follows:

$$P_o=\sum_{k_1=N-g}^{N+R_h}p(k_1,R_o)+\frac{1}{\lambda_o}\sum_{k_2=1}^{R_o}k_2\tau_o\sum_{k_1=N-g}^{N+R_h}p(k_1,k_2) \quad (5)$$

In equation (5), the first term of the sum represents the probability of event (i), whereas the second term represents the probability of event (ii). Similarily, an h-call might be dropped in the following cases: (iii) at the moment of the h-call's arrival the buffer is full; (iv) the degradation interval ends before the h-call is admitted to a channel. Therefore, this metric is calculated as

$$P_h=\sum_{k_2=0}^{R_o}p(N+R_h,k_2)+\frac{1}{\lambda_h}\sum_{k_1=N+1}^{N+R_h}(k_1-N)\tau_h\sum_{k_2=0}^{R_o}p(k_1,k_2) \quad (6)$$

The equation (6) is the analogue to equation (5), but applied to h-calls.. Further, by using a modification of Little's formula, we can obtain the following formulas to calculate the waiting time of o-calls (W_o) and h-calls (W_h) in queue:

$$W_x=\frac{L_x}{\lambda_x(1-P_x)},\ x\in\{o,h\} \quad (7)$$

The stationary distribution $p(\boldsymbol{k})$, with $\boldsymbol{k}\in S$, is determined by the solution of an appropriate set of equilibrium equations (SEE) of the given 2-D MC (see [10]). For this SEE, no analytic solution for state probabilities can be found, and the application of the method proposed in [10] has a number of computational difficulties, even at moderate values for R_o and R_h. To overcome the mentioned difficulties, a new approach for calculating QoS metrics of the investigated model is suggested below. The proposed approach is acceptable for models of micro-cells in which the following condition is fulfilled: $\lambda_h >> \lambda_o >> \mu$. With the distance power of radio signals fade away (fading or attenuation of signal occurs) which makes possible to use same frequencies over several cells, but in order to avoid interference, this process must be carefully planned. For better use of frequency recourse, existing carrier frequencies are grouped, and number of cells, in which this group of frequencies is used, defines so called frequency reuse factor. Therefore, in densely populated areas with large number of mobile subscribers small dimensioned cells (micro-cells and pico-cells) are to be used, because of limitations of volumes and frequency reuse factor.

Note that this condition is not extraordinary because it is fulfilled by many systems [12]. Moreover, it is shown below that the final results are independent of λ_h, λ_o and μ and are determined from $\nu_x:=\lambda_x/\mu$, $x\in\{o,h\}$.

In presentation (1), the transition intensities within classes S_i are essentially higher than those between states of different classes. Further, the state classes S_i combine into separate merged states <i>, and the following merging function in state space (1) is introduced:

$$U(\boldsymbol{k})=<i> \quad \text{if } \boldsymbol{k}\in S_i,\ i=0,1,2,\ \cdots \quad (8)$$

Function (8) determines a merged model, which is also a 1-D MC with state space $\tilde{S}:=\{<i>:\ i=0,1,2,\cdots,R_o\}$.. Then, according to [11], the stationary distribution of the initial model approximately equals:

$$p(i,j)\approx\rho_j(i)\pi(<j>),\ (i,j)\in S_j, \quad (9)$$

where $\{\rho_j(i):(i,j)\in S_j\}$ and $\{\pi(<j>):<j>\in\tilde{S}\}$ are stationary distributions within class S_j and the merged model, respectively.

The stationary distribution within class S_j is determined as the appropriate state probabilities of the one dimensional birth-death process (1-D BDP) with rates (see (2))

$$\lambda_j=\begin{cases}\lambda_o+\lambda_h, & \text{if } j<N-g,\\ \lambda_h, & \text{if } j\ge N-g;\end{cases}$$

$$\mu_j=\begin{cases}f(j)\mu, & \text{if } j\le N,\\ N\mu+(j-N)\tau_h, & \text{if } j>N.\end{cases}$$

Therefore, we have

$$\rho_0(i)=\begin{cases}\dfrac{\nu^i}{i!}\cdot\rho_0(0), & 1\le i\le N-g,\\ \left(\dfrac{\nu}{\nu_h}\right)^{N-g}\cdot\dfrac{\nu_h^i}{i!}\cdot\rho_0(0), & N-g+1\le i\le N,\\ \dfrac{\nu^{N-g}}{N!}\cdot\nu_h^g\cdot\prod_{j=N+1}^{i}\dfrac{\lambda_h}{N\mu+(j-N)\tau_h}\cdot\rho_0(0), & N+1\le i\le N+R_h,\end{cases} \quad (10)$$

where $\nu := \nu_o + \nu_h,\ \rho_0(0) = \left(\begin{array}{l} \sum_{i=0}^{N-g} \frac{\nu^i}{i!} + \left(\frac{\nu}{\nu_h}\right)^{N-g} \cdot \\ \sum_{i=N-g+1}^{N} \frac{\nu_h^i}{i!} + \frac{\nu^{N-g}}{N!} \cdot \nu_h^g \cdot \\ \sum_{j=N+1}^{N+R_h} \prod_{i=N+1}^{j} \frac{\lambda_h}{N\mu + (i-N)\tau_h} \end{array} \right)^{-1}$

All split models with state space S_i, $i \geq 1$ represent the same 1-D BDP in which birth rates are constant and equal λ_h, whereas death rates are state-dependent, and for state j, $j = N\text{-}g, \cdots, N+R_h$ is defined as $f(j)\mu+(j\text{-}N)^+\tau_h$. Thus, the stationary distribution within class S_i, $i \geq 1$ is defined as follows (because all models have the same distributions below, the subscripts are omitted):

$$\rho(j) = \begin{cases} \frac{\nu_h^j}{j!} \cdot \frac{(N-g)!}{\nu_h^{N-g}} \cdot \rho(N-g), & N-g+1 \leq j \leq N, \\ \nu_h^g \cdot \frac{(N-g)!}{N!} \cdot \prod_{i=N+1}^{j} \frac{\lambda_h}{N\mu+(i-N)\tau_h} \cdot \rho(N-g), & N+1 \leq j \leq N+R_h, \end{cases} \tag{11}$$

where

$$\rho(N-g) = \left(1 + \nu_h^g \cdot (N-g)! \left(\sum_{i=N-g+1}^{N} \frac{\nu_h^{i-N}}{i!} + \frac{1}{N!} \cdot \sum_{j=N+1}^{N+R_h} \prod_{i=N+1}^{j} \frac{\lambda_h}{N\mu+(i-N)\tau_h} \right)\right)^{-1}$$

Then, from (2), (10) and (11), the elements of the generating matrix of a merged model are found:

$$q(<i'>, <i''>) = \begin{cases} \tilde{\lambda}_o, & \text{if } i'=0, i''=1, \\ \lambda_o, & \text{if } i'>0, i''=i'+1, \\ \left((N-g)\mu + i'\tau_o\right)\rho(N-g) + i'\tau_o\left(1-\rho(N-g)\right), & \text{if } i''=i'-1, \\ 0 & \text{in other cases,} \end{cases} \tag{12}$$

where. $\tilde{\lambda}_o := \lambda_o \left(1 - \sum_{i=0}^{N-g-1} \rho_0(i)\right)$

The latter formula allows determining stationary distribution of a merged model. It coincides with an appropriate distribution of state probabilities of 1-D BDP for which state transition rates are defined in accordance with (12). Consequently, stationary distribution of a merged model is determined as follows:

$$\pi(<j>) = \frac{\tilde{\lambda}_o \lambda_o^{j-1}}{\prod_{i=1}^{j} q(<i>, <i-1>)} \cdot \pi(<0>),\ j = 1, \cdots, R_o, \tag{13}$$

where $\pi(<0>) = \left(1 + \tilde{\lambda}_o \sum_{i=1}^{R_o} \frac{\lambda_o^{i-1}}{\prod_{j=1}^{i} q(<i>, <i-1>)}\right)^{-1}$

Finally, by using (10)-(13), we can obtain the stationary distribution of an initial model from (9). Thus, from (3), we conclude that the average number of o-calls in the buffer can be approximated as follows:

$$L_o \approx \sum_{i=1}^{R_o} i \sum_{j=N-g}^{N+R_h} \rho(j)\pi(<i>) = \sum_{i=1}^{R_o} i\pi(<i>) \sum_{j=N-g}^{N+R_h} \rho(j) = \sum_{i=1}^{R_o} i\pi(<i>) \tag{14}$$

The approximate value of an average number of h-calls in buffer is calculated as (see (4)):

$$\begin{aligned} L_h &\approx \sum_{i=1}^{R_h} i \sum_{j=0}^{R_o} \rho_j(N+i)\pi(<j>) \\ &= \sum_{i=1}^{R_h} i \left(\rho_0(N+i)\pi(<0>) + \sum_{j=1}^{R_0} \rho(N+i)\pi(<j>) \right) \\ &= \sum_{i=1}^{R_h} i \left(\rho_0(N+i)\pi(<0>) + \rho(N+i) \sum_{j=1}^{R_0} \pi(<j>) \right) \\ &= \sum_{i=1}^{R_h} i \left(\rho_0(N+i)\pi(<0>) + \rho(N+i)\left(1-\pi(<0>)\right) \right). \end{aligned} \tag{15}$$

The blocking probability of o-calls can be approximated as (see (5))

$$\begin{aligned} P_o &\approx \sum_{i=N-g}^{N+R_h} \rho_{R_0}(i)\pi(<R_o>) + \frac{\tau_o}{\lambda_o} \sum_{j=1}^{R_o} j \sum_{i=N-g}^{N+R_h} \rho(i)\pi(<j>) \\ &= \pi(<R_o>) + \frac{\tau_o}{\lambda_o} \sum_{j=1}^{R_o} j\pi(<j>). \end{aligned} \tag{16}$$

By the same method we have (see (6))

$$\begin{aligned} P_h &\approx \sum_{i=0}^{R_o} \rho_i(N+R_h)\pi(<i>) + \frac{\tau_h}{\lambda_h} \sum_{i=N+1}^{N+R_h} (i-N) \sum_{j=0}^{R_o} \rho_j(i)\pi(<j>) \\ &= \rho_0(N+R_h)\pi(<0>) \\ &\quad + \rho(N+R_h) \sum_{i=1}^{R_o} \pi(<i>) + \frac{\tau_h}{\lambda_h} \sum_{i=1}^{R_h} i \sum_{j=0}^{R_o} \rho_j(N+i)\pi(<j>) \\ &= \rho_0(N+R_h)\pi(<0>) + \rho(N+R_h)\left(1-\pi(<0>)\right) \\ &\quad + \frac{\tau_h}{\lambda_h} \sum_{i=1}^{R_h} i \left(\rho_0(N+i)\pi(<0>) + \rho(N+i) \sum_{j=1}^{R_o} \pi(<j>) \right) \\ &= \rho_0(N+R_h)\pi(<0>) + \rho(N+R_h)\left(1-\pi(<0>)\right) \\ &\quad + \frac{\tau_h}{\lambda_h} \sum_{i=1}^{R_h} i \left(\rho_0(N+i)\pi(<0>) + \rho(N+i)\left(1-\pi(<0>)\right) \right). \end{aligned} \tag{17}$$

Further, by using (14)-(17), we can approximate the average waiting time of heterogeneous calls (see (7)).

Now, consider some special cases.

1. The model with infinitely patient o-calls. In this model, we set $\tau_o = 0$. For this model, the stationary distribution of splitting models is again calculated by relations (10) and (11). However, the stationary distribution of a merged model in this case is determined by the following simple formulas:

$$\pi(<i>) = \frac{\tilde{\lambda}_o}{\tilde{\mu}} \tilde{\nu}_o^{\,i-1} \pi(<0>),\ i = 1, \cdots, R_o, \tag{18}$$

where

$$\tilde{\mu} := (N-g)\mu\rho(N-g),\ \tilde{\nu}_o := \frac{\tilde{\lambda}_o}{\tilde{\mu}},\ \pi(<0>) = \left(1 + \frac{\tilde{\lambda}_o}{\tilde{\mu}} \cdot \frac{1-\tilde{\nu}_o^{R_o}}{1-\tilde{\nu}_o}\right)^{-1}$$

From (16), we conclude that for the given model, $P_o \approx \pi(<R_o>)$. Other QoS metrics are calculated from known formulas.

2. The model with an infinite degradation interval. In this model, we set $\tau_h=0$. In this model, the stationary distributions of splitting models are calculated as

$$\rho_0(i) = \begin{cases} \dfrac{\nu^i}{i!} \cdot \rho_0(0), & 1 \le i \le N-g, \\ \left(\dfrac{\nu}{\nu_h}\right)^{N-g} \cdot \dfrac{\nu_h^i}{i!} \cdot \rho_0(0), & N-g+1 \le i \le N, \\ \dfrac{\nu^{N-g}}{N!} \cdot \nu_h^g \cdot \tilde{\nu}_h^{i-N} \cdot \rho_0(0), & N+1 \le i \le N+R_h, \end{cases}$$

where

$$\rho_0(0) = \left(\sum_{i=0}^{N-g} \frac{\nu^i}{i!} + \left(\frac{\nu}{\nu_h}\right)^{N-g} \cdot \sum_{i=N-g+1}^{N} \frac{\nu_h^i}{i!} + \frac{\nu^{N-g}}{N!} \cdot \frac{\nu_h^{g+1}}{N} \cdot \frac{1-\tilde{\nu}_h^{R_h}}{1-\tilde{\nu}_h} \right)^{-1} \quad (19)$$

and

$$\rho(j) = \begin{cases} \dfrac{\nu_h^{\,j}}{j!} \cdot \dfrac{(N-g)!}{\nu_h^{N-g}} \rho(N-g), & j = \overline{N-g+1, N}, \\ \tilde{\nu}_h^{\,j-N} \cdot \nu_h^g \cdot \dfrac{(N-g)!}{N!} \rho(N-g), & N+1 \le j \le N+R_h, \end{cases} \quad (20)$$

where

$$\rho(N-g) = \left(1 + \nu_h^g (N-g)! \left(\sum_{i=N-g+1}^{N} \frac{\nu_h^{i-N}}{i!} + \frac{\tilde{\nu}_h}{N!} \cdot \frac{1-\tilde{\nu}_h^{R_h}}{1-\tilde{\nu}_h} \right)\right)^{-1}$$

The stationary distribution of a merged model in this case is determined in accordance with (13). It must be taken into account that the stationary distribution of splitting models are again calculated by relations (19) and (20). In this model, we have

$$P_h \approx \rho_0(N+R_h)\pi(<0>) + \rho(N+R_h)(1-\pi(<0>)) \quad (21)$$

3. The model with patient o-calls and an infinite degradation interval is considered in (Akimaru and Kawashima, 1993, 89-93). This model is the combination of the two previous models, i.e., in this case, we set $\tau_o = \tau_h = 0$. For this model, we have the following simple formulas to calculate QoS metrics:

$$L_o \approx \pi(<0>) \cdot \frac{\tilde{\lambda}_o}{\lambda_o} \cdot \sum_{i=1}^{R_o} i\tilde{\nu}_o^i, \quad (22)$$

$$L_h \approx \left(a\pi(<0>) + b(1-\pi(<0>))\right) \sum_{i=1}^{R_h} i\tilde{\nu}_h^i, \quad (23)$$

$$P_o \approx \pi(<R_o>), \quad (24)$$

$$P_h \approx \left(a\pi(<0>) + b(1-\pi(<0>))\right) \tilde{\nu}_h^{R_h} \quad (25)$$

where

$$a := \frac{\nu^{N-g}}{N!} \cdot \nu_h^g \cdot \rho_0(0),\ b := \frac{(N-g)!}{N!} \cdot \nu_h^g \cdot \rho(N-g) \quad (26)$$

Now consider the model with infinite queues. For the simplicity of intermediate expressions, consider the model with patient o-calls and an infinite degradation interval. Its state space is defined as follows

$$S = \bigcup_{i=0}^{\infty} S_i, \quad (27)$$

where

$$S_0 := \{\boldsymbol{k} : k_1 = 0, 1, \cdots;\ k_2 = 0\};$$
$$S_i := \{\boldsymbol{k} : k_1 = N-g, N-g+1, \cdots;\ k_2 = i\},\ i \ge 1.$$

Elements of the generating matrix of the appropriate 2-D MC are determined similarly to (2). The desired QoS metrics (3) and (4) can also be obtained from the indicated formulas in which the upper bounds of sums should be set to infinity (in this model, lost heterogeneous calls are impossible).

We will not repeat the description of the procedure from above but we will note that the scheme used to split the state space (27) is similar to (1). Then, the stationary distribution of the splitting model with state space S_0 is calculated as follows:

$$\rho_0(i) = \begin{cases} \dfrac{\nu^i}{i!} \cdot \rho_0(0), & 1 \le i \le N-g, \\ \left(\dfrac{\nu}{\nu_h}\right)^{N-g} \cdot \dfrac{\nu_h^i}{i!} \cdot \rho_0(0), & N-g+1 \le i \le N, \\ \dfrac{\nu^{N-g}}{N!} \cdot \nu_h^g \cdot \tilde{\nu}_h^{i-N} \cdot \rho_0(0), & i \ge N+1, \end{cases} \quad (28)$$

where

$$\rho_0(0) = \left(\sum_{i=0}^{N-g} \frac{\nu^i}{i!} + \left(\frac{\nu}{\nu_h}\right)^{N-g} \cdot \sum_{i=N-g+1}^{N} \frac{\nu_h^i}{i!} + \frac{\nu^{N-g}}{N!} \cdot \frac{\nu_h^{g+1}}{N} \cdot \frac{1}{1-\tilde{\nu}_h} \right)^{-1}$$

Here, we obtain the first condition for ergodicity of the given model:

$$\nu_h < N. \quad (29)$$

The stationary distributions of splitting models with state space S_j, $j \ge 1$ are calculated as follows

$$\rho(j) = \begin{cases} \dfrac{\nu_h^{\,j}}{j!} \cdot \dfrac{(N-g)!}{\nu_h^{N-g}} \rho(N-g), & j = \overline{N-g+1, N}, \\ \tilde{\nu}_h^{\,j-N} \cdot \nu_h^g \cdot \dfrac{(N-g)!}{N!} \rho(N-g), & j \ge N+1, \end{cases} \quad (30)$$

where

$$\rho(N-g)=\left(1+\nu_h^g(N-g)!\left(\sum_{i=N-g+1}^{N}\frac{\nu_h^{i-N}}{i!}+\frac{1}{N!}\cdot\frac{\tilde{\nu}_h}{1-\tilde{\nu}_h}\right)\right)^{-1}$$

For this model, elements of the generating matrix of the merged model are determined as follows:

$$q(<j'>,<j''>)=\begin{cases}\tilde{\lambda}_o, & \text{if } j'=0, j''=j'+1,\\ \lambda_o, & \text{if } j'>0, j''=j'+1,\\ \tilde{\mu}, & \text{if } j''=j'-1,\\ 0 & \text{in other cases.}\end{cases} \quad (31)$$

Thus, the stationary distribution of a merged model is determined by the following simple formulas:

$$\pi(<j>)=\frac{\tilde{\lambda}_o}{\tilde{\mu}}\cdot\tilde{\nu}_o^{\,j-1}\pi(<0>),\ j\ge 1, \quad (32)$$

where $\pi(<0>)=\left(1+\frac{\tilde{\lambda}_o}{\tilde{\mu}}\cdot\frac{1}{1-\tilde{\nu}_o}\right)^{-1}$

Here, we obtain second condition for ergodicity of the given model:

$$\nu_o<(N-g)\rho(N-g) \quad (33)$$

Note. Condition (33) has a simple probabilistic interpretation. So, because o-calls from the buffer are selected for service only when the number of busy channels is equal N-g, the total service intensity is equal to $\mu(N-g)\rho(N-g)$. Here, quantity $\rho(N-g)$ represents the probability that the number of busy channels is equal to N-g, and the buffer for o-calls is not empty. Therefore, the existence of the stationary mode requires that the arriving intensity of o-calls should be less than total service intensity, i.e., we obtain condition (33).

When ergodicity conditions (29) and (33) are fulfilled, we obtain the following closed-form expressions for calculating the QoS metrics of the model with infinite queues:

$$L_o\approx\frac{1}{(1-\tilde{\nu}_o)^2}\cdot\frac{\tilde{\lambda}_o}{\tilde{\mu}}\cdot\pi(<0>); \quad (34)$$

$$L_h\approx\frac{\tilde{\nu}_h}{(1-\tilde{\nu}_h)^2}\cdot\left(a\pi(<0>)+b(1-\pi(<0>))\right) \quad (35)$$

In this model, the approximate value of the mean waiting time in a queue is determined by the classical Little's formula, i.e., in (7), we set $P_o = P_h = 0$.

4. NUMERICAL RESULTS

The developed closed-form expressions for calculating the required QoS metrics permit easy analysis for any sizes of buffer stores for heterogeneous calls. To realize the algorithms mentioned above, we developed appropriate software in Delphi 7 and performed a large volume of computational experiments using a broad range of structural and load parameters for the cell. In the interest of brevity, only the results for the model with finite queues are shown in Figure 1~Figure 3. The behavior of the studied curves fully confirms all theoretical expectations.

In Figure 1, we plot the loss probabilities versus the number of guard channels for the model with parameters $N = 10$, $\lambda_o = 0.2$, $\lambda_h = 2.6$, $\mu = 5$, $\tau_o = 0.1$ and $\tau_h = 0.2$. As expected, the blocking probability for h-calls decreases when the number of guard channels increases Figure 1(a). On the other hand, the blocking probability of o-calls increases when the number of guard channels increases Figure 1(b). These facts are explained as follows: as the number of reserve channels increases, the chances of o-calls to access a channel are decreased; in contrast, as the number of reserve channels increases, the chances of h-calls to access a channel are increased. Morever, an increase in the size of buffers for calls of each type leads to a decrease in the respective blocking probability for each type of call. Note that an increase in the rate of each type of call leads to an increase in the blocking probability of calls of the appropriate type.

The average lenght of o-calls increases when the

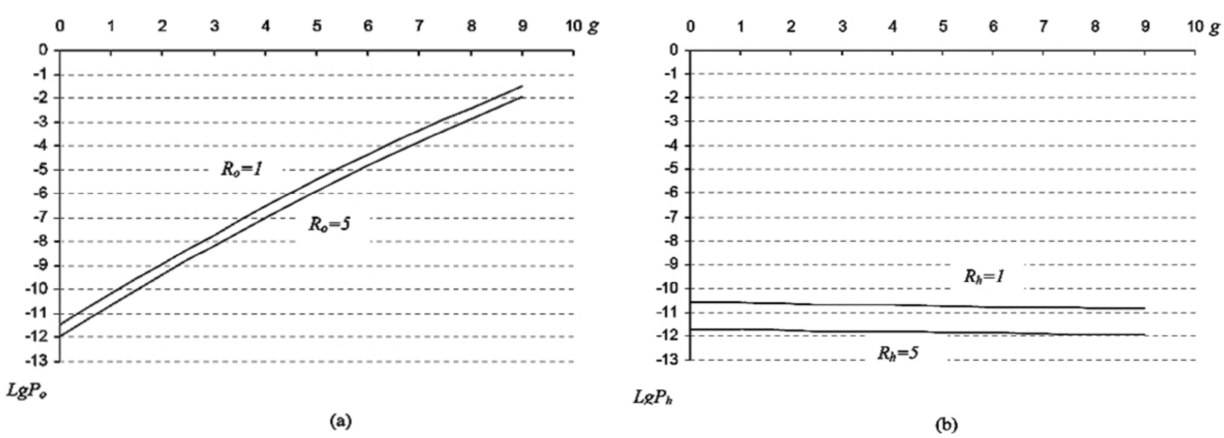

Figure 1. Blocking probabilities for o-calls and h-calls versus the number of guard channels at R_h=1 (a) and at R_o=5 (b).

number of guard channels increases Figure 2(a), whereas the analogous QoS metrics for h-calls decreases Figure 2(b). However, both functions are increasing with respect to the size of respective buffers. A similar form is seen for the waiting times of heterogeneous calls Figure 3.

Another direction of investigation attempts to define the accuracy of the suggested formulae for calculating the approximate values (AV) of examined QoS metrics. Their exact values (EV) are determined by using appropriate balance equations (as mentioned above, this last approach

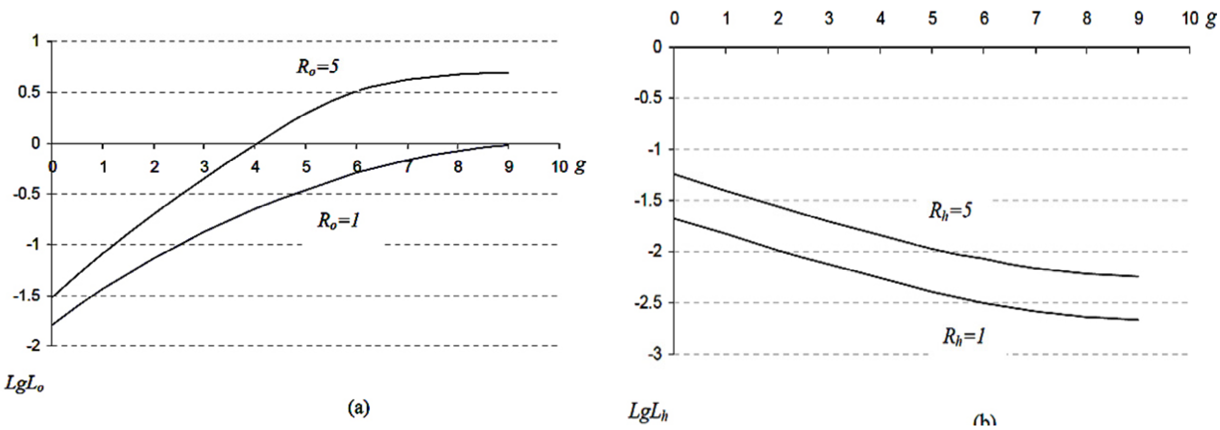

Figure 2. Average number of o-calls (a) at $R_h=1$ and h-calls (b) versus number of guard channels at $R_o=1$.

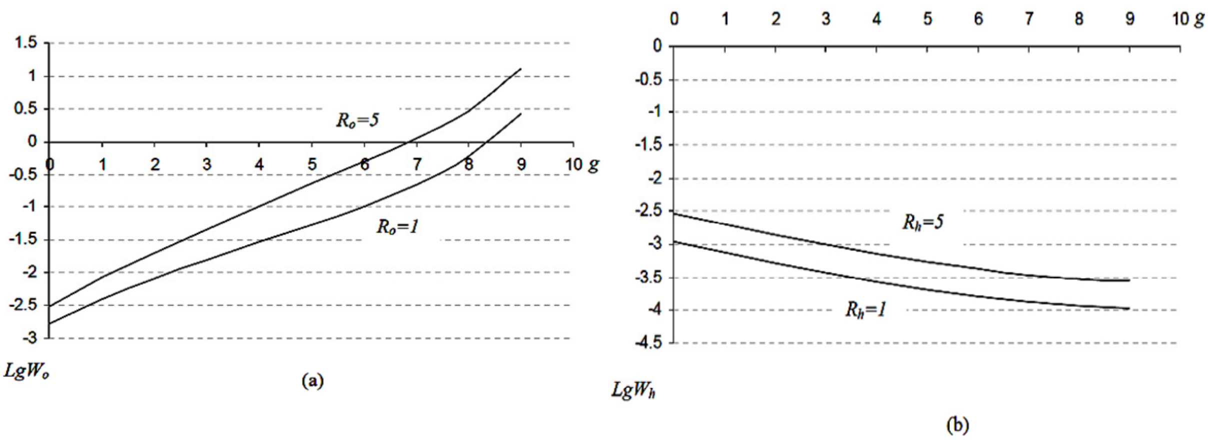

Figure 3. Mean waiting time of o-calls (a) at $R_h=1$ and h-calls (b) versus the number of guard channels at $R_o=1$.

Table 1. Comparison of QoS for o-calls.

g	P_o		L_o		W_o	
	EV	AV	EV	AV	EV	AV
0	3.16236E-12	2.43785E-12	1.62726E-02	1.62730E-02	1.65440E-03	1.65453E-03
1	6.30767E-11	7.86103E-11	3.73811E-02	3.73832E-02	3.88478E-03	3.88485E-03
2	1.15364E-09	2.44697E-09	7.42411E-02	7.42422E-02	8.02593E-03	8.02599E-03
3	1.91132E-08	5.78531E-08	1.34232E-01	1.34237E-01	1.55284E-02	1.55296E-02
4	2.84055E-07	3.14851E-07	2.24514E-01	2.24518E-01	2.90355E-02	2.90361E-02
5	3.74555E-06	3.77654E-06	3.49392E-01	3.49395E-01	5.39924E-02	5.39931E-02
6	4.33084E-05	4.35672E-05	5.06623E-01	5.06628E-01	1.03750E-01	1.03757E-01
7	4.34859E-04	4.35532E-04	6.82404E-01	6.82407E-01	2.19583E-01	2.19589E-01
8	3.80973E-03	3.80652E-03	8.45889E-01	8.45890E-01	5.80761E-01	5.80763E-01
9	3.17847E-02	3.17850E-02	9.54261E-01	9.54271E-01	2.63631E+00	2.63636E+00

Table 2. Comparison of QoS for h-calls.

g	P_h		L_h		W_h	
	EV	AV	EV	AV	EV	AV
0	2.67415E-11	4.04561E-11	2.12586E-02	2.12673E-02	1.08633E-03	1.08641E-03
1	2.48432E-11	3.77538E-11	1.48829E-02	1.48832E-02	7.55501E-04	7.55512E-04
2	2.30828E-11	3.72416E-11	1.04747E-02	1.04768E-02	5.29343E-04	5.29367E-04
3	2.14511E-11	3.68643E-11	7.47977E-03	7.47985E-03	3.76848E-04	3.76853E-04
4	1.99403E-11	2.99853E-11	5.45992E-03	5.45979E-03	2.74515E-04	2.74528E-04
5	1.85443E-11	2.92375E-11	4.10648E-03	4.10661E-03	2.06187E-04	2.0618E-04
6	1.72599E-11	2.83243E-11	3.21075E-03	3.21112E-03	1.61061E-04	1.61059E-04
7	1.60904E-11	2.73051E-11	2.63707E-03	2.63853E-03	1.32213E-04	1.32235E-04
8	1.50534E-11	2.61631E-11	2.30184E-03	2.30195E-03	1.15367E-04	1.15372E-04
9	1.41935E-11	2.58932E-11	2.14829E-03	2.14852E-03	1.07654E-04	1.07663E-04

is useful only for small size models). It is important to note the high accuracy of the suggested formulae under the above assumptions $\lambda_h >> \lambda_o >> \mu$, i.e., in these cases, the EV and AV almost completely coincide for all QoS metrics. Therefore, these comparisons are not presented here. It is obvious that the EV of QoS metrics are efficiently determined from the solution of SEE only for models with state space with a small number of dimensions (1). It is important to note that high accuracy in computing QoS metrics is observed even for loads that do not satisfy the above assumptions regarding the relationships between the rates of heterogeneous traffic. Thus, appropriate results for the initial parameters $N = 10$, $\lambda_o = 0.2$, $\lambda_h = 2.6$, $\mu = 5$, $\tau_o = 0.1$ and $\tau_h = 0.2$ are given in Table 1 and Table 2, where we assumed that $R_o = R_h = 1$ (note that $\lambda_o < \mu$ contradicts the assumption made above).

5. CONCLUSION

In this paper, we develop a simple-closed expression for the approximation of the QoS metrics for isolated cells of wireless networks with either finite or infinite queues of both new and handover calls. It is assumed that the degradation interval of h-calls is a random variable with finite mean and that o-calls in queue are impatient. The developed approach allows for the design of these networks without any computational difficulties. Also note that the approach is helpful for solving the problems related to finding both the optimal sizes of buffers for heterogeneous calls and the optimal number of guard channels to satisfy constraints on the QoS metrics. These kinds of problems are subject to further works.

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