Performance Analysis of Selection Relaying Schemes over Different Fading Environments in Wireless Networks

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Abstract

In this paper, two selection relaying schemes are proposed and compared with each other in terms of packet error rate (PER) and diversity order. Moreover, the comparison is performed over two different fading environments including fast and slow fading environments in which the diversity order is considered as the important factor to reach the decision which scheme is suitable for each fading environment. Numerical results validate the analytical results and indicate significant differences on the PER performance of two schemes over two fading environments.

Keywords: ARQ protocol, diversity order, selection relaying scheme, slow fading, fast fading, packet error rate

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1. Introduction

Nowadays, cooperative diversity has received a great deal of attention as an efficient solution to combat the detrimental effects of channels such as shadowing and deep fading by allowing single-antenna mobiles to gain several benefits of transmit diversity. Additionally, an alternative way to mitigate channel fading is to rely on the automatic repeat request (ARQ) protocol at the data link layer that requests retransmissions whenever the destination incorrectly receives the signal. As expected, combinations of the concept of cooperative diversity and the ARQ protocol to become the cooperative ARQ protocol can achieve higher performances on throughput, packet error rate than the traditional ARQ protocol [1][2][3][4][5][6][7]. In a simple network including a source, a relay, and a destination, the relay will retransmit the erroneous signal instead of the source when it correctly receives the signal [4][6][7]. So, what will happen if there is more than a relay between the source and the destination? In [1], the authors analyzed performances of the ARQ cooperative diversity in multi-hop wireless networks where all relay nodes that successfully decode their received signal jointly retransmit the erroneous signal. However, transmissions from all relay nodes as [1] consume more bandwidth and power than that from the best relay node [3][4][8]. In [3], the authors gave comparisons among five selection relaying schemes consisting of direct link, random relaying, instantaneous relaying, multi-hop, and Harbinger in which the Harbinger is the scheme requiring a retransmission from the closest relay whenever the destination cannot correctly receive the signal. From the simulation results, the authors proved that the Harbinger scheme can achieve the best performances on the throughput and the average delay among schemes. However, in [3] only simulation results were obtained.

By taking advantage of transmitting signals from the relay being the closest to the destination as the Harbinger scheme, in this paper, we propose two different selection relaying schemes where the relay having the closest distance to the destination is required to retransmit the erroneous signal. Differences from two proposed schemes with the Harbinger scheme are shown in following ways. First, the proposals do not transmit forward error correction (FEC) bits as the Harbinger to save power and bandwidth. Second, in the Harbinger scheme, the relays and the destination keep information about the message and do not flush away this information until the destination correctly receives the message. In contrast, the relays as well as the destination in two proposed schemes flush their memory whenever they receive the signal incorrectly or an acknowledge message from another relay node. Thus, the memory of nodes in the network can be saved. Those characteristics are similar to GeRaF [8][9]. However, in two proposals the analyses are performed in terms of packet error rate instead of the average number of hops to reach the destination as the GeRaF. Hence, it is more practical to estimate advantages of the schemes. Especially, the most difference from the GeRaF scheme, the Harbinger as well as that between two schemes is the different performances of the system when the closest relay may not correctly retransmit the signal to the destination. Moreover, the paper also focuses on analyzing the effect of different fading environments consisting of fast and slow fading environments to the performance of schemes. To the best of our knowledge, no paper has investigated this problem. As the result of the analysis, we suggest a suitable scheme in each fading environment where the diversity order is considered as the factor to reach the suggestion.

The goals of the paper include three contributions. First, we derive the closed form expressions for the PER performance of two schemes. Second, we provide a view on the performance of two schemes over different fading environments. Third, based on analyzing diversity order, we find the suitable scheme for each fading environment.

The remainder of this paper is organized as follows. In Section 2, we present a network model, describe the operation of schemes as well as the model for the data link layer. The packet error rate and diversity order are analyzed clearly in Section 3. Simulation results and discussion are given in Section 4. Finally, the paper is concluded in Section 5.

2. System Model

2.1 Protocol Description

We consider a network system consisting of a source (R_0), N relays and a destination (R_{N+1}) in which relays are numbered according to their distance to the destination, with R_1 being the furthest and R_N being the nearest as **Fig. 1** [3][8][9]. For simplicity, we reuse the list of assumptions about the flat channel, perfect channel state information at the receivers, and perfect error detection based CRC as [3].

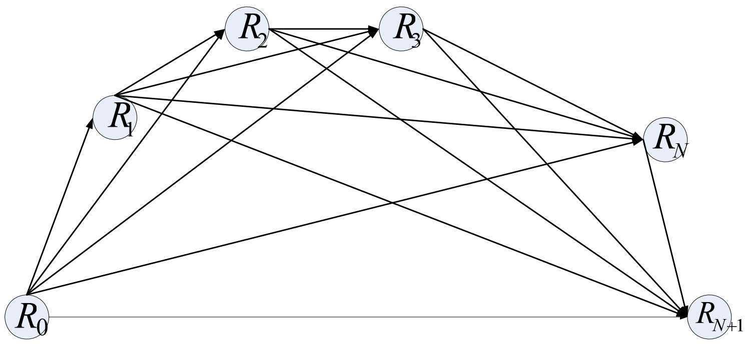

Fig. 1. System model for the proposals.

Each node in the network has a single half duplex radio and a single antenna. Whenever a node transmits, other nodes may receive the signal. Especially, a time division duplexing (TDD) mode is used in which during a time slot, a node may transmit or receive the signal, but not both. As [3][9], we assume that each node in the network knows its own position as well as that of the destination.

Two proposed schemes also base on the geographic information to choose the best relay and utilize advantages of cooperative ARQ schemes. Therefore, we call them as GeRaARQ I scheme and GeRaARQ II scheme. For simplicity, two schemes are referred as scheme I and scheme II. In both schemes, firstly, the source broadcasts a packet to the destination and all relays. After that, if the destination correctly receives the packet, it sends an acknowledge message (ACK) to inform other nodes that the packet is successfully received. Otherwise, the destination drops the erroneous packet and sends a non-acknowledge message (NACK) to invoke a retransmission. In here, we assume that the feedback channels from the destination to other nodes are errorless. Therefore, all relays and the source successfully receive the messages from the destination. In order to avoid collisions by retransmitting the packet from more than a node at the same time, we design a channel access as well as the way to choose the best relay for each scheme as follows.

Additionally, we use the finite retransmission model [10] to decide when the packet is dropped. It means that after the destination incorrectly receives the packet broadcasted from the source, retransmission from any node such as the relays or the source is limited to the maximum number of retransmissions m ($0 \le m < \infty$). The parameter is counted as: after receiving the erroneous packet from the source, whenever the destination receives the packet, it increases the number of retransmissions by one. After the *m*th erroneous packet is received, the destination sends a cancel message to all nodes to terminate transmitting this packet and to start a new one. Therefore, the retransmission terminates if the destination receives the packet correctly or the number of retransmissions reaches to m retransmissions.

In the scheme I, as [11], to reduce overall overhead by transmission of signaling among nodes, a method based on time is used to decide which node will retransmit the erroneous packet. After receiving the NACK message from the destination, each node R_i ($0 \le i \le N$) starts its delay time which is determined based on the distance from the node to the destination as the following equation:

$$T_i = \lambda d_{i,N+1} \tag{1}$$

where T_i is the delay time at the node R_i, λ has the units of time, and $d_{i,N+1}$ is the distance from the node R_i to the destination. From the equation, we can easily see that the closest relay R_N has the shortest time delay. When the delay time of the relay node reduces to zero (i.e., to expire), the relay will send the ACK message to inform other nodes and occupy the time slot to retransmit the packet if it satisfies two conditions as: its received packet is fully decoded; and it does not receive any ACK message from other nodes. Thus, when a node satisfies two conditions and sends the ACK message, all other nodes, while waiting in their delay time to reduce to zero, will receive the message. After that, they keep silent and drop their received packet. In the other hand, after the delay time expires, if the node may not satisfy the two above conditions, it drops its received packet and keeps silent.

As assumed in Section 2, whenever a node transmits, all nodes in the network may receive the signal. Thus, when the node transmits the signal, it is called as the broadcaster. In the scheme II, the determined delay time at each relay depends on the broadcaster. If the broadcaster is the source or the relay R_1, the delay time at each node is determined similarly as the scheme I in the equation (1). In the other hand, if the broadcaster R_i ($1 < i \le N$) may not correctly retransmit the packet to the destination, the delay time at each node R_j ($0 \le j \le N$) is defined as the following equations:

$$T_j = \begin{cases} \lambda d_{i,j} & if \left[\left(d_{j,N+1} > d_{i,N+1} \right) \cap \left(j \ne 0 \right) \right] \\ \lambda \left(d_{0,i} + d_{j,N+1} \right) & if \left(d_{j,N+1} \le d_{i,N+1} \right) \end{cases} \tag{2}$$

where $d_{i,j}$, $d_{i,N+1}$, and $d_{0,j}$ are the distances from the node R_i to the node R_j, from the node R_i to the destination and from the source to the node R_j, respectively. From (2), the shortest delay time is determined at the relay node (except the source) that has the shortest distance to the broadcaster R_i and the furthest distance to the destination (R_{N+1}). In this scheme, the source will not retransmit the packet after it receives the ACK message for the packet from any nodes in the network. A node only performs the retransmission after the delay time reduces to zero when it satisfies the two conditions defined in the scheme I. On the operation of the scheme II, in order to decide the delay time, each node in the network has to know the distance from the broadcaster to the source as well as that from it to the broadcaster. Thus, when the node satisfies two defined conditions and sends the ACK message to inform other nodes, the information about the its distance to the source and its position is included in the ACK message. The other nodes base on this information and their distance to the destination to decide the delay time as the equation II. In here, we assume that the included information in each ACK message is so small that it does not cause any effect to the performance of the schemes. However, by comparison with operation of the scheme I, the scheme II requires more complex performance than scheme I.

In two proposed schemes, all relay nodes only send the ACK message when they correctly receive the packet instead of both the ACK message and the NACK message as [1][3][4]. Thus, high bandwidth efficiency can be achieved.

2.2 Signal and Channel Model

Assume that the channels between two nodes are subject to Rayleigh fading including fast fading and slow fading chanels. In fast fading, the channel gain remains unchanged during a packet period, but able to change independently from the packet to packet while it is unchanged in a number of packet periods in slow fading.

It is also assumed that each node has a signal half duplex radio and a single antenna. The baseband equivalent symbol received at the node R_j from the transmitter R_i for a symbol n is given as:

$$y_{i,j}(n) = h_{i,j}\sqrt{P_p}\,x(n) + n_{i,j}(n) \tag{3}$$

where $y_{i,j}$ is the received symbol at the node R_j. $n_{i,j}$ is the noise at the node R_j and modeled as mutually independent complex Gausisan random variable with zero mean and unit variance N_1. $h_{i,j}$ captures the effect of the fading from the node R_i to the node R_j and is assumed to be i.i.d zero mean complex Gaussian r.v's with variance $V_{i,j}$. The $V_{i,j}$ expresses the packet energy decay which are modeled as $V_{i,j} = \left(d_0 / d_{i,j}\right)^{\mu}$ where d_0 is the reference distance, $d_{i,j}$ is the distance between the node R_i and the node R_j, and μ is the path loss coefficient with the values typically in the range $1 < \mu < 4$. In fast fading, $h_{i,j}$ remains unchanged in a packet but changed in the next packet. On the contrary, in slow fading, $h_{i,j}$ is unchanged in all packet periods. P_p is the transmitted power. $P_p = P_S$ if the transmitter is the source and $P_p = P_R$ if the transmitter is the relays.

We suppose the system in Rayleigh fading channels so the instantaneous SNR for the channel between the transmitter R_i and the receiver R_j is written as:

$$f_{\gamma_{i,j}}(\gamma) = \frac{1}{\sigma_{i,j}}\exp\left(-\frac{\gamma}{\sigma_{i,j}}\right) \tag{4}$$

where $\sigma_{i,j}$ is the expected value of the instantaneous received SNR of $\gamma_{i,j}$. In order to take path loss into account, we model the variance of channel coefficient between the transmitter R_i and the receiver R_j as the function of distance as:

$$\sigma_{i,j} = \frac{P_p}{N_1} d_{i,j}^{-\mu} = SNR \times d_{i,j}^{-\mu} \tag{5}$$

where μ is the path loss coefficient. The distance from the transmitter R_i and the receiver R_j is presented by $d_{i,j}$.

Following [4][6], we also can calculate the approximation physical packet error rate if FEC is not used as follows:

$$PER(\gamma) = \begin{cases} 1 & \text{if } \quad 0<\gamma<\gamma_t \\ \alpha \exp(-g\gamma) & \text{if } \gamma \geq \gamma_t \end{cases} \tag{6}$$

where (α, g, γ_t) parameters are found by least squares fitting method. The switching threshold γ_t is set such that:

$$\alpha \exp(-g\gamma_t) = 1 \tag{7}$$

Because of varying the channel gain from a transmission to the next one over the fast fading, the packet error rate of the channel R_i - R_j with z erroneous transmissions and y correct transmissions is calculated as:

$$\begin{aligned} P_{i,j}^{F,z,y} &= \underbrace{\int_0^\infty .. \int_0^\infty}_{(y+z)\ \text{integrals}} \begin{Bmatrix} \left[PER(\gamma_{y+1})...PER(\gamma_{y+z})\right]\left[1-PER(\gamma_1)\right]..\left[1-PER(\gamma_y)\right] \\ f_{\gamma_{i,j}}(\gamma_1) f_{\gamma_{i,j}}(\gamma_2)... f_{\gamma_{i,j}}(\gamma_{y+z}) d\gamma_1 d\gamma_2 ... d\gamma_{z+y} \end{Bmatrix} \\ &= \sum_{p=0}^{y} \binom{y}{p} (-1)^p \left[1 - \frac{\sigma_{i,j} g}{1+\sigma_{i,j} g} \exp\left(-\frac{\gamma_t}{\sigma_{i,j}}\right)\right]^{z+p} \end{aligned} \tag{8}$$

where $\{\gamma_k\}_{k=1}^{y+z}$ presents for the instantaneous SNR of the channel R_i - R_j in the kth transmission. Symbol F in above equation refers to calculating the packet error rate over fast fading.

In the slow fading, the channel gain remains unchanged in a number of transmissions. Hence, the packet error rate with y correct transmissions and z erroneous transmissions is written as:

$$\begin{aligned} P_{i,j}^{S,z,y} &= \int_0^\infty P^z(\gamma)\left[1-P(\gamma)\right]^y f_{\gamma_{i,j}}(\gamma) d\gamma \\ &= \sum_{p=0}^{y} \binom{y}{p} (-1)^p \left[1 - \frac{(p+z)\sigma_{i,j} g}{1+(p+z)\sigma_{i,j} g} \exp\left(-\frac{\gamma_t}{\sigma_{i,j}}\right)\right] \end{aligned} \tag{9}$$

where S presents for equations calculating the packet error rate over the slow fading.

3. Performance Analysis

3.1 Packet Error Rate

In the repetition based cooperative diversity algorithms, if the system has N relays, it consumes $N+1$ time slots to transmit signals to the destination [11][12][13]. In two proposed schemes, by using the TDD mode, each transmission from any node consumes a time slot. Thus, for a fair comparison in terms of time slots, the maximum number of time slots used in two proposals cannot exceed $N+1$ time slots. Not including a time slot when the source broadcasts the packet to the destination and all relays, the maximum number of time slots used for retransmissions equals N .

For the scheme I, the packet error rate is calculated as:

$$P_{SchI}^{R} = P_1\left(R_0^{R,N,0}\right) \tag{10}$$

Where

$$P_1(R_i^{R,t,0}) = P_{i,N+1}^{R,t+1,0}\prod_{h=i+1}^{N} P_{i,N-h}^{R,t,0} + \sum_{l=1}^{t} P_{i,N+1}^{R,l,0}\sum_{b=i+1}^{N}\prod_{e=i+1}^{N} P_{i,e}^{R,l-s(b),s(b)} P_1(R_e^{R,t-l,0}) \tag{11}$$

and

$$P_{i,e}^{R,l-s(b),s(b)} = \begin{cases} P_{i,e}^{R,l,0} & if\ e>b \\ P_{i,e}^{R,l-1,1} & if\ e=b \\ P_{i,e}^{R,l-1,0} & if\ e<b \end{cases}$$
$$P_1(R_N^{R,t,0}) = P_{i,N+1}^{R,t+1,0} \tag{12}$$
$$P_1(R_i^{R,0,0}) = 1$$

where $P_1(R_i^{R,t,0})$ presents for the probability that the transmitter R_i incorrectly transmits the packet to the destination in t times over R environment. When $R=F$, it means that the equation is used in the fast fading environments and the packet error rate from the transmitter R_i to other nodes R_j of the equation $P_{i,j}^{F,t,s(b)}$ follows as (8), otherwise the equation follows (9) and it represents for packet error rate over slow fading $R=S$. The proof is provided in Appendix.

For the scheme II, the packet error rate can be written as:

$$P_{SchII}^{R} = P_{0,N+1}^{R,N+1,0}\prod_{q=1}^{N} P_{0,q}^{R,N,0} + \sum_{l=1}^{t} P_{0,N+1}^{R,l,0} \times \sum_{b=1}^{N}\prod_{c=0}^{N-1} P_{i,N-c}^{R,l-s(b),s(b)} P_3\left(R_h^{R,N-l,0}\right) \tag{13}$$

where $P_{i,N-c}^{R,l-s(b),s(b)}$ also follows the equation (12) with $c=e$. $P_3\left(R_i^{R,t,0}\right)$ presents for the probability that the transmitter R_i incorrectly transmits the packet to the destination in t times over R environment. By following the operation of the schemes II, $P_3\left(R_i^{R,t,0}\right)$ can be calculated as:

$$\begin{aligned} P_3\left(R_i^{R,t,0}\right) &= Q_{i,N+1}^{R,t+1,0}\prod_{k_1=1}^{i-1} Q_{k_1,i}^{R,t,0}\prod_{k_2=i+1}^{N} Q_{i,k_2}^{R,t,0} \\ &+ \sum_{l=1}^{t} Q_{i,N+1}^{R,l,0} \times \left\{ \sum_{b_1=1}^{i-1}\left[\prod_{c=1}^{i-1} Q_{c,i}^{R,l-s(b_1),s(b_1)} P_3\left(R_{b_1}^{R,t-l,0}\right)\right] + \sum_{b_2=i+1}^{N}\left[\begin{array}{l}\prod_{d=1}^{i-1} Q_{i,d}^{R,l,0} \times \\ \prod_{c=0}^{N-i-1} Q_{i,N-c}^{R,l-s_2(b_2)+h,s(b_2)} P_3\left(R_{b_2}^{R,t-l,0}\right)\end{array}\right]\right\} \end{aligned} \tag{14}$$

and

$$Q_{i,j}^{R,t,v} = PER^{t}\left(\gamma_{i,j}\right)\left[1-PER\left(\gamma_{i,j}\right)\right]^{v} \tag{15}$$

$Q_{i,j}^{R,t,v}$ presents for the probability consisting of t correct transmissions and v erroneous transmissions from the broadcaster R_i to the receiver R_j. After multiplying all parts in the equation (14) and performing the integral as (8) or (9) following different fading environments, we can get the final equation $P_3\left(R_i^{R,t,0}\right)$.

3.2 Diversity Order

In this part, the spatial diversity of two proposed schemes is analyzed and compared over two fading environments. The diversity order is given by the magnitude of the slope of the PER as the function of high SNR where SNR is the function of the transmitter power over the noise variance [12]. As defined of $\sigma_{i,j}$ of the channel R_i - R_j in Section 2, when $SNR \to \infty$, it can be expressed as $\sigma_{i,j} \to \infty$. Let a be the diversity order of the schemes. The diversity order is limited by the weakest of all involved single links.

For z failed transmissions and y correct transmissions of the channel R_i - R_j over the slow fading, the diversity order can be written as:

$$\begin{aligned}\lim_{\sigma_{i,j}\to\infty} P_{i,j}^{S,z,y} &= \min\left\{\sum_{p=0}^{y}\binom{y}{p}(-1)^{p}\lim_{\sigma_{i,j}\to\infty}\left[1-\frac{(p+z)\sigma_{i,j}g}{1+(p+z)\sigma_{i,j}g}\exp\left(-\frac{\gamma_t}{\sigma_{i,j}}\right)\right]\right\} \\ &= \lim_{\sigma_{i,j}\to\infty}\left[1-\frac{z\sigma_{i,j}g}{1+z\sigma_{i,j}g}\exp\left(-\frac{\gamma_t}{\sigma_{i,j}}\right)\right]\end{aligned} \tag{16}$$

If $(z=0)$

$$\lim_{\sigma_{i,j}\to\infty}\left[1-\frac{z\sigma_{i,j}g}{1+z\sigma_{i,j}g}\exp\left(-\frac{\gamma_t}{\sigma_{i,j}}\right)\right]=\left(-\frac{\gamma_t}{\sigma_{i,j}}\right)^{0} \tag{17}$$

$$\Rightarrow a=0$$

If $(z\neq 0)$

$$\lim_{\sigma_{i,j}\to\infty}\left[1-\frac{z\sigma_{i,j}g}{1+z\sigma_{i,j}g}\exp\left(-\frac{\gamma_t}{\sigma_{i,j}}\right)\right]=\left(-\frac{\gamma_t}{\sigma_{i,j}}\right)^{1} \tag{18}$$

$$\Rightarrow a=1$$

Over the fast fading, the diversity order of z failed transmissions and y successful transmissions is calculated as:

$$\lim_{\sigma_{i,j}\to\infty} P_{i.j}^{F,z,y} = \min\left\{ \begin{array}{l} \sum_{p=0}^{y}\binom{y}{p}(-1)^p \\ \lim_{\sigma_{i,j}\to\infty}\left[1-\frac{\sigma_{i,j}g}{1+\sigma_{i,j}g}\exp\left(-\frac{\gamma_t}{\sigma_{i,j}}\right)\right]^{z+p} \end{array} \right\}$$

$$= \lim_{\sigma_{i,j}\to\infty}\left[1-\frac{\sigma_{i,j}g}{1+\sigma_{i,j}g}\exp\left(-\frac{\gamma_t}{\sigma_{i,j}}\right)\right]^{z} = \left(-\frac{\gamma_t}{\sigma_{i,j}}\right)^{z} \quad (19)$$

$$\Rightarrow a = z$$

To calculate the diversity order of the system, we can consider a random case happened for links to decide its diversity order. In the scheme I, we consider the case when the relay node R_N correctly receives the packet after the source broadcasts the packet to the destination and all nodes. And then, it will retransmit the packet until the retransmission terminates. The diversity order over the slow fading is shown as:

$$\begin{aligned} &\lim_{\sigma\to\infty} P_{0,N+1}^{S,1,0}P_{0,N}^{S,0,1}P_{N,N+1}^{S,N,0} \\ &= \lim_{\sigma_{0,N+1}\to\infty} P_{0,N+1}^{S,1,0} \lim_{\sigma_{0,N}\to\infty} P_{0,N}^{S,0,1} \lim_{\sigma_{N,N+1}\to\infty} P_{N,N+1}^{S,N,0} \\ &= \left(-\frac{\gamma_t}{\sigma_{0,N+1}}\right)^{1}\left(-\frac{\gamma_t}{\sigma_{0,N}}\right)^{0}\left(-\frac{\gamma_t}{\sigma_{N,N+1}}\right)^{1} \\ &\Rightarrow a = 2 \end{aligned} \quad (20)$$

Over the fast fading, the diversity order of the same case is calculated as:

$$\begin{aligned} &\lim_{\sigma\to\infty} P_{0,N+1}^{F,1,0}P_{0,N}^{F,0,1}P_{N,N+1}^{F,N,1} \\ &= \lim_{\sigma_{0,N+1}\to\infty} P_{0,N+1}^{F,1,0} \lim_{\sigma_{0,N}\to\infty} P_{0,N}^{F,0,1} \lim_{\sigma_{N,N+1}\to\infty} P_{N,N+1}^{F,N,1} \\ &= \left(-\frac{\gamma_t}{\sigma_{0,N+1}}\right)^{1}\left(-\frac{\gamma_t}{\sigma_{0,N}}\right)^{0}\left(-\frac{\gamma_t}{\sigma_{N,N+1}}\right)^{N} \\ &\Rightarrow a = N+1 \end{aligned} \quad (21)$$

From (20) and (21), we reach to the conclusion that over the fast fading, the scheme I can achieve the full diversity order while over the slow fading, its diversity order only equals the second order.

In the scheme II, as described in Section 2, after the relay R_N unsuccessfully retransmits the packet to the destination, it only repeats the packet in the next retransmission when the other nodes from the node R_{N-1} to the node R_1 cannot correctly receive the packet. So, the diversity order over different fading environments is calculated as, over fast fading:

$$\begin{aligned}
&\lim_{\sigma\to\infty} P_{0,N+1}^{F,1,0} P_{0,N}^{F,0,1} \left(\prod_{j=1}^{N-1} P_{N,N-j}^{F,1,0}\right)^{N-1} P_{N,N+1}^{F,N,0} \\
&= \lim_{\sigma_{0,N+1}\to\infty} P_{0,N+1}^{F,1,0} \lim_{\sigma_{0,N}\to\infty} P_{0,N}^{F,0,1} \times \left(\prod_{j=1}^{N-1} \lim_{\sigma_{N,N-j}\to\infty} P_{N,N-j}^{F,1,0}\right)^{N-1} \lim_{\sigma_{N,N+1}\to\infty} P_{N,N+1}^{F,N,0} \\
&= \left(-\frac{\gamma_t}{\sigma_{0,N+1}}\right)^1 \left(-\frac{\gamma_t}{\sigma_{0,N}}\right)^0 \times \left[\prod_{j=1}^{N-1}\left(-\frac{\gamma_t}{\sigma_{N,N-j}}\right)^1\right]^N \left(-\frac{\gamma_t}{\sigma_{N,N+1}}\right)^N \\
&\Rightarrow a > N+1
\end{aligned} \tag{22}$$

over slow fading:

$$\begin{aligned}
&\lim_{\sigma\to\infty} P_{0,N+1}^{S,1,0} P_{0,N}^{S,0,1} \left(\prod_{j=1}^{N-1} P_{N,N-j}^{S,1,0}\right)^{N-1} P_{N,N+1}^{S,N,0} \\
&= \lim_{\sigma_{0,N+1}\to\infty} P_{0,N+1}^{S,1,0} \lim_{\sigma_{0,N}\to\infty} P_{0,N}^{S,0,1} \times \left(\prod_{j=1}^{N-1} \lim_{\sigma_{N,N-j}\to\infty} P_{N,N-j}^{S,1,0}\right)^{N-1} \lim_{\sigma_{N,N+1}\to\infty} P_{N,N+1}^{S,N,0} \\
&= \left(-\frac{\gamma_t}{\sigma_{0,N+1}}\right)^1 \left(-\frac{\gamma_t}{\sigma_{0,N}}\right)^0 \times \left[\prod_{j=1}^{N-1}\left(-\frac{\gamma_t}{\sigma_{N,N-j}}\right)^1\right]^N \left(-\frac{\gamma_t}{\sigma_{N,N+1}}\right)^1 \\
&\Rightarrow a > N+1
\end{aligned} \tag{23}$$

From (22) and (23), if the system follows the case being similar with the scheme I, over both the fast and slow fading environments, the scheme II can get higher diversity order than $N+1$. However, if we consider another case as: after correctly receiving the packet from the source, the relay R_N becomes the broadcaster in the first retransmission, and then it transmits the packet unsuccessfully and successfully to the destination and the relay R_{N-1}, respectively. After that, the relay R_{N-1} becomes the broadcaster in the second retransmission. Consequently, in the qth ($1 \le q \le m$) retransmission, the relay R_{N-q-1} correctly receives the packet from the broadcaster R_{N-q} in the previous retransmission to become the broadcaster. In this case, the diversity order is calculated as,

over fast fading:

$$\begin{aligned}
&\lim_{\sigma\to\infty}\left(P_{0,N+1}^{F,1,0} P_{0,N}^{F,0,1} \prod_{j=1}^{N-1} P_{N,N-j}^{F,0,1} \prod_{j=1}^{N-1} P_{N-j,N+1}^{F,1,0}\right) \\
&= \lim_{\sigma_{0,N+1}\to\infty} P_{0,N+1}^{F,1,0} \lim_{\sigma_{0,N}\to\infty} P_{0,N}^{F,0,1} \prod_{j=1}^{N-1} \lim_{\sigma_{N,N-j}\to\infty} P_{N,N-j}^{F,0,1} \prod_{j=1}^{N-1} \lim_{\sigma_{N-j,N+1}\to\infty} P_{N-j,N+1}^{F,1,0} \\
&= \left(-\frac{\gamma_t}{\sigma_{0,N+1}}\right)^1 \left(-\frac{\gamma_t}{\sigma_{0,N}}\right)^0 \times \prod_{j=1}^{N-1}\left(-\frac{\gamma_t}{\sigma_{N,N-j}}\right)^0 \prod_{j=1}^{N-1}\left(-\frac{\gamma_t}{\sigma_{N-j,N+1}}\right)^1 \\
&\Rightarrow a = N+1
\end{aligned} \tag{24}$$

over slow fading:

$$\begin{aligned}&\lim_{\sigma\to\infty}\left(P_{0,N+1}^{S,1,0}P_{0,N}^{S,0,1}\prod_{t=1}^{N-1}P_{N,N-t}^{S,0,1}\prod_{k=1}^{N-1}P_{N-k,N+1}^{S,1,0}\right)\\&=\lim_{\sigma_{0,N+1}\to\infty}P_{0,N+1}^{S,1,0}\lim_{\sigma_{0,N}\to\infty}P_{0,N}^{S,0,1}\prod_{t=1}^{N-1}\lim_{\sigma_{N,N-t}\to\infty}P_{N,N-t}^{S,0,1}\prod_{k=1}^{N-1}\lim_{\sigma_{N-k,N+1}\to\infty}P_{N-k,N+1}^{S,1,0}\\&=\left(-\frac{\gamma_t}{\sigma_{0,N+1}}\right)^1\left(-\frac{\gamma_t}{\sigma_{0,N}}\right)^0\times\prod_{t=1}^{N-1}\left(-\frac{\gamma_t}{\sigma_{N,N-t}}\right)^0\prod_{k=1}^{N-1}\left(-\frac{\gamma_t}{\sigma_{N-k,N+1}}\right)^1\\&\Rightarrow a=N+1\end{aligned} \tag{25}$$

From (22)-(25), we can easily see that if we consider this case over both the slow fading and the fast fading, the diversity order of the scheme II equals $N+1$.

4. Simulation Results

In the simulation, we use BPSK modulation without FEC bits to simulate the performances of two proposed schemes and the traditional ARQ scheme. For a fair comparison in terms of consumed time slots, the number of time slots used for the traditional ARQ scheme, like two proposed schemes, also equals $N+1$. The length of a packet is set to be 1080 bits and the (α, g, γ_t) parameters in (7) are (67.7328, 0.9819, 6.3281 dB) [14]. The definition of the SNR in the following figures is the ratio of the transmitted power over the noise variance at the receiver that equals 1 ($N_1 = 1$). The source, the relays and the destination are corresponded to an equidistant line network in which the distance from a node to its adjacent node equals 1. Additionally, the reference distance d_0 and the path loss equal 1 and 3, respectively.

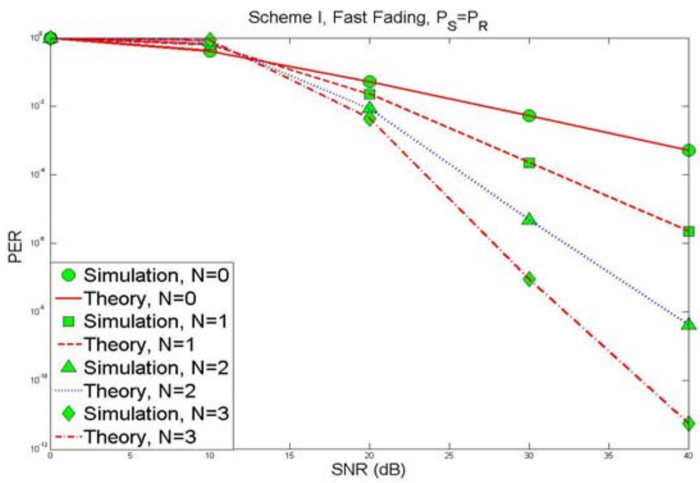

Fig. 2. PER performance of the scheme I over the fast fading with varying the number of relays in the network.

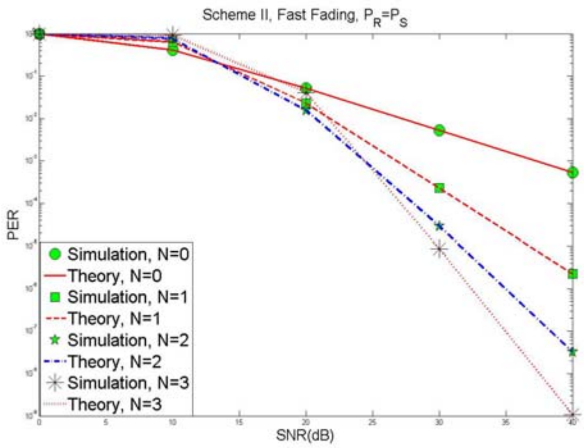

Fig. 3. PER performance of the scheme II over the fast fading when the transmitted powers at all nodes are similar.

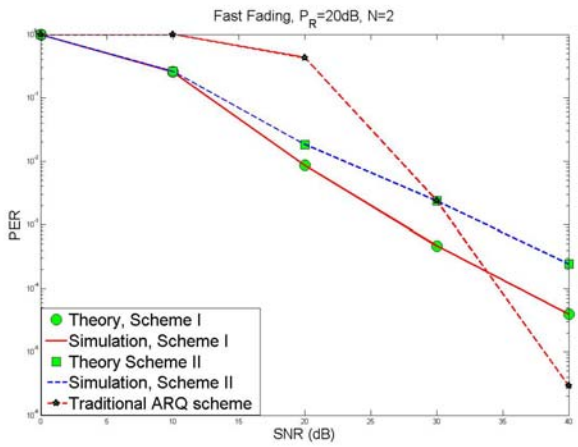

Fig. 4. PER comparison among two proposals and the traditional ARQ schemes over fast fading when the transmitted power at the relay is fixed at 20 dB.

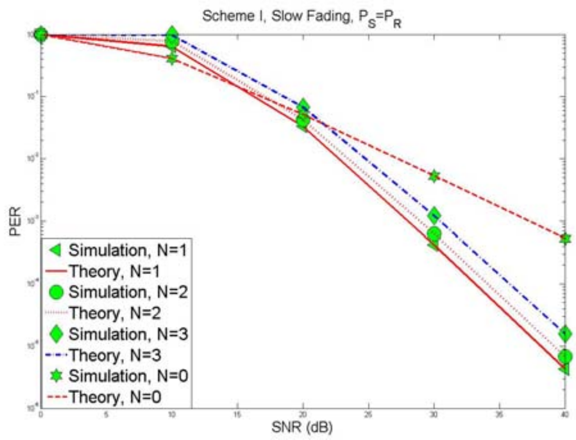

Fig. 5. PER performance of the scheme I over slow fading with varying number of relays in the network.

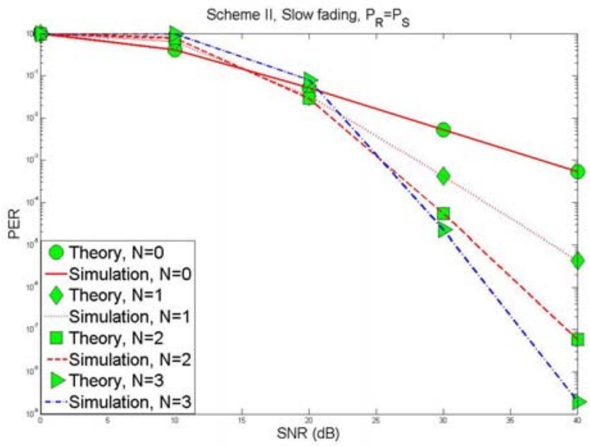

Fig. 6. PER performance of the scheme II where the transmitted powers at all nodes are same.

Fig. 2 and **Fig. 3** show the PER performance of two schemes over fast fading environments when the transmitted powers at the source and the relays are similar. The simulation results match exactly to the theoretical results, which verify the performance analysis in Section 3. As expected, over the fast fading, both two schemes including the scheme I and scheme II can achieve the full diversity order which can be seen the slopes of the PER performance curves which become steeper with increasing the number of relays. It is so obvious that when the system has only a relay or no relay, the performances of two schemes are similar.

The PER performance of different schemes is compared in **Fig. 4** when the transmitted power at the relays is fixed at 20 dB while varying the transmitted power at the source from 0 to 40 dB. From the figure, it is so easily to recognize that over the fast fading, the scheme I achieves better performance than the scheme II where the corresponding SNR gain is about 5 dB at $PER = 10^{-3}$ for the number of relays $N = 2$.

Over the slow fading, as analyzed in Section 3 and clearly shown in **Fig. 6** the scheme II can achieve the full diversity order while the diversity order of the scheme I only equals the second order in **Fig. 5**. Evidently, the scheme II outperforms the scheme I when the SNR is high enough. For example, the corresponding SNR gain is about 3 dB at $PER = 10^{-3}$ for the number of relays $N = 2$.

When we compare the PER performance of schemes over the fast and slow fading environments, there are some amazing results as: when the number of relays increases, the PER performances of the scheme I decrease over the slow fading but they increase over the fast fading; over both fading environments and when the SNR is high enough, the PER performance of the scheme II increases as the number of relays increases. Especially, as **Fig. 4** and **Fig. 7**, the traditional ARQ scheme achieves better performance than both two proposals when SNR is high enough ($SNR > 35$ dB) over fast fading but two proposals outperform the traditional ARQ scheme over slow fading. The reasons are explained as follows. First of all, from the equations (8, 9, 16, 18, 19) with $i = 0$, $j = N + 1$ and $z = N + 1$, $y = 0$, we can come to the conclusion that the traditional ARQ scheme achieves the one and full diversity order over the slow fading and the fast fading, respectively. Similarly, the scheme I takes advantage of the full diversity order over the fast fading and the second order in the slow fading while over both fading environments, the scheme II always takes advantage of the full diversity order. Thus, when SNR is high enough ($SNR > 25$ dB), the scheme II's performance increases as the number of relays increases but performances of the scheme I and the traditional ARQ scheme are changed over different fading environments. Additionally, the simulations are performed when nodes are corresponded to the equidistant line network. Thus, when the number of relays increases, the distances from the source to the destination or from the source to the closest relay also increase. By effects of the noise and fading over the long distance, the probability that the closest relay correctly receives the signal from the transmitter decreases. As the result of those operations, when the number of relays increases, the performances of the scheme I decrease over the slow fading but increase over the fast fading. In terms of performances of the traditional ARQ schemes, in **Fig. 4**, the simulation is performed with $P_S = P_R$ while P_R is fixed at 20 dB in **Fig. 7**. Thus, when the quality of the source-destination channel is too high compared to that of the relay-destination channel, the traditional ARQ protocol achieves better performance than two proposed schemes. However, the conclusion can not apply over the slow fading. From **Fig. 4**, we clearly recognize that although the channel from the source to the destination has better quality than the channel from the relays to the destination, the traditional ARQ still gets worse performance than two proposed schemes over slow fading.

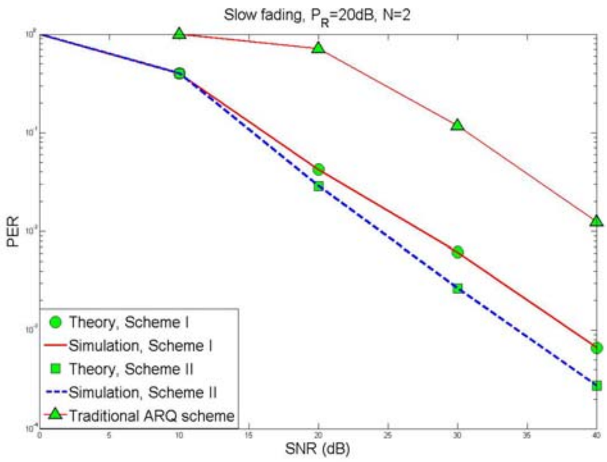

Fig. 7. PER comparison among two proposals and the traditional ARQ schemes over slow fading when the transmitted power at the source varies from 0 to 40 dB.

From simulation results, we may reach to the conclusion that over the fast fading, the scheme I is a better choice for the network while the scheme II is suitable over the slow fading.

5. Conclusions

In this paper, we have proposed and analyzed two different selection relaying schemes used in the cooperative ARQ protocol over two fading environments. Based on analyzing the diversity order, we can find the suitable scheme for each environment fading. Additionally, the closed form expressions for the PER performance of two schemes have been derived. The simulation results are also given, which verify the theoretical analysis and comparison.

Appendix

In this Appendix, the detailed equation of (10) and (13) is showed clearly.

Let $PER(\gamma_{i,j})$ is the packet error rate for the channel from $R_i - R_j$. $PER(\gamma_{i,j})$ follows as the equation (6).

As the operation of Scheme I, the average packet error rate can be calculated by recursive expression as:

$$P_{SchI} = P_1\left(R_0^N\right) \tag{26}$$

where

$$P_1\left(R_i^t\right) = PER\left(\gamma_{i,N+1}\right) \times \left[\Pr_{Incorrect}(i) P_1\left(R_i^{t-1}\right) + \sum_{h=i+1}^{N} \Pr_{Correct}(h) P_1\left(R_h^t\right)\right] \tag{27}$$

$P_1\left(R_i^t\right)$ denotes the average PER of the transmission from the node R_i to the destination with the maximum allowed number of retransmissions t.

$\Pr_{Correct}(h)$ is the probability of the event the relay R_h that is successfully decoded and is the closest to the destination. It is given by:

$$\Pr_{Correct}(h) = \left[1 - PER\left(\gamma_{i,h}\right)\right] \prod_{l=h+1}^{N} PER\left(\gamma_{i,l}\right) \tag{28}$$

$\Pr_{Incorrect}(i)$ is the probability of the event that no relay between the current transmitting relay R_i and the destination can decode correctly, given by:

$$\Pr_{Incorrect}(i) = \prod_{l=i+1}^{N} PER\left(\gamma_{i,l}\right) \tag{29}$$

After multiplying the equation (26), applying characteristics of fast fading or slow fading as (8) and (9), we integrate the equation following different channels. Finally, we can get equation in slow and fast fading as (10).

Similarly, in the scheme II, the packet error rate is written as:

$$P_{SchII} = P_3\left(R_0^N\right) \tag{30}$$

where

$$P_3\left(R_i^t\right) = PER\left(\gamma_{i,N+1}\right) \times \left[\begin{array}{l} \Pr_{In}(i) P_3\left(R_i^{t-1}\right) + \\ \sum_{h=i+1}^{N} \Pr_{H-Co}(h) P_3\left(R_h^t\right) + \sum_{l=1}^{i-1} \Pr_{L-Co}(l) P_3\left(R_l^t\right) \end{array}\right] \tag{31}$$

$\Pr_{H-Co}(h)$ is the probability of the event the relay R_h that is the successfully decoded and is the closest to the destination, given by:

$$\Pr_{H-Co}(h) = \left[1 - PER\left(\gamma_{i,h}\right)\right] \prod_{t=h+1}^{N} PER\left(\gamma_{i,t}\right) \tag{32}$$

$\Pr_{L-Co}(l)$ is the probability of the event the relay R_l that is the successfully decoded and is further to the destination, given by:

$$\Pr_{L-Co}(l) = \left[1 - PER_{i,l}\left(\gamma_{i,l}\right)\right] \times \prod_{p=l+1}^{N} PER\left(\gamma_{i,p}\right) \prod_{q=0}^{l-1} PER\left(\gamma_{i,q}\right) \tag{33}$$

$\Pr_{In}(i)$ is the probability of the event that no relay among the current transmitting relay R_i and the destination and the source can decode correctly, written as:

$$\Pr_{In}(i) = \prod_{q=i+1}^{N} PER(\gamma_{i,q}) \prod_{p=1}^{i-1} PER(\gamma_{i,p}) \quad (34)$$

Having the same method, we can get the equation as (13).

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