# An Application of Fuzzy Data Envelopment Analytical Hierarchy Process for Reducing Defects in the Production of Liquid Medicine 

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#### Abstract

This article demonstrated the application of the Fuzzy Data Envelopment Analytical Hierarchy Process (FDEAHP) to evaluate the root causes of critical defect problems occurring in the production of liquid medicine. The methodology of the research began by collecting the defect data by using Check Sheets, and ranking the significant problems by using a Pareto Diagram. Two types of major problems were found to occur, including glass fragments in the medicine and damaged lid threads. The causes of each problem were then analyzed by using Cause and Effect Diagrams. The significant causes were ranked by FDEAHP under three criteria, Severity (S), Occurrence (O) and Detection (D), followed by the framework of the FMEA Technique. Two causes with the highest Final Weight (FW) of each problem were selected to be improved, such as installing auxiliary equipment, using the Poka-Yoke system, setting the scale of the shaft and lathing the bushes of each bottle size. The results demonstrated a reduction in defects from $3.209 \%$ to $1.669 \%$ and showed that improving a few significant root causes, identified by an experienced decision maker, was sufficient to reduce the defect rate.


Keywords: Analytical Hierarchy Process, Data Envelopment Analysis, Fuzzy Set, Possibility Approach, Quality Improvement, Liquid Medicine Process

## 1. INTRODUCTION

Medicine is considered to be one of the fundamental essentials which are important for human life. The medicine industry is, therefore, extremely important for humanity. Both the social and economic turmoil of many countries and the current wave of 'new' diseases have rapidly increased the demand for medicine. The Drug Control Division of the Food and Drug Administration of Thailand stated that the quantity of modern medicine produced in 2007 has increased by $35.4 \%$ from 2006.

The Thai factory studied in this case study produces medicine and has, not only the competition to deal with, but also must deal with quality problems. It was discovered that the production of liquid medicine, which is the factory's main product, has a higher defect rate than the goal of less than $2 \%$ and increases unnecessary costs,
such as the cost of making the defective products, costs related to rework, and quality control costs. Furthermore, customer satisfaction has decreased. Consequently, the factory owner in the case study agreed with the importance of solving the problems of the manufacture of the medicine and reducing the defects. The improvement would not just reduce unnecessary costs, but would also respond to customer needs and increase the competitive capability.

## 2. BACKGROUND

### 2.1 Tools and Techniques for Quality Improvement

Quality leaders, namely Crosby (1979), Deming (1982), Ishikawa (1985), Juran (1988) and Feigenbaum (1991)

[^0]have published quality management principles. Subsequently, people have become more alert towards quality which is evidenced by the widespread use of Total Quality Management (TQM), both in theoretical and practical aspects (Tari and Sabater, 2004). Moreover, Evans and Lindsay (1999) have divided TQM into two dimensions: the management system consists of leadership, planning, human resources, etc., and the other dimension is about the technical system consisting of TQM tools and techniques. This is similar to how Wilkinson et al. (1998) divided TQM into two components: soft and hard. This article emphasizes the second part, the implementation of the tools and techniques for quality improvement. McQuater et al. (1995) stated that the tools and techniques would have a positive effect on the organization and would initiate continuous improvement. Similarly, Bunney and Dale (1997) and Stephens (1997) stated that the quality tools and techniques would lead to dramatic improvements in terms of quality. Dale and McQuater (1998) have divided quality tools and techniques frequently used by many organizations into four groups: 7QC tools, the new 7QC tools, other tools such as brainstorming and control plans, and lastly, techniques such as benchmarking, design of experiments (DOE), failure mode and effects analysis (FMEA). Furthermore, Pavletic et al. (2008) have researched and discovered that the 7QC tools have spread and succeeded in different areas, such as power plants, process industries, governments, health and tourism services. He et al. (1996) also pointed out that the 7 QC tools can solve up to $95 \%$ of quality-related problems. Therefore, this article targets quality improvement and control to reduce defects. Tools from groups 1, 3 and 4 were selected: Check Sheets, Pareto Diagrams, Cause and Effect Diagrams, Brainstorming, Framework of FMEA and Poka Yoke.

### 2.2 Fuzzy Data Envelopment Analytical Hierarchy Process (FDEAHP)

The analytical hierarchy process (AHP) was first proposed by Saaty (1980). It is a widely used decisionmaking analysis tool to deal with complicated, unstructured decision problems, especially in situations where there are important qualitative aspects that must be considered, in conjunction with various measurable quantitative factors based on hierarchical structures and the judgment of decision maker(s). It has unique advantages when important elements of the decision are difficult to quantify or compare, or where communication among team members is impeded by their different specializations, terminologies or perspectives. It has successfully been applied to many decision situations in areas such as selection, evaluation, planning and development, decision making and forecasting (Vaidya and Kumar, 2006).

In general, the AHP concept for decision making requires four steps. In the first step, the hierarchy structure of decision, must be constructed. The first layer of the
hierarchy structure is the main objective of the problem. The second is decision criteria. Sometimes, when the problem is complex, criteria can be divided further into sub-criteria and sub-sub-criteria and so on. The last layer is the alternative, which must be chosen. In the second step, the decision-maker(s) must build the judgment matrix by having pair-wise comparison criteria and alternatives in each criterion, based on discrete scales $1-9$, where $1,3,5,7$ and 9 respectively represent equally important, slightly more important, strongly more important, very strongly more important and extremely more important, and 2, 4, 6 and 8 represent intermediate values to reflect compromise. Each scale $\mathrm{a}_{\mathrm{ij}}$ of scale of the judgment matrix are the three rules: $\mathrm{a}_{\mathrm{ij}}>0, \mathrm{a}_{\mathrm{ij}}=1 / \mathrm{a}_{\mathrm{ji}}$, and $\mathrm{a}_{\mathrm{ii}}=1$ for all i . In the third step, the local weights (LW) of each judgment matrix are calculated. Based on Saaty (1980), the eigenvector method (EVM) is used to yield priorities for criteria and for alternative criteria. There are not only EVM, which is used to calculate LW, but also other methods are suggested for calculating weights, including the logarithmic least-square technique (LLST) and goal programming (GP). The last step is to synthesize the priorities of the alternative criteria into composite measures to arrive at a set of ratings for the alternatives or final weights (FW), based on the hierarchical arithmetic aggregation.

The data envelopment analytical hierarchy process (DEAHP) was first proposed by Ramanathan (2006). It used the concept of data envelopment analysis, which was first proposed by Charnes et al. (1978) for generating LW from the judgment matrices and aggregating them to be FW in AHP. The first DEA model is a CCR model. It used for evaluate relative efficiencies of decision making units (DMUs) in a case of constant returns to scale (CRS) of efficiency production frontiers in input or output oriented models in a form of linear programming (LP), and is extended to other models. One of the advantages of the DEA model is that it does not require either a priori weights or explicit specification of functional relations between the multiple outputs and inputs. Numerous research papers on efficiency measurement using DEA have been published on, for example, education systems, healthcare units, productions, and military logistics (See Seiford (1996) for a bibliography of more than 800 articles on DEA applications).

Since the judgment matrices of AHP are obtained using a suitable semantic scale, it is unrealistic to expect that the decision-maker(s) have either complete information or a full understanding of all aspects of the problem, which are represented as exact (or crisp, according to the fuzzy set terminology) numbers. So, the fuzzy set theory and possibility theory, which were proposed by Zadeh (1978), are used to confront the fuzzy uncertainty. References to possibility theory can be found in Dubois and Prade (1980) and Zimmermann (1996). It is called the fuzzy AHP (FAHP). Since the triangular fuzzy number has one discrete value at $=1$ and linear spread, then it is easier to model (Klir et al., 1997). In this paper, the fuzzy scales of fuzzy DEAHP (FDEAHP) are assumed to be the
triangular fuzzy number. Let minimum scale 1 be the crisp value, fuzzy judgments scale (2-8) be symmetry triangular fuzzy numbers with the lower and upper spreads $=1$, thus scale $2-8$ can be respectively rewritten in terms of $\alpha$-level set as follow; $2=[\alpha+1,3-\alpha], 3=[\alpha+$ $2,4-\alpha], \cdots, 8=[\alpha+7,9-\alpha]$, and maximum scale 9 be a triangular fuzzy number with the lower spreads $=1$, thus scale 9 can be rewritten in terms of $\alpha$-level set as $9=$ $[\alpha+8,9]$. Since 2-9 are positive fuzzy numbers, therefore $1 / 2-1 / 9$ can be calculated by extended division operator of fuzzy arithmetic (Zimmermann, 1990), e.g.,

$$
\begin{align*}
& 1(/) \tilde{\Lambda} \\
= & {\left[\min \left\{1 /(\tilde{\Lambda})_{\alpha}^{\mathrm{L}}, 1 /(\tilde{\Lambda})_{\alpha}^{\mathrm{U}}\right\}, \max \left\{1 /(\tilde{\Lambda})_{\alpha}^{\mathrm{L}}, 1 /(\tilde{\Lambda})_{\alpha}^{\mathrm{U}}\right\}\right] } \\
= & {\left[1 /(\tilde{\Lambda})_{\alpha}^{\mathrm{U}}, 1 /(\tilde{\Lambda})_{\alpha}^{\mathrm{L}}\right] } \tag{1}
\end{align*}
$$

where $\tilde{\Lambda}$ is a positive fuzzy number. Therefore, $1 / 2=[1 /(3-\alpha), 1 /(\alpha+1)], \cdots, 1 / 8=[1 /(9-\alpha), 1 /(\alpha+$ $7)]$, and $1 / 9=[1 / 9,1 /(\alpha+8)]$.

Focusing on FDEAHP, let A be a fuzzy judgment matrix of size $\mathrm{n} \times \mathrm{n}$ (compare n elements) and triangular fuzzy number $\tilde{\mathrm{a}}_{\mathrm{ij}}$ be entities of A . Thus, there are 1 dummy input, n outputs and n DMUs of the FDEAHP model. The FDEAHP model in a case of input oriented and constant return to scales (CRS) or FDEAHP-CCR-I and its dual problem or FDEAHP-DCCR-I is the following linear programming (LP) problem.

$$
\begin{align*}
& \text { (FDEAHP-CCR-I) Max } \theta=\sum_{\mathrm{r}=1}^{\mathrm{n}} \mathrm{u}_{\mathrm{i}} \tilde{\mathrm{a}}_{\mathrm{io}}  \tag{2}\\
& \text { Subject to } \mathrm{v}=1  \tag{3}\\
& \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{u}_{\mathrm{i}} \tilde{\mathrm{a}}_{\mathrm{ij}}-\mathrm{v}<0 \text { for } \mathrm{j}=1, \cdots, \mathrm{n}  \tag{4}\\
& \mathrm{u}_{\mathrm{i}}, \mathrm{v}>0  \tag{5}\\
& \text { (FDEAHP-DCCR-I) Min } \theta  \tag{6}\\
& \text { Subject to } \theta-\sum_{\mathrm{j}=1}^{\mathrm{n}} \lambda_{\mathrm{j}}>0  \tag{7}\\
& \tilde{\mathrm{a}}_{\mathrm{io}}-\sum_{\mathrm{j}=1}^{\mathrm{n}} \lambda_{\mathrm{j}} \tilde{\mathrm{a}}_{\mathrm{ij}}<0 ; \mathrm{i}=1, \cdots, \mathrm{n}  \tag{8}\\
& \theta \text { Unrestricted, } \lambda_{\mathrm{j}}>0 \tag{9}
\end{align*}
$$

where $u_{i}$ for $i=1, \cdots, n$ and $v$ are decision variables of the primal problem, $\theta$ and $\lambda_{\mathrm{j}}$ for $\mathrm{j}=1, \cdots, \mathrm{n}$ are dual variables.

Since the traditional DEA and DEAHP are formulated in the form of LP, then it basically requires exact crisp inputs and outputs of all DMUs. In Subsection 2.3, the concept of possibility theory and lemma, which is used to transform the FDEAHP to be the equivalent crisp DEAHP (E-CDEAHP), will be proposed.

### 2.3 Possibility Theory

Possibility theory in the context of the fuzzy set the-
ory was introduced by Zadeh (1978) to deal with nonstochastic imprecision and vagueness. Suppose that $\left(\Theta_{i}\right.$, $\left.\mathrm{P}\left(\Theta_{\mathrm{i}}\right), \pi_{\mathrm{i}}\right)$ for $\mathrm{i}=1, \cdots, \mathrm{n}$ is the possibility space with $\Theta_{\mathrm{i}}$ being the nonempty set of interest, $\mathrm{P}\left(\Theta_{\mathrm{i}}\right)$ is the collection of all subsets of $\Theta_{i}$, and $\pi_{i}$ is the possibility measure from $\mathrm{P}\left(\Theta_{\mathrm{i}}\right)$ to $[0,1]$, then $\pi(\phi)=0$ and $\pi\left(\Theta_{\mathrm{i}}\right)_{\xi}=1$, and $\pi\left(\cup_{\mathrm{i}} \mathrm{A}_{\mathrm{i}}\right)=$ $\sup _{\mathrm{i}}\left\{\pi\left(\mathrm{A}_{\mathrm{i}}\right)\right\}$ with each $\mathrm{A}_{\mathrm{i}} \in \mathrm{P}\left(\Theta_{\mathrm{i}}\right)$. Let ${ }^{\xi}$ be fuzzy variable as a real-value function defined over $\Theta_{\mathrm{i}}$. Therefore, the membership function of $\tilde{\xi}$ is given by

$$
\begin{align*}
\mu_{\tilde{\xi}}(\mathrm{s}) & =\pi\left(\left\{\theta_{\dot{j}} \in \Theta_{\mathrm{i}} / \tilde{\xi}\left(\theta_{\mathrm{j}}\right)=\mathrm{s}\right\}\right) \\
& =\sup _{\theta_{\mathrm{i}} \in \Theta_{\mathrm{i}}}\left\{\pi\left(\left\{\theta_{\mathrm{i}}\right\}\right) / \xi\left(\theta_{\mathrm{i}}\right)=\mathrm{s}\right\}, \forall s \in \Re . \tag{10}
\end{align*}
$$

Let $(\Theta, P(\Theta), \pi)$ be a product possibility space such that $\Theta=\Theta_{1} \mathrm{x} \cdots \mathrm{x} \Theta_{n}$ then

$$
\begin{equation*}
\pi(\mathrm{A})=\min \left\{\pi_{\mathrm{i}}\left(\mathrm{~A}_{\mathrm{i}}\right) / \mathrm{A}=\mathrm{A}_{1} \times \cdots \times \mathrm{A}_{\mathrm{n}}, \mathrm{~A}_{\mathrm{i}} \in \mathrm{P}\left(\Theta_{\mathrm{i}}\right)\right\} \tag{11}
\end{equation*}
$$

To compare fuzzy variables (Dubois and Prade, 1980), let $\tilde{a}_{1}, \cdots, \tilde{a}_{n}$ be fuzzy variables and $f_{j}: \Re^{n} \rightarrow \Re$ be a real-valued function for $j=1, \cdots, m$. The possibility measure of fuzzy event is given by

$$
\begin{align*}
& \pi\left(f_{j}\left(\tilde{a}_{1}, \cdots, \tilde{a}_{n}\right) \leq 0\right) \\
& \quad=\sup _{\mathrm{s}_{1}, \cdots, \mathrm{~s}_{\mathrm{n}} \in \mathfrak{M}}\left\{\min \left\{\mu_{\mathrm{a}_{\mathrm{i}}}\left(\mathrm{~s}_{\mathrm{i}}\right)\right\} / \mathrm{f}_{\mathrm{j}}\left(\mathrm{~s}_{1}, \cdots, \mathrm{~s}_{\mathrm{n}}\right) \leq 0\right\} \tag{12}
\end{align*}
$$

Possibility measures are adopted to prove Lemma 1 for solving the fuzzy multiplier form of input-oriented CCR (FCCR-I) model by Lertworasirikul et al. (2003).

Lemma 1: Let $\tilde{a}_{1}, \cdots, \tilde{a}_{n}$ be fuzzy variables with normal and convex membership functions and $b$ be a crisp variable. Let $(\cdot)_{\alpha_{j}}^{\mathrm{L}}$ and $(\cdot)_{\alpha_{i}}^{\mathrm{U}}$ denote the lower and upper bounds of the $\alpha$-level set of $\tilde{a}_{i}$ for all i. Then, for any given possibility levels $\alpha_{1}, \alpha_{2}$ and $\alpha_{3}$ with $0<\alpha_{1}, \alpha_{2}, \alpha_{3}<1$,
(i) $\pi\left(\tilde{\mathrm{a}}_{1}+\cdots+\tilde{\mathrm{a}}_{\mathrm{n}} \leq \mathrm{b}\right) \geq \alpha_{1}$ if and only if $\left(\tilde{a}_{1}\right)_{\alpha_{1}}^{\mathrm{L}}+\cdots+\left(\tilde{\mathrm{a}}_{\mathrm{n}}\right)_{\alpha_{1}}^{\mathrm{L}} \leq \mathrm{b}$,
(ii) $\pi\left(\tilde{a}_{1}+\cdots+\tilde{a}_{n} \geq b\right) \geq \alpha_{2}$ if and only if $\left(\tilde{a}_{1}\right)_{\alpha_{2}}^{U}+\cdots+\left(\tilde{a}_{n}\right)_{\alpha_{2}}^{U} \geq b$,
(iii) $\pi\left(\tilde{a}_{1}+\cdots+\tilde{a}_{n}=b\right) \geq \alpha_{3}$ if and only if $\left(\tilde{a}_{1}\right)_{\alpha_{3}}^{\mathrm{L}}+\cdots+\left(\tilde{a}_{n}\right)_{\alpha_{3}}^{\mathrm{L}} \leq \mathrm{b}$ and $\left(\tilde{a}_{1}\right)_{\alpha_{3}}^{U}+\cdots+\left(\tilde{a}_{n}\right)_{\alpha_{3}}^{U} \geq b$.

In the next Subsection, Lemma 1 will be used to transform the FDEAHP-CCR-I in equation (2)-(5) and FDEAHP-DCCR-I in equation (6)-(9) to respectively be the equivalent crisp DEAHP-CCR-I (E-CDEAHP-CCRI) and equivalent crisp DEAHP-DCCR-I (E-CDEAHP-DCCR-I).

### 2.4 Equivalent Crisp Data Envelopment Analytical Hierarchy Process (E-CDEAHP)

Based on possibility theory, the FDEAHP-CCR-I and FDEAHP-DCCR-I can be respectively rewritten in the form of possibility programming (PP), as follows:

$$
\begin{align*}
& \text { (PP-FDEAHP-CCR-I) Max } \theta=\Psi  \tag{13}\\
& \text { Subject to } \pi\left(\sum_{\mathrm{r}=1}^{\mathrm{n}} \mathrm{u}_{\mathrm{i}} \tilde{\mathrm{a}}_{\mathrm{io}} \geq \Psi\right) \geq \alpha  \tag{14}\\
& \pi\left(\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{u}_{\mathrm{i}} \tilde{\mathrm{a}}_{\mathrm{ij}} \leq 1\right) \geq \alpha \text { for } \mathrm{j}=1, \cdots, \mathrm{n}  \tag{15}\\
& \mathrm{u}_{\mathrm{i}}>0  \tag{16}\\
& \text { (PP-FDEAHP-DCCR-I) Min } \theta  \tag{17}\\
& \text { Subject to } \theta-\sum_{\mathrm{j}=1}^{\mathrm{n}} \lambda_{\mathrm{j}}>0  \tag{18}\\
& \pi\left(\tilde{\mathrm{a}}_{\mathrm{io}}-\sum_{\mathrm{j}=1}^{\mathrm{n}} \lambda_{\mathrm{j}} \tilde{\mathrm{a}}_{\mathrm{ij}} \leq 0\right) \geq \alpha \text { for } \mathrm{i}=1, \cdots, \mathrm{n}  \tag{19}\\
& \theta \text { Unrestricted, } \lambda_{\mathrm{j}}>0 \tag{20}
\end{align*}
$$

From (i) and (ii) in Lemma 1, the possibility constraints in PP-FDEAHP-CCR-I and PP-FDEAHP-DCCRI can be converted to the equivalent linear constraints at $\alpha$-level set. Thus, the E-CDEAHP-CCR-I and E-CDEAHP-DCCR-I will be transformed to be LP, as follows:
(E-CDEAHP-CCR-I) Max $\theta=\Psi$
Subject to $\sum_{\mathrm{r}=1}^{\mathrm{n}} \mathrm{u}_{\mathrm{i}}\left(\tilde{\mathrm{a}}_{\mathrm{io}}\right)_{\alpha}^{U} \geq \Psi$
$\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{u}_{\mathrm{i}}\left(\tilde{\mathrm{a}}_{\mathrm{ij}}\right)_{\alpha}^{\mathrm{L}} \leq 1$ for $\mathrm{j}=1, \cdots, \mathrm{n}$
$\mathrm{u}_{\mathrm{i}}>0$
(E-CDEAHP-DCCR-I) Min $\theta$
Subject to $\theta-\sum_{j=1}^{\mathrm{n}} \lambda_{\mathrm{j}}>0$
$\left(\tilde{a}_{i o}\right)_{\alpha}^{L}-\sum_{j=1}^{n} \lambda{ }_{j}\left(\tilde{a}_{i j}\right)_{\alpha}^{U} \leq 0$ for $\mathrm{i}=1, \cdots, n$
$\theta$ Unrestricted, $\lambda_{j}>0$.
Where $(\cdot)_{\alpha}^{\mathrm{L}}$ and $(\cdot)_{\alpha}^{\mathrm{U}}$ are the lower and upper bounds of the $\alpha$-level set of comparison entities. The fuzzy relative efficiencies $\left(\theta^{*}\right)$ from the E-CDEAHP-CCR-I or E-CDEAHP-DCCR-I must be converted to be the fuzzy LWs, and will be aggregated to be the FW by the concept of the fuzzy hierarchical arithmetic aggregation, based on the extension principle.

## 3. CASE STUDY OF THE REDUCTION OF DEFECTS IN THE PRODUCTION LINE OF LIQUID MEDICINE

This article demonstrates an application of the Fuzzy Data Envelopment Analytical Hierarchy Process (FDE

AHP) to evaluate the root causes of a liquid medicine production problem. The methodology of this article included five steps as illustrated in Figure 1.


Figure 1. The steps of the research methodology.

### 3.1 Using Pareto Diagrams to Choose the Significant Defect Attribute

The methodology in this paper started by designing the check sheets to gather the data of the defects, and the types of problems causing defects within a three month period, from September to November 2008. The results were that the problems could be divided into five categories: glass bits in the medicine, damaged lid threads, defects on the bottle lids, deformation of the bottle lids and others. The number of each defect and its percentages appear in Table 6. When the Pareto Diagram was used in assigning the priority of each problem with the $80 \%$ principle in selecting the problems with the highest priority to be solved, the results were that the broken glass bits in the bottles and the damaged lid threads were the problems selected, as illustrated in Figure 2.

### 3.2 Finding the Causes of the Problems by Using Cause and Effect Diagrams

Cause and Effect Diagrams were used and were combined with a brainstorming technique to analyze the causes of the occurrence of the two main problems, selected by rank, and done with the Pareto Diagram. Next, the uncontrollable causes were separated from the controllable causes of both problems. The results for the first problem of having glass bits in the bottles and the second problem of threaded lids are shown in Table 1 and Table 2 , respectively.


Figure 2. Pareto chart of the defect characteristics.
Table 1. The causes of the glass bits in the bottles.

| Symbol | Description |
| :---: | :--- |
| $\mathrm{A}_{1}$ | There are no scales to indicate the height of the <br> shaft of the machinery |
| $\mathrm{A}_{2}$ | Thread Bush Eroded |
| $\mathrm{A}_{3}$ | No standard in adjusting the bottle locks |
| $\mathrm{A}_{4}$ | Inappropriate height of the work table |
| $\mathrm{A}_{5}$ | Improper bottle washing |
| $\mathrm{A}_{6}$ | Employees did not properly put the bottles in the <br> slots. |
| $\mathrm{A}_{7}$ | The pressure in applying the lids onto the bottles. |
| $\mathrm{A}_{8}$ | Employees put the bottle racks over each other |

Table 2. The causes of the threaded lids.

| Symbols | Description |
| :---: | :--- |
| $\mathrm{B}_{1}$ | There are no scales to indicate the height of the <br> shaft of the machinery |
| $\mathrm{B}_{2}$ | Thread Bush Eroded |
| $\mathrm{B}_{3}$ | Improper thickness of the bush supporters |
| $\mathrm{B}_{4}$ | No standard in adjusting the bottle locks |
| $\mathrm{B}_{5}$ | Inappropriate height of the work table |
| $\mathrm{B}_{6}$ | Employees did not properly put the bottles in the <br> slots |

### 3.3 Creating a Structure of the Problems to be analyzed in a Form of Hierarchies within the FMEA Criteria

The Hierarchy based on the FMEA criteria consists of three levels: Level 0 are the problems being analyzed (glass bits in the bottles and the damage to lid threads), Level 1 are the criteria according to FMEA, consisting of Severity (S), Occurrence (O) and Detection (D), and Level 2 are the causes analyzed ( $\mathrm{A}_{1}-\mathrm{A}_{8}$ ) and ( $\mathrm{B}_{1}-\mathrm{B}_{6}$ ), as illustrated in Figure 3 and Figure 4.

### 3.4 Ranking the Root Causes of the Problems

In this paper, the controllable causes of each prob-


Figure 3. A hierarchy structure of glass bits in the bottles.


Figure 4. A hierarchy structure of the damaged lid threads.
lem were ranked, based on the FMEA framework. This information helped a decision maker to make a decision supported by machine operators, production leaders, quality control leaders and maintenance leaders. Pair the Severity (S), Occurrence (O), and Detection (D) in criteria level, and controllable causes of the first and the second problems, as following judgment matrices.

$$
\begin{align*}
& \tilde{\Theta}_{\text {Criteria/FMEA }}=\left[\begin{array}{ccc}
1 & \tilde{3} & \tilde{5} \\
1 / \tilde{3} & 1 & \tilde{3} \\
1 / \tilde{5} & 1 / \tilde{3} & 1
\end{array}\right]  \tag{29}\\
& \tilde{\Theta}_{1 / \mathrm{S}}=\left[\begin{array}{cccccccc}
1 & \tilde{9} & \tilde{9} & \tilde{9} & \tilde{5} & \tilde{3} & 1 & \tilde{7} \\
1 / \tilde{9} & 1 & 1 & 1 & 1 / \tilde{3} & 1 / \tilde{5} & 1 / \tilde{9} & 1 / \tilde{2} \\
1 / \tilde{9} & 1 & 1 & 1 & 1 / \tilde{3} & 1 / \tilde{5} & 1 / \tilde{9} & 1 / \tilde{2} \\
1 / \tilde{9} & 1 & 1 & 1 & 1 / \tilde{3} & 1 / \tilde{5} & 1 / \tilde{9} & 1 / \tilde{2} \\
1 / \tilde{5} & \tilde{3} & \tilde{3} & \tilde{3} & 1 & 1 / \tilde{3} & 1 / \tilde{5} & \tilde{3} \\
1 / \tilde{3} & \tilde{5} & \tilde{5} & \tilde{5} & \tilde{3} & 1 & 1 / \tilde{3} & \tilde{5} \\
1 & \tilde{9} & \tilde{9} & \tilde{9} & \tilde{5} & \tilde{3} & 1 & \tilde{7} \\
1 / \tilde{7} & \tilde{2} & \tilde{2} & \tilde{2} & 1 / \tilde{3} & 1 / \tilde{5} & 1 / \tilde{7} & 1
\end{array}\right]  \tag{30}\\
& \tilde{\Theta}_{1 / \mathrm{O}}=\left[\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 / \tilde{3} & 1 / \tilde{3} & 1 / \tilde{3} & 1 \\
1 & 1 & 1 & 1 & 1 / \tilde{3} & 1 / \tilde{3} & 1 / \tilde{3} & 1 \\
1 & 1 & 1 & 1 & 1 / \tilde{3} & 1 / \tilde{3} & 1 / \tilde{3} & 1 \\
1 & 1 & 1 & 1 & 1 / \tilde{3} & 1 / \tilde{3} & 1 / \tilde{3} & 1 \\
\tilde{3} & \tilde{3} & \tilde{3} & \tilde{3} & 1 & 1 & 1 & \tilde{3} \\
\tilde{3} & \tilde{3} & \tilde{3} & \tilde{3} & 1 & 1 & 1 & \tilde{3} \\
\tilde{3} & \tilde{3} & \tilde{3} & \tilde{3} & 1 & 1 & 1 & \tilde{3} \\
1 & 1 & 1 & 1 & 1 / \tilde{3} & 1 / \tilde{3} & 1 / \tilde{3} & 1
\end{array}\right]  \tag{31}\\
& \tilde{\Theta}_{1 / \mathrm{D}}=\left[\begin{array}{cccccccc}
1 & \tilde{5} & 1 & \tilde{5} & 1 & 1 / \tilde{3} & 1 / \tilde{3} & \tilde{5} \\
1 / \tilde{5} & 1 & 1 / \tilde{5} & 1 & 1 / \tilde{5} & 1 / \tilde{7} & 1 / \tilde{7} & 1 \\
1 & \tilde{5} & 1 & \tilde{5} & 1 & 1 / \tilde{3} & 1 / \tilde{3} & \tilde{5} \\
1 / \tilde{5} & 1 & 1 / \tilde{5} & 1 & 1 / \tilde{5} & 1 / \tilde{7} & 1 / \tilde{7} & 1 \\
1 & \tilde{5} & 1 & \tilde{5} & 1 & 1 / \tilde{3} & 1 / \tilde{3} & \tilde{5} \\
\tilde{3} & \tilde{7} & \tilde{3} & \tilde{7} & \tilde{3} & 1 & 1 & \tilde{7} \\
\tilde{3} & \tilde{7} & \tilde{3} & \tilde{7} & \tilde{3} & 1 & 1 & \tilde{7} \\
1 / \tilde{5} & 1 & 1 / \tilde{5} & 1 & 1 / \tilde{5} & 1 / \tilde{7} & 1 / \tilde{7} & 1
\end{array}\right]  \tag{32}\\
& \tilde{\Theta}_{2 / \mathrm{S}}=\left[\begin{array}{cccccc}
1 & \tilde{5} & \tilde{3} & \tilde{7} & \tilde{9} & \tilde{9} \\
1 / \tilde{5} & 1 & 1 / \tilde{3} & \tilde{3} & \tilde{5} & \tilde{5} \\
1 / \tilde{3} & \tilde{3} & 1 & \tilde{5} & \tilde{7} & \tilde{7} \\
1 / \tilde{7} & 1 / \tilde{3} & 1 / \tilde{5} & 1 & \tilde{3} & \tilde{3} \\
1 / \tilde{9} & 1 / \tilde{5} & 1 / \tilde{7} & 1 / \tilde{3} & 1 & 1 \\
1 / \tilde{9} & 1 / \tilde{5} & 1 / \tilde{7} & 1 / \tilde{3} & 1 & 1
\end{array}\right] \tag{33}
\end{align*}
$$

$$
\begin{align*}
& \tilde{\Theta}_{2 / \mathrm{O}}=\left[\begin{array}{cccccc}
1 & \tilde{3} & 1 & \tilde{3} & \tilde{5} & \tilde{5} \\
1 / \tilde{3} & 1 & 1 / \tilde{3} & 1 & \tilde{3} & \tilde{3} \\
1 & \tilde{3} & 1 & \tilde{3} & \tilde{5} & \tilde{5} \\
1 / \tilde{3} & 1 & 1 / \tilde{3} & 1 & \tilde{3} & \tilde{3} \\
1 / \tilde{5} & 1 / \tilde{3} & 1 / \tilde{5} & 1 / \tilde{3} & 1 & 1 \\
1 / \tilde{5} & 1 / \tilde{3} & 1 / \tilde{5} & 1 / \tilde{3} & 1 & 1
\end{array}\right]  \tag{34}\\
& \tilde{\Theta}_{2 / \mathrm{D}}=\left[\begin{array}{cccccc}
1 & 1 & 1 / \tilde{3} & 1 & \tilde{5} & \tilde{5} \\
1 & 1 & 1 / \tilde{3} & 1 & \tilde{5} & \tilde{5} \\
\tilde{3} & \tilde{3} & 1 & \tilde{3} & \tilde{7} & \tilde{7} \\
1 & 1 & 1 / \tilde{3} & 1 & \tilde{5} & \tilde{5} \\
1 / \tilde{5} & 1 / \tilde{5} & 1 / \tilde{7} & 1 / \tilde{5} & 1 & 1 \\
1 / \tilde{5} & 1 / \tilde{5} & 1 / \tilde{7} & 1 / \tilde{5} & 1 & 1
\end{array}\right] \tag{35}
\end{align*}
$$

From the judgment matrix of criteria in equation (29) and the E-CDEAHP-DCCR-I model, the fuzzy relative efficiencies of S, O, and D can be calculated by the following LP model,

$$
\begin{align*}
& \text { (E-CDEAHP-DCCR-I) Min } \theta_{\text {Criteria/FMEA }}  \tag{36}\\
& \quad \text { Subject to } \theta_{\text {Criteria/FMEA }}-\lambda_{1}-\lambda_{2}-\lambda_{3}>0  \tag{37}\\
& \quad\left(\tilde{a}_{1 \mathrm{o}}\right)_{\alpha}^{\mathrm{L}}-(1) \lambda_{1}-(1 /(\alpha+2)) \lambda_{2}-(1 /(\alpha+4)) \lambda_{3}<0  \tag{38}\\
& \quad\left(\tilde{\mathrm{a}}_{2 \mathrm{o}}\right)_{\alpha}^{\mathrm{L}}-(4-\alpha) \lambda_{1}-(1) \lambda_{2}-(1 /(\alpha+2)) \lambda_{3}<0  \tag{39}\\
& \quad\left(\tilde{a}_{3 \mathrm{o}}\right)_{\alpha}^{\mathrm{L}}-(6-\alpha) \lambda_{1^{-}}(4-\alpha) \lambda_{2}-(1) \lambda_{3}<0  \tag{40}\\
& \theta \text { Unrestricted, } \lambda_{\mathrm{j}}>0 \tag{41}
\end{align*}
$$

where $\left(\tilde{\mathrm{a}}_{10}\right)_{\alpha}^{\mathrm{L}} \in\{1,1 /(4-\alpha), 1 /(6-\alpha)\}, \quad\left(\tilde{\mathrm{a}}_{2 \mathrm{o}}\right)_{\alpha}^{\mathrm{L}} \in$ $\{\alpha+2,1,1 /(4-\alpha)\}$, and $\left(\tilde{a}_{30}\right)_{\alpha}^{\mathrm{L}} \in\{\alpha+4, \alpha+2,1\}$.

To obtain LW of each criterion, the E-CDEAHP-DC CR-I model must be solved at the specified $\alpha$-level set. In this paper, eleven levels of $\alpha$-level set, which were $0,0.1$, $0.2, \cdots, 1$, was specified. The relative efficiency of criteria $\mathrm{S}, \mathrm{O}$, and D at each $\alpha$-level set are shown in Table 3.

Table 3. Relative efficiency of criteria.

| $\alpha$-level | Relative Efficiency $\left(\theta_{\text {CriteriaFMEA }}\right)$ |  |  |
| :---: | :---: | :---: | :---: |
|  | DMU 1 (S) | DMU 2 (O) | DMU 3 (D) |
| 0.0 | 1 | 0.3333 | 0.1667 |
| 0.1 | 1 | 0.3559 | 0.1695 |
| 0.2 | 1 | 0.3793 | 0.1724 |
| 0.3 | 1 | 0.4035 | 0.1754 |
| 0.4 | 1 | 0.4286 | 0.1786 |
| 0.5 | 1 | 0.4545 | 0.1818 |
| 0.6 | 1 | 0.4815 | 0.1852 |
| 0.7 | 1 | 0.5094 | 0.1887 |
| 0.8 | 1 | 0.5385 | 0.1923 |
| 0.9 | 1 | 0.5686 | 0.1961 |
| 1.0 | 1 | 0.6000 | 0.2000 |

From Table 3, the relative efficiency of DMU 1 is crisp, and others are fuzzy. The membership functions of relative efficiency of DMU 2 and 3 can be approximated to be the one side triangular membership functions by regression analysis, $\theta_{\text {O/FMEA }}=[0.326+0.266 \alpha, 0.6]$ with $\mathrm{R}^{2}=99.7 \%$ for DMU 2, and $\theta_{\mathrm{D} / \mathrm{FMEA}}=[0.166+0.033 \alpha$, $0.2]$ with $\mathrm{R}^{2}=99.7 \%$ for DMU 3. By the extension principle, summation of fuzzy relative efficiency from DEA model is $[1.492+0.299 \alpha, 1.8]$. Therefore, the fuzzy LW in a form of traditional AHP can be rewritten as follows:

$$
\begin{align*}
\mathrm{LW}_{\text {S/FMEA }} & =\left[\frac{1}{1.8}, \frac{1}{1.492+0.299 \alpha}\right]  \tag{42}\\
\mathrm{LW}_{\text {O/FMEA }} & =\left[\frac{0.326+0.266 \alpha}{1.8}, \frac{0.6}{1.492+0.299 \alpha}\right]  \tag{43}\\
\mathrm{LW}_{\text {D/FMEA }} & =\left[\frac{0.166+0.033 \alpha}{1.8}, \frac{0.2}{1.492+0.299 \alpha}\right] \tag{44}
\end{align*}
$$

From brainstorming, it was discovered that the first problem has up to eight controllable root causes $\left(\mathrm{A}_{1}-\mathrm{A}_{8}\right)$. Each root cause affects the problems within the criteria of S, O and D differently. Therefore, the Judgment Matrix has a size of $8 \times 8$ of up to three matrices to compare the priorities of each cause according to the three criteria as expressed in equations (30)-(32).

The relative efficiency and LW are calculated similar to LW of criteria. Based on the criteria S in problem 1, $\theta_{\mathrm{A} 1 / \mathrm{S}}=\theta_{\mathrm{A} 7 / \mathrm{S}}=1, \theta_{\mathrm{A} 2 / \mathrm{S}}=\theta_{\mathrm{A} 3 / \mathrm{S}}=\theta_{\mathrm{A} 4 / \mathrm{S}}=0.111, \theta_{\mathrm{A} 5 / \mathrm{S}}=$ $[0.247+0.178 \alpha, 0.429]$ with $\mathrm{R}^{2}=99.9 \%, \theta_{\mathrm{A} 6 / \mathrm{S}}=[0.496$ $+0.214 \alpha, 0.714]$ with $\mathrm{R}^{2}=99.9 \%, \theta_{\mathrm{A} 8 / \mathrm{S}}=[0.117+$ $0.103 \alpha, 0.222]$ with $\mathrm{R}^{2}=99.0 \%$, and the summation of fuzzy LW from DEA model $=[3.193+0.495 \alpha, 3.698]$. Based on the criteria $O$ in problem 1, $\theta_{\mathrm{A} 1 / \mathrm{O}}=\theta_{\mathrm{A} 2 / \mathrm{O}}=\theta_{\mathrm{A} 3 / \mathrm{O}}$ $=\theta_{\mathrm{A} 4 / \mathrm{O}}=\theta_{\mathrm{A} 8 / \mathrm{O}}=[0.247+0.083 \alpha, 0.333]$ with $\mathrm{R}^{2}=$ $99.4 \%, \theta_{\mathrm{A} 5 / \mathrm{O}}=\theta_{\mathrm{A} 6 / \mathrm{O}}=\theta_{\mathrm{A} 7 / \mathrm{O}}=1$, and the summation of fuzzy LW from DEA model $=[4.235+0.415 \alpha, 4.665]$. Based on the criteria $D$ in problem 1, $\theta_{\mathrm{A} 1 / \mathrm{D}}=\theta_{\mathrm{A} 4 / \mathrm{D}}=\theta_{\mathrm{A} 5 / \mathrm{D}}$ $=[0.496+0.214 \alpha, 0.714]$ with $\mathrm{R}^{2}=99.9 \%, \theta_{\mathrm{A} 2 / \mathrm{D}}=\theta_{\mathrm{A} 3 / \mathrm{D}}$ $=\theta_{\mathrm{A} 8 / \mathrm{D}}=[0.125+0.018 \alpha, 0.143]$ with $\mathrm{R}^{2}=99.8 \%, \theta_{\mathrm{A} 6 / \mathrm{O}}$ $=\theta_{\mathrm{A} 7 / \mathrm{O}}=1$, and the summation of fuzzy LW from DEA model $=[3.863+0.696 \alpha, 4.571]$. In this paper, the fuzzy relative efficiency of each controllable cause is converted to be traditional fuzzy LWs by the concept extension principle. The results are shown as follows:

$$
\begin{align*}
\mathrm{LW}_{\mathrm{A} / \mathrm{S}} & =\mathrm{LW}_{\mathrm{A} 7 / \mathrm{S}}=\left[\frac{1}{3.698}, \frac{1}{3.193+0.495 \alpha}\right]  \tag{45}\\
\mathrm{LW}_{\mathrm{A} 2 / \mathrm{S}} & =\mathrm{LW}_{\mathrm{A} 3 / \mathrm{S}}=\mathrm{LW}_{\mathrm{A} 4 / \mathrm{S}} \\
& =\left[\frac{0.111}{3.698}, \frac{0.111}{3.193+0.495 \alpha}\right] \tag{46}
\end{align*}
$$

$$
\begin{align*}
\mathrm{LW}_{\mathrm{A} / / \mathrm{S}} & =\left[\frac{0.247+0.178 \alpha}{3.698}, \frac{0.429}{3.193+0.495 \alpha}\right]  \tag{47}\\
\mathrm{LW}_{\mathrm{A} / / \mathrm{S}} & =\left[\frac{0.496+0.214 \alpha}{3.698}, \frac{0.714}{3.193+0.495 \alpha}\right]  \tag{48}\\
\mathrm{LW}_{\mathrm{A} 8 / \mathrm{S}} & =\left[\frac{0.117+0.103 \alpha}{3.698}, \frac{0.222}{3.193+0.495 \alpha}\right]  \tag{49}\\
\mathrm{LW}_{\mathrm{A} 1 / \mathrm{O}} & =\mathrm{LW}_{\mathrm{A} 2 / \mathrm{O}}=\mathrm{LW}_{\mathrm{A} 3 / \mathrm{O}}=\mathrm{LW}_{\mathrm{A} 4 / \mathrm{O}}=\mathrm{LW}_{\mathrm{A} 8 / \mathrm{O}} \\
& =\left[\frac{0.247+0.083 \alpha}{4.665}, \frac{0.333}{4.235+0.415 \alpha}\right]  \tag{50}\\
\mathrm{LW}_{\mathrm{A} 5 / \mathrm{O}} & =\mathrm{LW}_{\mathrm{A} 6 / \mathrm{O}}=\mathrm{LW}_{\mathrm{A} 7 / \mathrm{O}} \\
& =\left[\frac{1}{4.665}, \frac{1}{4.235+0.415 \alpha}\right]  \tag{51}\\
\mathrm{LW}_{\mathrm{A} 1 / \mathrm{D}} & =\mathrm{LW}_{\mathrm{A} 3 / \mathrm{D}}=\mathrm{LW}_{\mathrm{A} / \mathrm{D}} \\
& =\left[\frac{0.496+0.214 \alpha}{4.571}, \frac{0.714}{3.863+0.696 \alpha}\right]  \tag{52}\\
\mathrm{LW}_{\mathrm{A} 2 / \mathrm{D}} & =\mathrm{LW}_{\mathrm{A} 4 / \mathrm{D}}=\mathrm{LW}_{\mathrm{A} 8 / \mathrm{D}} \\
& =\left[\frac{0.125+0.018 \alpha}{4.571}, \frac{0.143}{3.863+0.696 \alpha}\right]  \tag{53}\\
\mathrm{LW}_{\mathrm{A} / / \mathrm{D}} & =\mathrm{LW}_{\mathrm{A} 7 / \mathrm{D}}=\left[\frac{1}{4.571}, \frac{1}{3.863+0.696 \alpha}\right] \tag{54}
\end{align*}
$$

The FW of $\mathrm{A}_{1}-\mathrm{A}_{8}$ can be calculated by the fuzzy hierarchical arithmetic aggregation based on the fuzzy LWs in equations (42)-(54). The upper and lower of fuzzy FW of the controllable cause $\mathrm{A}_{1}-\mathrm{A}_{8}$ for all $\alpha$-level set can be respectively calculated by:

$$
\begin{align*}
& \left(\mathrm{FW}_{\vartheta}\right)_{\alpha}^{\mathrm{L}}=\sum_{\varphi \in\{\mathrm{S}, \mathrm{O}, \mathrm{D}\}}\left(\mathrm{LW}_{\varphi / \mathrm{FMEA}}\right)_{\alpha}^{\mathrm{L}}\left(\mathrm{LW}_{\vartheta / \varphi}\right)_{\alpha}^{\mathrm{L}}  \tag{55}\\
& \left(\mathrm{FW}_{\vartheta}\right)_{\alpha}^{\mathrm{U}}=\sum_{\varphi \in\{\mathrm{S}, \mathrm{O}, \mathrm{D}\}}\left(\mathrm{LW}_{\varphi / \mathrm{FMEA}}\right)_{\alpha}^{\mathrm{U}}\left(\mathrm{LW}_{\vartheta / \varphi}\right)_{\alpha}^{\mathrm{U}} \tag{56}
\end{align*}
$$

where $\vartheta \in\left\{\mathrm{A}_{1}, \cdots, \mathrm{~A}_{8}\right\}$. For example, the upper and lower of fuzzy FW of $\mathrm{A}_{1}$ can be calculated by:

$$
\begin{aligned}
\left(\mathrm{FW}_{\mathrm{A} 1} \mathrm{~L}_{\alpha}^{\mathrm{L}}=\right. & \left(\mathrm{LW}_{\mathrm{S} / \mathrm{FMEA}}\right)_{\alpha}^{\mathrm{L}}\left(\mathrm{LW}_{\mathrm{A} 1 / \mathrm{S}}\right)_{\alpha}^{\mathrm{L}} \\
& +\left(\mathrm{LW}_{\mathrm{O} / \mathrm{FMEA}}\right)_{\alpha}^{\mathrm{L}}\left(\mathrm{LW}_{\mathrm{A} 1 / \mathrm{O}}\right)_{\alpha}^{\mathrm{L}} \\
& +\left(\mathrm{LW}_{\mathrm{D} / \mathrm{FMEA}}\right)_{\alpha}^{\mathrm{L}}\left(\mathrm{LW}_{\mathrm{A} 1 / \mathrm{D}}\right)_{\alpha}^{\mathrm{L}} \\
= & \left(\frac{1}{1.8}\right)\left(\frac{1}{3.698}\right) \\
& +\left(\frac{0.326+0.266 \alpha}{1.8}\right)\left(\frac{0.247+0.083 \alpha}{4.665}\right)
\end{aligned}
$$

$$
\begin{align*}
+ & \left(\frac{0.166+0.033 \alpha}{1.8}\right)\left(\frac{0.496+0.214 \alpha}{4.571}\right) \\
= & 0.170+0.022 \alpha ; \forall \alpha \\
\left(\mathrm{FW}_{\mathrm{A} 1}\right)_{\alpha}^{\mathrm{U}}= & \left(\mathrm{LW}_{\mathrm{S} / \mathrm{FMEA}}\right)_{\alpha}^{\mathrm{U}}\left(\mathrm{LW}_{\mathrm{A} 1 / \mathrm{S}}\right)_{\alpha}^{\mathrm{U}} \\
& +\left(\mathrm{LW}_{\mathrm{O} / \mathrm{FMEA}}\right)_{\alpha}^{\mathrm{U}}\left(\mathrm{LW}_{\mathrm{A} 1 / \mathrm{O}}\right)_{\alpha}^{\mathrm{U}} \\
& +\left(\mathrm{LW}_{\mathrm{D} / \mathrm{FMEA}}\right)_{\alpha}^{\mathrm{U}}\left(\mathrm{LW}_{\mathrm{A} 1 / \mathrm{D}}\right)_{\alpha}^{\mathrm{U}} \\
= & \left(\frac{1}{1.492+0.299 \alpha}\right)\left(\frac{1}{3.193+0.495 \alpha}\right) \\
& +\left(\frac{0.6}{1.492+0.299 \alpha}\right)\left(\frac{0.333}{4.235+0.415 \alpha}\right) \\
& +\left(\frac{0.2}{1.492+0.299 \alpha}\right)\left(\frac{0.496+0.214 \alpha}{4.571}\right) \\
= & 0.266-0.073 \alpha ; \forall \alpha . \tag{58}
\end{align*}
$$

As the FW of controllable cause $\mathrm{A}_{1}$, the FWs of $\mathrm{A}_{2^{-}}$ $\mathrm{A}_{8}$ are shown as follows:

$$
\begin{align*}
& \mathrm{FW}_{\mathrm{A} 2}=[0.029+0.016 \alpha, 0.060-0.016 \alpha] ; \forall \alpha  \tag{59}\\
& \mathrm{FW}_{\mathrm{A} 3}=[0.036+0.022 \alpha, 0.080-0.021 \alpha] ; \forall \alpha  \tag{60}\\
& \mathrm{FW}_{\mathrm{A} 4}=[0.029+0.016 \alpha, 0.060-0.016 \alpha] ; \forall \alpha  \tag{61}\\
& \mathrm{FW}_{\mathrm{A} 5}=[0.086+0.070 \alpha, 0.210-0.055 \alpha] ; \forall \alpha  \tag{62}\\
& \mathrm{FW}_{\mathrm{A} 6}=[0.134+0.072 \alpha, 0.280-0.075 \alpha] ; \forall \alpha  \tag{63}\\
& \mathrm{FW}_{\mathrm{A} 7}=[0.209+0.040 \alpha, 0.340-0.092 \alpha] ; \forall \alpha  \tag{64}\\
& \mathrm{FW}_{\mathrm{A} 8}=[0.030+0.031 \alpha, 0.083-0.022 \alpha] ; \forall \alpha . \tag{65}
\end{align*}
$$

When substitute discrete $\alpha$-level set, which is $0,0.1$, $0.2, \cdots, 1$, the results are shown in Table 4.

Focusing on the second problem, damaged lid threads, it was discovered that the second problem has up to six controllable root causes $\left(B_{1}-B_{6}\right)$. Each root cause affects the problems within the criteria of $\mathrm{S}, \mathrm{O}$ and D differently. Therefore, the Judgment Matrix has a size of $6 x$ 6 of up to three matrices to compare the priorities of each cause, according to the three criteria as expressed in equations (42)-(44). Then, the relative efficiency and LW are calculated similar to LW of criteria.

Based on the criteria S in problem 2, $\theta_{\mathrm{B} 1 / \mathrm{S}}=1, \theta_{\mathrm{B} 2 / \mathrm{S}}$ $=[0.444+0.111 \alpha, 0.556]$ with $\mathrm{R}^{2}=100 \%, \theta_{\mathrm{B} 3 / \mathrm{S}}=[0.667$ $+0.111 \alpha, 0.778]$ with $\mathrm{R}^{2}=100 \%, \theta_{\mathrm{B} 4 / \mathrm{S}}=[0.222+$ $0.111 \alpha, 0.333]$ with $\mathrm{R}^{2}=100 \%, \theta_{\mathrm{B} 5 / \mathrm{S}}=\theta_{\mathrm{B} / \mathrm{S}}=0.111$, and the summation of fuzzy LW from DEA model $=[2.556+$ $0.333 \alpha, 2.889]$. Based on the criteria O in problem 2, $\theta_{\mathrm{B} 1 / \mathrm{O}}=\theta_{\mathrm{B} 3 / \mathrm{O}}=1, \theta_{\mathrm{B} 2 / \mathrm{O}}=\theta_{\mathrm{B} 4 / \mathrm{O}}=[0.326+0.266 \alpha, 0.6]$ with $\mathrm{R}^{2}=99.7 \%, \theta_{\mathrm{B} 5 / \mathrm{O}}=\theta_{\mathrm{B} 6 / \mathrm{O}}=[0.166+0.033 \alpha, 0.2]$ with $\mathrm{R}^{2}=99.7 \%$, and the summation of fuzzy LW from DEA model $=[2.984+0.598 \alpha, 3.6]$. Based on the criteria D in problem 2, $\theta_{\mathrm{B} 1 / \mathrm{D}}=\theta_{\mathrm{B} 2 / \mathrm{D}}=\theta_{\mathrm{B} 4 / \mathrm{O}}=[0.496+0.214 \alpha$, $0.714]$ with $\mathrm{R}^{2}=99.8 \%, \theta_{\mathrm{B} 3 / \mathrm{D}}=1, \theta_{\mathrm{B} 5 / \mathrm{O}}=\theta_{\mathrm{B} 6 / \mathrm{O}}=[0.125$ $+0.018 \alpha, 0.143]$ with $R^{2}=99.8 \%$, and the summation of
fuzzy LW from DEA model $=[2.738+0.678 \alpha, 3.428]$. In this paper, the fuzzy relative efficiency of each controllable cause is converted to be traditional fuzzy LWs by the concept extension principle. The results are shown as follows:
$\mathrm{LW}_{\mathrm{B} 1 / \mathrm{S}}=\left[\frac{1}{2.889}, \frac{1}{2.556+0.333 \alpha}\right]$
$\mathrm{LW}_{\mathrm{B} 2 / \mathrm{S}}=\left[\frac{0.444+0.111 \alpha}{2.889}, \frac{0.556}{2.556+0.333 \alpha}\right]$
$\mathrm{LW}_{\mathrm{B} 3 / \mathrm{S}}=\left[\frac{0.667+0.111 \alpha}{2.889}, \frac{0.778}{2.556+0.333 \alpha}\right]$
$\mathrm{LW}_{\text {B4/s }}=\left[\frac{0.222+0.111 \alpha}{2.889}, \frac{0.333}{2.556+0.333 \alpha}\right]$
$\mathrm{LW}_{\mathrm{B} 5 / \mathrm{S}}=\mathrm{LW}_{\mathrm{B} 6 / \mathrm{S}}=\left[\frac{0.111}{2.889}, \frac{0.111}{2.556+0.333 \alpha}\right]$
$\mathrm{LW}_{\mathrm{B} 1 / \mathrm{O}}=\mathrm{LW}_{\mathrm{B} 3 / \mathrm{O}}=\left[\frac{1}{3.6}, \frac{1}{2.984+0.598 \alpha}\right]$
$\mathrm{LW}_{\mathrm{B} 2 / \mathrm{O}}=\mathrm{LW}_{\mathrm{B} 4 / \mathrm{O}}=\left[\frac{0.326+0.266 \alpha}{3.6}, \frac{0.6}{2.984+0.598 \alpha}\right]$
$\mathrm{LW}_{\mathrm{B} 5 / \mathrm{O}}=\mathrm{LW}_{\mathrm{B} 6 / \mathrm{O}}=\left[\frac{0.166+0.033 \alpha}{3.6}, \frac{0.2}{2.984+0.598 \alpha}\right]$
$\mathrm{LW}_{\mathrm{B} 1 / \mathrm{D}}=\mathrm{LW}_{\mathrm{B} 2 / \mathrm{D}}=\mathrm{LW}_{\mathrm{B} 4 / \mathrm{O}}$

$$
\begin{equation*}
=\left[\frac{0.496+0.214 \alpha}{3.428}, \frac{0.714}{2.738+0.678 \alpha}\right] \tag{74}
\end{equation*}
$$

$\mathrm{LW}_{\mathrm{B} 3 / \mathrm{D}}=\left[\frac{1}{3.428}, \frac{1}{2.738+0.678 \alpha}\right]$
$\mathrm{LW}_{\mathrm{B} 5 / \mathrm{D}}=\mathrm{LW}_{\mathrm{B} 6 / \mathrm{D}}=\left[\frac{0.125+0.018 \alpha}{3.428}, \frac{0.143}{2.738+0.678 \alpha}\right]$
As the FW of controllable cause $A_{1}$, the FWs of $\mathrm{B}_{1^{-}}$ $\mathrm{B}_{6}$ are shown as follows:

$$
\begin{align*}
\mathrm{FW}_{\mathrm{B} 1} & =[0.256+0.056 \alpha, 0.432-0.122 \alpha] ; \forall \alpha  \tag{77}\\
\mathrm{FW}_{\mathrm{B} 2} & =[0.115+0.072 \alpha, 0.262-0.075 \alpha] ; \forall \alpha  \tag{78}\\
\mathrm{FW}_{\mathrm{B} 3} & =[0.205+0.073 \alpha, 0.388-0.111 \alpha] ; \forall \alpha  \tag{79}\\
\mathrm{FW}_{\mathrm{B} 4} & =[0.072+0.072 \alpha, 0.203-0.059 \alpha] ; \forall \alpha  \tag{80}\\
\mathrm{FW}_{\mathrm{B} 5} & =[0.033+0.012 \alpha, 0.063-0.018 \alpha] ; \forall \alpha  \tag{81}\\
\mathrm{FW}_{\mathrm{B} 6} & =[0.033+0.012 \alpha, 0.063-0.018 \alpha] ; \forall \alpha \tag{82}
\end{align*}
$$

When substitute discrete $\alpha$-level set, which is $0,0.1$, $0.2, \cdots, 1$, the results are shown in Table 5.

### 3.5 The Improvement and Adjustments according to the Priority of the causes

Taking the prioritized causes and adjusting them so that the top two causes were selected, it was discovered that the top causes of the first problem, having glass bits in the bottles, are $A_{7}$ the pressure applied when closing the lids and $\mathrm{A}_{6}$ the employees do not put the bottle into its slot properly. The top two causes of the problem of the damaged bottle lids are $B_{1}$ there are no scales to indicate the height of the shaft of the machinery and $B_{3}$ improper thickness of the bush supporters. The methodologies in solving these problems are stated in the following four Subsections.

### 3.5.1 Solving the problem caused by $\mathrm{A}_{7}$ the pressure applied

 when closing the lidsThe pressure applied in closing the threaded lids resulted in glass bits in the bottles. This can be solved by installing auxiliary equipment to absorb the pressure used in the production of the treaded lids.
3.5.2 Solving the problem caused by $\mathrm{A}_{6}$ employees did not properly put the bottle in the slot.
If the bottle is not properly placed in its slot, there would be a chance that there would be glass in the bottles and would lead to a damaged threaded lid and might become dangerous to the employees as well. The root cause of this problem is the carelessness of the employees themselves. Therefore, the improvement by using the Poka-Yoke system was applied by installing a Limit Switch, so that employees would arrange the bottles properly. The machine would still be operational when
the bottles touch the Limit Switch, but if no bottle touches it, then the machine would stop immediately.
3.5.3 Solving the problem caused by $\mathrm{B}_{1}$ no scales to indicate the height of the shaft of the machinery.
Without the scales to indicate the level of the shaft, the employees would have to use up to 30 minutes to install the machines with low reliability. This causes not only the damage of the lids, but also the glass bits in the bottles. Therefore, the cause was solved by installing a scale to indicate the height of the shaft.
3.5.4 Solving the problem caused by $\mathrm{B}_{3}$, improper thickness of the bush supporters.
The support rings for the bushes might not have the proper size when the bottle size changes. The adjustment requires the employees to apply a method of trial and error to find the right adjustment which wastes a large amount of time. If it was not adjusted properly, it would damage the lids. Therefore, the process may be improved by lathing the bushes, so that the lids can be threaded for the liquid medicine of each model.

## 4. RESULTS

Comparing the defects before and after the improvement by obtaining the information on the defects after the improvement for three months it was discovered that the percentage of the defects have decreased, as stated in Table 6 and the comparison of the characteristics of each problem, as illustrated in Figure 5.

Table 4. Final Weights of the glass bits in the bottles.

| $\alpha$-level | $\mathrm{A}_{1}$ |  | $\mathrm{A}_{2}$ |  | $\mathrm{A}_{3}$ |  | $\mathrm{A}_{4}$ |  | $\mathrm{A}_{5}$ |  | $\mathrm{A}_{6}$ |  | $\mathrm{A}_{7}$ |  | $\mathrm{A}_{8}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | L | U | L | U | L | U | L | U | L | U | L | U | L | U | L | U |
| 0 | 0.170 | 0.266 | 0.029 | 0.060 | 0.036 | 0.080 | 0.029 | 0.060 | 0.086 | 0.210 | 0.134 | 0.280 | 0.209 | 0.340 | 0.030 | 0.083 |
| 0.1 | 0.172 | 0.257 | 0.030 | 0.058 | 0.038 | 0.077 | 0.030 | 0.058 | 0.093 | 0.203 | 0.141 | 0.270 | 0.213 | 0.328 | 0.033 | 0.080 |
| 0.2 | 0.174 | 0.249 | 0.031 | 0.056 | 0.040 | 0.075 | 0.031 | 0.056 | 0.100 | 0.197 | 0.148 | 0.262 | 0.217 | 0.318 | 0.035 | 0.078 |
| 0.3 | 0.176 | 0.240 | 0.033 | 0.054 | 0.042 | 0.072 | 0.033 | 0.054 | 0.107 | 0.190 | 0.155 | 0.253 | 0.221 | 0.307 | 0.038 | 0.075 |
| 0.4 | 0.178 | 0.233 | 0.034 | 0.053 | 0.044 | 0.070 | 0.034 | 0.053 | 0.114 | 0.184 | 0.162 | 0.245 | 0.225 | 0.298 | 0.042 | 0.073 |
| 0.5 | 0.180 | 0.225 | 0.036 | 0.051 | 0.046 | 0.068 | 0.036 | 0.051 | 0.120 | 0.179 | 0.169 | 0.238 | 0.229 | 0.288 | 0.045 | 0.071 |
| 0.6 | 0.182 | 0.218 | 0.038 | 0.050 | 0.049 | 0.066 | 0.038 | 0.050 | 0.127 | 0.174 | 0.177 | 0.230 | 0.233 | 0.279 | 0.048 | 0.069 |
| 0.7 | 0.185 | 0.211 | 0.039 | 0.048 | 0.051 | 0.064 | 0.039 | 0.048 | 0.134 | 0.168 | 0.184 | 0.224 | 0.237 | 0.271 | 0.051 | 0.067 |
| 0.8 | 0.187 | 0.205 | 0.041 | 0.047 | 0.053 | 0.062 | 0.041 | 0.047 | 0.141 | 0.164 | 0.191 | 0.217 | 0.241 | 0.263 | 0.054 | 0.065 |
| 0.9 | 0.189 | 0.199 | 0.043 | 0.046 | 0.056 | 0.060 | 0.043 | 0.046 | 0.148 | 0.159 | 0.198 | 0.211 | 0.245 | 0.255 | 0.058 | 0.063 |
| 1 | 0.192 | 0.192 | 0.045 | 0.045 | 0.058 | 0.058 | 0.045 | 0.045 | 0.155 | 0.155 | 0.205 | 0.205 | 0.249 | 0.249 | 0.061 | 0.061 |
| Rank | 3 |  | 7 |  | 6 |  | 7 |  | 4 |  | 2 |  | 1 |  | 5 |  |

Table 5. Final Weights of the damaged lid threads.

| $\alpha$-level | $\mathrm{B}_{1}$ |  | $\mathrm{B}_{2}$ |  | $\mathrm{B}_{3}$ |  | $\mathrm{B}_{4}$ |  | $\mathrm{B}_{5}$ |  | $\mathrm{B}_{6}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Lower | Upper | Lower | Upper | Lower | Upper | Lower | Upper | Lower | Upper | Lower | Upper |
| 0 | 0.256 | 0.432 | 0.115 | 0.262 | 0.205 | 0.388 | 0.072 | 0.203 | 0.033 | 0.063 | 0.033 | 0.063 |
| 0.1 | 0.261 | 0.417 | 0.121 | 0.252 | 0.213 | 0.374 | 0.078 | 0.196 | 0.034 | 0.061 | 0.034 | 0.061 |
| 0.2 | 0.267 | 0.402 | 0.127 | 0.243 | 0.22 | 0.361 | 0.085 | 0.189 | 0.035 | 0.059 | 0.035 | 0.059 |
| 0.3 | 0.272 | 0.389 | 0.134 | 0.235 | 0.227 | 0.348 | 0.091 | 0.182 | 0.036 | 0.057 | 0.036 | 0.057 |
| 0.4 | 0.278 | 0.376 | 0.141 | 0.227 | 0.235 | 0.336 | 0.098 | 0.176 | 0.038 | 0.055 | 0.038 | 0.055 |
| 0.5 | 0.284 | 0.363 | 0.148 | 0.219 | 0.242 | 0.325 | 0.105 | 0.17 | 0.039 | 0.053 | 0.039 | 0.053 |
| 0.6 | 0.289 | 0.352 | 0.155 | 0.212 | 0.249 | 0.314 | 0.112 | 0.164 | 0.04 | 0.051 | 0.04 | 0.051 |
| 0.7 | 0.295 | 0.341 | 0.162 | 0.205 | 0.256 | 0.304 | 0.12 | 0.158 | 0.041 | 0.049 | 0.041 | 0.049 |
| 0.8 | 0.300 | 0.330 | 0.17 | 0.199 | 0.264 | 0.295 | 0.128 | 0.153 | 0.042 | 0.048 | 0.042 | 0.048 |
| 0.9 | 0.306 | 0.320 | 0.178 | 0.193 | 0.271 | 0.285 | 0.136 | 0.148 | 0.044 | 0.046 | 0.044 | 0.046 |
| 1 | 0.311 | 0.311 | 0.187 | 0.187 | 0.278 | 0.278 | 0.144 | 0.144 | 0.045 | 0.045 | 0.045 | 0.045 |
| Rank | 1 |  | 3 |  | 2 |  | 4 |  | 5 |  | 5 |  |

Table 6. The characteristics of defects before and after the improvement.

| No. | Problem | Before improvement <br> (production quantity 100,000 bottles) |  |  | After improvement <br> (production quantity 200,000 bottles) |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | defect <br> quantity | $\%$ defect | $\%$ cumulative <br> defect | defect <br> quantity | $\%$ defect | $\%$ cumulative <br> defect |
| 1 |  | 1,525 | 1.525 | 1.525 | 1,643 | 0.822 | 0.822 |
| 2 | damage bottle top threads | 1,315 | 1.315 | 2.840 | 1,694 | 0.847 | 1.669 |
| 3 | defect on the bottle lids | 196 | 0.196 | 3.036 | 0 | 0 | 1.669 |
| 4 | the deformation of the bottle lids | 121 | 0.121 | 3.157 | 0 | 0 | 1.669 |
| 5 | others | 52 | 0.052 | 3.209 | 0 | 0 | 1.669 |



Figure 5. Histogram of the percentage of defects before and after the improvement.

## 5. CONCLUSION

This paper used FDEAHP to rank the significance of causes of problems based on the framework of the FMEA technique. By using FDEAHP it was found that selecting the two priority causes to be improved reduced defects from $3.209 \%$ to $1.669 \%$ or by $47.99 \%$. Therefore, by applying the FDEAHP, it is possible to solve the problems more efficiently.

## 6. FUTURE STUDIES

As the FDEAHP model can be used to deal with only the fuzziness, it is efficient in a case where there is one decision maker. However, in many case the decision must be made by a team. Therefore, there is not only fuzziness uncertainty from a human judgment, but also randomness uncertainty from different of decision makers' experience. The fuzzy stochastic data envelopment analytical hierarchy process (FSDEAHP), which is used to confront both fuzziness and randomness, will be studied to support the team's decision.

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