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A Manufacturing/Remanufacturing System with the Consideration of Required Quality of End-of-used Products

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Abstract. A manufacturing/remanufacturing system is investigated with the consideration of required minimum quality of end-of-used products. A constant demand is satisfied by remanufacturing end-of-used products and manufacturing raw materials outsourced from outside. It is assumed in this system that the buyback price and remanufacturing cost are related to the different quality level of end-of-used products. For remanufacturing, only the used products that satisfy a required minimum quality level will be recycled. Thus, the returning rate is a function of the required minimum quality level. Functions of returning rate, buyback price and remanufacturing cost, which are closely connected to the quality level of end-of-used products, are investigated here. Treating the required minimum quality level of end-of-used products, the length of a cycle, the number of manufacturing lots and remanufacturing lots in a cycle as decision variables, the mathematical models with the objective of minimizing the average total cost are constructed. Through construction of a solution process based on Tabu Search algorithm and calculating examples, the validity of the models is illustrated.

Keywords: Buyback Cost, Manufacturing/Remanufacturing System, Quality Level, Remanufacturing Cost, Return Rate

1. INTRODUCTION

The current state and trend of environmental degradation (from regulatory, consumer, and moral standpoints) indicate a need for a change in manufacturing philosophy. That is, there must be a fundamental shift in the way production systems operate, (Alexander, 1996). During the past years, emerging terms such as industrial ecology, green supply chain, waste management, EPR (extending producers' responsibility) reflects endeavors that have been made on strategetic and operational aspects about this philosophy (Lindhqvist, 2001; Spicer and Johon, 2004; Sergio Rubio *et al*., 2008).

While remanufacturing activities are often motivated by environmental concerns or demands from customers or government authorities, the processing of returns has increasingly been viewed not simply as a cost of doing business, but as a profitable business model and source of competitive advantage (Galbreth, 2006; Yune, 2008). And remanufacturing system has unique advantage, unlike recycling and re-use, the process of remanufacturing does not degrade or even exceed the

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overall value of the materials used (Thierry *et al*., 1995; Beamon, 1999). Realizing the potential for profitable remanufacturing, many businesses have emerged to exploit specific remanufacturing opportunities (Wells and Seitz, 2004).

However, there exist differences and difficulties. The major difference between reverse logistics/remanufacturing and forward logistics/manufacturing involves the supply side (Fleischmann *et al*., 1997; Fleischmann, 2001). In a remanufacturing system, supply is largely exogenous, and the timing, quantity, and quality of supply are much more uncertain than in traditional production distribution systems (Trebilcock, 2002).

This study is mainly focused on the variable quality of end-of-used products. Generally, quality level of end-of-used products is uncertain and will affect remanufacturing cost and remanufacturing yield. If the quality is very low, remanufacturing process will be complex, and there should be more input than usual (e.g., expensive chemical pulp in newspaper manufacturing industry), and/or the output might not be assured, as can be seen in newspaper manufacturing industry and other industries, in which remanufacturing process have much relationship with the quality of input items, such as scrap metal recycling industry.

Since reverse logistic model (a deterministic EOQ-type model) was first analyzed by Schrady (Schrady, 1967), many studies have acknowledged this problem of variable supply quality for remanufacturing systems (Richter, 1996a; 1996b; Fleischmann *et al*., 2000; Teunter, 2001; Savaskan *et al*., 2004). Teunter (1998) has offered a model, where not all items can be remanufactured, but this model assumed a known return rate. Krikke *et al.* (1999) also acknowledge variability in returned product quality, but their MILP formulation of a reverse logistics network uses a constant unit cost of remanufacturing. Jayaraman *et al.* (1999) and Majumder and Groenevelt (2001) also model unit-remanufacturing cost as a constant. Guide *et al.* (2000) proposed assessment of conditions of end-of-used products, which is an important step in determining the optimal recovery action. Fleischmann (2001) built a simulation model that incorporates uncertainty in the quality of end-of-used products and uses this model to evaluate several reverse logistics configurations. Guide and Van Wassenhove (2001) proposed a model to manage the quality of cellular phone returns to help reduce variation in processing times. Salameh and Jaber (2000), Huang and Chang (2004) have investigated stochastic EOQ models with quality consideration and reselling defective items. Singer *et al.* (2003) have analyzed a game theoretic model in a supply chain with quality and disposable items. Aras *et al.* (2004) researched the effect of categorizing end-of-used products in remanufacturing. Dobos and Richter (2006) extended their previous work assuming that the quality of collected used/returned items is not always appropriate for further recycling. This paper main addressed the optimal production choice. Choi and Hwang (2007) extended their former work and investigated a generalized ordering and recovery policy for different quality of reusable items. Their paper assumed that a fixed proportion of the used products are collected from customers and later recovered for reuse. In Paper of Yune and Hwang (2008), quality and buy back price of returning items are both considered, but based on an assumption that they are independent.

In this research, a manufacturing/remanufacturing system is investigated with the consideration of required minimum quality of end-of-used products where a constant demand is satisfied by remanufacturing and manufacturing process. It is assumed that return rate, buyback price and remanufacturing cost are dependent on different quality level of end-of-used products. For remanufacturing, only the used products that satisfy a required minimum quality level will be recycled.

In contrast to the above literature, here returning rate, buyback price and remanufacturing cost are all unknown, and the quality of end-of-used products is assumed to be random variable. The remainder of this paper is organized as follows: Section 2 details the notations and assumptions. Section 3 introduces functions of return rate, buyback price and remanufacturing cost. In section 4 the mathematical models are developed with the objective function of minimizing the average total cost of two cases. In case 1, a production circle is supposed to be consisted of a single remanufacturing run and a single manufacturing run, both in EPQ settings. Here the relationship between the objective function and required minimum quality, return rate, buyback price, remanufacturing cost, as well as the length of a circle is investigated. And in case 2, a circle is extended to *m* remanufacturing runs and *n* manufacturing runs, the effect of this extension on the system is investigated. Section 5 proposes a solution methodology based on Tabu Search method. Numerical examples and sensitivity analysis are shown in section 6. In section 7 the obtained results are summarized and some directions for further research are proposed.

2. NOTATIONS AND ASSUMPTIONS

To model the manufacturing/remanufacturing system, the following notations and assumptions are introduced.

2.1 Notations

2.1.1 Decision variables
Q : The required minimum quality level of returned used products, $0 \le q \le 1$;
T : Length of a manufacturing and remanufacturing cycle [time];
m : Number of remanufacturing lots, positive integer;
n : Number of manufacturing lots, positive integer.

2.1.2 Input parameters

a : Parameter of the buyback cost function, $0 \leq a \leq 1$;
θ : Parameter of the buyback cost function;
b : Parameter of the return rate function, $0 \leq b \leq 1$;
φ : Parameter of the return rate function;
c : Parameter of the remanufacturing cost function, $0 \leq c \leq 1$;
δ : Parameter of the remanufacturing cost function;
h_S : Holding cost per unit per unit time of serviceable stock, [\$]/[unit][time];
h_R : Holding cost per unit per unit time of returned stock, [\$]/[unit][time];
h_{raw} : Holding cost per unit per unit time of raw material stock, [\$]/[unit][time];
C_M : Manufacturing cost per unit, [\$]/[unit];
D : Demand rate, [unit]/[time];
$(1/\beta)D$: Manufacturing rate ($\beta < 1$), [unit]/[time];
$(1/\gamma)D$: Remanufacturing rate ($\gamma < 1$), [unit]/[time];
S_R : Remanufacturing setup cost, [\$];
S_M : Manufacturing setup cost, [\$];
C_O : Ordering cost, [\$];
C_{raw} : Purchasing cost for raw material per unit, [\$]/[unit];

2.1.3 Parameters

$D = \alpha D$: Return rate, $0 \leq \alpha \leq 1$;
α : Marginal Return rate, $0 \leq \alpha \leq 1$;
p : Buyback price ratio (related to different quality level) to the unit production cost (unit manufacturing cost plus unit raw material cost, i.e., *CM+Craw*) of a new product, unit buyback price is $p(CM+Craw)$;
c_r : Remanufacturing cost ratio (related to different quality level) to manufacturing cost-C_M, unit remanufacturing cost is $c_r \cdot C_M$;
$H_{S,M}$: The average inventory cost of manufactured products, [\$]/[time];
$H_{S,R}$: The average inventory cost of remanufactured products, [\$]/[time];
H_R : The average inventory cost of end-of-used products, [\$]/[time];
H_{raw} : The average inventory cost of raw material, [\$]/[time];
T_R : Time interval of a remanufacturing lot;
T_M : Time interval of a manufacturing lot.

2.2 Assumptions

1) Demand rate is known and constant;
2) The quality of returned items is uniformly distributed;
3) Unit-buyback price is related to the quality level of end-of-used products;
4) Unit remanufacturing cost is affected by the quality level of end-of-used products;
5) Return rate of the used products is dependent on the required minimum quality level of end-of-used products, and return rate dose not exceed the demand rate;
6) There is no disposal of end-of-used products, i.e., all of the end-of-used products are remanufactured into new products;
7) "As good as new" policy is adopted;
8) Raw materials required for the remanufacturing of new products is purchased from outside in an EOQ setting;
9) Both manufacturing and remanufacturing setup cost are significant, i.e., they are not negligible and a simultaneous consideration of the involved trade-offs is required;
10) Both manufacturing and remanufacturing processes have negligible lead times, which is not a limitation since respective replenishments can be scheduled in advance to account for fixed delays;
11) No shortage is allowed;
12) For each item at the serviceable inventory, irrespective of being remanufactured or manufactured, a corresponding inventory holding cost rate is charged;
13) The inventory holding cost rate of serviceable items is bigger than that of recoverable items and raw materials.

3. FUNCTIONS

According to assumptions, return rate, buyback price and remanufacturing cost are related to quality level of used products. When recycling, if the required quality level is rather high, only little used product can be recycled and vice versa. As regard to buyback cost, because the quality of returned used products is different and it will affect remanufacturing cost, different price strategy (according to different quality level) is necessary. In reality, if the quality of used products is rather high, the remanufacturing/manufacturing companies are always willing to recycle them even with a little higher buyback price (this can be seen in newspaper manufacturing industry). The case of remanufacturing cost is similar. To construct formulations, the popular negative exponential functions were modified (Roberts and Urban, 1988; Smith and Achabal, 1998; Vörös, 2002).

3.1 Return rate function

According to analyses above, for a required minimum quality level q, $d = \alpha D$ is defined as return rate for a constant demand rate D, and α as: $\alpha = be^{-\varphi q}$, $q \in [0, 1]$;

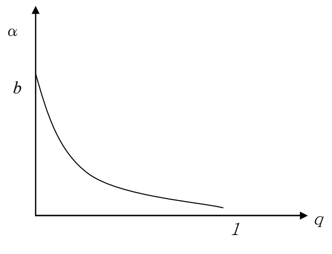

Figure 1. Marginal return rate α.

Here return rate is a function of required minimum quality level-q because the maximum supply of returned items is fixed during a circle time T.As can be seen from figure 1, if the required quality level is rised, the return rate will decrease, and vice versa. Note that parameters φ and b can be adjusted in different conditions.

3.2 Buyback price function

Buyback price ratio p (related to different quality level x) is defined as a function of the production cost of a new product (here production cost is defined as manufacturing cost plus raw material cost, i.e., $C_M + C_{raw}$) as: $p = ae^{-\theta(1-x)},\ \ x \in [q, 1]$;

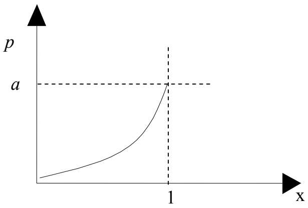

Figure 2. Buyback price ratio p.

Here buyback price is defined as a function of production cost because returned items, like raw materials, is deemed as a kind of production resource. If its price is higher than production cost, it will be not economical to return it. It should be mentioned that the quality of buyback items can affect remanufacturing cost. Note that parameters θ and a can be adjusted in different conditions.

With a required minimum quality level q, return rate can be defined as: $d = \alpha D$, then the average buyback cost is:

$$\alpha D \times (C_M + C_{raw}) \times E(p)\text{, Here}$$

$$E(p) = \int_q^1 \frac{1}{1-q} ae^{-\theta(1-x)} dx = \frac{a}{1-q} \times \frac{1}{\theta} \times [e^{-\theta(1-x)}]_q^1$$

$$= \frac{a}{(1-q)\theta} \times \left(1 - e^{-\theta(1-q)}\right);$$

So the average buyback cost is:

$$\alpha D \times (C_M + C_{raw}) \times E(p)$$

$$= \alpha D \times (C_M + C_{raw}) \times \frac{a}{(1-q)\theta} \times \left(1 - e^{-\theta(1-q)}\right);$$

3.3 Remanufacturing cost function

Similar to buyback price function, remanufacturing cost ratio c_r (related to different quality level x) is defined as: $c_r = ce^{\delta(1-x)},\ \ x \in [q, 1]$;

With the return rate $d = \alpha D$, all the returned/buyback items will be remanufactured, so the average remanufacturing cost is: $\alpha D \times C_M \times E(c_r)$, here

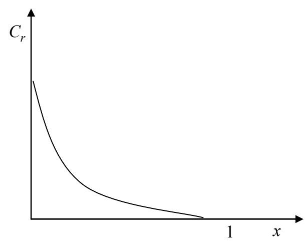

Figure 3. Remanufacturing cost ratio C_r.

$$E(c_r) = \int_q^1 \frac{1}{1-q} ce^{\delta(1-x)} dx = \frac{c}{1-q} \times \frac{1}{\delta} \times \left[-e^{\delta(1-x)}\right]_q^1$$

$$= \frac{c}{(1-q)\delta} \times \left(e^{\delta(1-q)} - 1\right)$$

So the average remanufacturing cost is:

$$\alpha D \times C_M \times E(c_r)$$

$$= \alpha D \times C_M \times \frac{c}{(1-q)\delta} \times \left(e^{\delta(1-q)} - 1\right)$$

Here remanufacturing cost is considered to be related with the quality of returned items, and remanufacturing cost be very low (less than the manufacturing cost) with the high quality of returned items and very high (more than the manufacturing cost) with the low quality of returned items. The reasons for this are listed as follows: If the quality of returned items is rather high, then the manufacturing process can be rather simple, the input can be greatly saved on, and/or the output can be assured, as stated above, which makes remanufacturing cost rather low, and vice versa.

Note that parameters δ and c can be adjusted in different conditions. As regard to continuous distributions of remanufacturing cost, please refer to Galbreth (2006), LI (2007), LI and DA (2008) and Yune (2008).

4. MODELING OF THE SYSTEMS

In this section, mathematical models with the objective function of minimizing the average total cost of two cases are developed. In case *1*, it is assumed that a production circle consists of a single remanufacturing run and a single manufacturing run, both in EPQ settings, i.e., $1T = 1T_R + 1T_M$. In case *2*, a circle is extended to m remanufacturing runs and n manufacturing runs in a circle, i.e., $1T = mT_R + nT_M$.

4.1 Case *1* ($1T = 1T_R + 1T_M$)

4.1.1 Materials flow and inventory status

Figure 4 shows the general framework of a manufacturing and remanufacturing process. In a circle time T (T is composed of a remanufacturing run-T_R and a manufacturing run-T_M), a constant demand DT is satisfied by remanufactured products and newly manufactured products. End-of-used products are bought back at a rate $d = \alpha D$ for remanufacturing while raw material required is purchased from outside in an EOQ setting. The quantity of bought back used products is $dT = \alpha DT$, which is remanufactured without any disposal. So in a circle T, the quantity of remanufactured products is $dT = \alpha DT = DT_R$. And the quantity of manufactured products is $DT-dT = (1-\alpha)DT = DT_M$. The system has three kinds of inventory stocks, serviceable stock (consists of remanufactured products and manufactured products), and returned stock and raw material stock (Figure 5). The remanufactured products inventory builds up at a rate of $(1/\gamma-1)\,D$ units per unit of time and stops at its peak $(1-\gamma)\,DT_R$. Then begins the manufacturing process, the manufactured products inventory builds up at a rate of $(1/\beta-1)\,D$ units per unit of time and stops at its peak $(1-\beta)\,DT_M$. At the same time, raw material ordered from outside in an EOQ setting to supply manufacturing process decreases with the rate of $(1/\beta)\,D$ from its highest level DT_M. Note that remanufacturing process begins when the inventory level of end-of-used products reaches the highest level of $\alpha D\,(T-\gamma T_R)$. After that, the inventory decreases with the rate of $\alpha D-(1/\gamma)D$.

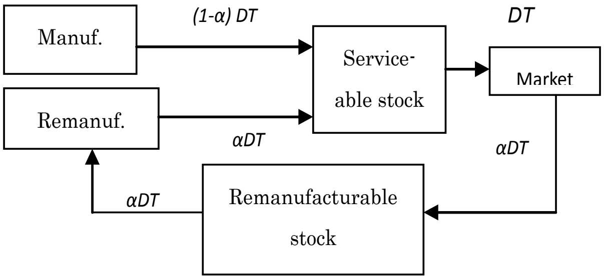

Figure 4. Material flow in a cycle.

According to above analysis, the related formulas are listed below:

$$1T = 1T_R + 1T_M;$$
$$DT_R = \alpha DT,\ T_R = \alpha T;$$
$$DT_M = (1-\alpha)DT,\ T_M = (1-\alpha)T;$$

$I_R = (1-\gamma)DT_R = (1-\gamma)\alpha DT;$ I_R is the maximum remanufactured products inventory;

$I_M = (1-\beta)DT_M = (1-\beta)(1-\alpha)DT;$ I_M is the maximum manufactured products inventory;

$I_r = (\frac{1}{\gamma}-\alpha)D\gamma T_R = (1-\alpha\gamma)DT_R = (1-\alpha\gamma)\alpha DT;$ I_r is the maximum end-of-used products inventory;

$I_{raw} = DT_M = (1-\alpha)DT;$ I_{raw} is the maximum raw material inventory.

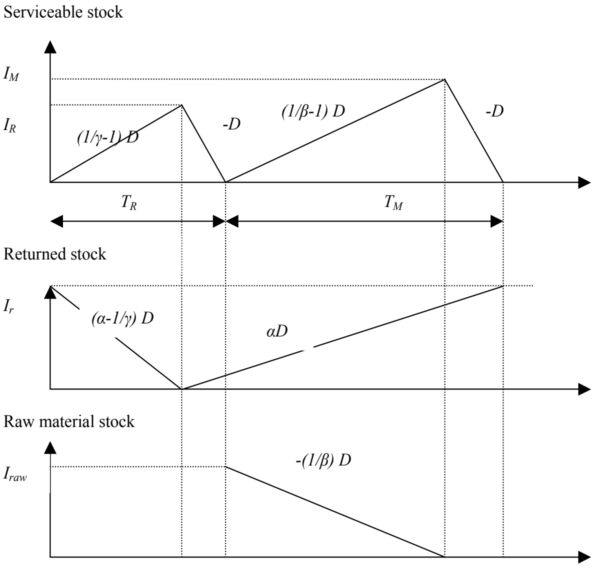

Figure 5. Inventory status of serviceable stock, returned stock and raw material stock ($1T = 1T_R + 1T_M$).

4.1.2 Average inventory holding cost

According to section 2, $H_{S,R}$, $H_{S,M}$, H_R, $Hraw$ denote the average inventory costs of serviceable stocks by manufactured products, serviceable stocks by remanufactured products, returned stock and raw material stock respectively.

The average inventory cost of remanufactured products is:

$$H_{S,R} = \frac{1}{2}h_S(1-\gamma)\alpha DTT_R\frac{1}{T} = \frac{1}{2}h_S(1-\gamma)\alpha^2 DT; \tag{1}$$

The average inventory cost of manufactured products is:

$$H_{S,M} = \frac{1}{2}h_S(1-\beta)(1-\alpha)DTT_M\frac{1}{T} = \frac{1}{2}h_S(1-\beta)(1-\alpha)^2 DT; \tag{2}$$

The average inventory cost of returned stock is:

$$H_R = \frac{1}{2}h_R(1-\alpha\gamma)\alpha DTT\frac{1}{T} = \frac{1}{2}h_R(1-\alpha\gamma)\alpha DT; \tag{3}$$

The average inventory cost of raw material stock is:

$$H_{raw} = \frac{1}{2}h_{raw}(1-\alpha)DT\beta T_M\frac{1}{T} = \frac{1}{2}h_{raw}(1-\alpha)^2\beta DT; \tag{4}$$

4.1.3 Average total cost

The average total cost consists of average inventory holding cost, setup cost, ordering cost, remanufacturing cost, manufacturing cost, raw material cost and buyback cost.

The average inventory holding cost is (1)+(2)+(3)+(4), i.e.,

$$\frac{1}{2}DT\{h_S[(1-\gamma)\alpha^2+(1-\beta)(1-\alpha)^2]+h_{raw}(1-\alpha)^2\beta+h_R(1-\alpha\gamma)\alpha\}; \tag{5}$$

The average setup cost is: $\frac{(S_R+S_M)}{T}$ (6)

The average ordering cost is: $\frac{C_O}{T}$ (7)

The average remanufacturing cost is:

$$\alpha D\times C_M\times E(c_r)=\alpha D\times C_M\times\frac{c}{(1-q)\delta}\times\left(e^{\delta(1-q)}-1\right) \tag{8}$$

The average manufacturing cost is: $(1-\alpha)DC_M$ (9)

The average buyback cost is:

$$\begin{aligned}&\alpha D\times(C_M+C_{raw})\times E(p)=\\&\alpha D\times(C_M+C_{raw})\times\frac{a}{(1-q)\theta}\times\left(1-e^{-\theta(1-q)}\right)\end{aligned} \tag{10}$$

The average raw material cost is: $(1-\alpha)DC_{raw}$ (11)

So, the average total cost (ATC) can be expressed as: (5)+(6)+(7)+(8)+(9)+(10)+(11);

Bring $\alpha=be^{-\varphi q}$ into the formula and after simple calculation: *ATC (T, q) =*

$$\begin{aligned}&\frac{1}{2}DT\{h_S[(1-\gamma)b^2e^{-2\varphi q}+(1-\beta)(1-be^{-\varphi q})^2]\\&+h_{raw}(1-be^{-\varphi q})^2\beta+h_R(1-\gamma be^{-\varphi q})be^{-\varphi q}\}\\&+D\times b\times C_M\times\frac{c}{\delta}\times\frac{1}{1-q}\times e^{-\varphi q}\times\left(e^{\delta(1-q)}-1\right)\\&+(1-be^{-\varphi q})(C_M+C_{raw})D+\frac{(S_R+S_M+C_O)}{T}\\&+Db\times(C_M+C_{raw})\times\frac{a}{\theta}\times\frac{1}{1-q}\times e^{-\varphi q}\times\left(1-e^{-\theta(1-q)}\right)\end{aligned} \tag{12}$$

4.2 Case *2* ($1T=mT_R+nT_M$)

Now a circle is extended to m remanufacturing runs and n manufacturing runs in a circle, i.e., $1T=mT_R+nT_M$. This kind of arrangement is generally called (*m*, *n*) strategy (Dobos and Richter, 2004; Dai Ying and Ma 2006; Dobos and Knut Richter, 2006). The effect of this extension on the system is investigated here.

4.2.1 Materials flow and inventory status

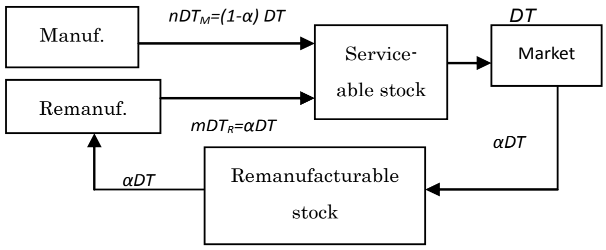

Figure 6. Material flow in a cycle.

Figure 6 shows the general framework of a manufacturing and remanufacturing process. In a circle time *T*(*T* is composed of *m* remanufacturing runs-mT_R and *n* manufacturing run-nT_M), a constant demand *DT* is satisfied by remanufactured products and newly manufactured products. End-of-used products are bought back at a rate $d=\alpha D$ for remanufacturing while raw material required is purchased from outside in an EOQ setting. The quantity of bought back used products is $dT=\alpha DT$, which is remanufactured without any disposal. So in a circle *T*, the quantity of remanufactured products is $dT=\alpha DT=m\,DT_R$. And the quantity of manufactured products is $DT-dT=(1-\alpha)\,DT=nDT_M$.

Similar to 4.1.1, the related formula are as follows (Figure 7):

$$\begin{aligned}&1T=mT_R+nT_M;\\&mDT_R=\alpha DT,\ mT_R=\alpha T;\\&nDT_M=(1-\alpha)DT,\ nT_M=(1-\alpha)T;\end{aligned}$$

$I_R=(1-\gamma)DT_R=\frac{1}{m}(1-\gamma)\alpha DT$; I_R is the maximum remanufactured products inventory;

$I_M=(1-\beta)DT_M=\frac{1}{n}(1-\beta)(1-\alpha)DT$; I_M is the maximum manufactured products inventory;

$I_r=m(\frac{1}{\gamma}-\alpha)D\gamma T_R-(m-1)\alpha D(T_R-\gamma T_R)$
$=[m(1-\gamma\alpha)-(m-1)\alpha(1-\gamma)]DT_R=\frac{[(1-\alpha)m+\alpha(1-\gamma)]}{m}\alpha DT$; I_r is the maximum end-of-used products inventory;

$I_{raw}=nDT_M=(1-\alpha)DT$; I_{raw} is the maximum raw material inventory.

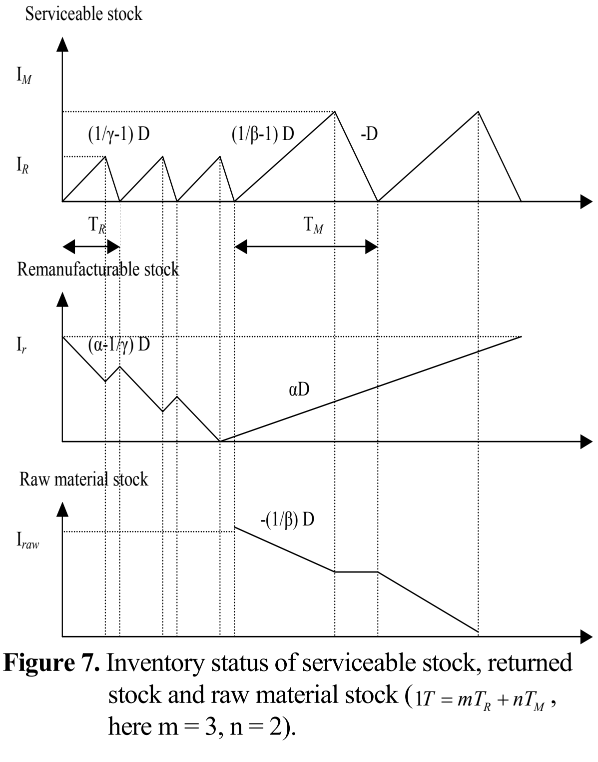

Figure 7. Inventory status of serviceable stock, returned stock and raw material stock ($1T=mT_R+nT_M$, here m = 3, n = 2).

4.2.2 Average inventory holding cost

The inventory status here is similar to 4.1.2.

The average inventory cost of remanufactured products is:

$$H_{S,R}=\frac{1}{2}h_S(1-\gamma)\alpha DTT_R\frac{1}{T}=\frac{1}{2}h_S\frac{1}{m}(1-\gamma)\alpha^2DT; \tag{13}$$

The average inventory cost of manufactured products is:

$$H_{S,M}=\frac{1}{2}h_S(1-\beta)(1-\alpha)DTT_M\frac{1}{T}=\frac{1}{2}h_S\frac{1}{n}(1-\beta)(1-\alpha)^2DT; \tag{14}$$

The average inventory cost of returned stock is:

$$H_R=\frac{1}{2}h_R[\frac{1}{m}(1-\gamma)\alpha^2+(1-\alpha)\alpha]DT; \tag{15}$$

The average inventory cost of raw material stock is:

$$H_{raw}=\frac{1}{2}h_{raw}(1-\alpha)^2[1-(1-\beta)\frac{1}{n}]\}DT; \tag{16}$$

4.2.3 Average total cost

Here the total average inventory cost is (13) + (14) + (15) + (16), i.e.

$$\begin{aligned}&\frac{1}{2}\{h_s[\frac{1}{m}(1-\gamma)\alpha^2+\frac{1}{n}(1-\beta)(1-\alpha)^2]+h_R[\frac{1}{m}(1-\gamma)\alpha^2\\&+(1-\alpha)\alpha]+h_{raw}(1-\alpha)^2[1-(1-\beta)\frac{1}{n}]\}DT;\end{aligned} \tag{17}$$

$$\textit{The average setup cost is}:\frac{(mS_R+nS_M)}{T}; \tag{18}$$

Other costs are same as 4.1.3, so the average total cost (ATC) can be expressed as

ATC (T, m, n, q) = (17)+(18)+(7)+(8)+(9)+(10)+ (11);

Bring $\alpha=be^{-\varphi q}$ into the formula and after simple calculation: *ATC (T, m, n, q) =*

$$\begin{aligned}&\frac{1}{2}DT\{h_S[\frac{1}{m}(1-\gamma)b^2e^{-2\varphi q}+\frac{1}{n}(1-\beta)(1-be^{-\varphi q})^2]\\&+h_{raw}(1-be^{-\varphi q})^2[1-(1-\beta)\frac{1}{n}]\\&+h_R[\frac{1}{m}(1-\gamma)b^2e^{-2\varphi q}+(1-be^{-\varphi q})be^{-\varphi q}]\}\\&+D\times b\times C_M\times\frac{c}{\delta}\times\frac{1}{1-q}\times e^{-\varphi q}\times\left(e^{\delta(1-q)}-1\right)\\&+(1-be^{-\varphi q})(C_M+C_{raw})D+\frac{(mS_R+nS_M+C_O)}{T}\\&+Db\times(C_M+C_{raw})\times\frac{a}{\theta}\times\frac{1}{1-q}\times e^{-\varphi q}\times\left(1-e^{-\theta(1-q)}\right)\end{aligned} \tag{19}$$

5. TABU SEARCH SOLUTION ALOGRITHM

In this study, decision variables are the required minimum quality level of returned used product-*q*, the length of a manufacturing and remanufacturing cycle-*T*, the number of remanufacturing lots-*m* and the number of manufacturing lots-*n*. In case 1, *m* and *n* are given and equal to 1 but in case 2, the value of *m*, *n* are unknown. Due to the complexity of the model, Tabu Search method (a kind of meta-heuristic solution approach) is used to find its solution.

5.1 Introduction of Tabu Search

Tabu search is a kind of meta-heuristic algorithm that can be used for solving wide variety of classical and practical problems such as traveling salesman problem (TSP), job shop problem (JSP) or vehicle routing problem (VRP). The basic concept of Tabu search is introduced by Glover (1990). Tabu search starts at some initial point and then moves to neighborhood point that gives the best value of the objective function at each iteration. This move continues until some stopping criterion has been satisfied. In tabu search, the most important feature is the tabu list that consists of the latest move made. With tabu list, revisiting the same point again is avoided, but can also restrict to move to the better point. To overcome this limitation of tabu list, aspiration criterion exists. Although the movement to best neighborhood point is in the tabu list, this point can be found if it satisfies aspiration criterion. A commonly used aspiration criterion is to allow solutions, which are better than the currently known best solution. In its simplest form, tabu search requires the following ingredients:

- Initial point;
- Neighborhood point generation method;
- Tabu list;
- Aspiration criterion;
- Stopping criterion.

5.2 Proposed algorithm

In case 1, the algorithm starts at some initial point (T^0, q^0), (in case 2, x^0 = (T^0, q^0, m^0, n^0), others are similar) and it goes through several iterations. At each iteration, *r* direction are generated randomly to move and line search is performed along the each generated direction. Among the *r* candidate neighborhood point, the non-tabu point that gives the best objective function value (minimum value of the average total cost in this study) or tabu point that satisfies the aspiration criterion is selected to the next point and its associated direction is stored in the tabu list. This procedure is repeated until stop criterion has been satisfied. In this study, aspiration criterion is to allow solutions which are better than the currently known best solution and the stop criterions are give as the maximum number of iteration.

Step 1: Initialization

Choose the number of random search direction to be used at each iteration (*r*);

```
        Choose the maximum number of iteration
        ($MAX\_ITER$)
        Choose an initial point $x^0 = (T^0, q^0)$;
        Let $TL = \emptyset$, Best value = $ATC(x^0)$, j = $0$.

Step 2: Perform Line Search
        (2.1) Generate r random direction, $d^1, d^2 \cdots d^r$;
              Let $\lambda^*$ and $d^*$ be such that

              $ATC(x_j + \lambda^* d^*) = \min_{1 \le i \le r} ATC(x_j + \lambda^i d^i)$;

        (2.2) Check tabu list
              If ($d^* \notin TL$) or ($d^* \in TL$ and $ATC(x_j + \lambda^* d^*)$
              < Best value), go to step 2.3;
              Otherwise choose second best solution and
              repeat step 2.2;

        (2.3) Update current point
              Let $x_{j+1} = x_j + \lambda^* d^*$ and update tabu list.
              If $ATC(x_j + \lambda^* d^*)$ < Best value then Best
              value = $ATC(x_j + \lambda^* d^*)$;
              $j = j+1$ and go to step 3;

Step 3: Check stopping criterion
        If $j = MAX\_ITER$, stop;
        Else go to Step 2.
```

6. NUMERICAL EXAMPLES

The proposed algorithm (Tabu search) is programmed in Java language. The performance of tabu search is compared with the near-optimal solutions obtained by Lingo calculating software. The numerical values of the parameters are as follow: $h_S = 2$, $h_R = 0.2$, $hraw = 0.2$, $C_M = 30$, $S_R = 1500$, $S_M = 1500$, $C_O = 1000$, $D = 1000$, $C_{raw} = 20$, $a = 0.9$, $b = 0.9$, $c = 0.1$, $\gamma = 0.6$, $\beta = 0.5$ and $\varphi = 2$. With three different values of $\theta = \{4, 5, 6\}$ and $\delta = \{3.5, 4, 5\}$, different cases were examined.

6.1 Numerical example for case 1

Table 1. Solution by Tabu search method.

θ	δ	q	T	m	n	ATC
4	3.5	0.143	3.793	1	1	39800.11
	4	0.269	3.840	1	1	42954.65
	5	0.444	3.638	1	1	46405.41
5	3.5	0.131	3.752	1	1	38203.40
	4	0.258	3.852	1	1	41593.04
	5	0.436	3.656	1	1	45337.47
6	3.5	0.124	3.741	1	1	37064.59
	4	0.251	3.856	1	1	40598.52
	5	0.427	3.664	1	1	44517.99

Table 2. Solution by Lingo software.

θ	δ	q	T	m	n	TC (q)
4	3.5	0.143	3.775	1	1	39800.09
	4	0.268	3.847	1	1	42954.62
	5	0.444	3.640	1	1	46405.40
5	3.5	0.131	3.751	1	1	38203.39
	4	0.257	3.852	1	1	41592.95
	5	0.433	3.658	1	1	45336.74
6	3.5	0.124	3.736	1	1	37064.57
	4	0.250	3.854	1	1	40598.48
	5	0.426	3.668	1	1	44517.95

Table 1 and Table 2 are comparisons of solutions of case1 (Formula 12) of proposed algorithm (Tabu search) and those of Lingo calculating software. Here *m* and *n* are all assumed as *1*, generally, this kind of case is called (*1*, *1*) case.

6.2 Numerical example for case 2

Table 3. Solution by Tabu search method.

θ	δ	q	T	m	n	ATC
4	3.5	0.134	5.520	2	1	39673.40
	4	0.269	3.840	1	1	42954.65
	5	0.449	5.085	1	2	46368.31
5	3.5	0.121	5.544	2	1	38053.86
	4	0.258	3.853	1	1	41593.04
	5	0.439	5.092	1	2	45308.57
6	3.5	0.117	5.540	2	1	36916.09
	4	0.256	3.846	1	1	40601.09
	5	0.434	5.094	1	2	44497.87

Table 4. Solution by Lingo software.

θ	δ	q	T	m	n	ATC
4	3.5	0.133	5.544	2	1	39662.48
	4	0.268	3.847	1	1	42954.62
	5	0.449	5.084	1	2	46368.27
5	3.5	0.122	5.559	2	1	38045.72
	4	0.257	3.852	1	1	41592.95
	5	0.438	5.090	1	2	45307.98
6	3.5	0.115	5.566	2	1	36894.96
	4	0.250	3.854	1	1	40598.48
	5	0.431	5.093	1	2	44493.99

Table 3 and 4 are comparisons of solutions of case2 (Formula 19) of proposed algorithm (Tabu search) and those of Lingo calculating software. Here *m* and *n* are variables; generally, this kind of case is called (m, n) case.

6.3 Analysis for the calculating results

From the results of the calculation, it can be seen that,

- The performance of calculating methodology is rather good, because the results of tabu search method and that of lingo computing software are highly consistent in the two cases (the biggest gap between the two is less than *0.06%*).
- Through the calculating examples, the validity of the models is illustrated. The results imply that if the buyback price can be lowered, i.e., if company can acquire end-of-used products at rather low price (when $\theta = 6$), then it tends to remanufacture more (q is smaller) with lower average total cost; and if the remanufacturing cost is higher (when $\delta = 5$), then the average total cost will be higher and company is unwilling to recycle the used products (q is bigger). These implications are consistent with expectations.
- From the results of case 1 and case 2, it can be verified that the extension is valuable. Generally m and n are used to optimize the EOQ (EPQ)-related cost, others being equal, choosing m and n is helpful to reduce average total cost and optimize the production decision. The average total cost in case 2 are uniformly lower than that of case 1 and; the return rate in case 2 are uniformly higher (q is smaller) than that of case 1 if the remanufacturing cost is rather low (when $\delta = 3.5$); however if the remanufacturing cost is rather high (when $\delta = 5$), the return rate in case 2 are uniformly lower (q is bigger) than that of case 1. All these are adequate proof of the value of this extension.

7. CONCLUSIONS AND FURTHER RESEARCH

This research studied a mixed manufacturing and remanufacturing system through the development of mathematical models where the stationary demand is satisfied by either remanufactured products or newly manufactured products. In contrast to other literature, here returning rate, buyback price and remanufacturing cost are all considered to be unknown. And the nonlinear functions of returning rate, buyback price and remanufacturing cost that dependent on the quality level of end-of-used products were also investigated.

The mathematical models of this system are developed with the objective function of minimizing the average total cost of two cases. In case 1, a production circle is considered to be consisted of a single remanufacturing run and a single manufacturing run, both in EPQ settings. Here the relationship of objective function between required minimum quality, return rate, buyback price, remanufacturing cost and the length of a circle are investigated. And in case 2, a circle is extended to m remanufacturing runs and n manufacturing runs, and the effect of this extension on the system is investigated.

Due to the complexity of the model, Tabu Search method, a kind of meta-heuristic solution approach is developed to find its solution. To examine the performance of the proposed method, the solutions are compared with near-optimal solutions obtained by Lingo. The results show that the proposed algorithm generates almost the same results as that of Lingo.

This research could be extended to the following fields:

The quality distribution of end-of-used products is non-uniform distribution, i.e., normal distribution or/and gamma distribution, etc. In fact, some end-of-used products, such as consuming electronic products, their quality distribution have been researched as gamma distribution (Galbreth, 2006; LI and DA 2008).

With the consideration of ERP (extending producers' responsibility), the mathematical model of this system can be adapted to describe that case.

Under the condition of stochastic demand and/or lead-time, how will the model turn out to be? These are open for further research.

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