Commun. Korean Math. Soc. **25** (2010), No. 2, pp. 185–191
DOI 10.4134/CKMS.2010.25.2.185

SUMMATION FORMULAS DERIVED FROM THE SRIVASTAVA'S TRIPLE HYPERGEOMETRIC SERIES H_C

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Abstract. Srivastava noticed the existence of three additional complete triple hypergeometric functions H_A, H_B and H_C of the second order in the course of an extensive investigation of Lauricella's fourteen hypergeometric functions of three variables. In 2004, Rathie and Kim obtained four summation formulas containing a large number of very interesting reducible cases of Srivastava's triple hypergeometric series H_A and H_C. Here we are also aiming at presenting two unified summation formulas (actually, including 62 ones) for some reducible cases of Srivastava's H_C with the help of generalized Dixon's theorem and generalized Whipple's theorem on the sum of a ${}_3F_2$ obtained earlier by Lavoie et al.. Some special cases of our results are also considered.

1. Introduction

The triple hypergeometric series H_A and H_C introduced by Srivastava (see [9, pp. 68–69, Eqs. (36) and (38)]) are defined as follows:

$$H_A(\alpha,\beta,\beta';\gamma,\gamma';x,y,z)=\sum_{m,n,p=0}^{\infty}\frac{(\alpha)_{m+p}(\beta)_{m+n}(\beta')_{n+p}}{(\gamma)_m(\gamma')_{n+p}m!n!p!}x^m y^n z^p \tag{1.1}$$
$$(|x|<r,\ \ |y|<s,\ \ |z|<t,\ \text{ and }\ r+s+t=1+st);$$

$$H_C(\alpha,\beta,\beta';\gamma;x,y,z)=\sum_{m,n,p=0}^{\infty}\frac{(\alpha)_{m+p}(\beta)_{m+n}(\beta')_{n+p}}{(\gamma)_{m+n+p}m!n!p!}x^m y^n z^p \tag{1.2}$$
$$(|x|<1,\ \ |y|<1,\ \text{ and }\ |z|<1),$$

where $(\alpha)_n$ denotes the Pochhammer symbol defined, in terms of the well-known Gamma function $\Gamma(z)$, by

$$(\alpha)_n=\frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}\quad (n=0,\,1,\,2,\,\ldots).$$

Received September 21, 2007.
2000 *Mathematics Subject Classification.* Primary 33C20, 33C60, 39A10; Secondary 33C70, 33C65.
Key words and phrases. triple hypergeometric series H_A and H_C, Appell's function, generalized Dixon's theorem, generalized Whipple's theorem.

We recall only two identities for the Pochhammer symbol (see [8, pp. 6–8]):

$$(\alpha)_m\,(\alpha+m)_n=(\alpha)_{m+n} \tag{1.3}$$

and

$$\frac{\Gamma(\alpha-n)}{\Gamma(\alpha)}=\frac{(-1)^n}{(1-\alpha)_n}. \tag{1.4}$$

In [7], Rathie and Kim obtained a large number of summation formulas for the series H_A and H_C. In fact they have presented explicit Gamma function-expressions for the following cases:

(i)

$$H_A(\alpha,\beta,\beta^{'};1+\beta-\alpha+\frac{1}{2}\beta^{'}+i,1+\beta+\beta^{'}-\alpha;-1,1,-1)$$
$$(i=0,\pm1,\pm2,\pm3,\pm4,\pm5);$$

(ii)

$$H_A(\alpha,\beta,\beta^{'};\frac{1}{2}(1+\alpha+\beta-\frac{1}{2}\beta^{'}+i),1+\beta+\beta^{'}-\alpha;\frac{1}{2},1,-1)$$
$$(i=0,\pm1,\pm2,\pm3,\pm4,\pm5);$$

(iii)

$$H_A(\alpha,\beta,2\alpha+2\beta-2-2i;\gamma,3\beta+\alpha-2i-1;\frac{1}{2},1,-1)$$
$$(i=0,\pm1,\pm2,\pm3,\pm4,\pm5);$$

(iv)

$$H_C(\alpha,\beta,\beta+i+j;3\beta+2j+i;1,1,1)$$
$$(i=0,\pm1,\pm2).$$

These results are derived with the help of generalized Kummer's theorem, generalized Gauss' second theorem, generalized Bailey's theorem and generalized Watson's theorem obtained earlier by Lavoie et al. [4, 5].

Here we are aiming at providing 62 further new reducible cases in the form of two unified formulas for the triple hypergeometric series H_C by using, mainly, generalized Dixon's theorem and generalized Whipple's theorem obtained earlier by Lavoie et al. [5, 6]. Those easily established results in this paper may be interesting and useful.

2. Results required

In this section we just recall some known results which will be used for our present investigation. Appell and Kampé de Fériet [9, p. 55, Eq. (15)]:

$$F_1(a,b,b^{'};c;1,1)=\frac{\Gamma(c)\Gamma(c-a-b-b^{'})}{\Gamma(c-a)\Gamma(c-b-b^{'})} \tag{2.1}$$
$$(\Re(c-a-b-b^{'})>0;\;\; c\neq0,-1,-2,\ldots).$$

Generalized Dixson's theorem [6]:

(2.2)
$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix} a, & b, & c \\ a-b+i+1, & 1+a-c+i+j \end{matrix};1\right)\\
&=\frac{2^{-2c+i+j}\Gamma(a-b+i+1)\Gamma(1+a-c+i+j)\Gamma(b-\frac{1}{2}|i|-\frac{1}{2}i)\Gamma(c-\frac{1}{2}(i+j+|i+j|))}{\Gamma(b)\Gamma(c)\Gamma(a-2c+i+j+1)\Gamma(a-b-c+i+j+1)}\\
&\quad\times\Bigg\{A_{i,j}\frac{\Gamma(\frac{1}{2}a-c+\frac{1}{2}+[\frac{i+j+1}{2}])\Gamma(\frac{1}{2}a-b-c+1+i+[\frac{j+1}{2}])}{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}a-b+1+[\frac{i}{2}])}\\
&\qquad+B_{i,j}\frac{\Gamma(\frac{1}{2}a-c+1+[\frac{i+j}{2}])\Gamma(\frac{1}{2}a-b-c+\frac{3}{2}+i+[\frac{j}{2}])}{\Gamma(\frac{1}{2}a)\Gamma(\frac{1}{2}a-b+\frac{1}{2}+[\frac{i+1}{2}])}\Bigg\}\\
&\quad(\Re(a-2b-2c)>-2-2i-j;\ i=-3,-2,-1,0,1,2;\ j=0,1,2,3).
\end{aligned}
$$

Here and in what follows, $[x]$ is the greatest integer less than or equal to x and $|x|$ denotes the usual absolute value of x. The coefficients $A_{i,j}$ and $B_{i,j}$ are given in the tables at the end of the paper [6].

It is interesting to observe here that if $f_{i,j}$ is the left-hand side of (2.2), the natural symmetry

$$f_{i,j}(a,b,c)=f_{i+j,-j}(a,c,b) \tag{2.3}$$

makes it possible to extend the result to $j=-1,-2,-3$.

The special case of (2.2) when $i=j=0$ reduces, immediately, to the following classical Dixon's theorem [1]:

$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix} a, & b, & c \\ 1+a-b, & 1+a-c \end{matrix};1\right)\\
&=\frac{\Gamma(1+\frac{1}{2}a)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+\frac{1}{2}a-b-c)}{\Gamma(1+a)\Gamma(1+\frac{1}{2}a-b)\Gamma(1+\frac{1}{2}a-c)\Gamma(1+a-b-c)}
\end{aligned}
\tag{2.4}
$$

provided $\Re(a-2b-2c)>-2$.

Generalized Whipple's theorem [5]:

(2.5)
$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix} a, & b, & c \\ & e, & f \end{matrix};1\right)\\
&=\frac{2^{-2a+i+j}\Gamma(e)\Gamma(f)\Gamma(c-\frac{1}{2}(j+|j|))\Gamma(e-c-\frac{1}{2}(i+|i|))\Gamma(a-\frac{1}{2}(i+j+|i+j|))}{\Gamma(a)\Gamma(c)\Gamma(e-a)\Gamma(f-a)\Gamma(e-c)}\\
&\quad\times\Bigg\{C_{i,j}\frac{\Gamma(\frac{1}{2}e-\frac{1}{2}a+\frac{1}{4}(1-(-1)^i))\Gamma(\frac{1}{2}f-\frac{1}{2}a)}{\Gamma(\frac{1}{2}e+\frac{1}{2}a-\frac{1}{2}i+[\frac{-j}{2}])\Gamma(\frac{1}{2}f+\frac{1}{2}a-\frac{1}{2}i+\frac{(-1)^j}{4}((-1)^i-1)+[-\frac{j}{2}])}\\
&\quad+D_{i,j}\frac{\Gamma(\frac{1}{2}e-\frac{1}{2}a+\frac{1}{4}(1+(-1)^i))\Gamma(\frac{1}{2}f-\frac{1}{2}a+\frac{1}{2})}{\Gamma(\frac{1}{2}e+\frac{1}{2}a-\frac{1}{2}-\frac{1}{2}i+[-\frac{j}{2}+\frac{1}{2}])\Gamma(\frac{1}{2}f+\frac{1}{2}a-\frac{1}{2}-\frac{1}{2}i+\frac{(-1)^j}{4}(1-(-1)^i)+[-\frac{j}{2}+\frac{1}{2}])}\Bigg\},
\end{aligned}
$$

where $a+b=i+j+1$ and $e+f=2c+i+1$, i, j takes values in a subset of $i,\ j=0,\pm1,\pm2,\pm3$, and the coefficients $C_{i,j}$ and $D_{i,j}$ are given in the tables in [5].

The special case $i=j=0$ of (2.5) yields the following classical Whipple's theorem [1]:

$$(2.5)\qquad {}_3F_2\left(\begin{matrix}a, & b, & c\\ e, & f & \end{matrix};1\right)=\frac{2^{-2c+1}\pi\Gamma(e)\Gamma(f)}{\Gamma(\frac{1}{2}e+\frac{1}{2}a)\Gamma(\frac{1}{2}f+\frac{1}{2}a)\Gamma(\frac{1}{2}e+\frac{1}{2}b)\Gamma(\frac{1}{2}f+\frac{1}{2}b)},$$

where $a+b=1$ and $e+f=2c+1$.

3. Main summation formulas

The 62 summation formulas established in this paper can be presented in the following two unified forms. If $\Re(2\beta-\alpha)>2i+j$ for convergence, we have

$$(3.1)\qquad \begin{aligned}&H_C(\alpha,\beta,4\beta-\alpha-1-3i-j;3\beta-2i-j;1,1,1)\\ &=\frac{2^{4\beta-2\alpha-3i-j-2}\Gamma(2\beta-i)\Gamma(\beta-\frac{i}{2}-\frac{|i|}{2})\Gamma(1+i-2\beta)}{\Gamma(\beta)\Gamma(\beta-i)\Gamma(1+\alpha-2\beta+2i+j)}\\ &\quad\times\frac{\Gamma(3\beta-2i-j)\Gamma(1+\alpha-2\beta+\frac{3}{2}i+\frac{1}{2}j-\frac{1}{2}|i+j|)}{\Gamma(2\beta-\alpha-2i-j)\Gamma(4\beta-\alpha-1-3i-j)}\\ &\quad\times\left\{\overline{A}_{i,j}\frac{\Gamma(\beta-\frac{1}{2}\alpha-i-j+[\frac{j+1}{2}])\Gamma(2\beta-\frac{1}{2}\alpha-\frac{1}{2}-2i-j+[\frac{i+j+1}{2}])}{\Gamma(\frac{1}{2}\alpha+\frac{1}{2})\Gamma(\frac{1}{2}\alpha-\beta+1+[\frac{i}{2}])}\right.\\ &\qquad\left.+\overline{B}_{i,j}\frac{\Gamma(\beta-\frac{1}{2}\alpha+\frac{1}{2}-i-j+[\frac{j}{2}])\Gamma(2\beta-\frac{1}{2}\alpha-2i-j+[\frac{i+j}{2}])}{\Gamma(\frac{1}{2}\alpha)\Gamma(\frac{1}{2}\alpha-\beta+\frac{1}{2}+[\frac{i+1}{2}])}\right\},\end{aligned}$$

where $i=0,\pm1,\pm2,\pm3$, $j=0,1,2,3$, and the coefficients $\overline{A}_{i,j}$ and $\overline{B}_{i,j}$ are obtained from the tables of A_{ij} and B_{ij} in (2.2) by replacing a, b, and c by α, β, and $1+\alpha-2\beta+2i+j$, respectively. Moreover, in view of the observation (2.3), the results in (3.1) can be extended to $j=-1,-2,-3$.

$$(3.2)\qquad \begin{aligned}&H_C(\alpha,1-\alpha+i+j,\beta';\frac{3}{2}+i+\frac{1}{2}j;1,1,1)\\ &=\frac{2^{-2\alpha+i+j}\Gamma(\frac{3}{2}+i+\frac{1}{2}j)\Gamma(\frac{1}{2}-\frac{1}{2}j-\beta')\Gamma(\frac{1}{2}+\frac{1}{2}j+\beta')}{\Gamma(\frac{1}{2}-\frac{1}{2}j)\Gamma(\frac{1}{2}+\frac{1}{2}j)\Gamma(\alpha)}\\ &\quad\times\frac{\Gamma(\frac{1}{2}-\frac{1}{2}|j|)\Gamma(1+\frac{1}{2}i-\beta'-\frac{1}{2}|i|)\Gamma(\alpha-\frac{1}{2}(i+j+|i+j|))}{\Gamma(\frac{3}{2}+i+\frac{1}{2}j-\beta'-\alpha)\Gamma(1-\beta'+i)\Gamma(\frac{1}{2}+\frac{1}{2}j+\beta'-\alpha)}\\ &\quad\times\left\{\overline{C}_{i,j}\frac{\Gamma(\frac{1}{4}+\frac{1}{4}j+\frac{1}{2}\beta'-\frac{1}{2}\alpha)\Gamma(\frac{3}{4}+\frac{1}{2}i+\frac{1}{4}j-\frac{1}{2}\beta'-\frac{1}{2}\alpha+\frac{1}{4}(1-(-1)^i))}{\Gamma(\frac{3}{4}+\frac{1}{4}j-\frac{1}{2}\beta'+\frac{1}{2}\alpha+[-\frac{j}{2}])\Gamma(\frac{1}{2}\alpha+\frac{1}{4}+\frac{1}{4}j-\frac{1}{2}i+\frac{1}{2}\beta'+\frac{(-1)^j}{4}((-1)^i-1)+[-\frac{j}{2}])}\right.\\ &\quad\left.+\overline{D}_{i,j}\frac{\Gamma(\frac{3}{4}+\frac{1}{4}j+\frac{1}{2}\beta'-\frac{1}{2}\alpha)\Gamma(\frac{3}{4}+\frac{1}{2}i+\frac{1}{4}j-\frac{1}{2}\beta'-\frac{1}{2}\alpha+\frac{1}{4}(1+(-1)^i))}{\Gamma(\frac{1}{4}+\frac{1}{4}j-\frac{1}{2}\beta'+\frac{1}{2}\alpha+[-\frac{j}{2}+\frac{1}{2}])\Gamma(\frac{1}{2}\alpha+\frac{1}{2}\beta'+\frac{1}{4}j-\frac{1}{2}i-\frac{1}{4}+\frac{(-1)^j}{4}(1-(-1)^i)+[-\frac{j}{2}+\frac{1}{2}])}\right\},\end{aligned}$$

where $i, j = 0, \pm 1, \pm 2, \pm 3$, and the coefficients $\overline{C}_{i,j}$ and $\overline{D}_{i,j}$ are obtained from the tables of C_{ij} and D_{ij} in (2.5) by replacing a, b, c, e, and f by α, $1-\alpha+i+j$, $\frac{1}{2}+\frac{1}{2}j$, $\frac{3}{2}+i+\frac{1}{2}j-\beta^{'}$, and $\frac{1}{2}+\frac{1}{2}j+\beta^{'}$, respectively.

Now let us start with the proof of (3.1). By using the definition of H_C in (1.2), we have

$$\begin{aligned} H_C :&= H_C(\alpha, \beta, 4\beta-\alpha-1-3i-j; 3\beta-2i-j; 1,1,1) \\ &= \sum_{m,n,p=0}^{\infty} \frac{(\alpha)_{m+n}(\beta)_{m+p}(4\beta-\alpha-1-3i-j)_{n+p}}{(3\beta-2i-j)_{m+n+p}m!n!p!}, \end{aligned} \tag{3.3}$$

which, upon using (1.3), becomes

$$\begin{aligned} H_C &= \sum_{m,n,p=0}^{\infty} \frac{(\alpha)_m(\alpha+m)_n(\beta)_m(\beta+m)_p(4\beta-\alpha-1-3i-j)_{n+p}}{(3\beta-2i-j)_m(3\beta-2i-j+m)_{n+p}m!n!p!} \\ &= \sum_{m=0}^{\infty} \frac{(\alpha)_m(\beta)_m}{(3\beta-2i-j)_m m!} \sum_{n,p=0}^{\infty} \frac{(\alpha+m)_n(\beta+m)_p(4\beta-\alpha-1-3i-j)_{n+p}}{(3\beta-2i-j+m)_{n+p}n!p!}. \end{aligned} \tag{3.4}$$

By making use of the Apell function of two variables (see [9, p. 53, Eq. (4)] defined by:

$$F_1[a, b, b^{'}; c; x, y] = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}(b)_m(b^{'})_n}{(c)_{m+n}m!n!} x^m y^n \tag{3.5}$$

in (3.4), we obtain

$$\begin{aligned} &H_C \\ &= \sum_{m=0}^{\infty} \frac{(\alpha)_m(\beta)_m}{(3\beta-2i-j)_m m!} F_1(4\beta-\alpha-1-3i-j, \alpha+m, \beta+m; 3\beta-2i-j+m; 1,1). \end{aligned}$$

If we use the result (2.1), we have

$$H_C = \sum_{m=0}^{\infty} \frac{(\alpha)_m(\beta)_m \Gamma(3\beta-2i-j)\Gamma(1+i-2\beta-m)}{\Gamma(1+\alpha-\beta+i+m)\Gamma(2\beta-\alpha-2i-j-m)m!}. \tag{3.6}$$

By employing (1.4) in (3.6), we get

$$H_C = \frac{\Gamma(3\beta-2i-j)\Gamma(1+i-2\beta)}{\Gamma(1+\alpha-\beta+i)\Gamma(2\beta-\alpha-2i-j)} \sum_{m=0}^{\infty} \frac{(\alpha)_m(\beta)_m(1+\alpha-2\beta+2i+j)_m}{(1+\alpha-\beta+i)_m(2\beta-i)_m m!}, \tag{3.7}$$

which is rewritten as follows:

$$H_C = \frac{\Gamma(3\beta-2i-j)\Gamma(1+i-2\beta)}{\Gamma(1+\alpha-\beta+i)\Gamma(2\beta-\alpha-2i-j)} {}_3F_2\left[\begin{matrix} \alpha, \beta, 1+\alpha-2\beta+2i+j \\ 2\beta-i, \quad 1+\alpha-\beta+i \end{matrix} ; 1\right]. \tag{3.8}$$

Now if we use the generalized Dixon's theorem (2.2) with $a = \alpha$, $b = \beta$, $c = 1+\alpha-2\beta+2i+j$ ($i = -3, -2, -1, 0, 1, 2$, and $j = 0, 1, 2$), and after a little simplification, we arrive at the right-hand side of (3.1).

For the proof of the result (3.2), proceeding in the same manner as in the proof of (3.1), we get

(3.9)
$$H_C(\alpha, 1-\alpha+i+j, \beta'; \frac{3}{2}+i+\frac{1}{2}j; 1,1,1)$$
$$= \frac{\Gamma(\frac{3}{2}+i+\frac{1}{2}j)\Gamma(\frac{1}{2}-\frac{1}{2}j-\beta')}{\Gamma(\frac{3}{2}+i+\frac{1}{2}j-\beta')\Gamma(\frac{1}{2}-\frac{1}{2}j)}\, {}_3F_2 \left[\begin{matrix} \alpha, & 1-\alpha+i+j, & \frac{1}{2}+\frac{1}{2}j \\ \frac{3}{2}+i+\frac{1}{2}j-\beta', & \frac{1}{2}+\frac{1}{2}j+\beta' & \end{matrix} ;1 \right],$$

and if we use the generalized Whipple's theorem (2.5) with $a = \alpha$, $b = 1-\alpha+i+j$, $c = \frac{1}{2}+\frac{1}{2}j$, $e = \frac{3}{2}+i+\frac{1}{2}j-\beta'$, and $f = \frac{1}{2}+\frac{1}{2}j+\beta'$ ($i, j = 0, \pm1, \pm2, \pm3$), and after a little simplification, we arrive at the right-hand side of (3.2).

Finally, we present simple special cases of (3.1) and (3.2) by taking $i = j = 0$ there:

(3.10)
$$H_C(\alpha, \beta, 4\beta-\alpha-1; 3\beta; 1,1,1) = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}+\beta)\Gamma(3\beta)\Gamma(1-2\beta)}{\Gamma(\frac{1}{2}+\frac{1}{2}\alpha)\Gamma(1+\frac{1}{2}\alpha-\beta)\Gamma(2\beta-\frac{1}{2}\alpha)\Gamma(\frac{1}{2}-\frac{1}{2}\alpha+\beta)},$$

which is recorded in [2].

(3.11)
$$H_C(\alpha, 1-\alpha, \beta'; \frac{3}{2}; 1,1,1) = \frac{2^{-2\alpha-1}\Gamma(\frac{1}{2}-\beta')\Gamma(\frac{1}{2}+\beta')\Gamma(\frac{1}{4}+\frac{1}{2}\beta'-\frac{1}{2}\alpha)\Gamma(\frac{3}{4}-\frac{1}{2}\beta'-\frac{1}{2}\alpha)}{\Gamma(\frac{3}{2}-\beta'-\alpha)\Gamma(\frac{1}{2}+\beta'-\alpha)\Gamma(\frac{3}{4}-\frac{1}{2}\beta'+\frac{1}{2}\alpha)\Gamma(\frac{1}{2}\alpha+\frac{1}{2}\beta'+\frac{1}{4})},$$

which is (presumably) new.

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