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Problem Posing by Mathematically Gifted Middle School Students: A Case Study[1)]

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This study involves investigating problem posing practices for mathematically gifted first year middle school students in Korea. The overall purpose of this study is twofold: to examine the students' preferences on problem posing resources on the division algorithm and to analyze the approaches of the students' posing problems related to specific solution methods. To this end, the patterns of the problems are classified into 6 types such as 'routine' and 'nonroutine' problems associated with 3 levels of the original version of problems. Based on the analysis on the problems, we provide some implications about the nature of mathematically gifted students' problem posing practices in gifted education.

Ⅰ. Introduction

National Council of Teachers of Mathematics [NCTM] (1980) acknowledged that the students most neglected, in terms of realizing full potential, are the gifted students of mathematics. It is well recognized that the selection and construction of worthwhile mathematical tasks is considered as one of the most important decisions teachers need to make (NCTM, 1991). The tasks teachers pose in their classrooms deserve important consideration because they open or close the students' opportunity for meaningful mathematical learning (Crespo, 2003). Thus when teachers in gifted education are in such a position to pose worthwhile mathematical problems for their students, it is essential to pose them so that they would meet the needs of the mathematically gifted. NCTM (1989) recommended that students should have some experience recognizing and formulating their own problems. Furthermore, NCTM (1991) stated that a concept of problem posing such as: "students should be given opportunities to formulate problems from given situations and create new problems by modifying the conditions of a given problem."

Although current interest in mathematical problem posing can be seen as representing a new facet of a longstanding interest in mathematical problem solving (Stanic and Kilpatrick, 1988), less is known about instructional strategies that can be effectively promote productive problem posing (Silver and Cai, 1996). A few researchers have examined the mathematical problems posed by children (Ellerton, 1986; English, 1998), by middle school or prospective secondary school teachers (Silver et al., 1996), by middle school students (Silver and Cai, 1996), by preservice teachers

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(Crespo, 2003), or by teachers in gifted education (Paek and Yi, 2009). As a part of these studies on problem posing, we intend to develop and use specific solution methods on the division algorithm to examine the tendency of the problem posing of mathematically gifted first year middle school students.

In this study we consider two basic questions with respect to the students' problem posing practices: What kinds of problem posing resources do they prefer? and How do they pose problems related to specific solution methods?

In order to illuminate the tendency of the students' problem posing processes, it would be appropriate to investigate the problems based on their problem types. Hence in the study reported here, we analyze the patterns of the 'student-posed problems [problems]' which are classified into 6 types such as 'routine' and 'nonroutine' problems associated with 3 levels of the original version of problems. Based on the analysis on the problems, we then provide some implications in the nature of the students' problem posing practices in gifted education.

II. Background

According to Brown and Walter (1990), problem posing is deeply embedded in the activity of problem solving in two very different ways. First, it is impossible to solve a new problem without first reconstructing the task by posing new problems in the very process of solving. Second, it is frequently the case that after we have supposedly solved a problem, we do not fully understand the significance of what we have done, unless we begin to generate and try to analyze a completely new set of problems. However, while problem solving is easily identified as an important aspect of learning mathematics, problem posing has long been considered a neglected aspect of mathematical inquiry.

Silver (1994) concluded that students' activities such as generating their own problems or solving preformulated problems have added benefit of providing insight into students' understanding of important mathematical concepts as well as into the nature of their school mathematics activities. In addition, he pointed out the term 'problem posing' has been used to refer both to the generation of new problems and to the reformulation of given problems. 'Problem posing' referred in this paper is that involves generating new problems or reformulating given problems in the sense of Silver (1994).

When it now comes to the literature on secondary school students' posing problems, Ellerton (1986) found that the more able children of 11 to 13-year-olds made up problems of greater computational difficulty, with more complex number systems and with more operations than their less able peers. In addition, there was evidence to suggest that the more able students planned their problems and were able to work out the answer, while their less able peers had difficulty with both the planning and the solution of their own problems. Silver and Cai (1996) studied on the mathematical problems generated by middle school students who were given a brief written 'story-problem' description and asked to pose questions that could be answered using the information. The problems were examined for solvability, linguistic and mathematical complexity, and relationships within the set of posed problems. It was found that students were able to generate a large number of solvable mathematical problems and that about half the students generated sets of related problems. Moreover, they found that students' problem solving performance was highly correlated with their problem posing performance.

The present problem posing practices are part of such studies. In this study consideration is now given to mathematically gifted first year middle school students' problem posing activity involving the generation of new problems by adapting or modifying the structures of the original version of problems.

Ⅲ. Method

1. Subjects

The subjects were 53 mathematical problems posed by 53 students out of 76 'mathematically gifted first year middle school students [students]' in 'A' metropolitan city in Korea participated in a week-long intensive learning program at the institute for gifted education in science in the city in 2009. The institute has run a week-long intensive learning program for science and mathematics twice a year. As far as it concerned about the mathematics program, it aimed to improve students' creative problem solving abilities and self-directed learning in gifted education. All 53 participants were enrolled either in the institute of gifted education in science or in regional gifted education centers in the city. However, little was known to the authors about the students' mathematical beliefs, mathematical competence, mathematical knowledge, and previous problem posing experiences in gifted education.

2. Tasks

'Posing problems related to specific solution methods on the division algorithm' was the theme of the intensive learning program for mathematics given at the institute in 2009. In the mathematics program comprising two 1.5 hour classes and one 40 minute class, learning resources on problem posing related to specific solution methods on the division algorithm including 3 levels of the original version of mathematical problems which illustrate how to generate such problems were given to the participating students. The tasks began with a lecture and discussion where the nature of tasks was explained and the students' interpretations of the problem posing and understanding of the solution methods used in the original version of problems were shared for approximately 45 minutes. They were also given 45 minute individual work to pose problems they would most like to generate in the first phase and 45 minutes to solve their own generated problems in the second phase. The final phase consisted of 45 minute small group (4 or 5 students) discussion and an additional 40 minute whole group discussion to share ideas.

In addition, the students were expected to generate problems individually and hence it is of interest to note that 3 levels of the original version of problems were carefully selected as both self-directed learning resources and instructional examples to help the students to work alone. Based on the original version of problems, each student was asked to pose a problem related to specific solution methods using the division algorithm:

> Let a and b be integers with $b>0$. Then there are unique integers q and r such that $a=bq+r$ with $0 \le r < b$.

Recall that the students were asked to pose problems they would most like to generate.

Needless to say, it is important to pose problems well, however it is clearly more important for the students to develop understandings of their solution methods. That was one reason that the students were asked to pose problems with their solutions and the other was that critical oversights in posing problems are most likely due to either the fact that they are posed without being solved beforehand or lacks of understanding of mathematical concepts.

From now on, unless stated otherwise, we assume that all the numbers in the problems are integers throughout this study. The following is the list of 3 levels of the original version of problems given to the students.

Problem A. Show that every perfect square is of the form $3n$ or $3n+1$.
Problem B. Show that if a^2 is divisible by 3, then so is a.
Problem C. Find all twin primes (p, q) such that $pq+4$ is prime.

Problems A, B, and C are denoted by [A], [B], and [C], respectively. Their solution methods using the division algorithm were also instructed to the students as follows: For [A], let a^2 be a perfect square. Then, by the division algorithm, a is of the form $3b$ or $3b+1$ or $3b+2$. It follows that a^2 is of the form $3n$ or $3n+1$. As an immediate consequence of [A], it is worthwhile to note the fact that 11111111111 is not a perfect square because it is of the form $3n+2$. It was given to the students as a corollary of [A]. For [B], we suppose that a^2 is divisible by 3 and, for contradiction, assume that a is not divisible by 3. Then a is of the form $3n+1$ or $3n+2$. It follows that a^2 is not divided by 3, a required contradiction. Meanwhile, [C] is one of the Korean mathematical olympiad problems in 1987. It can be solved by using the fact that every prime number greater than 4 is of the form $6n-1$ or $6n+1$. We may assume that $q=p+2$. If $p=6n+1$, then q is not prime; if $p=6n-1$, then $pq+4$ is not prime. Hence $p \leq 4$, which implies that $p=3$ and $q=5$.

Note that [A] is regarded as a computational exercise. For, if the students know how to apply the division algorithm to their solution methods, then their solutions involve simple arithmetic computations. As shown above, [B] can be justified by 'proof by contradiction,' however, the students might not get used to that kind of logical argument. Hence the specific solution method of [B] requires somewhat deeper understanding of mathematical reasoning than that of [A]. It is now evident that the solution method of [C] is much harder than that of [B]. Note also that since twin primes are not within the scope of school mathematics curriculum, it follows that the students might not have substantial experiences in learning such topics. Therefore, it is reasonable to rate the difficulty levels of [A], [B], and [C] as being moderately easy, intermediate, and challenging, respectively.

3. Problem Selection

We examined all 76 problems and ruled out 23 of them as our research subjects because neither their solution methods were related properly to the division algorithm nor they modified the conditions of the given problems. As a consequence of our initial problem selection procedure, we selected 53 problems as our research subjects. Such 53 problems were first classified into 3 categories according to their problem statement contents associated with [A], [B], and [C]. It turned out that 32, 19, and 2 problems were related to [A],

[B], and [C], respectively. The next step involved categorizing the problems as 'routine' or 'nonroutine' problems. These features are to be discussed later in further detail in the next section. Moreover, there were 28, 14, and 1 routine problems; whereas 4, 5, and 1 nonroutine problems associated with [A], [B], and [C], respectively. Routine and nonroutine problems associated with [A], [B], and [C] were denoted by RA, RB, RC, NA, NB, and NC, respectively. Hence all 53 problems were completely classified into 6 types so that there were 28, 14, 1, 4, 5, and 1 problems of type RA, RB, RC, NA, NB, and NC, respectively. We then identified problems of the same type with others according to their problem statement contents and problem solving strategies related to the division algorithm so that there were 3, 2, 1, 4, 4, and 1 prototypical problems of type RA, RB, RC, NA, NB, and NC, respectively. These 15 prototypical problems were denoted by RA1, RA2, RA3, NA1, NA2, NA3, NA4, RB1, RB2, NB1, NB2, NB3, NB4, RC1, and NC1. Here, for example, we meant RA1 by the first problem of type RA and NB2 the second problem of type NB. Therefore, there really were 15 problems to consider altogether and they were the main sources of data for this study.

4. Data Analysis

For the analysis of the patterns of the problems in this study, the very nature of the 3 original version of problems [A], [B], and [C] and the initial overview of the problems were considered before we identified problem types (routine, nonroutine; single answer, computational) and problem features (textbook-like, investigation-like) among several aspects of problem posing discussed in NCTM (1991), and also the ways of approaches such as 'making problems easy to solve,' 'posing familiar problems,' 'posing unfamiliar problems,' and 'posing problems blindly' stated in the study of Crespo (2003). Hence we adapted the aspects of the problem types and features discussed in NCTM (1991) and Crespo (2003) so that the patterns of the problems were classified into 6 problem types described in Section 3. In addition, 'routine' and 'nonroutine' problems were characterized by the following features.

> Problems are considered to be 'routine' if they modify the conditions of the given problem to somewhat less different conditions or can be solved by simple arithmetic computations.
> Problems are considered to be 'nonroutine' if they modify or extend the conditions of the given problem to different conditions or require somewhat deeper understanding of mathematical concepts or reasoning to solve the problems.

Ⅳ. Analysis

We recall that 3 levels of the original version of problems are: [A] Show that every perfect square is of the form $3n$ or $3n+1$; [B] Show that if a^2 is divisible by 3, then so is a; [C] Find all twin primes (p, q) such that $pq+4$ is prime.

The following is the list of 15 prototypical problems characterized by their problem types. Recall that, unless stated otherwise, all the numbers in the problems are assumed to be integers throughout this study.

RA1. Show that every perfect square is of the form $5n$ or $5n+1$ or $5n+4$.
RA2. Is 135442 a perfect square?
RA3. Show that every cubic number is of the form $4n$ or $4n+1$ or $4n+3$.

RB1. Show that if a^2 is divisible by 5, then so is a.

RB2. Show that if a^2+b is even, then so is $a+b$.

RC1. Find all twin primes (p, q) such that $pq+5$ is prime.

NA1. What is the probability that a perfect square n is divisible by 3 and 4?

NA2. Show that the product of two consecutive odd integers cannot be a perfect square.

NA3. Find the number of positive integers $n \leq 100$ such that $7 \mid n^2-2$ and $7 \mid n^3-1$.

NA4. Let $p \geq 5$ be a prime number. Then show that p^3 is of the form $18n+1$ or $18n-1$.

NB1. Show that $3 \mid 4a(4a+1)(5a+1)$.

NB2. Find all x such that $7 \mid 3^x-2$.

NB3. Prove that at least one of $a-b$, ab, and $a+b$ is divisible by 3.

NB4. If $9 \mid n^3-n$, then show that n cannot be a multiple of 3 which is not a multiple of 9.

NC1. Show that there are no twin primes (p, q) such that $(p-1)(q-1)$ is prime.

All 53 problems are now completely categorized as 15 prototypical problems with their frequencies as shown in Table Ⅳ-1.

<Table Ⅳ-1> Frequency of Problems in Each Type

Type	Problem	Frequency		Percent(%)	
RA	RA1	17	28	32	52
	RA2	7		13	
	RA3	4		7	
RB	RB1	13	14	24	26
	RB2	1		2	
RC	RC1	1	1	2	2
NA	NA1	1	4	2	8
	NA2	1		2	
	NA3	1		2	
	NA4	1		2	
NB	NB1	2	5	4	10
	NB2	1		2	
	NB3	1		2	
	NB4	1		2	
NC	NC1	1	1	2	2

As can be seen in Table Ⅳ-1, approximately 80% of the students generated routine problems whereas about 20% of the students generated nonroutine ones. As a whole about 60%, 36%, and 4% of the problems were related to [A], [B], and [C], respectively. Since [A], [B], and [C] can be rated as moderately easy, intermediate, and challenging problems, respectively, it is reasonable to conclude that almost all students (about 96%) prefer to choose less challenging problems like [A] and [B] rather than [C] for their problem posing resources. The main reason for only 4% (2 out of 53) of the students' choosing [C] as their problem posing resources is most likely due to the fact that [C] is neither within the scope of school mathematics curriculum nor easily accessible resource by the students. It is of some interest to note that slightly more than half (about 52%) of the students posed problems of type RA, whereas about 8% of them posed problems of type NA. On the other hand, about 26% of the problems are of type RB and about 10% of them are of type NB. Thus it would be much harder for the students to generate nonroutine problems than routine ones even though they choose the same problem posing resources.

By comparing the pairs of problem types (RA, NA), (RB, NB), and (RC, NC), it can be concluded that: If the original problem gets easier, then it gets easier to generate routine problems; Even though the original problem gets easier, it gets harder to generate nonroutine problems; If the original problem gets harder, then it gets harder to generate both routine and nonroutine problems. Although most of these results are not surprising when we consider the simple task in this study, understanding the depth of the relationship between the difficulty level of the given problems and generating (non)routine problems should be lent by further empirical support.

1. Routine problems

We are now ready to investigate 6 prototypical routine problems RA1, RA2, RA3, BB1, RB2, and RC1. First of all, RA1 is considered as routine because no actual adaptations are made to [A]. In the problems obtained in this study, nearly one of every three problems (about 32%) is similar to RA1. Moreover, the other 16 problems similar to RA1 have somewhat less different conditions such as:

> Show that every perfect square is of the form $6n$ or $6n+1$ or $6n+3$ or $6n+4$.
> Show that every perfect square is of the form $7n$ or $7n+1$ or $7n+2$ or $7n+4$.
> Show that every perfect square is of the form $8n$ or $8n+1$ or $8n+4$.
> Show that every perfect square is of the form $11n$ or $11n+1$ or $11n+3$ or $11n+4$.

RA2 is an immediate consequence of [A]. Out of 6 problems similar to RA2, it turns out that 2 problems' solution methods are using the result of [A] and 4 problems' solution methods are using the fact that every perfect square is of the form $4n$ or $4n+1$. RA3 is considered as routine because no actual adaptations are made to [A] except the condition of perfect squares. The other 3 problems analogous to RA3 are related to either cubic or biquadratic numbers such as:

> Every cubic number is of the form $4n$ or $4n+1$ or $4n+3$.
> Every biquadratic number is of the form $16n$ or $16n+1$.

RB1 is a routine problem because no substantial adaptations are made to [B]. Moreover, the other 12 problems similar to RB1 have somewhat less different conditions on the divisibility of a when a^2 is divided by 6, 7, and 17, respectively. RB2 is regarded as routine, however its statement is slightly different from that of RB1. In order solve it, one might need to consider two cases: a is even and a is odd. Since the solution of RB1 follows directly from that of [B] and the solution of RB2 involves slightly more arithmetic computations than that of [B], it can be concluded that RB2 is somewhat more difficult to be generated than RB1. This may partly explain the reason that there are 12 problems similar to RB1, but there is none similar to RB2.

RC1 is the only routine problem related to [C]. It asks to find all twin primes (p, q) such that $pq+5$ is prime. However, there does not exist a pair of twin primes (p, q) satisfying $pq+5$ is prime. Although the student's solution was concluded that there were no such twin primes, it is appropriate to rewrite RC1 to understand what is to be proven. Or else, it might be not easy to conclude that such twin primes do not exist. Hence we may rewrite RC1 as follows:

> Show that there are no twin primes (p, q) such that $pq+5$ is prime.

In fact, the result can be easily verified by the following well-known property of twin primes.

> Every twin prime pair except $(3, 5)$ is of the form $(6n-1, 6n+1)$.

Of course, this particular property is helpful in posing problems such as RC1. Hence the students' lack of mathematical content knowledge on the properties of twin primes would caused some constraint in their posing problems on twin primes.

2. Nonroutine Problems

We are now in position to consider 9 prototypical nonroutine problems NA1, NA2, NA3, NA4, NB1, NB2, NB3, NB4, and NC1. Note that except NB1, all nonroutine problems are unique in the sense of their modifications and extensions of the given problems. It is of interest to note that NA1 is the only problem associated with the concept of probability. In statistics, a (theoretical) probability is used when each outcome in a sample space is equally likely to occur and so the probability for an event is usually given by 'number of outcomes in the event divided by total number of outcomes in sample space.' Hence it might be rather easier to solve NA1 if the sample space is stated by adding the specific range for n so that we have a finite number of outcomes. An interesting finding shown in the student's solution of NA1 is that it is the only problem with wrong answer. The reason is most likely that the student considered, by mistake, the particular form of a, but not a^2 when a is divided by 3 or 4. NA2 is written as negative statements and its condition 'the product of two consecutive odd integers' is somewhat different from that of [B]. Moreover, its solution processes are related to the following properties:

The product of two consecutive odd integers is of the form $4n+3$.
Every perfect square is of the form $4n$ or $4n+1$.

The statement of NA3 is rather complicated because it involves two conditions $7 \mid n^2-2$ and $7 \mid n^3-1$. It turns out that such n should be of the form $7k+4$. It then remains to count the number of positive integers of the form $7k+4$ up to 100. Thus it is reasonable to regard NA3 as nonroutine because of such two conditions, even though they involve simple arithmetic computations. NA4 involves a prime number $p \geq 5$ such that p^3 is of the form $18n+1$ or $18n-1$. It is of interest to note that the solution requires multiple steps to be taken. First step is to show that p^3 is of the form $4a-1$ or $4a+1$ or $9b-1$ or $9b+1$. Second step is to lead the fact that p^3 is of the form $36c-1$ or $36c+1$ or $36c+17$ or $36c+19$ by arithmetic computations. Last step is to verify that p^3 has the required form. Hence, in some sense, the process of solving NA4 demonstrates the student's problem posing ability to extend the problem structures and solution methods of the given problems.

NB1 is to show that $3 \mid 4a(4a+1)(5a+1)$. Since 3 and 4 are relatively prime, it is equivalent to show that $3 \mid a(4a+1)(5a+1)$. The key factor to solve this is to substitute a for $3k-1$ or $3k$ or $3k+1$. Although its solution method needs less logical reasoning than that of [B], it is appropriate to regard it as nonroutine because of its somewhat different structure of the dividend. Another problem similar to NB1 is to show that $6 \mid n(n+1)(2n+1)$ for every positive integer n. NB2 involves the divisibility of $3^{r}-2$ by 7 and the structure of the dividend is rather different from that of NB1. Even though 'congruences' are not within the scope of school mathematics curriculum, the student used congruences in the solution such as $3^2 \equiv 2$ (mod 7) and $3^6 \equiv 1$ (mod 7). Meanwhile, NB3 is to show that at least one of $a-b$, ab, and $a+b$ is divisible by 3. The student's solution is supported by logical arguments by considering two cases, depending on whether or not at least one of a and b is divisible by 3. Note also that, in the solution of NB3, the student used the relation whether or not $a \equiv b$ (mod 3). Hence it is worthwhile to note that those students already have some college level knowledge such as congruences. NB4 is written as negative statements and its condition 'n cannot be a multiple of 3 which is not a multiple of 9' is not written clearly enough to be understood. Hence it is somewhat poorly stated and so it could be rewritten as:

If $3 \mid n$ and $9 \mid n^3 - n$, then show that $9 \mid n$.

NC1 is also written as negative statements. Considering its modification of the structure of the given problem, NC1 can be considered as nonroutine even though the result comes directly from simple arithmetic computations.

V. Conclusion

The present study explored a couple of issues concerning first year middle school students' problem posing practices in a week-long intensive learning program. The main goal of the discussion here is to reveal the students' tendency to pose problems in terms of their preferences on resources and approaches of specific solution methods. Hence two basic issues were investigated: What kinds of problem posing resources do they prefer? and How do they pose problems related to specific solution methods? To investigate these issues, we selected 53 problems related to specific solution methods on the division algorithm. The patterns of the problems are classified into 6 types altogether such as 'routine,' and 'nonroutine' problems associated with 3 levels of the original version of problems.

1. Routine Problems

As can be discerned from Table Ⅳ-1, about 80% of the problems are routine. In fact, slightly more than half (about 52%) of the problems are of type RA, whereas about 26% of them are of type RB and only about 2% of them are of type RC. Recall that the difficulty levels of [A], [B], and [C] were rated as being moderately easy, intermediate, and challenging, respectively. Hence, based on the analyses conducted in this study, it can be concluded that if the original problem gets easier (harder), then it gets easier (harder, respectively) to generate routine problems.

RA1, RA2, and RA3 involve perfect squares or cubic numbers or biquadratic numbers, whereas RB1 and RB2 involve divisibility satisfying certain conditions. They are classified into routine problems because basically no adaptations are made to [A] or [B]. It is of interest to note that RA1, RA2, and RA3 could be regarded as computational exercises because their answers are easy to be found by arithmetic computations. Moreover, in the solution processes of RB1 and RB2, the students used the argument of 'proofs by contradiction.' Hence the given solution of [B] turned out helpful for the students in generating such problems. When it comes to the problems of type RC, it is be evident that the students' lack of mathematical content knowledge on the properties of twin primes would caused some constraint in their posing problems on twin primes.

2. Nonroutine Problems

As the data in Table Ⅳ-1 show, about 20% of the problems are nonroutine. In fact, about 8% of the problems are of type NA, whereas about 10% of them are of type NB and only about 2% of them are of type NC. Considering the difficulty levels of the given problems, it is possible to conclude that if the original problem gets harder, then it gets harder to generate nonroutine problems. Note that the total number of the nonroutine problems is only about a quarter of that of the routine problems. As a whole, it can be concluded that even though the original problem gets easier, it gets harder to generate nonroutine problems in the task studied here.

NA1 is the only problem associated with the concept of probability and also it is the only problem with wrong answer. NA2, NB4, and NC1 are written as negative statements and, in addition, NB4 is the only poorly stated nonroutine problem. The statements of NA2, NA3, and NB3 require more than two conditions to consider such as 'the product of two consecutive odd integers,' '$7 \mid n^2-2$ and $7 \mid n^3-1$,' and 'at least one of $a-b$, ab, and $a+b$ is divisible by 3,' respectively. In particular, the process of solving NA4 may demonstrate the student's problem posing ability to extend the problem structures and solution methods.

NB1 involves the divisibility of $4a(4a+1)(5a+1)$ by 3, whereas NB2 involves the divisibility of 3^x-2 by 7. Even though there are quite a few problems on divisibility, NB1 and NB2 have their own features because of the uncommon patterns of their dividends. It is also of interest to note that the students solved NB2 and NB3 by means of congruences, even though they are not within the scope of school mathematics curriculum.

3. Overview of Problems

As indicated in Table Ⅳ-1, the problems are compared to each other with respect to their problem types. Although it is not possible to know precisely the tendency of the student's problem posing practices, the accumulated results in this study suggest that almost all students are able to pose appropriate mathematical problems with suitable solution methods.

Regardless of the routine and nonroutine problems, RC1 and NB4 are somewhat poorly stated. Thus it can be concluded that nearly 96% of the problems are well-posed in terms of problem structures and solution methods. Since the problems in 'routine' and 'nonroutine' categories do not have the same features, it is difficult to compare them based on the same criterion mentioned in Section 4, Chapter 3. However, it is evident that there are some relationships between the distribution of the problems and their problem types. The distribution of routine problems is biased by RA1, RA2, and RB1 because about 69% of all the problems in this study are similar to them, whereas nonroutine problems except NA1 are somewhat fairly distributed with respect to their problem types. It is of interest to note that nearly 96% of the problems are of types RA or RB or NA or NB and only about 4% of the problems are of types RC or NC. The exact reason of the imbalance of the distribution of the problems is not revealed by this study, however it might be reasonable to conclude that it is most likely due to the fact that the contents of [A] and [B] are easily discussed topics of mathematics curriculum in gifted education, whereas the content of [C] is neither within the scope of school mathematics curriculum nor easily accessible resource by the students. Hence the students' lack of mathematical content knowledge or substantial learning experiences on the properties of twin primes would caused some constraint in their problem posing.

Now as far as students' solution methods concerned, most approaches turn out suitable even though some of them involve somewhat complicated arithmetic computations. Since NA1 is the only problem with wrong answer, we conclude that about 98% of the students solved their own generated problems correctly.

4. Implications

This study investigates the tendency of the students' problem posing activities characterized by the preference and adaptations of problem resources. Although it is not possible to know precisely the patterns of the student's problem posing practices in the study reported here, an effective students' problem posing practices can be suggested based on the results of our investigation. According to Silver et al. (1996) middle school teachers and prospective secondary school teachers' lack of substantial educational experience with problem posing was not a barrier to their being able to use problem posing with their students. Hence, it is necessary for teachers in gifted eduction to pose problems so that their students also would have substantial experiences with problem posing within school mathematics curriculum.

While the simple task used in this study provides some aspects of the students' problem posing activities in a week-long intensive learning program, there are several ways in which it could be improved:

First, since the original version of problems related to specific solution methods on the division algorithm played crucial roles in this problem posing activity, it seems important to provide the students with suitable opportunities for generating their own problems based on well selected original version of problems or resources with respect to the problem types and features indicated as aspects of problem posing discussed in NCTM (1991).

Second, the fact that more than four of every five problems (43 out of 53 problems) are routine suggests that most students need to enhance flexibility in their adaptations or modifications of given problems to construct or generate their own problems. Hence it would be beneficial to run problem posing activities not only in the intensive learning and but in regular in-class learning so that the students have more substantial opportunities to pose their own problems as English (1997) pointed out that students' problem posing should come an important learning process within total mathematics curriculum not just within a single program of activities.

Third, since the tendency of the students' problem posing performances in this study is evaluated by examining the patterns of the problems related to specific solution methods on the division algorithm, it is evident that students need to broaden the types of problem posing experiences in dealing with more general and informal situations.

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수학 영재의 문제만들기: 사례 연구

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중학교 1학년 수학 영재의 문제만들기 활동에 관한 본 연구의 전반적인 두 가지 목적은 문제만들기 활동을 위하여 나눗셈 정리와 관련하여 소재로 제시한 문제에 대한 선호도를 조사하고 구체적인 해결 방법과 관련된 문제를 만드는데 나타난 접근 방법을 분석하는 것이다. 이를 위하여 수학 영재가 만든 문제를 '정형적인' 문제와 '비정형적인' 문제로 구분하고 나눗셈 정리와 관련하여 제시한 3단계 수준의 문제와 결합하여 모두 6가지 문제 유형으로 분류하였다. 문제 분석 결과를 바탕으로 수학 영재의 문제 만들기 활동에 대한 시사점을 제시하였다.

*key words : mathematically gifted students (수학 영재), problem posing (문제만들기)

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