대한수학교육학회지 수학교육학연구 제 20 권 제 3 호
Journal of Educational Research in Mathematics
Vol. 20, No. 3, 357-371. Aug 2010

Knowledge Variation of Teachers in Middle-School Mathematics Classrooms[1)]

Kim, Goo Yeon*

This study aimed to investigate how two eighth-grade mathematics teachers' pedagogical content knowledge is manifested in their classroom instruction. A case study was conducted for the study. The results revealed similarities and differences in the teachers' knowledge manifested in their mathematics classrooms and that the two teachers had slightly different structures of pedagogical content knowledge. One teacher is more dependent on his knowledge of mathematics and knowledge of students' understanding and the other teacher on her knowledge of instructional process and, to a lesser extent, knowledge of mathematics.

I. Introduction

Teacher knowledge is one source of teachers' actions and decision-making. Teacher knowledge provides them with the flexibility that makes it possible for them to reason, to make a decision, to contemplate alternatives, and to reflect. The knowledge that mathematics teachers hold determines their conduct and their potential efficacy (Clark & Yinger, 1978).

For over decades, research studies in education have focused on teachers' knowledge. Scholars in the field have tried to characterize teacher knowledge to ascertain what knowledge is needed to teach effectively. In mathematics education, various research studies have, in general, focused on teachers' mathematical knowledge. Mathematics teachers' knowledge plays a key role in proficient mathematics teaching. It affects both the mathematics they teach and how they teach it (Ball, 1991, 2000; Ball & Bass, 2000, 2003; Ma, 1999). Knowing mathematics, however, is not sufficient for teaching effectively. For their mathematics teaching to be effective, mathematics teachers should know and deeply understand mathematics, students as learners, and pedagogical strategies and should be capable of using that knowledge flexibly while teaching mathematics (Kilpatrick, Swafford & Findell, 2001; NCTM, 1991, 2000). Such knowledge has been identified and defined as pedagogical content knowledge by Shulman (1986) and mathematical knowledge for teaching (Ball, Lubienski, & Mewborn, 2001; Ball, Hill & Bass, 2005) in mathematics education.

In particular, pedagogical content knowledge strongly supports a perspective of teaching as a profession and teachers as professionals (Grossman, 1995). The teaching practices of mathematics teachers in the classroom would be evidence of their pedagogical

* Sogang University, gokim@sogang.ac.kr
1) This work was partially supported by the Sogang University Research Grant of 2009.

content knowledge. Nevertheless, we lack data that show how teachers' pedagogical content knowledge is manifested and how it can be characterized in classroom teaching practice. It is necessary to fill the gap in the literature by investigating the pedagogical content knowledge of mathematics teachers in the mathematics classroom.

Pedagogical content knowledge is essential for mathematics teachers to teach mathematics effectively. Pedagogical content knowledge in mathematics is much more than simple knowledge of mathematics (Davis & Simmt, 2006; Grossman, 1990; Kahan, Cooper, & Bethea, 2003; Kilpatrick, Swafford & Findell, 2001; Marks, 1990). There have been relatively few studies, however, on the pedagogical content knowledge of mathematics teachers and how it might be characterized in their teaching practice. One reason for so few studies is the difficulties and ambiguities that, by its nature, pedagogical content knowledge contain (Mewborn, 2000). Even though the concept of pedagogical content knowledge might be difficult to identify distinctly, such identification would be very useful because pedagogical content knowledge "represents a class of knowledge that is central to teachers' work and that would not typically be held by nonteaching subject matter experts or by teachers who know little of that subject" (Marks, 1990, p. 9).

The study reported here examines the pedagogical content knowledge of two eighth-grade mathematics teachers in terms of how it is manifested in their classroom instruction and thus identifies similarities and differences in their pedagogical content knowledge. I investigated the pedagogical content knowledge of the teachers by (a) analyzing and articulating their pedagogical content knowledge in the context of their teaching practice and (b) scrutinizing what elements contribute most to their pedagogical content knowledge in teaching mathematics.

II. Conceptual Framework

Pedagogical content knowledge is "a unique kind of knowledge that intertwines content with aspects of teaching and learning" (Ball, Lubienski, & Mewborn, 2001, p. 448) and is commonly considered to be a transformation of two constituent knowledge domains: general pedagogical knowledge and subject matter knowledge (Gess-Newsom, 1999; Marks, 1990). In introducing the term into the literature, Shulman (1986) argued that pedagogical content knowledge is specific to particular subject matter and that learning to teach requires not only understanding the subject itself but also developing a wide repertoire of pedagogical content knowledge. Ball et al. (2001) noted that the concept of pedagogical content knowledge in mathematics implies that teachers must not only know the content of mathematics in depth and conceptually but also know the common representations of particular ideas or concepts that students hold at various stages of development of their mathematical knowledge. Pedagogical content knowledge refers to knowledge that allows the subject matter content to be taught. The concept of pedagogical content knowledge "was originally construed as a form of content knowledge composed of subject matter transformed for the purpose of teaching" (Munby, Russell, & Martin, 2001, p. 881). Hence, teachers must find ways to adapt and represent subject matter by reflecting on it in accordance with the needs of students. As a result,

> the idea of pedagogical content knowledge substantially

> improves our understanding of the knowledge required for teaching. The concept implies that not only must teachers know content deeply, know it conceptually, and know the connections among ideas, but also [they] must know the representations for and the common student difficulties with particular ideas. (Ball, Lubienski, & Mewborn, 2001, p. 449)

Ball, Hill, and Bass (2005) elaborated Shulman's notion of pedagogical content knowledge to be specific to mathematics education and defined mathematical knowledge for teaching, which consists of two key elements: "'common' knowledge of mathematics that any well-educated adults should have and mathematical knowledge that is 'specialized' to the work of teaching and that only teachers need know"(p.22). Also, in order to enhance students' learning mathematics, An, Kulm, and Wu (2004) claim that mathematics teachers need profound pedagogical content knowledge, which is beyond Ma's(1999) notion of profound understanding of fundamental mathematic. In their study of comparing pedagogical content knowledge of mathematics teachers of the U. S. and China, An et al. identified profound pedagogical content knowledge as comprising knowledge of content, teaching, and curriculum in an interactive network for effective teaching.

Investigating 8 fifth-grade teachers' teaching of equivalence of fractions, Marks (1990) presented a structure of pedagogical content knowledge: (a) subject matter for instructional purposes; (b) students' understanding of the subject matter; (c) media for instruction in the subject matter (texts and materials); and (d) instructional processes for the subject matter. He included itemized subcategories in each component. That is, subject matter knowledge consists of the purposes of math instruction, justification for learning a given topic, important ideas to teach in a given topic, prerequisite knowledge for a given topic, and typical "school math" problems. Likewise, students' understanding involves students' learning processes, students' typical understanding, students' common errors, things that are hard/easy for students, and particular students' understanding. Moreover, Marks proposed three possible derivations of pedagogical content knowledge: an interpretation of subject matter knowledge, a specification of general pedagogical knowledge, and a synthesis of both general content knowledge and subject matter knowledge. Marks indicated that the categories of pedagogical content knowledge could be almost equally derived from subject matter knowledge and pedagogical knowledge; for instance, learning activities, understanding students' misconceptions, and the use of teaching strategies.

This study aims to investigate two mathematics teachers' pedagogical content knowledge manifested in their mathematics instruction in middle-school level. For the purpose, the guiding research question is: How is two eighth-grade mathematics teachers' pedagogical content knowledge manifested in their teaching practice?

III. Methods

Participants and Contexts

This study involves a case study among qualitative research methods. For participants for the study, I sought middle-school teachers with approximately 5 to 10 years of teaching experience in the middle grades. This study investigated two eighth-grade mathematics teachers, Mr. Duncan and Ms. Campbell (pseudonyms), in two different middle schools in the Southeastern state in the U. S. Mr. Duncan had been teaching mathematics, primarily eighth grade, at a school of small Southern suburban city for 13 years. Mr. Duncan was one of two mathematics teachers at that grade level in his school. Ms. Campbell was the only eighth-grade mathematics teacher in the school system of a small rural city in the same Southeastern state. She had previously participated in a summer institute that was held as part of a professional development project at a nearby research university in the south and was recommended to me. She enthusiastically agreed to participate in the study. She had about 18 years of teaching experience in elementary and middle grades in the city where she had been born and had grown up; however, this was her first year of teaching mathematics to eighth graders at the time of this study. She had started her teaching career as an elementary teacher and gradually moved to teaching middle grades.

The school contexts where the two teachers taught were different in several respects. First, although both teachers were teaching eighth-grade mathematics, the topics they taught during my observations were different, and second, the way students were grouped in the two schools was different. For example, Mr. Duncan taught six groups of students who were homogeneously grouped. He taught Algebra to the most advanced group, Pre-Algebra to the next-most-advanced groups, and General Mathematics to the lowest group. My classroom observations focused on the advanced groups, the Algebra and Pre-Algebra classes. While Mr. Duncan was one of two eighth-grade mathematics teachers in his school, in contrast, Ms. Campbell was the only eighth-grade mathematics teacher in her school and taught Pre-Algebra to all of the eighth graders, who were heterogeneously grouped. All of the groups were observed in both schools on multiple occasions.

Data Collection and Analysis

Data sources consisted of teacher interviews, classroom observation field notes, and documents. Mr. Duncan participated in two 60-minute interviews and was observed for successive 8 class periods over 2 weeks. To provide further insights into his practice, he responded to a questionnaire. Mr. Duncan audio recorded his answers to the questionnaire and sent the audiotape to me. Classroom observations were recorded and transcribed later. Similarly, Ms. Campbell participated in three interviews; most interviews lasted 40−60 minutes. I observed her for 10 class periods in a raw for about 3 weeks. All interviews and classroom observations were audio taped and later transcribed. For both teachers, I took field notes during the classroom observations, and collected classroom artifacts such as copies of textbooks, review sheets, tests, lesson plans, state curriculum objectives, and handouts.

I analyzed the data using the constant comparative method (Glaser & Strauss, 1967). The process of data analysis began with finding key words from the interview transcripts and categorizing those words into themes emerging from the interviews. Every problem, comment, question, representation, or symbol was classified into categories derived from the theoretical framework, which included knowledge of subject matter, knowledge of students' understanding, and knowledge of pedagogy. In addition, I analyzed the field notes according to the theoretical framework. Then, I organized the data with regard to the above characterizations. Each category consisted of subcategories that emerged from the data. The interview data were constantly compared with the field notes, the textbooks, and classroom artifacts.

IV. Findings

Mr. Duncan's Mathematics Classroom

Mr. Duncan taught the mathematical topics of radicals and radial equations in the Algebra class during my observations. He viewed those topics are "pieces of the puzzle in Algebra I" and thus, the students must understand the concepts of what square root means, how to take a square root, what the sequence of a number is, and how to get rid of a square root sign. Further, he valued that the students should be able to apply the concepts and to transfer the concept of a radicals to radical equations. In addition, Mr. Duncan said that there are a variety of ways of solving radical equations; there is always one more tool or method to solve the equation. By making connections to prior learning, Mr. Duncan used various examples of fraction concepts to teach how to simplify radicals. For example, he reminded and showed the "rules" for simplifying fractions and extended the method of multiplying fractions such as multiplying denominators and numerators and then doing cancellation and vice versa.

Mr. Duncan seemed to emphasize to his students that there are many ways to get answers in solving mathematical problems. During his instruction, he repeatedly told the students that they needed to learn different approaches to solving a problem and to understand those approaches. Although he did not ask them to try a different method for every problem in his mathematics classrooms, he appeared to make efforts to draw various methods of solution from the students. He would get a student to show his or her own way; sometimes students would volunteer to present their approach on the board if they had a different one from Mr. Duncan's presentation or from their peers' methods. In doing this, he urged them to look for different methods and be able to find a lot of ways to arrive an answer. In particular, it was obvious that Mr. Duncan anticipated what the students would feel confused about and what they would find difficult about the topic by mentioning "One student may do it in this way," "Another student might do it using cross cancel later," or "One student may confuse it with cross cancel later."

Mr. Duncan used questions to elicit the students' thoughts, misconceptions, and errors as examples. He reviewed his students' common errors that were made on a test after the test on radicals; he picked out examples from the test explained "the rules" of simplifying and factoring skills. In doing that, he again asked questions to draw out students' misconceptions in simplifying square roots. Moreover, he extended such examples to specify students' confusions and difficulties in solving radical equations. By exemplifying fractions, he showed several more examples that might be confusing to the students. Rather than giving an explanation, he posed questions, which provided the students with an opportunity to see and reflect upon their reasoning and confusions about the solving radical equations.

When introducing new concepts or topics, Mr. Duncan used examples of real world applications of such concepts or topics in order to motivate students. For instance, when introducing the Pythagorean theorem, he posed a problem: A teenager gets a 5-foot pole at a store and has to take a bus to get home. But it is only permissible to carry a maximum of a 4-foot-wide object on the bus. How could the teenager resolve the situation? Although no students got the correct answer, the students seemed to be very engaged in working on the problem.

It was noteworthy that Mr. Duncan seemed to follow the organization of a textbook in teaching radicals, the Pythagorean theorem, and radical equations. However, the organization of each lesson at times did not correspond to that suggested in the textbook. Although he used example problems and practice problems from the textbook, Mr. Duncan reorganized the concepts and made connections between the concepts and students' prior knowledge. For example, he created a handwritten handout for a lesson on operations with radicals, which included a brief review of simplifying "rules" and a list of practice problems. Some of the problems were from the textbook, but others were not. On occasions, Mr. Duncan appeared to ignore an important concept or a fact that should have been explained to the students and was suggested in the textbook.

Surprisingly, Mr. Duncan regarded himself as still developing his knowledge about mathematics:

> As I have taught more, as I've read more, I think I have also been challenged by students more, and I just think I increased my knowledge through courses that I have taken. This is my, I think, thirteenth year of teaching, and [I] certainly am just challenged and grow in my math abilities each and every year. I think staff development [activities] is another way that I can become more confident in my knowledge of mathematics.

On the other hand, it was interesting that he also saw himself as having "limited knowledge" or "more of a textbook knowledge" of the concept of radicals; he described that knowledge as follows:

> [A] Teacher who may be asked to teach one yea, who maybe teachers science, for example, who [is] asked to teach history. Their knowledge is probably limited with just what they have been exposed to. They don't have the background knowledge or the depth of knowledge. And I think that I feel that way with some of my content. Radicals would probably be that way. I know what I have read from a book, and I know that I've never been taught as a student myself. But probably having a great depth of knowledge of how radicals can be used in research or the scientific world or in different areas of development [is needed]. I just feel that I would be confident in saying that I'm limited in that.

Ms. Campbell's Mathematics Classroom

Ms. Campbell taught rational numbers and probability and integers for a review in her algebra class during my observations. She seemed to understand somewhat deeply as she created "a wheel of rational numbers" showing the concepts of number system, properties, compare, convert, simplify, evaluate, graph, and proportions are all related to rational numbers and used the wheel to explain rational numbers. This was connected to Ms. Campbell's teaching the definition of probability, which was described as a ratio "that is basically a fraction." In reviewing integers for review, she seemed to seek to associate integers with realistic situations by providing an example of temperature. In doing so, she appeared to connect the students' real lives to the concept of negative integers so that the students could appreciate that the concept is not isolated and they are using the concept in their lives.

Ms. Campbell seemed to focus on following and covering goals of a curriculum map derived from the state standard curriculum for the eighth grade. Using the curriculum map, she elected and topics and concepts to teach and organized them by considering the flow and connections with previous topics or concepts. Although Ms. Campbell said that she was trying to help her students to build new knowledge on their prior knowledge, she often appeared to just repeat what she had done before in review rather than to add something new information. For example, when teaching probability, she might have covered compound probability; however, she did not go beyond the basic concept of probability.

Considering many different students' learning styles such as "concrete" and "abstract" learners, Ms. Campbell seemed to attempt to have various ways for her students to actively participate in her classroom. With the realization, she involved various manipulatives, visuals, and hands-on activities. It was evident that she realized students' difficulties in learning equations with fractions and decimals and inequalities having a negative sign with a variable due to lack of their understanding on the concepts. In addition, Ms. Campbell incorporated mistakes and errors that students commonly made during the lessons on fractions. She presented an approach that would less confuse the students when they could not grasp the concept in subtracting mixed numbers. For instance, Ms. Campbell had her students convert mixed numbers to improper fractions in learning of subtraction of mixed numbers because she found that the process of regrouping and then rewriting confused the students. Also, Ms. Campbell recognized students' common misconceptions on using exponents involving negative numbers and thus, she encouraged to connect the process of with what they had cone with whole numbers.

Ms. Campbell had various ways of representing mathematical ideas and concepts in the lessons. The representation varied in explanations, examples, counter-examples, and demonstrations conveyed by symbols, words, and pictorial forms. The representations were combined with one another most of the time. By the same token, she seemed to provide many examples of mathematical concepts in real-life situations. For instance, when teaching probability concept, she exemplified it by referring to a weather forecast. She also employed word problems for almost every concept and topic. Furthermore, Ms. Campbell presented more than one example for a concept. For a sample space of probability, she asked the students to list all the possibilities when rolling a die, choosing a letter from the alphabet, selecting a student from among those in the fourth-period class, and flipping a coin. While teaching the Pythagorean Theorem, she showed several ways in which it can be applied to find the third side of a triangle.

Ms. Campbell drew on materials that enable students to better understand mathematical concepts by doing hands-on activities. Among the materials and activities, a mathematics journal was a very important material for the students' learning in Ms. Campbell's class. All students had to have their own mathematics journal as a resource and future reference. She said,

> The mathematics journal is a manual that the students could use. It is a manual, and that's what I have tried to explain to them. Just like a mechanic doesn't know everything; he has a book that he can go to. If it is a particular job that he hasn't done in a long time, he might need a little reminder of how to do this particular job. [It is the] same with an engineer. He doesn't know everything, but he's got a manual or a resource that he can go to. And what I've tried to emphasize to the kids is that this is just a resource so that [if] by chance you have forgotten this particular concept, you can go into your resource and look at it and see what it means and see an example or two. And then maybe that will be, 'Oh, that's how you do it.'

Whereas the textbooks were never explicitly used, students' mathematics journals were used to take notes as needed in every lesson. What the students kept in their journals included the essential question that Ms. Campbell posed at the beginning of each lesson, answers to the question, examples, properties, formulas, tables, and charts. At times, the students "transferred" items from worksheets when the concepts or problem types were important. What is more, the students were strongly encouraged to consult their mathematics journals when taking tests and exams, which was another purpose Ms. Campbell supported the students to keep their journals in her classroom.

V. Discussion

Similarities

There exist similarities in Mr. Duncan and Ms. Campbell's pedagogical content knowledge manifested in their mathematics teaching (See Figures 1 and 2). Not surprisingly, first, they appeared to be influenced by knowledge of mathematics involving the purpose of teaching mathematics, making connections among topics, and understanding concepts to teach. For example, Mr. Duncan applied his knowledge of fractions to teaching radicals and radical equations as to explain how to simplify with fractional form of radicals. Similarly, Ms. Campbell's understanding of rational numbers and the connections between them was reflected in her decisions about the ordering of topics to teach.

Both Mr. Duncan and Ms. Campbell seemed to focus very strongly on students' understanding in their mathematics instruction. Their knowledge of students' understanding seemed to consist of similar elements: knowledge of students' common errors and misconceptions and of students' difficulties and confusions. In particular, both teachers showed their understanding of students' misconceptions of the multiplication and division of fractions, which hindered the students in understanding new concepts and in solving problems. Ms. Campbell recognized students' common mistakes with computation and linear equation involving mixed numbers or improper fractions and students' misconceptions about using exponents. Likewise, Mr. Duncan repeatedly revisited students' work that involved misconceptions and errors from tests and frequently. In doing that, he asked questions to elicit the students' thoughts, difficulties, misconceptions, and confusions on simplifying and computation of radicals and radical equations.

By using realistic examples and applications, Mr. Duncan tried to motivate the students and to promote their thinking, which illustrated his knowledge of instructional process and simultaneously demonstrated his knowledge of mathematics. For instance, when introducing the Pythagorean Theorem, Mr. Duncan presented a problem in which the Pythagorean Theorem is applied in the real world in order for the students to enthusiastically engage

in working on the problem. Ms. Campbell also tried to use realistic examples and applications. Although she felt uncomfortable solving word problems herself,

she frequently incorporated word problems into her lessons to help them see how mathematical topics and ideas can be applied and thus motivate their learning mathematics.

Differences

The two teachers showed differences in their manifestation of pedagogical content knowledge (See Figures 1 and 2). First of all, knowledge of students' understanding played a key role in Mr. Duncan's teaching. His major concerns in teaching lay in the

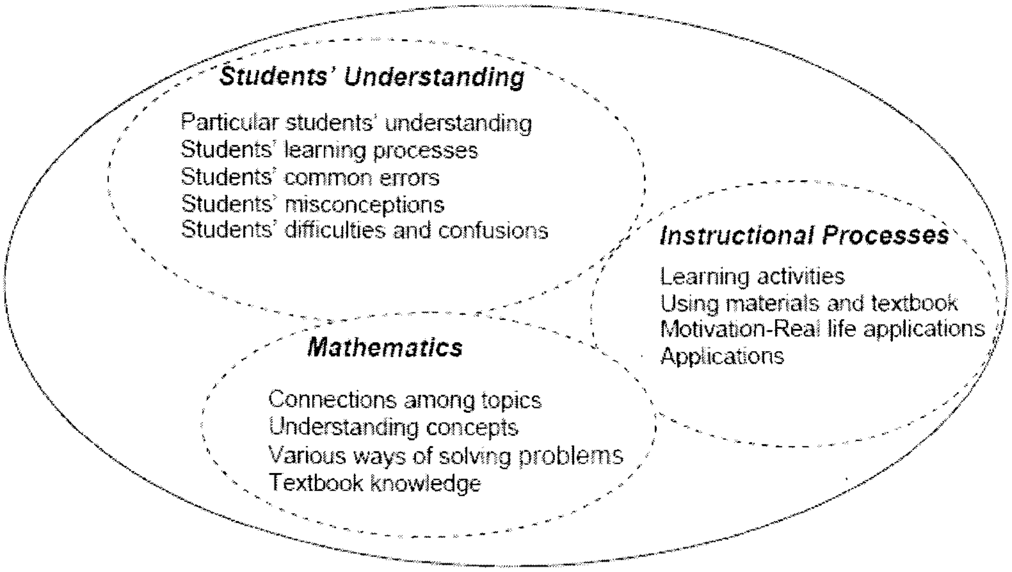

Figure 1. Salient elements of Mr. Duncan's pedagogical content knowledge.

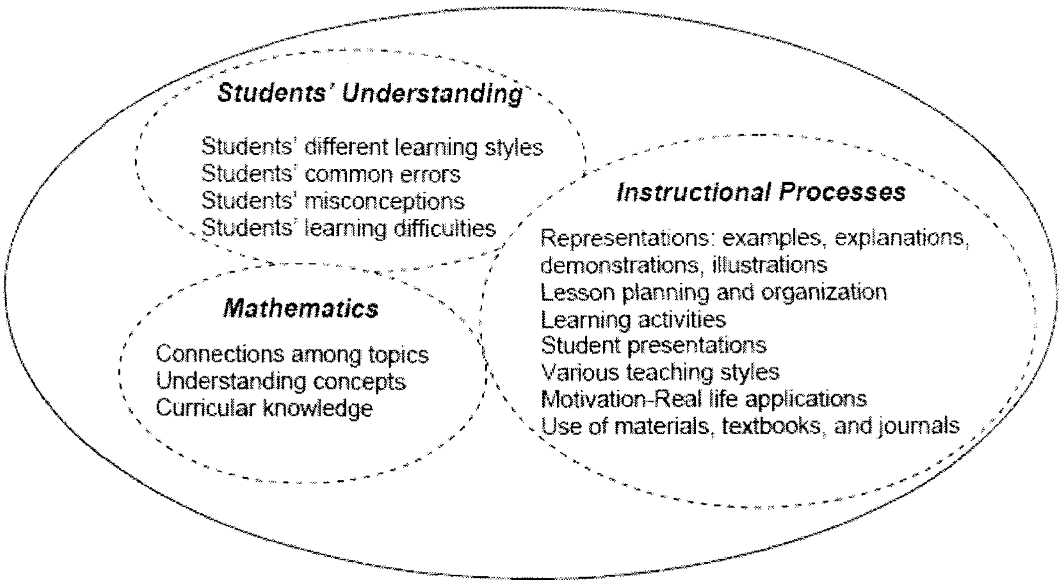

Figure 2. Salient elements of Ms. Campbell's pedagogical content knowledge.

students' learning for both academic and practical purposes. In his instruction, he appeared to particularly, apply his knowledge about students' learning processes and understanding, and particular students' topic-specific understanding. For instance, by paying attention to a student' common mistakes and difficulties in simplifying radicals, he used his knowledge to refine his explanations and to elicit the students' thinking so that the students would better understand the topic and correct themselves. Mr. Duncan's knowledge made it possible for him to improve his students' thinking and to develop appropriate activities for them (Ball, 2000; Ball & Bass, 2000; Franke & Kazemi, 2001; Llinares, 2000). Furthermore, he used many examples obtained from the students' solutions or approaches to problems.

In contrast, Ms. Campbell's knowledge of students' understanding seemed general rather than topic-specific; she was aware of how students learn differently, but her awareness was related to visual, auditory, and kinesthetic learning styles, not to subject-specific knowledge. Although Ms. Campbell had more opportunities than Mr. Duncan to obtain knowledge of students' understanding because her lessons were organized in a greater variety of pedagogical styles, she did not appear to apply much of that knowledge to her lessons. She responded to and corrected errors according to the students' needs when they expressed their confusions and misconceptions or made mistakes while solving problems; however, she did not use that knowledge to adapt her instruction.

Most important, Mr. Duncan's mathematics teaching seemed to based on his knowledge of mathematics; he appeared to have a profound understanding of some concepts or topics so that he was able to use alternative approaches, give counter-examples, and make connections within the algebra chapter of the book and across chapters and grade levels. Also, his decisions about whether a particular student's answer and approach were correct were rooted in his knowledge of mathematics (Ball, 1999, 2000; Even & Tirosh, 1995; Fernandez, 1997).

Whereas Mr. Duncan's pedagogical content knowledge appeared to be relatively dependent on his knowledge of students' learning and his knowledge of mathematics, Ms. Campbell's pedagogical content knowledge relied heavily on her knowledge of pedagogy. Unlike Mr. Duncan, whose use of instructional strategies appeared to be rather narrow during my observations, Ms. Campbell seemed to depend on a variety of pedagogical techniques. Although both of them considered realistic applications important in their teaching, they used them differently. Mr. Duncan used realistic examples and problems to motivate students; in contrast, Ms. Campbell used them to help students better understand concepts and practice solving problems. For example, I observed Mr. Duncan using a story problem in a real world context when introducing the Pythagorean Theorem. Ms. Campbell also used various instructional tasks including word problems; hands-on activities; games; auditory, visual, and kinesthetic presentations; and student presentations. She either created or adapted an activity for almost every lesson, which seemed to be rooted in her beliefs that students learn better when they are actively involved. Whereas Mr. Duncan covered most of the topics of a chapter in the textbook, Ms. Campbell omitted some topics. This omission occurred because she used the textbook as a resource rather than as a curriculum guide. This might be related to her feeling of weakness and discomfort with certain topics of mathematics.

Ms. Campbell's use of mathematics journals was significant. She had students keep journals to facilitate their learning. They could revisit their journals to see what they had done. I assumed that her use of journals was related to her knowledge of students' understanding, but she apparently did not use much of the knowledge she obtained from reading her students' journals. By assessing student presentations, she became aware of students' difficulties, confusions, common errors, and misconceptions; unlike Mr. Duncan, however, Ms. Campbell did not apply much of that knowledge to her lessons.

Ms. Campbell's lesson planning and organization appeared to come from her knowledge of mathematics. When planning her lessons, she considered whether a topic could be connected to previous topics. This consideration showed her sound understanding of certain concepts, such as rational numbers. Although she could see connections among topics in algebra, she apparently did not see the connections between algebra and geometry until she participated in a professional development program. She did not consider herself an expert in mathematics. She felt comfortable in some areas of mathematics teaching, such as solving algebraic equations, divisibility rules, how to factor, and finding the greatest common factor and uncomfortable in other areas such as working on word problems and problems with percentages.

VI. Conclusions and Implications

This case study of the pedagogical content knowledge of two eighth-grade mathematics teachers suggests slightly different structures of pedagogical content knowledge. Mr. Duncan's pedagogical content knowledge sometimes appears to be a combination of his knowledge of mathematics and knowledge of students' learning, and at other times a combination of his knowledge of mathematics and knowledge of instructional process, or a combination of all the components. Ms. Campbell's pedagogical content knowledge at times is manifested as a union of knowledge of instructional process and knowledge of mathematics or of all components and on occasion as a combination of knowledge of students' understanding and knowledge of instructional process.

Mr. Duncan was more dependent on knowledge of mathematics and knowledge of students' understanding, whereas Ms. Campbell was more dependent on knowledge of pedagogy and, to some degree, knowledge of mathematics. Consequently, no single model fits the pedagogical content knowledge of both teachers, perhaps because they used their knowledge differently in teaching mathematics. Or they might have focused their knowledge differently in teaching. This study did not explore the relationships among the components of pedagogical content knowledge. The results, however, imply that the components are interactive and interrelated; each component affects the others.

Finally, this study supports the notion that pedagogical content knowledge is dynamic (Cochran et al., 1993). The components of pedagogical content knowledge seemed to be subsets of a whole; pedagogical content knowledge could not exist without its various components, and the union of those components constitutes pedagogical content knowledge.

The findings of this study suggest that the two teachers' knowledge of mathematics is reflected in their different approaches to teaching mathematics

(Ball, 1991; Ball et al., 2001; Even & Tirosh, 1995; Fernandez, 1997; Leinhardt et al., 1991; Ma, 1999). The teachers not only realized the strengths, weaknesses, and limitations in their knowledge of mathematics but also saw them as developing their knowledge through teaching experiences (Hiebert & Carpenter, 1992).

The findings also suggest that pedagogical content knowledge varies by teachers, student groups, and topics. As shown in the previous chapters, the two teachers' pedagogical content knowledge did not look the same; further, the components comprising the pedagogical content knowledge were manifested in different ways. The teachers were more dependent on some types of knowledge than on others. Also, not only did their pedagogical content knowledge vary according to whom they taught but it also was differently revealed by the topics they taught.

This study supports the view that pedagogical content knowledge is quite dynamic (Cochran et al., 1993). That pedagogical content knowledge varied by teacher, student group, and topic proves its dynamics. Pedagogical content knowledge is neither static nor unchangeable. The teachers seemed to be developing their pedagogical content knowledge through their own learning. Their increase in one type of knowledge influenced their pedagogical content knowledge. Moreover, the findings suggest that the components of pedagogical content knowledge are combined rather than isolated and interact with one another. In their instruction, the teachers' pedagogical content knowledge was often manifested in a combination of its components.

Future studies of pedagogical content knowledge could explore the relationships among its components for particular mathematics teachers. Within the framework of a particular mathematics teacher's pedagogical content knowledge, the association between the components would explain how each affects the others. In other words, one could investigate how a teacher's improved knowledge of mathematics might influence his or her knowledge of students' understanding and of pedagogy.

References

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U. S. *Journal of Mathematics Teacher Education, 7*, 145-172.

Ball, D. L. (1991). Research on teaching mathematics: Making subject matter knowledge part of the equation. In J. Brophy (Ed.), *Advances in research on teaching: Vol. 2. Teachers' knowledge of subject matter as it relates to their teaching practice* (pp. 1-48). Greenwich, CT: JAI.

Ball, D. L. (1999). Crossing boundaries to examine the mathematics entailed in elementary teaching. *Contemporary Mathematics, 243*, 15-36.

Ball, D. L. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education, 51*, 241-247.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 83-104). Westport, CT: Ablex.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433-456). Washington, DC: American Educational Research Association.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(3), 14-17, 20-22, 43-44.

Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 17-36). New York: Routledge.

Clark, C. M., Yinger, R. J. (1978). *Research on teacher thinking*. East Lansing, MI: Institute for Research on Teaching.

Cochran, K. F., DeRuiter, J. A., & King, R. A. (1993). Pedagogical content knowing: An integrative model for teacher preparation. *Journal of Teacher Education, 44*, 263-272.

Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics, 61*, 293-319.

Even, R., & Tirosh, D. (1995). Subject-matter knowledge and knowledge about students as sources of teacher presentations of the subject-matter. *Educational Studies in Mathematics, 29*, 1-20.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-147). New York: Macmillan.

Fernandez, E. (1997). *The "Standards-like" role of teachers' mathematical knowledge in responding to unanticipated student observations*. Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory into Practice, 40*, 102-109.

Gess-Newsome, J. (1999). Pedagogical content knowledge: An introduction and orientation. In J. Gess-Newsome & N. G. Lederman (Eds.), *Pedagogical content knowledge and science education: The construct and its implications for science education* (pp. 3-17). Dordrecht, Netherlands: Kluwer.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Hawthorne, NY: Adline de Gruyter.

Grossman, P. L. (1995). Teachers' knowledge. In L. W. Anderson (Ed.), *International encyclopedia of teaching and teacher education* (pp. 20-24). New York: Elsevier.

Hashweh, M. Z. (1987). Effects of subject-matter knowledge in the teaching of biology and physics. *Teaching and Teacher Education, 3*(2), 109-120.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on* *mathematics teaching and learning* (pp. 65-97). New York: Macmillan.

Hill, H. C. (2007). Mathematical knowledge of middle school teachers: Implications for the No Child Left Behind Policy Initiative. *Educational Evaluation and Policy Analysis, 29*, 95-114.

Kahan, J. A., Cooper, D. A., & Bethea, K. A. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education, 6*, 223-252.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Leinhardt, G., Putnam, R., Stein, M. K., & Baxter, J. (1991). Where subject knowledge matters. In J. Brophy (Ed.), *Advances in research on teaching: Vol. 2. Teachers' knowledge of subject matter as it relates to their teaching practice* (pp. 87-114). Greenwich, CT: JAI.

Llinares, S. (2000). Secondary school mathematics teachers' professional knowledge: A case from the teaching of the concept of function. *Teacher and Teaching: Theory and Practice, 6*(1), 41-62.

Lodholz, R. D. (1990). The transition from arithmetic to algebra. In E. L. Edwards, Jr. (Ed.), *Algebra for everyone* (pp. 24-33). Reston, VA: National Council of Teachers of Mathematics.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of Teacher Education, 41*(3), 3-11.

Mewborn, D. S. (2000). *An analysis of the research on K-8 teachers' mathematical knowledge*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans.

Munby, H., Russell, T., & Martin, A. K. (2001). Teachers' knowledge and how it develops. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 877-904). Washington, DC: American Educational Research Association.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Philipp, R. A., & Schappelle, B. P. (1999). Algebra as generalized arithmetic: Starting with the known for a change. *Mathematics Teacher*, 92, 310-316.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

Tamir, P. (1988). Subject matter and related pedagogical knowledge in teacher education. *Teaching and Teacher Education, 4*, 99-110.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A. F. Coxford (Ed.), *The ideas of algebra, K-12* (pp. 8-19). Reston, VA: National Council of Teachers of Mathematics.

중학교 수학수업에 나타난 다양한 형태의 교사지식 분석 및 고찰

김 구 연 (서강대학교)

이 연구의 목적은 중학교 수학교사의 교수내용지식이 수업을 통하여 어떻게 나타나는지 다양한 형태의 교사 지식을 분류하고 분석하는 것이다. 연구 결과, 연구에 참여한 두 명의 수학 교사의 지식은 유사하면서도 다른 구조의 형태를 띠고 있었다. 한 교사는 수학 내용 지식과 학생에 대한 지식을 더 많이 사용하는 것으로, 다른 교사는 교수적인 전략과 수학내용 지식을 더 많이 활용하는 것으로 나타났다.

* **Key Words** : teacher knowledge (교사 지식), mathematics teaching (수학 수업), middle grades mathematics teaching (중등 수업), pedagogical content knowledge (교수 내용 지식), knowledge variation (다양한 유형의 지식 활용)

논문접수: 2010. 7. 23
논문수정: 2010. 8. 12
심사완료: 2010. 8. 20