

A Novel Scheme for Sliding-Mode Control of DC-DC Converters with a Constant Frequency Based on the Averaging Model

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Abstract

A new scheme for sliding-mode control (SMC) of DC-DC converters with a constant switching frequency is proposed. The scheme is based on the averaging model and the output signal of the controller is d^+ or d^- instead of the on or off signal of a direct sliding-mode (SM) controller or the continuous signal $d = u_{eq}$ of an indirect SM controller. Two approaches using the new scheme are also proposed and the design procedures for a buck converter are given in detail. The first approach called constant d^+ and d^- SMC is simple, cost effective and dynamically fast. In order to improve the dynamic characteristics of the reaching phase and to alleviate chattering, the second approach called reaching law SMC is also presented. Analyses and simulation results demonstrate the feasibility of the proposed scheme.

Key Words: DC-DC converter, Sliding mode control, Constant switching frequency, Averaging model, Reaching law

I. INTRODUCTION

The use of sliding-mode control (SMC) techniques in variable structure systems (VSS) makes these systems very robust to parameter variations and external disturbances. Switching converters constitute an important case of VSS, and various sliding mode strategies to control this class of circuits have been reported in the past [1]–[7].

The main advantages of SMC over conventional Pulse Width Modulation (PWM) control are stability (even for large supply and load variations), robustness, good dynamic response and simple implementation.

An ideal sliding-mode (SM) requires an infinite switching frequency in order to maintain the state on the sliding manifold. Obviously, an infinite switching frequency is not acceptable in practice and therefore one must implement a technique that can ensure a finite and possibly constant switching frequency [4]. In many applications, having a constant switching frequency is desirable and sometimes it is mandatory [5]. Due to this fact, methods to limit the switching frequency have been proposed [4]–[10]. In Section 2, these methods are discussed and a new scheme for the SMC of DC-DC converters with a constant switching frequency is proposed.

In Section 3, two approaches using the new scheme for a buck converter are proposed and the design procedures are

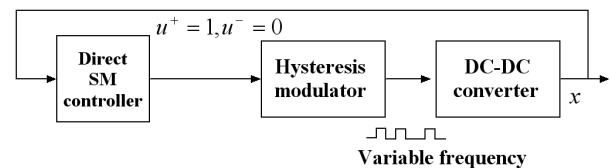


Fig. 1. Block diagram of the direct SMC scheme.

given in detail. The first approach called constant d^+ and d^- SMC is easy to implement and the second called reaching law SMC has good static and dynamic properties. It should be noted that the scheme is also suitable for other types of converters. In Section 4, the simulation results and discussions of a buck converter controlled by the two above mentioned approaches are given in detail.

II. TRADITIONAL AND PROPOSED SMC SCHEMES FOR DC-DC CONVERTERS

Basically, there are two familiar schemes of SMC of DC-DC converters. One is called the direct SMC scheme, since the output signal of the hysteresis circuit directly controls the switching of the power transistor. In the other scheme, the output of the SM controller, similar to PWM switching, is first compared with an external triangular waveform. Then the output signal of the comparator is employed to switch the power transistor. In this case, the control signal is not applied directly to the power switch. Therefore, it is referred to as the indirect SMC scheme.

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A. Direct SMC

Power converters can be modeled by:

$$\dot{x} = Ax + Bu + D, \quad (1)$$

where $x \in R^n$ is the system state vector, usually consisting of the inductor currents and capacitor voltages. The matrices A, B and D are all continuous and $u \in \{0, 1\}$ is the switch position which makes the system discontinuous. The dot stands for the derivative with respect to time.

The SM controller design process aims at determining the switch position u , which generally has the form:

$$\begin{cases} u = 1 & \text{for } s(x) > 0 \\ u = 0 & \text{for } s(x) < 0 \end{cases}, \quad (2)$$

where $s(x)$ is a smooth scalar function. A converter controlled by a direct SM controller is shown in Fig1.

The function of a hysteresis modulator is to ensure a finite and possibly constant switching frequency. The control law in Eq. (2) is redefined as:

$$u = \begin{cases} 1 = 'on' & \text{if } s(x) > k \\ 0 = 'off' & \text{if } s(x) < -k \\ \text{unchanged} & \text{otherwise} \end{cases} \quad (3)$$

Thus, by introducing a region $-k < s(x) < k$ where no switching occurs, the maximum switching frequency of the SM controller can be controlled. Additionally, it is now possible to control the frequency of the operation by varying the magnitude of k . However, if k is constant, SM controlled converters generally suffer from significant switching frequency variation when the input voltage or output load is varied [6].

In order to ensure a constant switching frequency for all operating conditions, there are two common approaches that can be used. One approach is to incorporate a constant ramp or timing function directly into the controller [7]–[9]. The main advantage of this approach is that the switching frequency is constant under all operating conditions, and can be easily controlled through varying the ramp/timing signal. However, due to the imposition of a ramp or timing function onto the SM switching function, the resulting converter system suffers from deteriorated transient response [6]. The other approach utilizes an adaptive hysteresis band, but this leads to a more complicated controller implementation [4].

B. Equivalent control based indirect SMC

Indirect SMC is realized by changing the modulation method of the SM controller from hysteresis modulation to PWM, also known as duty cycle control. The advantage of PWM is that by fixing the frequency of the ramp, the frequency of the output switching signal will be constant, regardless of how the duty cycle varies with the variation of the control signal [10]. A converter controlled by an indirect SM controller is shown in Fig. 2. To achieve such a controller, a relationship between direct SMC and duty cycle control is required.

If a direct SM controller works in the ideal case of infinite switching frequency, at some time, the system trajectory will

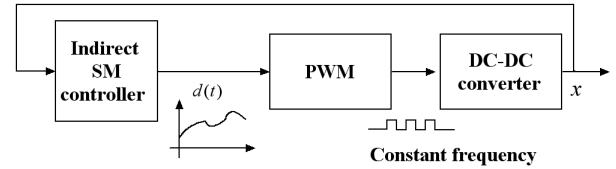


Fig. 2. Block diagram of the indirect SMC scheme.

evolve on the surface $s(x) = 0$. When this happens, the system trajectory is described by means of the equivalent dynamics [5]:

$$\dot{x} = Ax + Bu_{eq} + D, \quad (4)$$

where u_{eq} , referred to as the equivalent control, is the solution for u in the equation $\dot{s}(x) = 0$.

It has been proven in [11] that u_{eq} is the average of u . Hence, the performance of a SM controller u should be similar to the performance of an average controller d provided that $d = u_{eq}$ [12]. Consequently, it is possible to design a SM controller like the one defined in Eq. (2) and implement it with a PWM by making $d = u_{eq}$.

In practice, indirect SMC is similar to classical PWM control schemes in which the control signal is compared to the ramp waveform to generate a discrete gate pulse signal. However, the implementation of u_{eq} is usually far more difficult than $s(x)$ [5] and it is not applicable to all kinds of sliding surfaces. To simplify SM controller implementation, an alternative method is proposed in [13]. The advantages of this method are that it does not need additional hardware circuitries since the switching function is performed by the PWM modulator, and that its transient response is not deteriorated. However, the implementation is nontrivial in order to preserve the original SM control law, especially when both current and voltage state variables are involved [6].

C. Proposed SMC scheme

Using the state space averaging method, Eq. (1) can be transformed to:

$$\dot{\tilde{x}} = A\tilde{x} + Bd + D. \quad (5)$$

Vector \tilde{x} is the moving average of vector x ; the matrices A, B and D are the same as those appearing in Eq. (1); $d \in [0, 1]$ is the duty cycle.

The idea presented in this paper is rooted in the Pulse Adjustment Control Technique for DC-DC converters operating in DCM proposed in [14]. This technique achieves output voltage regulation based on generating high and low power pulses, rather than employing the PWM control technique. When the output voltage is lower than the desired voltage level, the controller chooses d_H as the switching duty ratio until the desired voltage level is reached. On the other hand, if the voltage is higher than the desired voltage level, the controller chooses d_L as the duty ratio to reduce the level of the output voltage to its reference value [14].

The merit of this technique is that it needs only a few logic gates and comparators to implement this control, thus, making

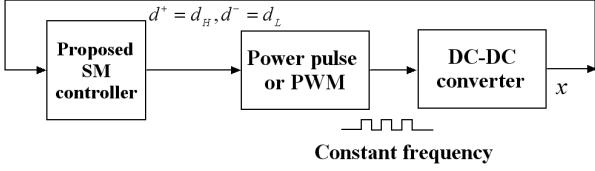


Fig. 3. Block diagram of the proposed scheme.

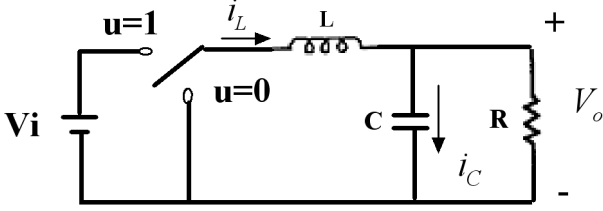


Fig. 4. Simplified diagram of a buck converter.

it simple, cost effective and dynamically fast. It is similar to an averaging model based SMC with a sliding surface of $s = V_{ref} - v_0$. However, its process of analysis and synthesis is cumbersome, tedious and not from the viewpoint of SMC. Furthermore, it has been proven in [1] that buck and boost converters in CCM can not use $s = V_{ref} - v_0$ as a sliding surface because the equilibrium point for the inductor current is unstable.

The proposed scheme in this paper is based on a model of Eq. (5). Analysis and synthesis methods similar to direct SMC are used to design the controller. However, the output signal of the controller is d^+ or d^- instead of the 1(ON) or 0(OFF) signals of a direct SM controller or the continuous signal $d = u_{eq}$ of the traditional indirect SM controller. The power switch can be driven by a power pulse or PWM for different situations, so it has a constant switching frequency naturally. Fig.3 is a block diagram of a converter controlled by the proposed SM controller.

In this paper, the scheme is realized by two approaches in which d^+ and d^- are constant or variable. The first approach has the same advantages as the Pulse Adjustment Control Technique mentioned earlier. Furthermore, other types of sliding surface can be used. However, it is difficult to guarantee the dynamic characteristics in the reaching phase and the chattering problem is not negligible. In order to improve the performance of the reaching segment and alleviate chattering, the concept of a reaching law was introduced in [16]. The second approach is based on the reaching law theory. In this paper, we refer to the first approach as constant d^+ and d^- SMC and the second as reaching law SMC.

III. EXAMPLE: VOLTAGE CONTROL FOR A BUCK CONVERTER OPERATING IN CCM

A. Modeling of a buck converter

A simplified diagram of a buck converter is shown in Fig. 4.

To design the control system of a converter, it is necessary to model the converter's dynamic behavior. Unfortunately, the modeling of this behavior is hampered by the nonlinear time-varying nature of the switching and pulse-width modulation

processes. The formulation, in the form of a bilinear system, of a buck converter is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{V_i}{L} \\ 0 \end{bmatrix} u. \quad (6)$$

where x_1 and x_2 are the inductor current i_L and the capacitor (output) voltage $v_c(v_0)$ respectively; u is a scalar control taking values from the discrete set $\{0,1\}$ which defines the switch positions; C , L and R are the capacitance, inductance and instantaneous load resistance respectively; V_i is instantaneous input voltage.

The state space averaging technique is an approximation technique used to analyze switch mode power converters. The behavior of a circuit during each switching state is taken into account and an average is obtained. This method can be used for any of the converter topologies by monitoring the state variable waveforms for each state and then averaging them over time. Using the state space averaging method, the averaging model of a buck converter in CCM can be obtained by:

$$\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} + \begin{bmatrix} \frac{V_i}{L} \\ 0 \end{bmatrix} d \quad (7)$$

where \bar{x}_1 and \bar{x}_2 are the moving averages of x_1 and x_2 respectively; d is the duty cycle.

B. Design of a constant d^+ and d^- SM controller

SMC have been used to improve the robustness and the dynamic response in different converters. Many papers have presented a variety of SM controller design steps, which could be summarized as follows:

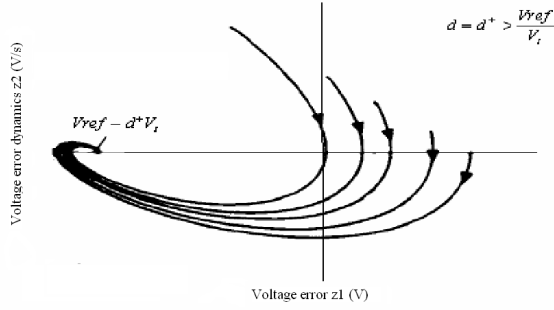
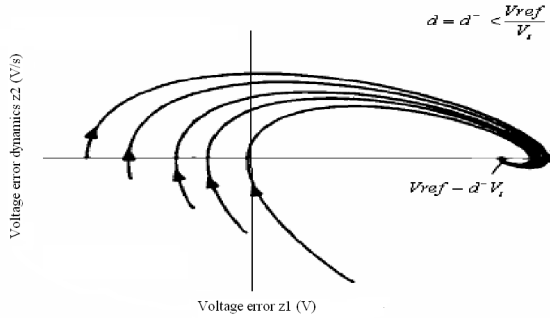
- 1) To relate the steady state and the dynamic requirements with an appropriate sliding surface.
- 2) To establish a structure-control law with reference to the selected sliding surface, such that the conditions of the reaching and the existence of a sliding regime are satisfied.

1) *Design of the sliding surface:* Typically, the sliding surface is a linear combination of the state variables. This is due to its easy implementation and theoretical analysis. The SM controller forces the system to be held in the mentioned surface and then the system is driven to the equilibrium point. The sliding surface must include the equilibrium point.

The inductor current x_1 and the output voltage x_2 are both continuous variables. The derivative of the output voltage x_2 is proportional to the capacitor current, which is also a continuous variable. Let the desired output voltage be V_{ref} . Then the moving averaging of the output voltage error is defined as $z_1 = V_{ref} - \bar{x}_2$ and its derivative is $z_2 = \dot{z}_1 = -\bar{i}_C/C$. As a result the averaging state-space model in phase canonical form, derived from Eq. (7), is:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -\frac{1}{LC}z_1 - \frac{1}{RC}z_2 - \frac{V_i}{LC}d + \frac{V_{ref}}{LC} \end{cases} \quad (8)$$

A buck converter is a second order system with one control input. Therefore the dynamic response obtainable under a SMC is of the first order. The desired steady state and dynamic

Fig. 5. Phase trajectories of $d = d^+$ for different starting positions.Fig. 6. Phase trajectories of $d = d^-$ for different starting positions.

response may then be expressed as a one dimensional sliding surface (line) in the phase plane, which is expressed as:

$$s(z) = \alpha z_1 + z_2 = 0 \quad \alpha > 0, \quad (9)$$

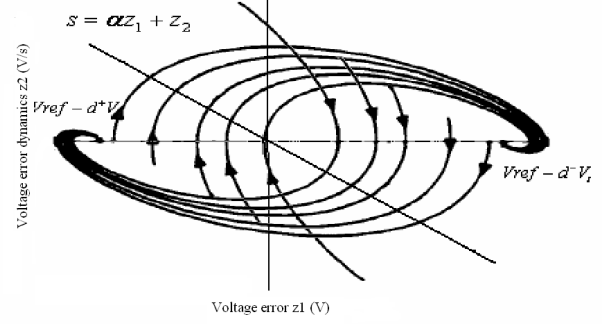
where α is the control parameter (termed the sliding coefficient) to be designed. Eq. (9) describes a stable trajectory in the phase plane with a steady state operating point $v_0 = V_{ref}$ and a first order transient recovery with a time constant α . The purpose of this sliding line is to serve as a boundary to split the phase plane into two regions. Each of these regions is specified with a switching state to direct the phase trajectory towards the sliding line. The system is considered to be stable only when the phase trajectory reaches and tracks the sliding line towards the origin [17].

2) *Design of the control law:* Because of the reaching condition, the system trajectory must eventually reach the sliding line. The following control law is considered:

$$d = \begin{cases} d^+ > V_{ref}/V_i & s(z) > 0 \\ d^- < V_{ref}/V_i & s(z) < 0 \end{cases}, \quad (10)$$

For $d = d^+$ and the system described by Eq. (8), the equilibrium point is given by: $(V_{ref} - d^+V_i, 0)$. The phase trajectories of $d = d^+$ for different starting positions are shown in Fig.5. For $d = d^-$ and the system described by Eq. (8), the equilibrium point is given by: $(V_{ref} - d^-V_i, 0)$. The phase trajectories for $d = d^-$ are shown in Fig.6.

Fig.7 is a combination of the two plots from Fig.5 and 6. It can be observed that if the phase trajectory is at any arbitrary position above the sliding line, according to Eq. (10), $d = d^+$ must be employed so that the trajectory is directed towards the

Fig. 7. Phase trajectories of $d = d^+$ and $d = d^-$ for different starting positions.

sliding line. Conversely, when the phase trajectory is at any position below the sliding line, $d = d^-$ must be employed for the trajectory to be directed towards the sliding line. Therefore, the reaching condition is satisfied.

It should be noticed that d^+ and d^- should not be equal to 1 or 0, otherwise the system turns into a direct SMC. Considering the fluctuations of the input voltage and the physical meaning of the duty cycle, d^+ and d^- should satisfy the following inequalities:

$$\begin{cases} 1 > d^+ > V_{ref}/V_{i \min} \\ 0 < d^- < V_{ref}/V_{i \max} \end{cases}, \quad (11)$$

where $V_{i \min}$ and $V_{i \max}$ are the possible minimum and maximum values of the input voltage.

3) *Existence condition of sliding regime:* The control law in Eq. (10) only provides the general requirement that the trajectories will be driven towards the sliding line. To ensure that the trajectory is maintained on the sliding line, the existence condition, which is derived from Lyapunov's direct method to determine asymptotic stability, must be obeyed [10]:

$$\lim_{s \rightarrow 0} s \cdot \dot{s} < 0 \quad (12)$$

Substituting Eq. (9) into (12), Eq. (12) becomes:

$$\dot{s} = \begin{cases} \alpha z_1 + \dot{z}_2 < 0 & \text{for } 0 < s < \xi \\ \alpha z_1 + \dot{z}_2 > 0 & \text{for } -\xi < s < 0 \end{cases} \quad (13)$$

where ξ is an arbitrarily small positive quantity. From Eqs. (8), (10) and (13) we have:

$$\begin{cases} \lambda_1 = -\frac{1}{LC}z_1 + (\alpha - \frac{1}{RC})z_2 \\ -\frac{V_i}{LC}d_H + \frac{V_{ref}}{LC} < 0, & 0 < s < \xi \\ \lambda_2 = -\frac{1}{LC}z_1 + (\alpha - \frac{1}{RC})z_2 \\ -\frac{V_i}{LC}d_L + \frac{V_{ref}}{LC} > 0, & -\xi < s < 0 \end{cases} \quad (14)$$

To compare the sliding region with the direct SMC, the existence condition of the direct SMC is rewritten as follows [10]:

$$\begin{cases} \lambda'_1 = -\frac{1}{LC}z_1 + (\alpha - \frac{1}{RC})z_2 \\ -\frac{V_i}{LC} + \frac{V_{ref}}{LC} < 0, & 0 < s < \xi \\ \lambda'_2 = -\frac{1}{LC}z_1 + (\alpha - \frac{1}{RC})z_2 \\ + \frac{V_{ref}}{LC} > 0, & -\xi < s < 0 \end{cases} \quad (15)$$

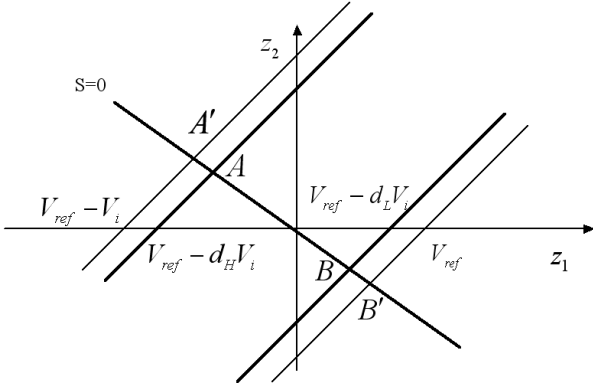


Fig. 8. Regions of existence of the SM for the direct SMC and the constant d^+ and d^- SMC.

The above conditions are depicted in Fig.8 for the proposed approach and the direct SMC. For the proposed approach, the intersections of lines $\lambda_1 = 0$, $\lambda_2 = 0$, and axis z_1 are $(V_{ref} - d^+ V_i, 0)$ and $(V_{ref} - d^- V_i, 0)$. The sliding mode will only occur on the portion of the sliding line $s = 0$. This portion is within A and B, where A is the intersection of $s = 0$ and $\lambda_1 = 0$; and B is the intersection of $s = 0$ and $\lambda_2 = 0$.

For direct SMC, the intersections of lines $\lambda'_1 = 0$, $\lambda'_2 = 0$ and axis z_1 are $(V_{ref} - V_i, 0)$ and $(V_{ref}, 0)$. The sliding mode will only occur on the portion of the sliding line $s = 0$ (provided that the sliding line is the same as the former). This portion is within A' and B' , where A' is the intersection of $s = 0$ and $\lambda'_1 = 0$; and B' is the intersection of $s = 0$ and $\lambda'_2 = 0$. Since $d^+ < 1$ and $d^- > 0$, the line segment AB is shorter than $A'B'$ and the sliding region of the constant d^+ and d^- SMC is smaller than that of the direct SMC when the sliding coefficients are identical. The quantity of the reduced portion of the sliding region depends on the values of d^+ , d^- and α .

C. Design of a reaching law SM controller

Although the constant d^+ and d^- approach is easy to analyze, synthesize and implement, some problems still exist.

First, because a SMC itself has a non-continuous switch feature, it can cause high frequency chattering of a system, which can excite non-modeled high frequency components and cause a remarkable disturbance and even make the system unstable. Second, in the stage from the initial states to the sliding surface (i.e. the reaching stage), the system is only an ordinary feedback control, which has no self-adaptation to parameter variation or external disturbance. It weakens the robustness of the system to a certain degree.

In order to improve the character of the reaching segment and alleviate the chattering, the concept of a reaching law was introduced in [16].

The reaching law is a differential equation which specifies the dynamics of the switching function $s(x)$. The differential equation of an asymptotically stable $s(x)$ is itself a reaching condition. In addition, by the choice of the parameters in the differential equation, the dynamic quality of a VSC system in the reaching mode can be controlled [18]. A practical general form of the reaching law for a single input system is:

$$\dot{s} = -\varepsilon \text{sgn}s - f(s) \quad (16)$$

where $\varepsilon > 0$, $f(0) = 0$, $sf(s) > 0$.

It is known from (16) that the existence condition and the reaching condition of the sliding mode are satisfied automatically, that is:

$$s \bullet \dot{s} = -\varepsilon |s| - sf(s) < 0 \quad (17)$$

In the following we will show the constant rate reaching and the constant plus proportion rate reaching given in [16].

1) Constant rate reaching:

Constant rate reaching is defined as:

$$\dot{s} = -\varepsilon \text{sgn}(s) \quad (18)$$

Suppose the initial value of the switching function is $s(0)$, after the time interval $t_r = \varepsilon^{-1} s(0)$, the system trajectory will hit the switching surface at a constant rate ε . The merit of this reaching law is its simplicity. But if ε is too small, the reaching time will be too long. On the other hand, an ε that is too large will cause severe chattering.

2) Constant plus proportion rate reaching:

Constant plus proportion rate reaching is defined as:

$$\dot{s} = -\varepsilon \text{sgn}(s) - ks, \quad \varepsilon > 0, k > 0 \quad (19)$$

After the time interval $t_r = k^{-1} [\ln(s(0) + k^{-1}\varepsilon) - \ln(k^{-1}\varepsilon)]$, the system trajectory will hit the switching surface, then move across the switching surface and finally reach the equilibrium of the system. Clearly, when adding the proportional rate term $-ks$, the state is forced to approach the switching manifolds faster when s is large. Therefore, a shorter reaching time t_r and a smaller chattering amplitude can be obtained by choosing appropriate parameters for ε and k . Thus, constant plus proportion rate reaching has better dynamic characteristics than constant rate reaching for SMC systems. As a result, the constant plus proportion rate reaching law is adopted in the proposed SMC scheme.

Having selected the reaching law equation, the control law can now be determined. Computing the time derivative of $s(x)$ along the reaching mode trajectory, we have:

$$\dot{s} = \alpha z_1 + \dot{z}_2 = -\frac{1}{LC} z_1 + \left(\alpha - \frac{1}{RC}\right) z_2 - \frac{V_i}{LC} d + \frac{V_{ref}}{LC} \quad (20)$$

Then from Eqs. (19) and (20), we can get:

$$-\frac{1}{LC} z_1 + \left(\alpha - \frac{1}{RC}\right) z_2 - \frac{V_i}{LC} d + \frac{V_{ref}}{LC} = -\varepsilon \text{sgn}(s) - ks \quad (21)$$

This equation is solved for the control law, resulting in:

$$d = \frac{LC}{V_i} \varepsilon \text{sgn}(\alpha z_1 + z_2) + \frac{V_{ref}}{V_i} + \left(\frac{k\alpha LC}{V_i} - \frac{1}{V_i}\right) z_1 + \frac{LC}{V_i} \left(\alpha - \frac{1}{RC} + k\right) z_2 \quad (22)$$

For designing the PWM modulator, the feed-forward adaptive method proposed in [10] can also be utilized to make

the system robust to input voltage. In terms of PWM-based controlled systems, the instantaneous duty cycle d is expressed as:

$$d = \frac{V_c}{\hat{v}_{ramp}} \quad (23)$$

where V_c is the control signal for the PWM and \hat{v}_{ramp} is the peak magnitude of the constant frequency ramp signal. From Eqs. (22) and (23), we have:

$$V_c = LC\varepsilon \text{sgn}(\alpha z_1 + z_2) + V_{ref} + (k\alpha LC - 1)z_1 + LC\left(\alpha - \frac{1}{RC} + k\right)z_2 \quad (24)$$

and:

$$\hat{v}_{ramp} = V_i \quad (25)$$

for the practical implementation of the reaching law SM controller. Therefore, the line regulation robustness can be maintained by varying the peak magnitude of the ramp signal with the input voltage.

It is worthwhile to mention that the control signal V_c is actually load dependent. As a result, for the controller to be regulation-robust to load changes, the instantaneous value of R should be fed back. However, this would require additional sensors and computations, which complicate the controller. Fortunately, the dependence and sensitivity of V_c on the load can be minimized by the proper design of α , ε and k such that $(\alpha + k) \gg 1/RC$. Therefore, the design value of the load resistance can be made a constant parameter R_{nom} , the nominal value of load resistance [10].

A final but important note is that the control law (22), obtained via a reaching law, automatically leads to a free-order switching scheme. From a practical point of view, this scheme appears to be the most efficient [16].

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, a simulation is conducted to verify the validity of the analysis results given above. The parameters of buck converter are: $L = 0.11mH$, $C = 100\mu F$, $R = 6\Omega$, $V_i = 24V$, $V_{ref} = 12V$ and $f_s = 200kHz$.

For the constant d^+ and d^- SM controller, the coefficients α , d^+ and d^- are chosen as $3/RC$, 0.8 and 0.2 respectively. For the reaching law SM controller, by setting $\alpha = 3/RC$ and $k = \varepsilon = 0.0001V_i/LC$, we obtain the control signal feeding the pulse width modulator as follows:

$$V_c = v_0 + 40z_1 + 0.024z_2 + 0.024\text{sgn}(s). \quad (26)$$

The simulation results of the approaches previously mentioned during start up and load change are shown in Fig.9 and 10.

It can be observed that the reaching law SM controller has a shorter settling time than the constant d^+ and d^- SM controller because of the improved dynamic characteristics in the reaching phase. Both of the figures indicate controllers with first order responses and no overshoot of output voltage. It shows order reduction which is one of the SMC properties.

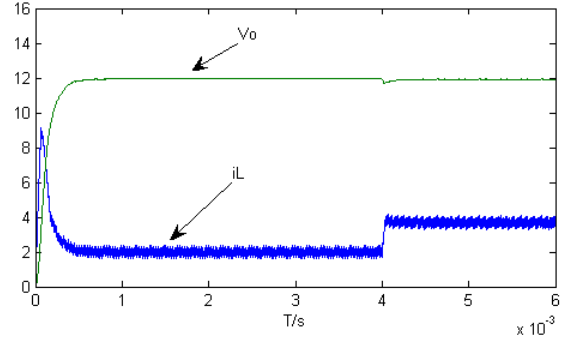


Fig. 9. Response in start up and load change (constant d^+ and d^- SMC).

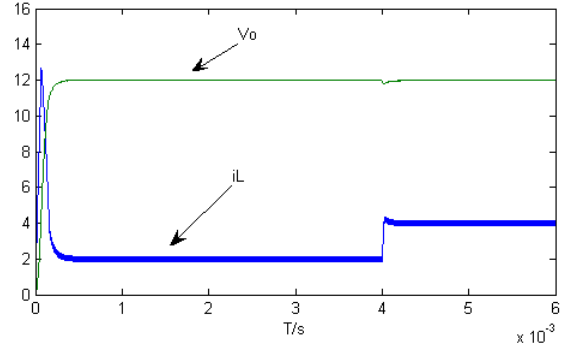


Fig. 10. Response in start up and load change (reaching law SMC).

In order to show the robustness of the controllers, a sudden load change is introduced at $t = 4ms$. Fig.11 shows a comparison of the two controller's responses to a load resistance decrease from 6Ω to 3Ω . Fig.12 presents a comparison of a load resistance increase from 6Ω to 12Ω . Moreover, the state trajectories on the phase plane of the two controllers in start up and load change are shown in Fig.13 and 14, respectively. It can be seen that the controllers are highly robust under load variations and that the reaching law SM controller has better performance than the constant d^+ and d^- SM controller. In addition, the output voltage ripple of the reaching law SM controller is lower than that of the constant d^+ and d^- SM controller due to the alleviation of chattering. It should be noticed that there is a steady state error for both controllers. This problem and the non-constant operating frequency problem of the SMC are two obstacles to the practical adoption of SMC in power converters [19]. They can be alleviated by choosing some other kind of sliding surface which is not the focus of this paper.

V. CONCLUSIONS

A new scheme for sliding mode control with a constant switching frequency for DC-DC converters is proposed. The design procedures for a constant d^+ and d^- SM controller and a reaching law SM controller for buck converters are given in detail. To show the feasibility of the proposed control methodologies, both of them are applied to buck converters in simulation. The simulation results show that the reaching law SMC exhibits superior static and dynamic properties than the

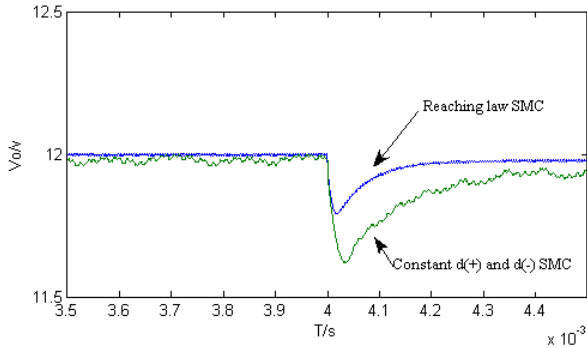


Fig. 11. Comparison of the response under load increase.

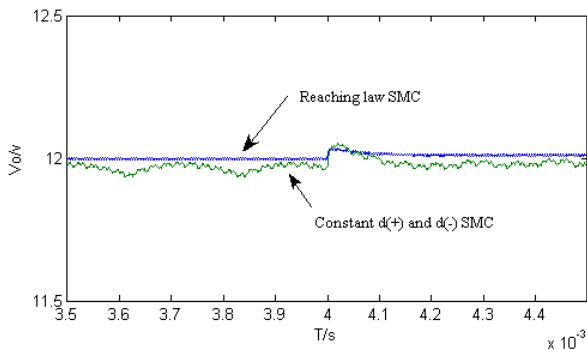


Fig. 12. Comparison of the response under load decrease.

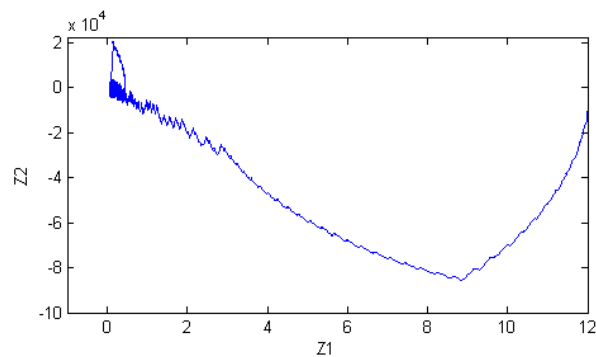


Fig. 13. Phase plane trajectory of start up and load change (constant d^+ and d^- SMC).

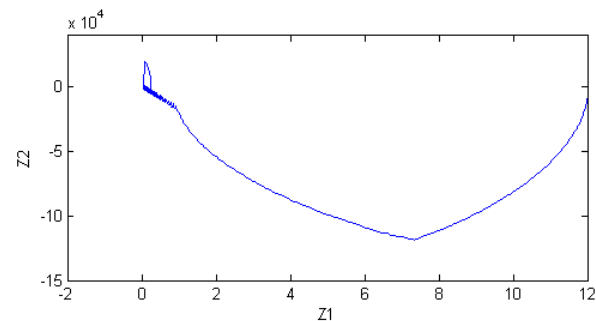


Fig. 14. Phase plane trajectory of start up and load change (reaching law SMC).

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constant d^+ and d^- SMC. However, the latter is simple, cost effective and easy to implement.



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