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Design of Front Lower Control Arm Considering Buckling Strength and Durability Strength

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<Abstract>

Recently, the concept of structural design against instability has been proposed in the chassis parts. The design considerations of lower control arm of chassis parts under the buckling and durability strengths are the general. More precisely, this paper considers a specific application and associated optimization problem for two strengths, where the design variables are the physical or geometric dimensions for skins and stiffeners. The objective is the minimization of the total weight, while optimization constrains involve reserve or improve factors for the buckling and durability strengths. The most important features are related to the numerical simulations for the estimation of buckling factor and their sensitivities by means of nonlinear and linear finite element analyses. The bucking and durability strength analyses, and the morping geometries are directly included in the optimization problem and the modified design is formulated. As a result, the optimal structure with stable behavior is obtained or increases the buckling and durability strengths of parts. Most of design problems for structures exposed to elastic instability can be formulated and solved

Keywords : Buckling strength, Buckling factor, Differential stiffness, Elastic instability, Lower control arm

1. Introduction

The loading configurations of the automotive chassis parts under the dynamic environments are not fixed but they are assumed to be described by piecewise linear functions defined along the time history. Hence dynamic loading distributions with piecewise static view may be approximated through consideration of more service conditions. One of them is the instability and the instability can be considered through the buckling condition. In particular, the buckling condition under the dynamic environments should be considered for the nonlinearities. Instability load for a dynamically moving structure is the standard problems of optimization under stability constraints. The nonlinear buckling behaviors and the influence of imperfections are not included in such a standard formulation and

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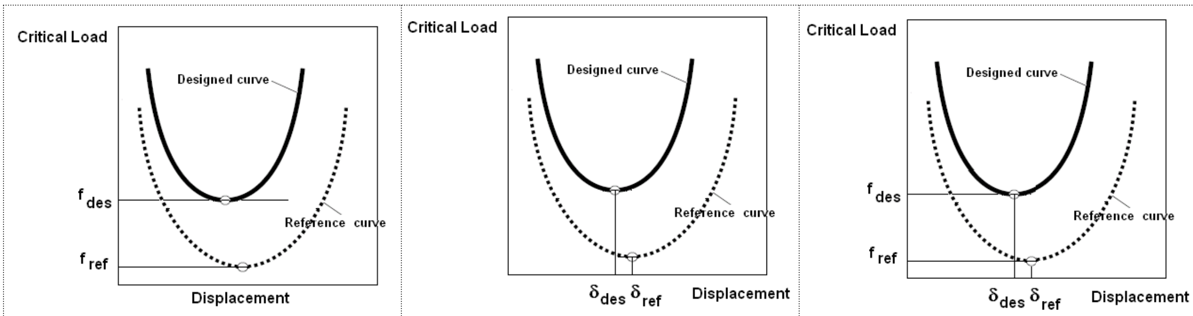
the important information about behavior of a designed structure after buckling is not provided. The standard optimal structure represents unstable buckling behavior under the dynamic environments such as the running mode. It indicates that a modification of such a standard optimization is necessary from practical point of view. Various classifications of the modified design problems have been proposed for the concept of modified optimization imposed on stability of buckling behaviors.[1~3]



(a) Maximization of critical load. (b) Minimization of critical buckling displacement. (c) Maximization of critical load and minimization of critical buckling displacement.

Fig. 1. Optimization schemes.

The standard optimization problem cannot be formulated but modified design against nonlinear buckling problem can be performed in the instability. The numerical approaches are based on the discretization and nonlinear programming. It occurs that modified designs are sensitive to geometry changes in the buckling state. This paper presents the design considerations against instability in the formulations and numerical solutions for the given current problem, which use the geometric dimensioning routine of structure, nonlinear buckling analysis routine and optimization design routine under the feasible design constraints with the target critical buckling load. The structure instability can be characterized by the minimum value of the deformation curve which refers to lower critical load. In what follows the optimization problem can be formulated either as maximization of lower critical loading [Fig.1 (a)] or minimization of generalized displacement for lower critical load[Fig.1(b)] or maximization of generalized displacement for which unstable equilibrium begins[Fig.1(c)].

Most of the modified optimal solutions can be obtained using the numerical approach based on discretization and non-linear progra-mming.

Some structures optimized for minimal design zone under a given critical loading show unstable buckling path. Such problems should be avoided from engineering point of view. The present paper gives both numerical and discretized treatment in the views of the manufacturing environments.

2. Theoretical considerations

2.1 Durability strength in geometric and material nonlinearities

All nonlinear elements[4,5] use the co-rotational formulation. The average rigid body motion of a finite element is condensed out of the total deformations and the remaining net deformations are considered to calculate strains in the element. If we assume that the element net deformations remain small, a linear strain measure in the element is sufficient for large overall deformations. The virtual works of the internal forces can be given by;

$$\delta W_{\text{int}} = \delta\{u\}^T\{f\} \qquad (1)$$

where $\delta\{u\}$ are the virtual displacements and rotations and $\{f\}$ are the normal forces an moments as the internal forces. In the corotational formulation, the internal forces $\{\bar{f}\}$ are evaluated from element net deformations $\{\bar{u}\}$. The net deformations are the total deformations minus the element rigid body motion. The virtual work in terms of the internal element forces $\{\bar{f}\}$ may be given by;

$$\delta W_{\text{int}} = \delta\{u\}^T[T][P_d]^T\{\bar{f}_d\} \qquad (2)$$

with $$[P_d] = \partial\{\bar{u}_d\} / \partial\{u_d\} \qquad (3)$$

where $[T]$ is the transformation from the basic to the deformed element system, and the subscript d indicates components in the deformed element system. The tangent stiffness is defined to be the derivative of the internal forces with respect to the deformations.

$$[K] = \partial\{f\} / \partial\{u\} \qquad (4)$$

The tangent stiffness matrix is derived from the second variation of the virtual work eq.(2) and calculated by adding the linear or nonlinear material stiffness $\lceil\bar{K}\rceil$ to the differential stiffness $\lfloor K^D \rfloor$,

$$[K] = [T]\left([P_d]^T[\bar{K}_d][P_d] + [K^D{}_d]\right)[T]^T \qquad (5)$$

with $$[\bar{K}_d] = \partial\{\bar{f}_d\} / \partial\{\bar{u}_d\} \qquad (6)$$

The linear or nonlinear material stiffness contains the terms due to the variation of the stresses. The differential stiffness contains the terms due to the variation of the strains, it is called the geometric stiffness. For the geometric linear problems, the correction matrix $[P_d]$ becomes the identity matrix. To find an instability point within a small range of nonlinear domain, two methods of idealization can be contemplated: The tangent stiffness matrix is proportional to the external loads, which implies that the critical load may be linearly interpolated. The tangent stiffness matrix is proportional to the displacement increments, which implies that the critical displacements may be obtained by extrapolating from the current state. Since the tangent matrix is assumed to change linearly, the internal loads are quadratic function of displacements.

2.2 Buckling strength in nonlinear buckling mode

The nonlinear buckling analysis[1,6] will provide a more accurate buckling load and furthermore we get the buckling shape and higher order buckling loads and shapes. In nonlinear buckling, the following eigenvalue problem is solved;

$$\left([K_n] + \lambda_i[\Delta K]\right)\{u_i\} = \{0\} \qquad (7)$$

with $$[\Delta K] = [K_n] - [K_{n-1}] \qquad (8)$$

where $[K_n]$, $[K_{n-1}]$ are the tangent stiffness matrices at load step n and n-1. The critical buckling displacement and load under the given load can be given by;

$$\{u_{crit}\} = \{u_n\} + \lambda\{\Delta u\} \quad \{f_{crit}\} = \{f_n\} + \alpha\{\Delta f\} \qquad (9)$$

with

$$\{\Delta u\} = \{u_n\} - \{u_{n-1}\} \quad \{\Delta f\} = \{f_n\} - \{f_{n-1}\}$$

where

$$f_{crit} = f_n + \int_{u_n}^{u_{crit}} K\,du = f_n + \int_{u_n}^{u_{crit}} K(\lambda)\Delta u\,du \cong f_n + \lambda\left[K_n + \tfrac{1}{2}\lambda\Delta K\right]\Delta u \qquad (10)$$

The factor α can be calculated by;

$$\alpha = \frac{\lambda_i\{\Delta u\}^T\left([K_n] + \tfrac{1}{2}\lambda_i[\Delta K]\right)\{\Delta u\}}{\{\Delta u\}^T\{\Delta f\}} \qquad (11)$$

with $$\{\Delta u\} = \{u_n\} - \{u_{n-1}\} \qquad (12)$$

For the computational model, we assume von-Mises elastic-plastic material law with isotropic hardening, see Fig. 2.

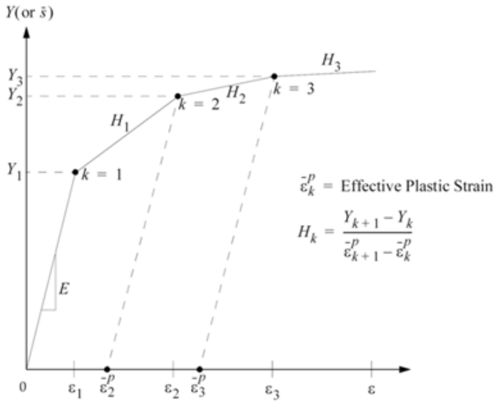


Fig. 2. Stress-Strain curve definition for the durability analysis.

The objective of the design problem is to maximize the lowest buckling load factor using the gradient based techniques and the buckling load factor sensitivities should be computed in an efficient way. The design variables are termed x_i and the direct approach to obtain the eigenvalue sensitivity is to differentiate eq.(7) with respect to a design variable, pre-multiplying by $\{u_j\}^T$ and make use of eq.(7), then the following expression is obtained for the eigenvalue sensitivity in case of a eigenvalue λ_i,

$$\frac{d\lambda_i}{d\,x_i} = \{u_j\}^T \left(\frac{d[K_n]}{d\,x_i} + \lambda_i \frac{d[\Delta K]}{d\,x_i} \right) \{u_j\} \tag{13}$$

where the eigenvectors have been $[\Delta K]$ orthonormalized, such that $\{u_j\}^T[\Delta K]\{u_j\} = 1$. The stress stiffness matrix is an implicit function of the displacement field, $[\Delta K] = [\Delta K(D(x), x)]$, which can be taken into account as

$$\frac{d[\Delta K]}{d\,x_i} = \frac{\partial[\Delta K]}{\partial\,x_i} + \frac{\partial[\Delta K]}{\partial D}\frac{dD}{d\,x_i} \tag{14}$$

The displacement sensitivities $dD/d\,x_i$ should be computed which is done efficiently using the direct differentiation approach, the static equilibrium equation is differentiated with respect to a design variable x_i.

$$[K]\frac{d\{D\}}{dx_i} = \frac{\partial\{f\}}{\partial x_i} - \frac{\partial[K]}{\partial x_i}\{D\} \tag{15}$$

where $\partial f/\partial\,x_i$ is the load sensitivity. The stress stiffness matrix sensitivities $d[\Delta K]/d\,x_i$ are computed by central difference approximations at the element level.

In this study, we perform the analysis in two steps. Nonlinear static analysis with arc-length method is done to examine the buckling and post-buckling path of the load deflection curve. Potential nonlinear effects are the yielding of the material and the geometric softening effect due to the curved shape of the control arm. Therefore, we run nonlinear static analysis with material and geometric nonlinearities turned on. A restart is made into the nonlinear static solution to perform a nonlinear buckling analysis. The nonlinear buckling analysis will provide a more accurate buckling load and furthermore we get the buckling shape and higher order buckling loads and shapes.

2.3 Design schemes

The structural changes concerning the nonlinearities[7,8] can be divided into element changes. Many elements are required to change with geometric dimensions, manufacturability and add-ons.

$$[\Delta K] = \sum_{i=1}^{m}[\Delta K]_i \tag{16}$$

Each element change can be given by a sum of terms needed to separate bending, torsion and stretching. These relationships may be linear or nonlinear. $[\Delta K]$ is provided by the structural shape and size changes. The geometric changes used in the structural shape and size changes, $\{G\}$ can be given by;

$$\{\Delta G\} = [\{T_1\}\{T_2\}\cdots\{T_k\}]\begin{Bmatrix}\{\Delta x_1\}\\ \{\Delta x_2\}\\ \vdots \\ \{\Delta x_k\}\end{Bmatrix} \tag{17}$$

where $\{T\}$ is the direction vector of $\{\Delta x\}$ and $\{\Delta x\}$ is the design variable of geometric dimension or physical properties. The piecewise linear changes of geometric dimensions are represented by a combination of basis functions defined at specific points. Notice that the basic functions used to represent the loading distribution must not necessarily be defined at nodal locations.

The procedure can be divided into two parts. The first part is an outer iteration improving design parameters by sequential quadratic programming; the second part is a deterministic analyzer for linear and nonlinear structural performances. In the optimization procedure, the objective function to minimize is the total elastic strain energy with a constraint on the total available volume. Generally, a typical optimization design problem of minimizing an objective function subject to a set of constraints can be written by;

$$\begin{aligned} &\min \quad \Psi(x) \\ &subject\ \ to \quad G_i(x),\ i = 1, \ldots, g \\ &x_j^{\ L} \le x_j \le x_j^{\ U} \qquad j = 1, \ldots, k \end{aligned} \tag{18}$$

In a realistic environment, the design parameters and state parameters may fluctuate about their nominal values. Thus, the distributions in the objective function and constraints due to the random parameter must be considered in the feasible design stages. The above optimization processes can be described as an iterative search process that uses the following steps:

① Define an initial design $x^{i=0}$

② Analyze the linear and nonlinear char -acteristics using a linear and nonlinear solver routine.

③ Compare the results of the analysis with such requirements as allowable elastic or plastic specifications.

④ If the requirements are not met, perform the optimization routine in order to set δx

⑤ Correct the new design variables δx^{i+1} based on the old design variables with $\beta = 0.9$,

$$\left(\delta x^{i+1}\right)_{new} = \left(\delta x^{i+1}\right)_{old} \left(\frac{\lambda^i}{\lambda^{l-1}}\right)^{\beta} \tag{19}$$

$$\left(\delta x^{i+1}\right)_{old} = G^i \cdot \alpha = \left(-\nabla\Psi^i + \frac{\left|\nabla\Psi^i\right|^2}{\left|\nabla\Psi^{i-1}\right|^2} S^{i-1}\right) \cdot \left(\frac{\partial\Psi^i}{\left(\nabla\Psi^i\right)^T G^i}\right)$$

where,

Objective $\Delta f_i(x)$ can be approximated for each design x^i using the series expansion,

$$\Delta\Psi = \Delta\Psi\left(x^i\right) + \sum_{j=1}^{n} \frac{\partial(\Delta\Psi)}{\partial x_j} \Delta x_j \tag{20}$$

The gradient $\partial(\Delta\Psi)/\partial x_j$ can be obtained directly from the results of finite element analysis. If the gradient is known, the search direction Δx can be obtained from the solution of an approximate optimization problem.

Change the design parameters using $x^{i+1} = x^i + \delta x^{i+1}$

If the requirements are satisfied, perform the discrete design for the design parameters with consideration of manufacturability.

Otherwise, go to ②

From the optimization results, the verification and validation shown in figure 3 should be made through the process of determining that a model implementation represents the designer's conceptual description of the model and the solution to the model and the process of determining the degree to which a model is a representation of the real world from the perspective of the intended uses of the model. The predictive capability of models is limited to specific load ranges and has been proven by showing the correlation with tests.

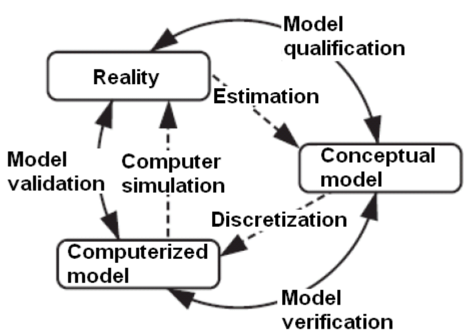


Fig. 3. Phases of modeling and simulation.[9)]

3. Applications

The front lower control arm is under the dynamic environments. The dynamic environ-ments are represented by the B/G(Belgium Ground) durability and buckling mode.

Fig. 4 shows the finite element model of front lower control arm, which is composed of 15,000 elements and 10,000 nodes. Table 1 shows the boundary and load conditions of Belgium ground durability strength and buckling strength. The material properties are shown in Table 2.

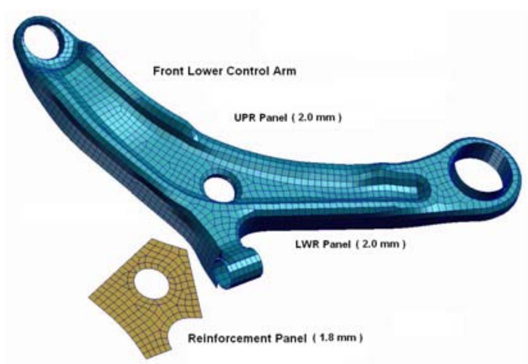


Fig. 4 Finite element model of FRT LWR C/ARM.

Table 1. Analysis conditions of FRT LWR C/ARM

Analysis type	Boundary condition	Load condition
B/G durability strength	- xyz fixed at the vertical MTG point - yz fixed at the horizontal MTG point - z fixed at the ball joint point	1G backward at the ball joint point
Buckling strength	- xyz fixed at the vertical MTG point with bush - xyz fixed at the horizontal MTG point with bush - z fixed at the ball joint point	Load backward at the ball joint point

* 1G is 325 kgf on the base of G.V.W. (Gross Vehicle Weight)

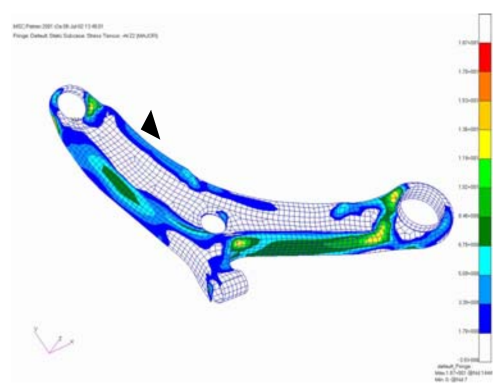
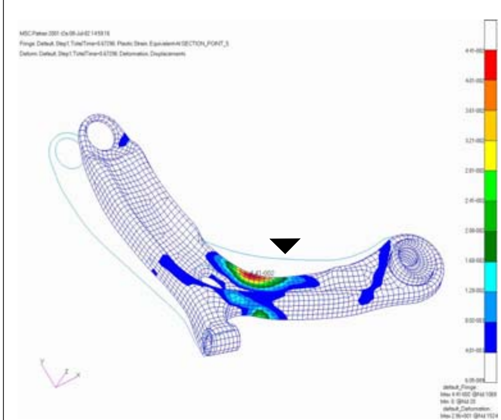
Table 2. Material properties of FRT LWR C/ARM

	UTS (kg_f/mm^2)	E (kg_f/mm^2)	ρ ($10^{-6}kg/mm^3$)	ν
SAPH 38	41	21,087	8.00	0.3190

The durability strength and buckling strength of the initial model are shown in Table 3 on the design specifications.

The initial design model meets the Belgium Ground specifications, but it's critical buckling load is below the request specification (5G) which don't pass through the Kerb Impact Test.

Table 3. Comparisons of durability strength and buckling strength of FRT LWR C/ARM

	Design specifications	
	Durability strength	Buckling strength
Results	 Maximum principal stress: 20.3 kgf/mm^2 Static stiffness: 256 kgf/mm	 Critical buckling load: 1,024 kgf (3.1G)
Estimations	Meets 3000PKM Belgian Road	Not meet the Kerb Impact Test

* PKM (Passenger-Kilometer) is a unit of passenger transportation quantity

The initial design model meets the Belgium Ground specifications, but it's critical buckling load is below the request specification (5G) which don't pass through the Kerb Impact Test.

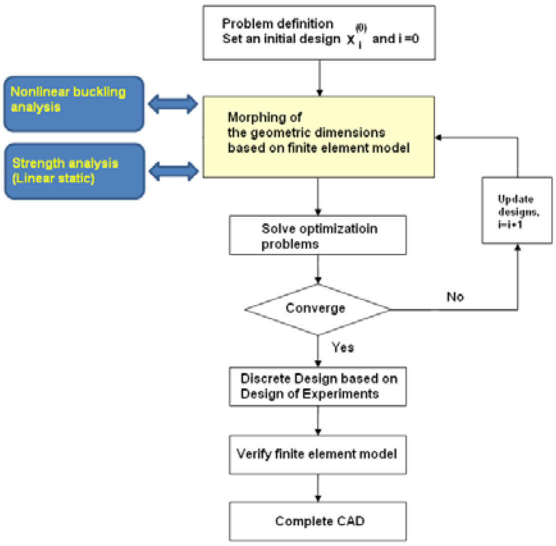


Fig. 5. Optimization flows for the size an shape of control arm.

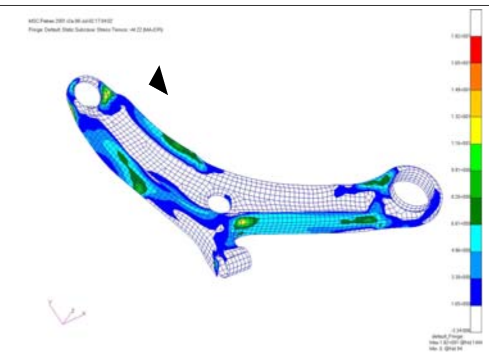
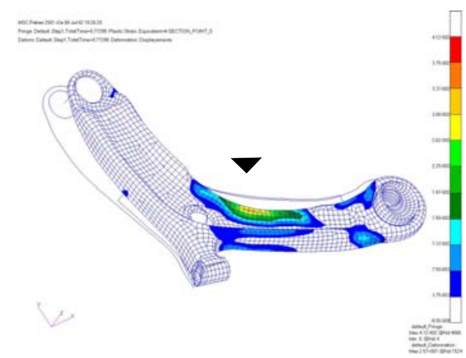
For the durability and critical buckling load specifications, the design optimization processes for the static strength and nonlinear buckling strength are applied to this model on the size and shape of reinforcement panels in the control arm.

The Table 4 shows the durability and critical buckling strengths of optimized control arm and the critical buckling load meets the specifications.

4. Conclusion

For the nonlinear buckling analysis, knowing the experimental buckling load, we load the structure up to about 80% of the observed buckling load and use arc-length method. The purpose is to make the analysis more efficient by avoiding small load steps for load levels lower than the buckling load. If there is no experimental data available, we recommend to run the nonlinear static analysis up to 80% of linear buckling load and to watch for the first occurrence of a negative factor diagonal in the stiffness.

Table 4. Comparisons of durability strength and buckling strength after the design optimization process of FRT LWR C/ARM

	Design specifications	
	Durability strength	Buckling strength
Results	 Maximum principal stress: 14.2 kgf/mm^2 Static stiffness: 439 kgf/mm	 Critical buckling load: 1,654 kgf (5.1G)
Estimations	Meets 3000PKM Belgian Road	Meets the Kerb Impact Test
Remarks	-	Shape change of reinforcement panel. Thickness change of UPR Panel (2.6 mm) Thickness change of LWR Panel (2.4 mm) Thickness change of Reinf. Panel (1.3 mm)

It has been shown that we can match the critical buckling load with linear buckling analysis. The critical load from linear buckling analysis is much higher that the experimental buckling loads. The lower control arm behaves strongly nonlinear. Both nonlinear static and nonlinear buckling analysis give a buckling load which is close to the experimental buckling load. The finite element model overestimates the buckling load by about 9%. As the example, the simulation model for buckling and post-buckling analysis of lower control arm has developed through the optimization design. All load conditions that are the typical running environments are included in the model. Energy principles are applied and geometrical nonlinearities are accounted for using large deflection plate theory.

The most important advantage is a combination of nonlinear finite element methods and optimization algorithm in a large gain of computational efficiency.

In addition, the geometric modeling and meshing models are created automatically. Compared to conventional design technique, the major advantage of the method is the more direct calculation strategy which gives increased accuracy.

The procedure presented in this paper may serve as a general guideline for the optimization design of nonlinear buckling rigidities and outlines roughly aspects of the validation and verification process applied to simulations in the automotive fields. The main focus is on design approach validation since majority of verification activities is not the physical phenomena. The strategy proposed is applicable in the optimization of structures against buckling environments when multiple load conditions are present or when the loadings are unpredictable.

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