IEMS Vol. 9, No. 2, pp. 107-120, June 2010.

Rigidity Evaluation under Uncertainties for Multiple Investment Alternatives over Multiple Periods

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Received, February 26, 2010; Revised, April 16, 2010; Accepted, May 17, 2010

Abstract. In today's uncertain economic environment, the evaluation of safety for investment alternatives is of practical importance in manufacturing companies. This paper examines a method of quantitatively evaluating profitability and risk for multiple alternatives using the total-cost unit-cost domain. The paper assumes such factors as unit sales price, sales and production volume, unit variable cost, fixed cost, and yield for each alternative. The paper incorporates the relationship between production capacity and demand, distinguishing between cases of production capacity surplus and shortage for each year over the entire planning horizon. The paper investigates the case in which the values of each factor independently move in the direction of decreasing profit each year, and clarifies the procedure of comparing safety among multiple investment alternatives on a single consolidated total-cost unit-cost domain. The difficulty of the problem lies in the method of consolidating multiple total-cost unit-cost domains into a single domain since the combination of years of capacity surplus and shortage depends upon the change values in each factor under consideration. A systematic method of evaluating profitability as well as risk is presented, and the validity of the proposed method is verified using a numerical example.

Keywords: Safety, Total-cost Unit-cost Domain, Capacity Surplus, Capacity Shortage, Yield, Breakeven Point

1. INTRODUCTION

With the recent downturn in global economic conditions, a method for quantitatively evaluating risks under uncertainties would be of practical use in management. There are a number of research outcomes related to economic risk (Ruefli, Collins and Lacugna, 1999). But most of them assume stochastic uncertainties (Kira, Kusy and Rakita, 1997; Leung, Wu and Lai, 2006). Some research results are limited to specific situations (Modiano, 1987). In this stream, research outcomes to support practical decision making are not sufficient.

Research findings related to uncertainties in the area of engineering economy, whose main objective is to support practical decision making under uncertainties, are also limited. Senju *et al*. (1982, 1994) were first to develop tools and concepts to deal with risk under uncertainties. A graphical representation method for evaluating risk was later developed (Kono and Mizumachi, 2006). The sensitivity coefficient was presented and its application was discussed (Nakamura, 2002; Kono, 2003, 2006). In this stream of research, production volume and demand are assumed to be equivalent. This assumption is not always true in practical situations (Kono and Mizumachi 2009).

Where new product sales surpass expectation, production capacity falls short against demand. In contrast, during periods of downturn in the case of seasonal product items, production capacity goes into surplus against demand. Although production capacity is adjusted to be in balance with demand in the long run, this imbalance of production capacity against demand can be observed frequently in practical situations.

Thus, in a simple profit equation composed of unit sales price, unit variable cost, and fixed cost, it is assumed that income is proportional to sellable product quantity and expenditure is proportional to production volume. From the viewpoint of balance between sellable

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product quantity and production volume, capacity surplus and shortage situations are defined. In the case of capacity surplus, increase in capacity does not bring about income increase, while in the case of capacity shortage, increase in capacity results in increase in contribution margin.

The aim of this paper is to present a method of evaluating and comparing risks for multiple economic investment alternatives whose life cover multiple periods. Such factors as sales price, demand volume, unit variable cost, fixed cost, and yield may change independently from expected values in the direction of decreasing profit. During a period when capacity is in shortage, decrease in demand may turn the capacity into surplus. If yield decreases, quality products decrease and years of capacity surplus may turn into years of capacity shortage. Thus, dynamic evaluation of capacity surplus and shortage for each year is required.

This paper intentionally analyzes the problem with uncertainties on a simple model of profit equation. The paper proposes a consolidated total-cost (TC) unit-cost (UC) domain as a tool of analysis. As a result, rigidity evaluation against uncertainties for multiple alternatives, against changes in factors, becomes possible. This result is an extension of a previous result that is restricted to a single period problem or a problem that does not consider the factor of yield. The result can be applied to a real world situation to determine which product should be selected for investment given uncertainties in relevant factors.

Chapter 2 presents the model, with fundamental assumptions and notations. Chapter 3 defines the cases of capacity surplus and shortage. Chapter 4 proposes the total-cost unit-cost domain, on which the breakeven point is presented. Chapter 5 presents a method for evaluating risk and safety, which is verified by a numerical example in Chapter 6.

2. MODEL FORMULATION

The following conditions and notations are assumed.

2.1 Assumptions and Notations

(1) There are multiple products k = A, B, C, $\cdots$, which are denoted by superscript k.
(2) The life of product k is composed of n^k years. Each year is denoted by j, $j = 1, 2 \ \cdots, n^k$.
(3) Production requires variable cost and fixed cost. Unit variable cost for year j of product k is denoted by v_j^k, and annual fixed cost for year j of product k is denoted by F_j^k.
(4) Product k is sold at a stable price in year j, which is denoted by p_j^k.
(5) Annual sales volume for year j of product k is denoted by S_j^k, and production volume for year j of product k is represented by Q_j^k.
(6) Annual demand quantity for product k is predetermined for year j and represented by D_j^k. Production capacity is also given as a stable value over the entire planning horizon, denoted by Q^*. Because of production machinery and operational personnel, capacity Q^* is predetermined and fixed over the entire period.
(7) Yield of product k, i.e., the quality product ratio against total production volume, is represented for year j by g_j^k.
(8) Annual interest rate is assumed to be i percent over the entire planning horizon.

Among the above notations, the following conditions are satisfied:

$$Q_j^k \le Q^*, \ j = 1, \ 2, \ \cdots, \ n^k. \tag{1}$$

$$D_j^k \ge S_j^k, \ j = 1, \ 2, \ \cdots, \ n^k. \tag{2}$$

$$S_j^k = g_j^k \times Q_j^k, \ \ j = 1, \ 2, \ \cdots, \ n^k. \tag{3}$$

2.2 Profit Equation

From assumptions (2) through (4), annual profit by cash-flow basis for year j of product k is given by:

$$\pi_j^k = p_j^k \times S_j^k - v_j^k \times Q_j^k - F_j^k, \quad j = 1, \ 2, \ \cdots, \ n^k. \tag{4}$$

This statement can be converted as follows:

$$\begin{aligned} \pi_j^k &= p_j^k \times \frac{S_j^k}{D_j^k} \times D_j^k - v_j^k \times \frac{Q_j^k}{Q^*} \times Q^* - F_j^k \\ &= p_j^k \times a_j^k \times D_j^k - v_j^k \times b_j^k \times Q^* - F_j^k, \\ & j = 1, \ 2, \ \cdots, \ n^k, \end{aligned} \tag{5}$$

where

$$a_j^k = \frac{S_j^k}{D_j^k}, \ \ j = 1, \ 2, \ \cdots, \ n^k, \tag{6}$$

and

$$b_j^k = \frac{Q_j^k}{Q^*}, \ \ j = 1, \ 2, \ \cdots, \ n^k. \tag{7}$$

Notation a_j^k refers to demand satisfaction ratio and b_j^k denotes capacity utilization ratio of product k for year j. From statements (1) and (2), it is clear that $a_j^k \le 1$ and $b_j^k \le 1$ for any j, $j = 1, 2, \cdots, n^k$.

Based on these assumptions, the purposes of this paper are to determine:

1) Profit equation for each case of production capacity surplus and shortage, and

2) Effect of change in each factor on profit, utilizing the breakeven point represented on the TC-UC domain.

3. CAPACITY SURPLUS AND SHORTAGE SITUATIONS

This chapter examines the balance between demand quantity and production capacity and clarifies the conditions of capacity surplus and shortage.

Where $b_j^k < 1$ for year j of product k, namely, $Q_j^k < Q^*$, quantity of sellable product (S_j^k) becomes equal to demand (D_j^k) because it is obviously profitable to increase production volume if demand is larger than S_j^k. Therefore, $a_j^k = 1$ (i.e., $D_j^k = S_j^k$) is satisfied in the case of $b_j^k < 1$. This situation is referred to as capacity surplus for year j of product k. The relationship among D_j^k, S_j^k, Q_j^k, and Q^* in this situation is represented in Figure 1.

Next, where $a_j^k < 1$ for year j of product k, i.e., $D_j^k > S_j^k$, production volume equals capacity, since it would be a loss of profit-making opportunity if the demand were not met by an intentional restriction of production volume. Therefore, $b_j^k = 1$ (namely, $Q_j^k = Q^*$) is satisfied where $a_j^k < 1$. This situation is referred to as capacity shortage for year j of product k. The balance among D_j^k, S_j^k, Q_j^k, and Q^* in this situation is represented in Figure 2.

Theoretically, as a special case, $a_j^k = 1$ and $b_j^k = 1$ can be satisfied simultaneously. This is referred to as capacity balance and is illustrated in Figure 3. Even though this appears rare in theory, practical capacity balance may fall into this category over time, since companies adjust production capacity to the quantity of demand in the long run.

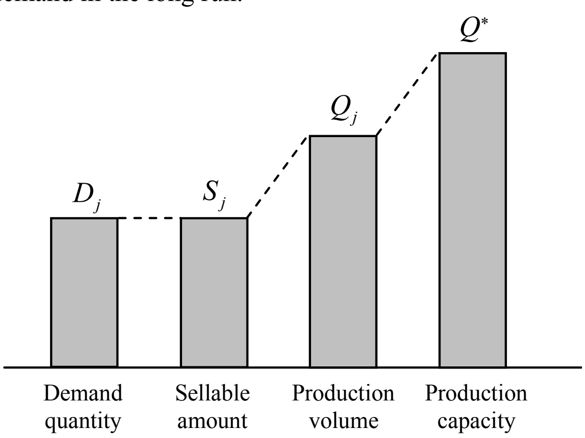

Figure 1. Capacity surplus situation for year j of product k.

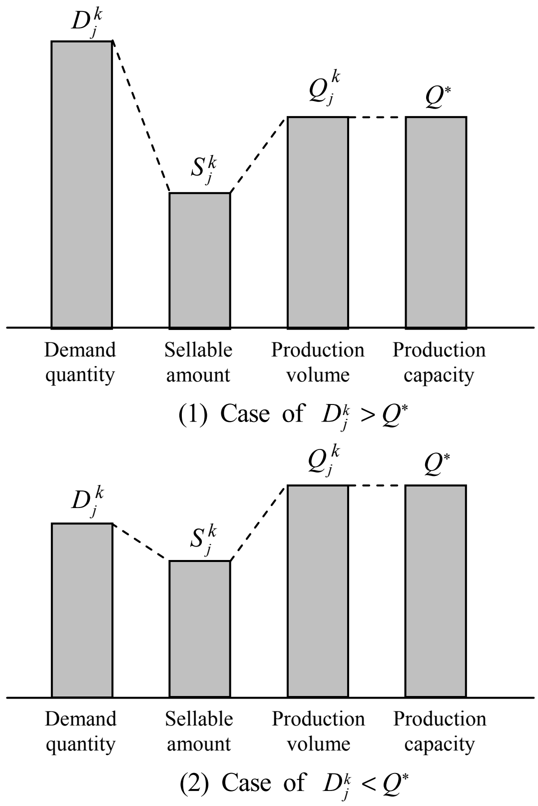

Figure 2. Capacity shortage situation for year j of product k.

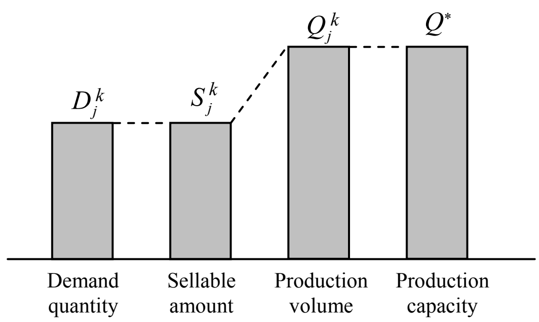

Figure 3. Capacity balance situation for year j of product k.

Summarizing the above discussions, the following conditions are valid:

capacity surplus: $a_j^k = 1$ and $b_j^k < 1$

capacity shortage: $a_j^k < 1$ and $b_j^k = 1$

capacity balance: $a_j^k = 1$ and $b_j^k = 1$

Putting these conditions into equation (5), we get the following statements:

capacity surplus:

$$\pi_j^k = p_j^k \times D_j^k - v_j^k \times Q_j^k - F_j^k. \quad (8)$$

capacity shortage:

$$\pi_j^k = p_j^k \times S_j^k - v_j^k \times Q^* - F_j^k. \quad (9)$$

capacity balance:

$$\pi_j^k = p_j^k \times S_j^k - v_j^k \times Q_j^k - F_j^k. \tag{10}$$

Further, putting equation (3) into these statements:
capacity surplus:

$$\begin{aligned}\pi_j^k &= p_j^k \times D_j^k - v_j^k \times Q_j^k - F_j^k \\ &= p_j^k \times D_j^k - v_j^k \times \frac{S_j^k}{g_j^k} - F_j^k \\ &= p_j^k \times D_j^k - v_j^k \times \frac{D_j^k}{g_j^k} - F_j^k.\end{aligned} \tag{11}$$

capacity shortage:

$$\begin{aligned}\pi_j^k &= p_j^k \times S_j^k - v_j^k \times Q^* - F_j^k \\ &= p_j^k \times g_j^k \times Q_j^k - v_j^k \times Q^* - F_j^k \\ &= p_j^k \times g_j^k \times Q^* - v_j^k \times Q^* - F_j^k.\end{aligned} \tag{12}$$

capacity balance:

$$\begin{aligned}\pi_j^k &= p_j^k \times S_j^k - v_j^k \times Q_j^k - F_j^k \\ &= p_j^k \times g_j^k \times Q_j^k - v_j^k \times Q_j^k - F_j^k.\end{aligned} \tag{13}$$

The total planning horizon of n^k years for the life of product k is composed of some years of capacity surplus and capacity shortage in remaining years. Years of capacity balance could be categorized as either. Therefore, the total n^k years are divided into those of capacity surplus and capacity shortage.

Present value of profit for year j of product k is denoted by $\overline{\pi}_j^k$, which is given by, if year j of product k is of capacity surplus,

$$\overline{\pi}_j^k = \frac{\pi_j^k}{(1+i)^j} = \frac{p_j^k \times D_j^k}{(1+i)^j} - \frac{v_j^k \times D_j^k / g_j^k}{(1+i)^j} - \frac{F_j^k}{(1+i)^j}, \tag{14}$$

or, if year j of product k is of capacity shortage,

$$\overline{\pi}_j^k = \frac{\pi_j^k}{(1+i)^j} = \frac{p_j^k \times g_j^k \times Q^*}{(1+i)^j} - \frac{v_j^k \times Q^*}{(1+i)^j} - \frac{F_j^k}{(1+i)^j}. \tag{15}$$

Denoting the set of those years of capacity surplus for product k by S^k and that of capacity shortage for product k by H^k, the present value of profit over the entire planning horizon is given by:

$$\begin{aligned}\pi^k &= \sum_{j=1}^{n^k}{}_{j\in S^k}\overline{\pi}_j^k + \sum_{j=1}^{n^k}{}_{j\in H^k}\overline{\pi}_j^k \\ &= \sum_{j=1}^{n^k}{}_{j\in S^k}\frac{p_j^k \times D_j^k}{(1+i)^j} - \frac{v_j^k \times D_j^k / g_j^k}{(1+i)^j} - \frac{F_j^k}{(1+i)^j} \\ &+ \sum_{j=1}^{n^k}{}_{j\in H^k}\frac{p_j^k \times g_j^k \times Q^*}{(1+i)^j} - \frac{v_j^k \times Q^*}{(1+i)^j} - \frac{F_j^k}{(1+i)^j}.\end{aligned} \tag{16}$$

The problem considered below clarifies the breakeven point at which profit becomes zero for each factor under consideration.

4. THE TC-UC DOMAIN

As a tool for representing the breakeven point, this paper considers a domain in which the horizontal axis represents total cost and the vertical axis represents unit cost. From equations (11) and (12), where year j of product k is in capacity surplus, the TC-UC domain is represented as in Figure 4. Where year j of product k is in capacity shortage, the domain is represented as in Figure 5.

This paper refers to the present value of total income for year j of product k as R_j^k, which is given by

$$R_j^k = \frac{p_j^k \times D_j^k}{(1+i)^j}, \ if\ j \in S^k. \tag{17}$$

$$R_j^k = \frac{p_j^k \times g_j^k \times Q^*}{(1+i)^j}, \ if\ j \in H^k. \tag{18}$$

Then, from equations (14) and (15),

$$\overline{\pi}_j^k = R_j^k - \frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k - \frac{F_j^k}{p_j^k \times D_j^k} \times R_j^k, \quad if\ j \in S^k. \tag{19}$$

$$\overline{\pi}_j^k = R_j^k - \frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k - \frac{F_j^k}{p_j^k \times g_j^k \times Q^*} \times R_j^k, \quad if\ j \in H^k \tag{20}$$

Since the value of R_j^k is dependent on j and k, equations (19) and (20) are further converted as follows:

$$\begin{aligned}\frac{\overline{\pi}_j^k}{R_1^k} &= \frac{\overline{\pi}_j^k}{R_j^k} \times \frac{R_j^k}{R_1^k} \\ &= \frac{R_j^k}{R_1^k} - \frac{v_j^k}{p_j^k \times g_j^k} \times \frac{R_j^k}{R_1^k} - \frac{F_j^k}{p_j^k \times D_j^k} \times \frac{R_j^k}{R_1^k}, \\ & if\ j \in S^k.\end{aligned} \tag{21}$$

$$\begin{aligned}\frac{\overline{\pi}_j^k}{R_1^k} &= \frac{\overline{\pi}_j^k}{R_j^k} \times \frac{R_j^k}{R_1^k} \\ &= \frac{R_j^k}{R_1^k} - \frac{v_j^k}{p_j^k \times g_j^k} \times \frac{R_j^k}{R_1^k} - \frac{F_j^k}{p_j^k \times g_j^k \times Q^*} \times \frac{R_j^k}{R_1^k}, \\ & if\ j \in H^k.\end{aligned} \tag{22}$$

Then, from equations (21) and (22), the following results can be obtained:

$$\sum_{j=1}^{n^k}{}_{j\in S^k} \frac{\bar{\pi}_j^k}{R_1^k} = \frac{\sum_{j=1}^{n^k}{}_{j\in S^k} R_j^k}{R_1^k} - \sum_{j=1}^{n^k}{}_{j\in S^k} \frac{v_j^k}{p_j^k \times g_j^k} \times \frac{R_j^k}{R_1^k} - \sum_{j=1}^{n^k}{}_{j\in S^k} \frac{F_j^k}{p_j^k \times D_j^k} \times \frac{R_j^k}{R_1^k}. \tag{23}$$

$$\sum_{j=1}^{n^k}{}_{j\in H^k} \frac{\bar{\pi}_j^k}{R_1^k} = \frac{\sum_{j=1}^{n^k}{}_{j\in H^k} R_j^k}{R_1^k} - \sum_{j=1}^{n^k}{}_{j\in H^k} \frac{v_j^k}{p_j^k \times g_j^k} \times \frac{R_j^k}{R_1^k} - \sum_{j=1}^{n^k}{}_{j\in H^k} \frac{F_j^k}{p_j^k \times g_j^k \times Q^*} \times \frac{R_j^k}{R_1^k}. \tag{24}$$

By representing sales volume for year j of product k by S_j^k, equations (23) and (24) can be consolidated into the next statement.

$$\sum_{j=1}^{n^k} \frac{\bar{\pi}_j^k}{R_1^k} = \frac{\sum_{j=1}^{n^k} R_j^k}{R_1^k} - \sum_{j=1}^{n^k} \frac{v_j^k}{p_j^k \times g_j^k} \times \frac{R_j^k}{R_1^k} - \sum_{j=1}^{n^k} \frac{F_j^k}{p_j^k \times S_j^k} \times \frac{R_j^k}{R_1^k} \tag{25}$$

It follows that we get a consolidated TC-UC domain for the total of n^k years for product k, neglecting the distinction of capacity surplus and shortage, as shown in Figure 6.

It is to be noted that the maximum values of both axes in Figure 6 differ from product to product, making it difficult to compare multiple items whose total incomes are not identical. For the purpose of comparing the safety of multiple product items on the TC-UC domain, both axes in Figure 6 are further normalized to the scale of 1 by multiplying $\sum_{j=1}^{n^k} \frac{R_1^k}{R_j^k}$ to both axes. Then, from statement (25), the next statement is obtained.

$$\frac{\sum_{j=1}^{n^k} \bar{\pi}_j^k}{\sum_{j=1}^{n^k} R_j^k} = 1 - \frac{\sum_{j=1}^{n^k} \frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k}{\sum_{j=1}^{n^k} R_j^k} - \frac{\sum_{j=1}^{n^k} \frac{F_j^k}{(1+i)^j}}{\sum_{j=1}^{n^k} R_j^k}. \tag{26}$$

Therefore, the profit ratio over the present value of total income $\sum_{j=1}^{n^k} R_j^k$ is demonstrated on the horizontal axis of the TC-UC domain as in Figure 7. Since the scales of both axes are normalized to 1, it becomes feasible to compare the safety and risk of multiple items on this domain, as they are not affected by the life of each product n^k nor by the difference in income structure R_j^k, $j=1, 2, \cdots, n^k$, among multiple products. This implies that the variable cost and fixed cost ratio is important in the total cost and revenue structure over the entire planning horizon, in evaluating rigidity against uncertainties in practice.

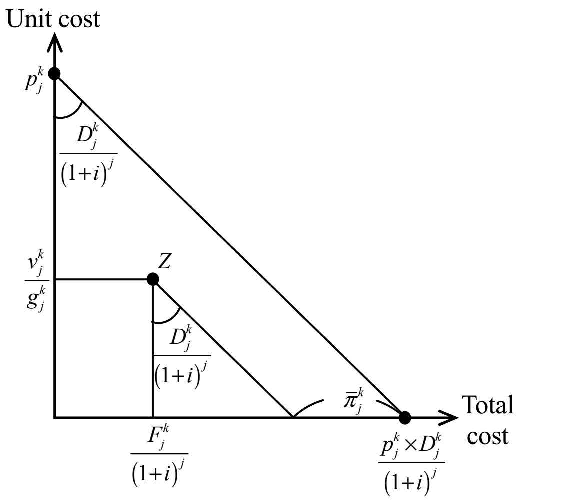

Figure 4. TC-UC domain for year k of capacity surplus.

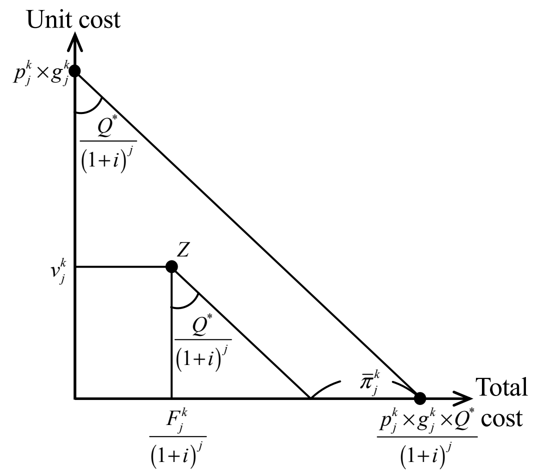

Figure 5. TC-UC domain for year k of capacity shortage.

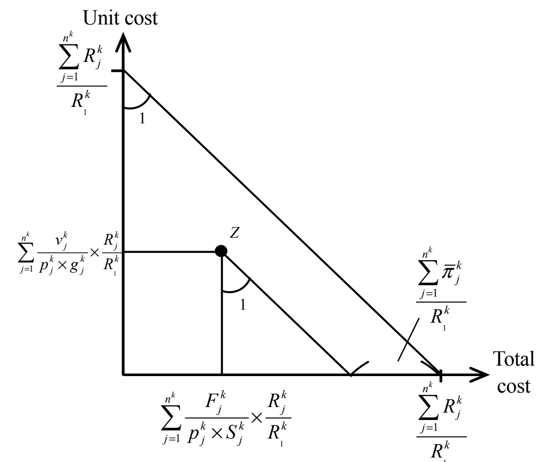

Figure 6. Consolidated TC-UC domain.

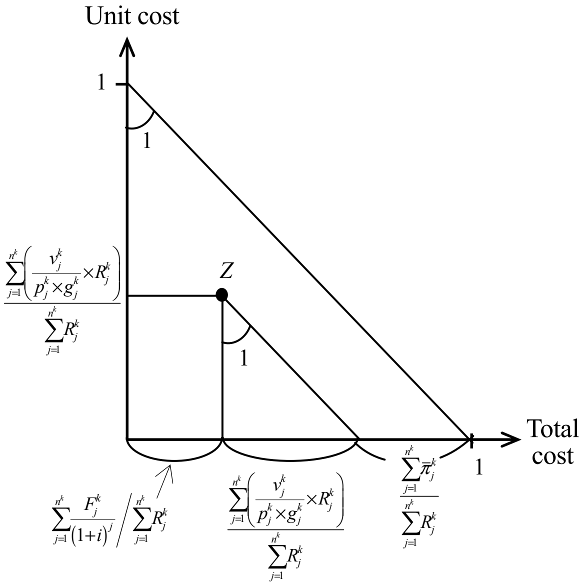

Figure 7. Consolidated TC-UC domain to the scale of $1\Big/\sum_{j=1}^{n^k} R_j^k$.

5. ANALYSIS OF SAFETY AGAINST CHANGE IN FACTORS

5.1 Change in Parameters and Breakeven Points

This chapter analyzes the case in which the value of each factor under consideration has independently become worse, moving in the direction of decreasing profit. Taking the case of product k, these changes can be categorized into the following cases:

Decrease in Price:

$$p_j^k \to \alpha^k p_j^k,\ 0<\alpha^k<1,\quad j=1,2,\cdots,n^k.$$

Decrease in Demand:

$$D_j^k \to \beta^k D_j^k,\ 0<\beta^k<1,\quad j=1,2,\cdots,n^k.$$

Increase in Unit Variable Cost:

$$v_j^k \to \gamma^k v_j^k,\ \gamma^k>1,\quad j=1,2,\cdots,n^k.$$

Increase in Fixed Cost:

$$F_j^k \to \delta^k F_j^k,\ \delta^k>1,\quad j=1,2,\cdots,n^k.$$

Decrease in Yield:

$$g_j^k \to \varepsilon^k g_j^k,\ 0<\varepsilon^k<1,\quad j=1,2,\cdots,n^k.$$

The breakeven point at which profit becomes zero for each factor is denoted by α^*, β^*, γ^*, δ^*, and ε^*, respectively. In the section below, each case is analyzed utilizing the TC-UC domain represented in Figure 7.

5.2 The Case of Decrease in Price

The first case is where sales price is decreased from p_j^k to $\alpha^k p_j^k$, $0<\alpha^k<1$ for all years $j = 1, 2 \cdots, n^k$. Along with the decrease in sales price, the line with incline 1 moves down and to the left, as indicated in Figure 8.

It is clear from Figure 8 that the length of c is calculated as follows:

$$c=\alpha^k-\frac{\sum_{j=1}^{n^k}\left(\frac{v_j^k}{p_j^k\times g_j^k}\times R_j^k\right)}{\sum_{j=1}^{n^k}R_j^k}-\frac{\sum_{j=1}^{n^k}\frac{F_j^k}{(1+i)^j}}{\sum_{j=1}^{n^k}R_j^k} \tag{27}$$

$$=\alpha^k-\frac{\sum_{j=1}^{n^k}\frac{v_j^k\times S_j^k+F_j^k}{(1+i)^j}}{\sum_{j=1}^{n^k}R_j^k}.$$

This result indicates that c in Figure 8 represents the total profit ratio over the present value of total income under the decreased price $\alpha^k p_j^k$, j=1, 2, $\cdots$, n^k.

The breakeven point α^* is obtained from the condition $c=0$. Therefore,

$$\alpha^*=\frac{\sum_{j=1}^{n^k}\frac{v_j^k\times S_j^k+F_j^k}{(1+i)^j}}{\sum_{j=1}^{n^k}R_j^k}. \tag{28}$$

It can be confirmed that the breakeven point α^* is represented on the TC-UC domain by the line with incline 1 which crosses the plot Z in Figure 8.

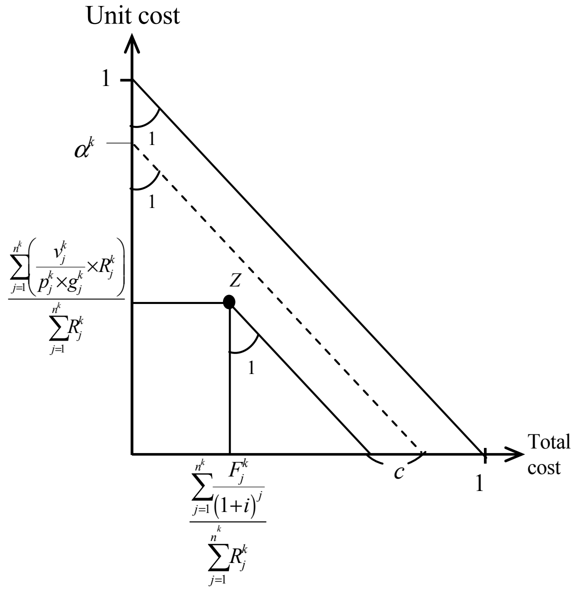

Figure 8. Decrease in price.

5.3 The Case of Decrease in Demand

The case where demand is uncertain and decreased from the originally expected value of D_j^k to $\beta^k D_j^k$, $0<\beta^k<1$, j = 1, 2, ⋯, n^k, is analyzed. It should be noted that for a certain year j under a capacity shortage situation, the next condition is satisfied.

$$D_j^k > g_j^k \times Q^*. \tag{29}$$

As demand volume decreases, the left-hand side of (29) becomes smaller, and when demand reaches the value at which the inequality in (29) turns to equality, the situation of year j of product k turns into capacity surplus. Thus, decrease in demand affects the distinction of capacity surplus and shortage.

For years of capacity shortage, profit is not a function of demand D_j^k, as described in statement (15). Therefore, profit is stable against decrease in demand. For years of capacity surplus, profit decreases together with decrease in demand, as shown in statement (14). Therefore, we must determine at which range of decrease in demand exists the breakeven point of demand.

For those years j under capacity shortage, i.e., $j \in H^k$, the decrease ratio at which year j turns into one of capacity surplus is denoted by $\tilde{\beta}_j^k$, which is given from (29) by

$$\tilde{\beta}_j^k = \frac{g_j^k \times Q^*}{D_j^k}. \tag{30}$$

For years that belong to H^k, all values of $\tilde{\beta}_j^k$, $j \in H^k$, should be calculated and arranged in decreasing order. Calculating $\pi\left(\tilde{\beta}_j^k\right)$, for all $j \in H^k$, we can find the range that satisfies

$$\pi\left(\tilde{\beta}_j^k\right) > 0 \ and \ \pi\left(\tilde{\beta}_l^k\right) \le 0. \tag{31}$$

This means that the breakeven point β^* exists in the range of $\left[\tilde{\beta}_l^k,\ \tilde{\beta}_j^k\right)$. Since the set of capacity shortage years H^k and surplus years S^k are determined, β^* can be calculated as follows. First,

$$\pi(\beta^k) = \sum_{j=1}^{n^k}{}_{j\in S^k}\left[\frac{p_j^k \times \beta^k D_j^k}{(1+i)^j} - \frac{v_j^k \times \beta^k D_j^k \times / g_j^k}{(1+i)^j} - \frac{F_j^k}{(1+i)^j}\right] + \sum_{j=1}^{n^k}{}_{j\in H^k}\left[\frac{p_j^k \times g_j^k \times Q^*}{(1+i)^j} - \frac{v_j^k \times Q^*}{(1+i)^j} - \frac{F_j^k}{(1+i)^j}\right]. \tag{32}$$

Then,

$$\beta^* = \frac{\displaystyle\sum_{j=1}^{n^k}{}_{j\in S^k}\frac{F_j^k}{(1+i)^j} - \sum_{j=1}^{n^k}{}_{j\in H^k}\left[\frac{p_j^k \times g_j^k \times Q^*}{(1+i)^j} - \frac{v_j^k \times Q^*}{(1+i)^j} - \frac{F_j^k}{(1+i)^j}\right]}{\displaystyle\sum_{j=1}^{n^k}{}_{j\in S^k}\left[\frac{p_j^k \times D_j^k}{(1+i)^j} - \frac{v_j^k \times D_j^k / g_j^k}{(1+i)^j}\right]}$$
$$= \frac{\displaystyle\sum_{j=1}^{n^k}{}_{j\in S^k}\frac{F_j^k}{p_j^k \times D_j^k} \times R_j^k - \sum_{j=1}^{n^k}{}_{j\in H^k}\left[R_j^k - \frac{v_j^k}{P_j^k \times g_j^k} \times R_j^k - \frac{F_j^k}{p_j^k \times S_j^k} \times R_j^k\right]}{\displaystyle\sum_{j=1}^{n^k}{}_{j\in S^k}\left[R_j^k - \frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k\right]}. \tag{33}$$

It is to be noted that, if the breakeven point is obtained under the condition that all periods are in capacity surplus, i.e., where $H^k = \phi$, then statement (33) is converted to

$$\beta^* = \frac{\displaystyle\sum_{j=1}^{n^k}\frac{F_j^k}{p_j^k \times D_j^k} \times R_j^k}{\displaystyle\sum_{j=1}^{n^k}\left[R_j^k - \frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k\right]}. \tag{34}$$

This condition is equivalent in form to the case of a single-year problem (Kono and Mizumachi 2008). It should also be noted that β^* in statement (34) is represented on the consolidated TC-UC domain of Figure 8, but β^* cannot be represented on a single TC-UC domain if $H^k \neq \phi$. This point reflects a property of the multi-period problem where capacity surplus and shortage coexist for the same product over the entire planning horizon.

5.4 The Case of Increase in Unit Variable Cost

The next case is where unit variable cost increases from v_j^k to $\gamma^k v_j^k$, $\gamma^k > 1$, j = 1, 2, ⋯, n^k. Together with the increase in unit variable cost, the line with incline 1 moves upward, as shown in Figure 9. Then, the length of c in Figure 9 is calculated as follows:

$$c = 1 - \gamma^k \times \frac{\displaystyle\sum_{j=1}^{n^k}\left(\frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k\right)}{\displaystyle\sum_{j=1}^{n^k} R_j^k} - \frac{\displaystyle\sum_{j=1}^{n^k}\frac{F_j^k}{(1+i)^j}}{\displaystyle\sum_{j=1}^{n^k} R_j^k}. \tag{35}$$

It follows that γ^* is given from the condition $c=0$ that

$$\gamma^* = \frac{\displaystyle\sum_{j=1}^{n^k} R_j^k - \sum_{j=1}^{n^k}\frac{F_j^k}{(1+i)^j}}{\displaystyle\sum_{j=1}^{n^k}\left(\frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k\right)} = \frac{\displaystyle\sum_{j=1}^{n^k}\left\{\frac{p_j^k \times S_j^k}{(1+i)^j} - \frac{F_j^k}{(1+i)^j}\right\}}{\displaystyle\sum_{j=1}^{n^k}\frac{v_j^k \times S_j^k}{(1+i)^j}}. \tag{36}$$

It can be confirmed that the breakeven point can be represented on the TC-UC domain in Figure 9.

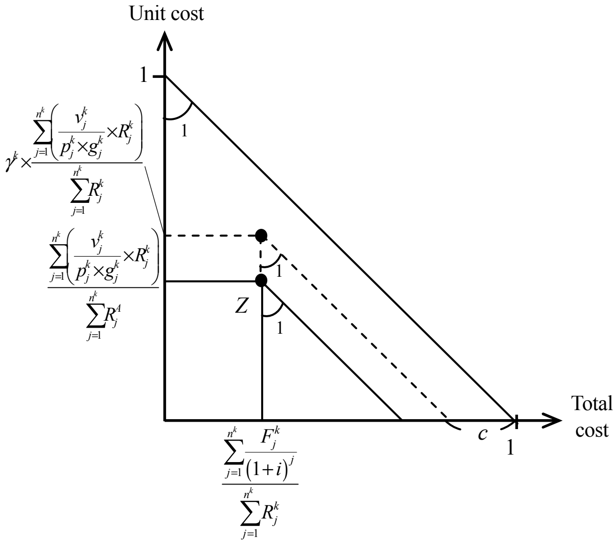

Figure 9. Increase in unit variable cost.

5.5 The Case of Fixed Cost Increase

The case in which fixed cost increases from the originally expected value of F_j^k to $\delta^k F_j^k$, $\delta_j^k > 1$, j = 1, 2, $\cdots$, n^k is then analyzed. Along with the increase in fixed cost, the plot Z of the alternative moves to the right, as shown in Figure 10. Then, the length of c is calculated as follows:

$$c = 1 - \frac{\sum_{j=1}^{n^k}\left(\frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k\right)}{\sum_{j=1}^{n^k} R_j^k} - \delta^k \times \frac{\sum_{j=1}^{n^k}\frac{F_j^k}{(1+i)^j}}{\sum_{j=1}^{n^k} R_j^k}. \tag{37}$$

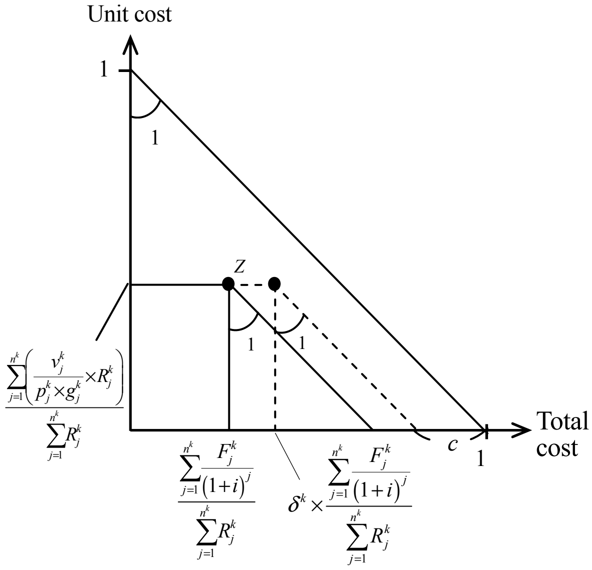

Figure 10. Increase in fixed cost.

In Figure 10, profit becomes zero when the plot Z moves to the right to reach the income line with incline 1. Therefore,

$$\delta^* = \frac{\sum_{j=1}^{n^k} R_j^k - \sum_{j=1}^{n^k}\left(\frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k\right)}{\sum_{j=1}^{n^k}\frac{F_j^k}{(1+i)^j}} = \frac{\sum_{j=1}^{n^k}\frac{p_j^k \times S_j^k}{(1+i)^j} - \sum_{j=1}^{n^k}\frac{v_j^k \times S_j^k}{(1+i)^j}}{\sum_{j=1}^{n^k}\frac{F_j^k}{(1+i)^j}}. \tag{38}$$

5.6 The Case of Decrease in Yield

As yield decreases from g_j^k to $\varepsilon^k g_j^k$, $0 < \varepsilon^k < 1$, for each year, the amount of sellable product, which is given by $\varepsilon^k g_j^k \times Q^*$, decreases. Therefore, during years of capacity surplus, the production volume necessary to meet the demand, given by $\frac{D_j^k}{\varepsilon^k g_j^k}$, increases and profit decreases. For years of capacity shortage, sellable volume out of full production capacity, given by $\varepsilon^k g_j^k \times Q^*$, decreases and profit does as well. The case is somewhat complicated in years of capacity surplus because they turn into capacity shortage years if yield decreases beyond a certain limit.

We shall denote, for years j of product k under capacity surplus, i.e., $j \in S^k$, the decrease ratio at which the year j of product k turns into capacity shortage, by $\tilde{\varepsilon}_j^k$.

A year $j \in S^k$ satisfies

$$g_j^k \times Q^* > D_j^k. \tag{39}$$

Together with the decrease in yield from g_j^k to $\varepsilon^k g_j^k$, the left-hand side of (39) decreases, so that $\tilde{\varepsilon}_j^k$ is given by

$$\tilde{\varepsilon}_j^k = \frac{D_j^k}{g_j^k \times Q^*}, \quad j = 1, 2, \cdots, n^k. \tag{40}$$

It is clear that if $D_j^k > g_j^k \times Q^*$, then $\tilde{\varepsilon}_j^k > 1$, which means that the year j of product k is originally a year of capacity shortage and never turns into capacity surplus even if yield is decreased.

For years that belong to S^k, all values of $\tilde{\varepsilon}_j^k$, should be calculated and arranged in decreasing order. Calculating $\pi\left(\tilde{\varepsilon}_j^k\right)$ for all $j \in S^k$, e can find the range hat satisfies

$$\pi\left(\tilde{\varepsilon}_j^k\right) > 0 \text{ and } \pi\left(\tilde{\varepsilon}_l^k\right) \le 0. \tag{41}$$

This means that the breakeven point ε^* exists in the range of $\left[\tilde{\varepsilon}_l^k,\ \varepsilon_j^k\right)$. Since the set of capacity surplus years S^k and shortage years H^k are determined, ε^* can be calculated by applying profit equation (16) as follows:

$$\pi\left(\varepsilon^k\right) = \sum_{j=1}^{n^k}{}_{j\in S^k}\left[\frac{p_j^k \times D_j^k}{(1+i)^j} - \frac{v_j^k \times D_j^k / \varepsilon^k g_j^k}{(1+i)^j} - \frac{F_j^k}{(1+i)^j}\right] + \sum_{j=1}^{n^k}{}_{j\in H^k}\left[\frac{p_j^k \times \varepsilon^k g_j^k \times Q^*}{(1+i)^j} - \frac{v_j^k \times Q^*}{(1+i)^j} - \frac{F_j^k}{(1+i)^j}\right]. \tag{42}$$

From the condition $\pi\left(\varepsilon^*\right) = 0$,

$$\begin{aligned}&\sum_{j=1}^{n^k}{}_{j\in S^k}\frac{v_j^k \times D_j^k / \varepsilon^* g_j^k}{(1+i)^j} - \sum_{j=1}^{n^k}{}_{j\in H^k}\frac{p_j^k \times \varepsilon^* g_j^k \times Q^*}{(1+i)^j}\\ &= \sum_{j=1}^{n^k}{}_{j\in S^k}\left[\frac{p_j^k \times D_j^k}{(1+i)^j} - \frac{F_j^k}{(1+i)^j}\right]\\ &- \sum_{j=1}^{n^k}{}_{j\in H^k}\left[\frac{v_j^k \times Q^*}{(1+i)^j} + \frac{F_j^k}{(1+i)^j}\right].\end{aligned} \tag{43}$$

It follows,

$$\begin{aligned}&\sum_{j=1}^{n^k}{}_{j\in S^k}\frac{v_j^k}{p_j^k \times \varepsilon^* g_j^k} \times R_j^k - \sum_{j=1}^{n^k}{}_{j\in H}\varepsilon^* R_j^k\\ &= \sum_{j=1}^{n^k}{}_{j\in S^k}\left[R_j^k - \frac{F_j^k}{p_j^k \times D_j^k} \times R_j^k\right]\\ &- \sum_{j=1}^{n^k}{}_{j\in H^k}\left[\frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k + \frac{F_j^k}{p_j^k \times g_j^k \times Q^*} \times R_j^k\right].\end{aligned} \tag{44}$$

Then,

$$\begin{aligned}&\sum_{j=1}^{n^k}{}_{j\in S^k}\frac{v_j^k}{p_j^k \times \varepsilon^* g_j^k} \times R_j^k - \sum_{j=1}^{n^k}{}_{j\in H^k}\varepsilon^* R_j^k\\ &= \sum_{j=1}^{n^k}{}_{j\in S^k}\left[R_j^k - \frac{F_j^k}{p_j^k \times S_j^k} \times R_j^k\right]\\ &- \sum_{j=1}^{n^k}{}_{j\in H^k}\left[\frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k + \frac{F_j^k}{p_j^k \times S_j^k} \times R_j^k\right].\end{aligned} \tag{45}$$

It must be noted that if the breakeven point ε^* is obtained under the condition that all years are in capacity shortage, i.e., where $S^k = \phi$, then statement (45) is converted to

$$\sum_{j=1}^{n^k}\varepsilon^* R_j^k = \sum_{j=1}^{n^k}\left[\frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k + \frac{F_j^k}{p_j^k \times g_j^k \times Q^*} \times R_j^k\right]. \tag{46}$$

Then,

$$\varepsilon^* = \frac{\sum_{j=1}^{n^k}\left[\frac{v_j^k}{p_j^k \times g_j^k} \times R_j^k + \frac{F_j^k}{p_j^k \times g_j^k \times Q^*} \times R_j^k\right]}{\sum_{j=1}^{n^k} R_j^k}. \tag{47}$$

This condition is equivalent in form to the case of a single-year problem (Kono and Mizumachi 2008). It should also be noted that ε^* in statement (47) is represented in the consolidated TC-UC domain as shown in Figure 11, but ε^* in (45) cannot be represented on a single TC-UC domain. This reflects the nature of the multi-period problem where capacity surplus and shortage coexist for the same product over the entire planning horizon.

To summarize the discussions above, the breakeven points are listed in Table 1. These values are represented on the TC-UC domain as in Figure 11.

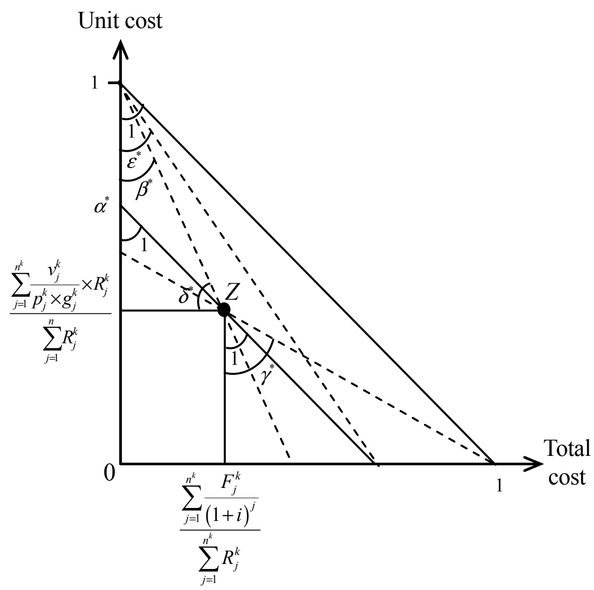

Figure 11. BEP on the TC-UC domain.

Note: β^* and ε^* are represented only in a special case, $H^k = \phi$ and $S^k = \phi$, respectively.

Table 1. Summary of breakeven points.

Factor	Notation	Breakeven Point
Price (p)	α^*	$\dfrac{\sum_{j=1}^{n^k} \dfrac{v_j^k}{p_j^k \times D_j^k} + \sum_{j=1}^{n^k} \dfrac{F_j^k}{p_j^k \times S_j^k} \times R_j^k}{\sum_{j=1}^{n^k} R_j^k}$
Demand (D)	β^*	$\dfrac{\sum_{j=1}^{n^k} \dfrac{F_j^k}{p_j^k \times S_j^k} \times R_j^k}{\sum_{j=1}^{n^k} R_j^k - \sum_{j=1}^{n^k} \dfrac{v_j^k}{p_j^k \times g_j^k} \times R_j^k}$
Unit Variable Cost (v)	γ^*	$\dfrac{\sum_{j=1}^{n^k} R_j^k - \sum_{j=1}^{n^k} \dfrac{F_j^k}{p_j^k \times S_j^k} \times R_j^k}{\sum_{j=1}^{n^k} \dfrac{v_j^k}{p_j^k \times g_j^k} \times R_j^k}$
Fixed Cost (F)	δ^*	$\dfrac{\sum_{j=1}^{n^k} R_j^k - \sum_{j=1}^{n^k} \dfrac{v_j^k}{p_j^k \times g_j^k} \times R_j^k}{\sum_{j=1}^{n^k} \dfrac{F_j^k}{p_j^k \times S_j^k} \times R_j^k}$
Yield (g)	ε^*	$\dfrac{\sum_{j=1}^{n^k} \dfrac{v_j^k}{p_j^k \times g_j^k} + \sum_{j=1}^{n^k} \dfrac{F_j^k}{p_j^k \times S_j^k} \times R_j^k}{\sum_{j=1}^{n^k} R_j^k}$

Note: β^* represents the value under the special condition that all years are of capacity surplus. ε^* is obtainned under the special case in which all years are of capacity shortage.

6. A NUMERICAL EXAMPLE

This paper considers two products, A and B, whose lives are both five years. Product B has higher yield, higher variable cost, and lower fixed cost.

Product B can be produced utilizing existing facilities, which can reduce the burden of fixed cost. The two products A and B target a similar market, and therefore the company has to choose which product to select as a better candidate for investing management resources in. This sort of situation is often observed in practice.

Fundamental values are listed in Tables 2(1) and 2(2). Profit and total income for each year are obtained as shown in Tables 3(1) and 3(2). Values on the axes of the TC-UC domain are obtained as shown in Tables 4(1) and 4(2). The consolidated TC-UC domain for the two products are shown in Figures 12.

Table 2 (1). A numerical example for product A.

j	Demand (D_j^A)	Yield (g_j^A)	Net capacity ($g_j^A Q^*$)	Surplus/ Shortage	$\tilde{\beta}_j^A = \dfrac{g_j^A \times Q^*}{D_j^A}$
1	700	0.75	750	Surplus	>1
2	800	0.84	840	Surplus	>1
3	1,000	0.9	900	Shortage	0.9
4	1,200	0.96	960	Shortage	0.8
5	900	0.99	990	Surplus	>1

j	p_j^A	S_j^A	$p_j^A \times S_j^A$	v_j^A	Q_j^A	$v_j^A \times Q_j^A$	F_j^A
1	60	700	42,000	40	934	37,360	5,000
2	70	800	56,000	50	953	47,650	7,000
3	70	900	63,000	50	1,000	50,000	7,000
4	60	960	57,600	40	1,000	40,000	8,000
5	50	900	45,000	30	910	27,300	7,000

Capacity $Q^* = 1{,}000$ units/year. Interest rate i = 10%.

Table 2 (2). A numerical example for product B.

j	Demand (D_j^B)	Yield (g_j^B)	Net capacity ($g_j^B \times Q^*$)	Surplus/ Shortage	$\tilde{\beta}_j^B = \dfrac{g_j^B \times Q^*}{D_j^B}$
1	700	0.8	880	surplus	>1
2	800	0.9	990	surplus	>1
3	1000	0.9	990	shortage	0.99
4	1200	0.99	1089	shortage	0.825
5	900	1.0	1100	surplus	>1

j	p_j^B	S_j^B	$p_j^B \times S_j^B$	v_j^B	Q_j^B	$v_j^B \times Q_j^B$	F_j^B
1	60	700	42,000	45	875	39,375	2,500
2	70	800	56,000	50	889	44,450	5,000
3	70	990	69,300	60	1,100	66,000	5,000
4	60	990	59,400	50	1,100	55,000	3,000
5	50	900	45,000	30	900	27,000	4,000

Capacity $Q^* = 1{,}100$ units/year. Interest rate i = 10%.

Table 3 (1). Profit and total income for product A.

j	π_j^A	$\bar{\pi}_j^A = \dfrac{\pi_j^A}{(1+i)^j}$	$p_j^A \times S_j^A$	$R_j^A = \dfrac{p_j^A \times S_j^A}{(1+i)^j}$	$\dfrac{R_j^A}{\sum_{j=1}^{5} R_j^A}$
1	-360	-327	42,000	38,182	0.192
2	1,350	1,116	56,000	46,281	0.232
3	6,000	4,508	63,000	47,333	0.238
4	9,600	6,557	57,600	39,342	0.198
5	10,700	6,644	45,000	27,941	0.140
Σ		18,498		199,079	1.000

Table 3 (2). Profit and total income for product B.

j	π^B_j	$\bar{\pi}^B_j = \frac{\pi^B_j}{(1+i)^j}$	$p^B_j \times S^B_j$	$R^B_j = \frac{p^B_j \times S^B_j}{(1+i)^j}$	$\frac{R^B_j}{\sum_{j=1}^{5} R^B_j}$
1	125	113.6	42,000	38,182	0.186
2	6550	5413.2	56,000	46,281	0.226
3	-1700	-1277.2	69,300	52,066	0.254
4	1400	956.2	59,400	40,571	0.198
5	14000	8692.9	45,000	27,941	0.136
Σ		13898.7		205,041	1.000

On the basis of Figure 12, we can evaluate the safety for the two products against decrease in price, increase in unit variable cost, and increase in fixed cost as follows:

A is safer against decrease in price
A is safer against increase in unit variable cost
B is safer against increase in fixed cost

From Figure 12, the following values are obtained for product A.

$$\alpha^*_A = 0.779 + 0.128 = 0.907,$$

$$\gamma^*_A = \frac{1-0.128}{0.779} = 1.119,$$

and

$$\delta^*_A = \frac{1-0.779}{0.128} = 1.727.$$

The following values are obtained for product B.

$$\alpha^*_B = 0.844 + 0.074 = 0.918,$$

$$\gamma^*_B = \frac{1-0.074}{0.844} = 1.097,$$

and

$$\delta^*_B = \frac{1-0.844}{0.074} = 2.108.$$

From these values, we can confirm the results described above.

Regarding the decrease in demand, taking the case of product A, decreasing ratio β^A_j, at which years of capacity shortage turns into capacity surplus, is obtained as:

Table 4 (1). Values on the axes of the TC-UC domain for product A.

j	$\frac{R^A_j}{\sum_{j=1}^{5} R^A_1}$	$p^A_j \times g^A_j$	$\frac{v^A_j}{p^A_j \times g^A_j}$	$\frac{v^A_j}{p^A_j \times g^A_j} \times \frac{R^A_j}{\sum_{j=1}^{5} R^A_j}$	$p^A_j \times S^A_j$		$\frac{F^A_j}{p^A_j \times S^A_j}$	$\frac{F^A_j}{p^A_j \times S^A_j} \times \frac{R^A_j}{\sum_{j=1}^{5} R^A_j}$
					$p^A_j \times D^A_j$	$p^A_j \times g^A_j \times Q^*$		
1	0.192	45	0.889	0.171	42,000	-	0.119	0.023
2	0.232	58.8	0.850	0.197	56,000	-	0.125	0.029
3	0.238	63	0.794	0.189	-	63,000	0.111	0.026
4	0.198	57.6	0.694	0.137	-	57,600	0.139	0.028
5	0.140	49.5	0.606	0.085	45,000	-	0.156	0.022
Σ				0.779				0.128

Table 4 (2). Values on the axes of the TC-UC domain for product B.

j	$\frac{R^B_j}{\sum_{j=1}^{5} R^B_1}$	$p^B_j \times g^B_j$	$\frac{v^B_j}{p^B_j \times g^B_j}$	$\frac{v^B_j}{p^B_j \times g^B_j} \times \frac{R^B_j}{\sum_{j=1}^{5} R^B_j}$	$p^B_j \times S^B_j$		$\frac{F^B_j}{p^B_j \times S^B_j}$	$\frac{F^B_j}{p^B_j \times S^B_j} \times \frac{R^B_j}{\sum_{j=1}^{5} R^B_j}$
					$p^B_j \times D^B_j$	$p^B_j \times g^B_j \times Q^*$		
1	0.186	48	0.938	0.174	42000	-	0.060	0.011
2	0.226	63	0.794	0.179	56000	-	0.089	0.020
3	0.254	63	0.953	0.242	-	69300	0.079	0.020
4	0.198	59.4	0.842	0.167	-	59400	0.051	0.010
5	0.136	50	0.6	0.082	45000	-	0.089	0.012
Σ				0.844				0.074

$$\tilde{\beta}_3^A = \frac{g_3^A \times Q^*}{D_3^A} = \frac{0.9 \times 1000}{1000} = 0.9,$$

and

$$\tilde{\beta}_4^A = \frac{g_4^A \times Q^*}{D_4^A} = \frac{0.96 \times 1000}{1200} = 0.8.$$

We confirm that $\pi(\beta) > 0$ at $\beta = 0.8$. Therefore, β_A^* exists in the range smaller than 0.8, which implies that all years are of capacity surplus under β_A^*. Then, from Figure 12:

$$\beta_A^* = \frac{0.128}{1 - 0.779} = 0.579,$$

For product B, in the same manner,

$$\tilde{\beta}_3^B = \frac{g_3^B \times Q^*}{D_3^B} = \frac{0.9 \times 1100}{1000} = 0.99,$$

and

$$\tilde{\beta}_4^B = \frac{g_4^B \times Q^*}{D_4^B} = \frac{0.99 \times 1100}{1200} = 0.9075.$$

We can confirm that $\pi(\beta) > 0$ at $\beta = 0.9075$. Therefore, β_B^* is smaller than 0.9075, which means that β_B^* exists when all years are of capacity surplus. Then from Figure 12,

$$\beta_B^* = \frac{0.074}{1 - 0.884} = 0.638$$

Comparing values of β_A^* and β_B^*, we can conclude that A is safer against decrease in demand.

The next case is decrease in yield. For the case of product A, in years of capacity surplus (years 1, 2, and 5), the yield decreasing ratio $\tilde{\varepsilon}_j^A$, at which time those years turn into capacity shortage, is given as follows:

$$\tilde{\varepsilon}_1^A = \frac{D_1^A}{g_1^A \times Q^*} = \frac{700}{750} = 0.933,$$

$$\tilde{\varepsilon}_2^A = \frac{D_2^A}{g_2^A \times Q^*} = \frac{800}{840} = 0.952,$$

and

$$\varepsilon_5^A = \frac{D_5^A}{g_5^A \times Q^*} = \frac{900}{990} = 0.909.$$

We confirm that, for minimum $\tilde{\varepsilon}_j^A = 0.909$, $\pi\left(\tilde{\varepsilon}_j^A\right) > 0$ is satisfied. This means that ε_A^* is obtained under the condition that all years fall into capacity shortage, which means $\varepsilon_A^* < 0.909$.

Then, with a parameter ε, the profit for each year is obtained, as shown in Table 5. From the column of $\overline{\pi}_j$ in this table, we obtain the condition where ε_A^* satisfies

$$206915\varepsilon_A^* - 186600 = 0.$$

Therefore,

$$\varepsilon_A^* = 0.902.$$

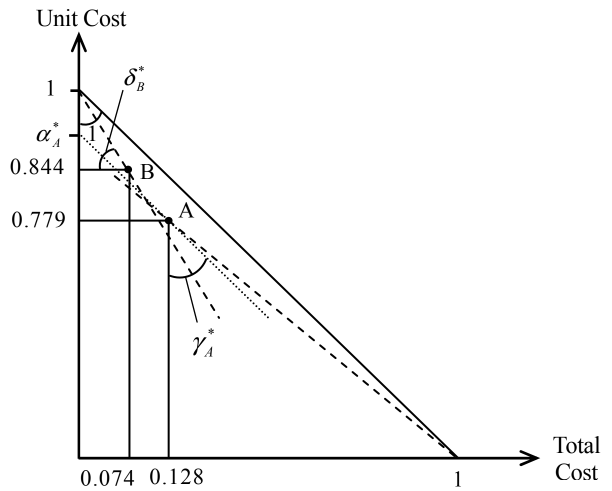

Figure 12. Consolidated TC-UC domain for the two products.

It is to be noted that, in the range of $0.952 < \varepsilon^A < 1$, years 1, 2, and 5 are in capacity surplus, where decreasing yield increases required production volume $D_j^A / \varepsilon^A g_j^A$, resulting in an increase in total variable cost and a decrease in profit. At the same time, sellable quantity $\varepsilon^A g_j^A \times Q^*$ decreases in years 3 and 4, which also results in a decrease in profit. In the range of $0.933 < \varepsilon^A < 0.952$, a capacity surplus is found in years 1 and 5, whereas a capacity shortage is found in years 2, 3, and 4. In the range of $0.909 < \varepsilon^A < 0.933$, there is surplus capacity in year 5, while there is a shortage in years 1, 2, 3, and 4. Thus, in the case of decreasing yield, sets of years of capacity surplus and shortage fluctuate depending upon the range of ε^A, which makes the analysis and calculation of ε_A^* somewhat complicated.

In the same manner, for product B, years 1, 2, and 5 turn into capacity shortage at

$$\tilde{\varepsilon}_1^B = \frac{700}{880} = 0.795$$

$$\tilde{\varepsilon}_2^B = \frac{800}{990} = 0.808$$

and

$$\tilde{\varepsilon}_5^B = \frac{900}{1100} = 0.818.$$

Profit $\pi(\varepsilon) < 0$ at $\varepsilon = 0.818$. Therefore, $0.818 < \varepsilon_B^* < 1$, which means that years 1, 2, and 5 are capacity surplus under ε_B^*. Then, from Table 6, we can get the condition:

$$123500 + 14850\varepsilon_B^* - \frac{111425}{\varepsilon_B^*} = 0.$$

Therefore, we get that $\varepsilon_B^* = 0.821$. From the values of ε_A^* and ε_B^*, we can conclude that B is safer against yield decrease.

Summarizing the above analysis, product A is safer against price decrease, demand decrease, and unit variable cost increase, while product B is safer against fixed cost increase and yield decrease. This result can help product planning in terms of which product to select, considering rigidity against uncertainties, when a company has to choose a single product out of multiple candidates under uncertain situations in practice.

7. CONCLUSION

This paper analyzed the problem of evaluating the risk and safety of investment alternatives over a multi-year planning horizon, in order to select the most rigid alternative. Therefore, the result can first be applied to product planning situations. Of course, the comparison of existing multiple alternatives falls under the scope of this research. The paper assumed that each investment alternative was composed of the following factors: sales price, demand volume, unit variable cost, fixed cost, and yield. It analyzed the case in which each of these factors independently changed for the worse and moved in the direction of decreasing profit. The breakeven point of each factor under consideration was clarified.

The balance between demand and production was

Table 5. Profit with parameter ε^A for product A.

j	$\varepsilon_A^* \times g_j^A \times Q^*$	$p_j^A \times \varepsilon_A^* \times g_j^A \times Q^*$	$v_j^A \times Q^*$	F_j^A	π_j^A	$\bar{\pi}_j^A$
1	$750\varepsilon_A^*$	$45{,}000\varepsilon_A^*$	40,000	5,000	$45{,}000\varepsilon_A^* - 45{,}000$	$40{,}909\varepsilon_A^* - 40{,}909$
2	$840\varepsilon_A^*$	$58{,}800\varepsilon_A^*$	50,000	7,000	$58{,}800\varepsilon_A^* - 57{,}000$	$48{,}595\varepsilon_A^* - 47{,}107$
3	$900\varepsilon_A^*$	$63{,}000\varepsilon_A^*$	50,000	7,000	$63{,}000\varepsilon_A^* - 57{,}000$	$47{,}333\varepsilon_A^* - 42{,}825$
4	$960\varepsilon_A^*$	$57{,}600\varepsilon_A^*$	40,000	8,000	$57{,}600\varepsilon_A^* - 48{,}000$	$39{,}342\varepsilon_A^* - 32{,}785$
5	$990\varepsilon_A^*$	$49{,}500\varepsilon_A^*$	30,000	7,000	$49{,}500\varepsilon_A^* - 37{,}000$	$30{,}736\varepsilon_A^* - 22{,}974$
Σ						$206{,}915\varepsilon_A^* - 186{,}600$

Table 6. Profit with parameter ε^B for product B.

j	$\varepsilon_B^* g_j^B$	Q_j^B		$v_j^B \times Q_j^B$	S_j^B	$p_j^B \times S_j^B$	F_j^B	π_j^B
		$\varepsilon_B^* g_j^B \times Q^*$	$\frac{D_j^B}{\varepsilon_B^* g_j^B}$					
1	$0.8\varepsilon_B^*$	-	$\frac{875}{\varepsilon_B^*}$	$\frac{39375}{\varepsilon_B^*}$	700	42000	2500	$39500 - \frac{39375}{\varepsilon_B^*}$
2	$0.9\varepsilon_B^*$	-	$\frac{889}{\varepsilon_B^*}$	$\frac{44450}{\varepsilon_B^*}$	800	56000	5000	$51000 - \frac{44450}{\varepsilon_B^*}$
3	$0.9\varepsilon_B^*$	$990\varepsilon_B^*$	-	$59400\varepsilon_B^*$	$990\varepsilon_B^*$	$69300\varepsilon_B^*$	5000	$9900\varepsilon_B^* - 5000$
4	$0.99\varepsilon_B^*$	$1089\varepsilon_B^*$	-	$54450\varepsilon_B^*$	$1089\varepsilon_B^*$	$59400\varepsilon_B^*$	3000	$4950\varepsilon_B^* - 3000$
5	$1.0\varepsilon_B^*$	-	$\frac{900}{\varepsilon_B^*}$	$\frac{27000}{\varepsilon_B^*}$	900	45000	4000	$41000 - \frac{27000}{\varepsilon_B^*}$

examined and cases of production capacity surplus and shortage were distinguished. Characteristic of a multi-year problem, the total years of product life are generally composed of those of capacity surplus and those of capacity shortage.

As a tool for analysis, this paper proposed a domain called the TC-UC domain, along with a method of representing the present value-based profit ratio over the entire planning horizon on a single domain termed the consolidated TC-UC domain. It also examined the values of each breakeven point depicted on the consolidated TC-UC domain. As a property of a multi-year problem, decrease in demand or yield affects the balance of production and demand, altering the distinction between capacity surplus and capacity shortage years. A method of comparing the safety of multiple alternatives using breakeven points on the consolidated TC-UC domain was proposed. This method of calculating the breakeven point for each factor offers a quantitative guide for evaluating rigidity under uncertainties in product planning situations, enabling the selection of the most rigid product among all alternatives.

The paper assumed independent change for each factor under consideration. In practice, however, dependent changes are observed, for example, variable cost increase results in price increase and price decrease may result in demand increase. This sort of dynamic change remains an issue to examine in future research.

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