J. Appl. Math. & Informatics Vol. **28**(2010), No. 5 - 6, pp. 1561-1571
Website: http://www.kcam.biz

FUZZY PAIRWISE STRONG PRE-IRRESOLUTE CONTINUOUS MAPPINGS

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ABSTRACT. We define and characterize a fuzzy pairwise strong pre-irresolute continuous mapping and a fuzzy pairwise strong pre-irresolute open mapping on a fuzzy bitopological space.

AMS Mathematics Subject Classification : 54A40
Key words and phrases : (τ_i, τ_j)-fuzzy strongly preinterior, fuzzy pairwise strong pre-irresolute continuous mapping and fuzzy pairwise strong pre-irresolute open mapping.

1. Introduction

Singal and Prakash [9] introduced a fuzzy preopen set and studied characteristic properties of a fuzzy precontinuous mapping on a fuzzy topological space. Later, Sampath Kumar [7] defined a (τ_i, τ_j)-fuzzy preopen set and characterized a fuzzy pairwise precontinuous mapping on a fuzzy bitopological space as a natural generalization of a fuzzy topological space. Also, Im [2] characterized a fuzzy pairwise pre-irresolute mapping on a fuzzy bitopological space.

Krsteska [3, 4] also defined a fuzzy strongly preopen set and studied a fuzzy strong precontinuous mapping (a fuzzy strong preopen mapping) on a fuzzy topological space. In particular, he defined and characterized a fuzzy strong pre-irresolute mapping and a fuzzy strong pre-irresolute open mapping on a fuzzy topological space.

Recently, Park, Lee and Im [8] characterized a fuzzy pairwise strong precontinuous mapping and a fuzzy pairwise strong preopen(preclosed) mapping on a fuzzy bitopological space. One purpose of this paper is to find more stronger mapping than we studied in [8].

Received February 9, 2010. Accepted May 27, 2010. *Corresponding author.
†The second author was supported by Dongguk University Research Plan 2010.

In this paper, we define a fuzzy pairwise strong pre-irresolute continuous mapping and a fuzzy pairwise pre-irresolute open mapping (fuzzy pairwise pre-irresolute closed mapping) on a fuzzy bitopological space and study their properties. We also give an example is a fuzzy pairwise strong precontinuous mapping but not a fuzzy pairwise strong pre-irresolute continuous mapping.

2. Preliminaries

Let X be a set and let τ_1 and τ_2 be fuzzy topologies on X. Then we call (X, τ_1, τ_2) a *fuzzy bitopological space* [*fbts*].

A mapping $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ is *fuzzy pairwise continuous* [*fpc*] if the induced mapping $f : (X, \tau_k) \to (Y, \tau_k^*)$ is fuzzy continuous for $k = 1, 2$.

A mapping $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ is *fuzzy pairwise open* [*fp open*] (*fuzzy pairwise closed* [*fp closed*]) if the induced mapping $f : (X, \tau_k) \to (Y, \tau_k^*)$ is fuzzy open (fuzzy closed) for $k = 1, 2$.

Notations. (1) Throughout this paper, we take an ordered pair (τ_i, τ_j) with $i, j \in \{1, 2\}$ and $i \neq j$.

(2) For simplicity, we abbreviate a τ_i-fuzzy open set μ and a τ_j-fuzzy closed set μ with a $\tau_i - fo$ set μ and a $\tau_j - fc$ set μ respectively. Also, we denote the interior and the closure of μ for a fuzzy topology τ_i with $\tau_i - \mathrm{Int}\,\mu$ and $\tau_i - \mathrm{Cl}\,\mu$ respectively.

Definition 2.1. [7] Let μ be a fuzzy set on a *fbts* X. Then we call μ;

(1) a (τ_i, τ_j)-*fuzzy preopen* [$(\tau_i, \tau_j) - fpo$] *set* on X if

$$\mu \leq \tau_i - \mathrm{Int}(\tau_j - \mathrm{Cl}\,\mu) \text{ and}$$

(2) a (τ_i, τ_j)-*fuzzy preclosed* [$(\tau_i, \tau_j) - fpc$] *set* on X if

$$\tau_i - \mathrm{Cl}(\tau_j - \mathrm{Int}\,\mu) \leq \mu.$$

Definition 2.2. [7] Let μ be a fuzzy set on a *fbts* X.

(1) The (τ_i, τ_j)-*preinterior of* μ, [$(\tau_i, \tau_j) - pInt\,\mu$] is

$$\bigvee \{\nu \mid \nu \leq \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - fpo \text{ set}\}.$$

(2) The (τ_i, τ_j)-*preclosure of* μ, [$(\tau_i, \tau_j) - pCl\,\mu$]*)* is

$$\bigwedge \{\nu \mid \nu \geq \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - fpc \text{ set}\}.$$

Definition 2.3. [8] Let μ be a fuzzy set on a $fbts$ X. Then we call μ;
(1) a (τ_i, τ_j)-*fuzzy strongly preopen* $[(\tau_i, \tau_j) - fspo]$ *set* on X if

$$\mu \leq \tau_i - \mathrm{Int}((\tau_j, \tau_i) - \mathrm{pCl}\,\mu) \quad \text{and}$$

(2) a (τ_i, τ_j)-*fuzzy strongly preclosed* $[(\tau_i, \tau_j) - fspc]$ *set* on X if

$$\tau_i - \mathrm{Cl}((\tau_j, \tau_i) - \mathrm{pInt}\,\mu) \leq \mu.$$

It is clear that a $\tau_i - fo$ set is a $(\tau_i, \tau_j) - fspo$ set and a $(\tau_i, \tau_j) - fspo$ set is a $(\tau_i, \tau_j) - fpo$ set on a $fbts$ X. But the converses are not true in general [8].

Proposition 2.4. [8] *(1) A union of* $(\tau_i, \tau_j) - fspo$ *sets is a* $(\tau_i, \tau_j) - fspo$ *set.*
(2) An intersection of $(\tau_i, \tau_j) - fspc$ *sets is a* $(\tau_i, \tau_j) - fspc$ *set.*

We remark an intersection of two $(\tau_i, \tau_j) - fspo$ sets need not be a $(\tau_i, \tau_j) - fspo$ set and a union of two $(\tau_i, \tau_j) - fspc$ sets need not be a $(\tau_i, \tau_j) - fspo$ set [8].

Definition 2.5. [8] Let μ be a fuzzy set on a $fbts$ X.
(1) The (τ_i, τ_j)-*strongly preinterior of* μ, $[(\tau_i, \tau_j) - spInt\,\mu]$ is

$$\bigvee\{\nu \mid \nu \leq \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - fspo \text{ set}\}.$$

(2) The (τ_i, τ_j)-*strongly preclosure of* μ, $[(\tau_i, \tau_j) - spCl\,\mu]$ is

$$\bigwedge\{\nu \mid \nu \geq \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - fspc \text{ set}\}.$$

Obviously, $(\tau_i, \tau_j) - \mathrm{spCl}\,\mu$ is the smallest $(\tau_i, \tau_j) - fspc$ set which contains μ, and $(\tau_i, \tau_j) - \mathrm{spInt}\,\mu$ is the largest $(\tau_i, \tau_j) - fspo$ set which is contained in μ. Therefore, $(\tau_i, \tau_j) - \mathrm{spCl}\,\mu = \mu$ for every $(\tau_i, \tau_j) - fspc$ set μ and $(\tau_i, \tau_j) - \mathrm{spInt}\,\mu = \mu$ for every $(\tau_i, \tau_j) - fspo$ set μ.

Moreover, we have

$$\tau_i - \mathrm{Int}\,\mu \leq (\tau_i, \tau_j) - \mathrm{spInt}\,\mu \leq (\tau_i, \tau_j) - \mathrm{pInt}\,\mu \leq \mu,$$
$$\mu \leq (\tau_i, \tau_j) - \mathrm{pCl}\,\mu \leq (\tau_i, \tau_j) - \mathrm{spCl}\,\mu \leq \tau_i - \mathrm{Cl}\,\mu.$$

We state the following lemma from the above definition, which will be used later.

Lemma 2.6. [8] *Let* μ *be a fuzzy set on a* $fbts$ X*. Then*

$$(\tau_i, \tau_j) - spInt(\mu^c) = ((\tau_i, \tau_j) - spCl\,\mu)^c$$

and

$$(\tau_i, \tau_j) - spCl(\mu^c) = ((\tau_i, \tau_j) - spInt\,\mu)^c.$$

Definition 2.7. [8] Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then f is called a *fuzzy pairwise strong precontinuous [fpspc] mapping* if $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - fspo$ set on X for each $\tau_i^* - fo$ set ν on Y.

It is clear that every fpc mapping is a $fpspc$ mapping and every $fpspc$ mapping is a $fppc$ mapping on $fbts$. But the converses are not true in general [8].

Definition 2.8. [8] Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then f is called;

(1) a *fuzzy pairwise strong preopen [fpsp open] mapping* if $f(\mu)$ is a $(\tau_i^*, \tau_j^*)-fspo$ set on Y for each $\tau_i - fo$ set μ on X and

(2) a *fuzzy pairwise strong preclosed [fpsp closed] mapping* if $f(\mu)$ is a $(\tau_i^*, \tau_j^*)-fspc$ set on Y for each $\tau_i - fc$ set μ on X.

It is clear that every fp *open*(fp *closed*) mapping is a $fpsp$ *open*($fpsp$ *closed*) mapping and every $fpsp$ *open*($fpsp$ *closed*) mapping is a fpp *open*(fpp *closed*) mapping on $fbts$. But the converses are not true in general [8].

3. Fuzzy pairwise strong pre-irresolute continuous mappings

In this section, we introduce a fuzzy pairwise strong pre-irresolute continuous mapping and a fuzzy pairwise strong pre-irresolute open mapping which are stronger than a fuzzy pairwise strong precontinuous mapping and a fuzzy pairwise strong preopen mapping respectively. And we characterize a fuzzy pairwise strong pre-irresolute continuous mapping and a fuzzy pairwise pre-irresolute open mapping.

Definition 3.1. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then f is called a fuzzy pairwise strong pre-irresolute continuous [$fpsp$-irresolute continuous] mapping if $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - fspo$ set on X for each $(\tau_i^*, \tau_j^*) - fspo$ set ν on Y.

It is clear that every $fpsp$-irresolute continuous mapping is $fpspc$ from the above definitions. But the converse is not true in general as the following example shows.

Example 3.2. Let μ_1, μ_2, μ_3, μ_4, μ_5 and μ_6 be fuzzy sets on $X = \{a, b, c\}$ with

$$\mu_1(a) = 0.9, \mu_1(b) = 0.5, \mu_1(c) = 0.9,$$
$$\mu_2(a) = 0.5, \mu_2(b) = 0.7, \mu_2(c) = 0.5,$$
$$\mu_3(a) = 0.8, \mu_3(b) = 0.5, \mu_3(c) = 0.8,$$
$$\mu_4(a) = 0.8, \mu_4(b) = 0.5, \mu_4(c) = 0.7,$$
$$\mu_5(a) = 0.5, \mu_5(b) = 0.5, \mu_5(c) = 0.5 \text{ and}$$
$$\mu_6(a) = 0.3, \mu_6(b) = 0.4, \mu_6(c) = 0.3.$$

Let

$$\tau_1 = \{0_X, \mu_4, \mu_6, 1_X\}, \tau_2 = \{0_X, \mu_3, \mu_6, 1_X\} \text{ and}$$
$$\tau_1^* = \{0_X, \mu_1, 1_X\}, \tau_2^* = \{0_X, \mu_2, 1_X\}.$$

be fuzzy topologies on X.

Then we can show that the identity mapping $i_X : (X, \tau_1, \tau_2) \to (X, \tau_1^*, \tau_2^*)$ is $fpspc$ but not $fpsp$-irresolute continuous and μ_5 is a $(\tau_i^*, \tau_j^*) - fspo$ set but not a $(\tau_i, \tau_j) - fspo$ set. □

Theorem 3.3. *Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then the following statements are equivalent:*

(1) f is $fpsp$-irresolute continuous.

(2) The inverse image of each $(\tau_i^*, \tau_j^*) - fspc$ set on Y is a $(\tau_i, \tau_j) - fspc$ set on X.

(3) $f((\tau_i, \tau_j) - spCl\,\mu) \leq (\tau_i^*, \tau_j^*) - spCl(f(\mu))$ for each fuzzy set μ on X.

(4) $(\tau_i, \tau_j) - spCl(f^{-1}(\nu)) \leq f^{-1}((\tau_i^*, \tau_j^*) - spCl\,\nu)$ for each fuzzy set ν on Y.

(5) $f^{-1}((\tau_i^*, \tau_j^*) - spInt\,\nu) \leq (\tau_i, \tau_j) - spInt(f^{-1}(\nu))$ for each fuzzy set ν on Y.

Proof. (1) implies (2): Let ν be a $(\tau_i^*, \tau_j^*) - fspc$ set on Y. Then ν^c is a $(\tau_i^*, \tau_j^*) - fspo$ set on Y. Since f is $fpsp$-irresolute continuous, $f^{-1}(\nu^c) = (f^{-1}(\nu))^c$ is a $(\tau_i, \tau_j) - fspo$ set on X. Hence $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - fspc$ set on X

(2) implies (3): Let μ is a fuzzy set on X. Then $f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spCl}(f(\mu)))$ is a $(\tau_i, \tau_j) - fspc$ set on X. Thus

$$\begin{aligned}(\tau_i, \tau_j) - \mathrm{spCl}\,\mu &\leq (\tau_i, \tau_j) - \mathrm{spCl}(f^{-1}(f(\mu)))\\ &\leq (\tau_i, \tau_j) - \mathrm{spCl}(f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spCl}(f(\mu))))\\ &= f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spCl}(f(\mu))).\end{aligned}$$

Hence

$$\begin{aligned}f((\tau_i, \tau_j) - \mathrm{spCl}\,\mu) &\leq f(f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spCl}(f(\mu))))\\ &\leq (\tau_i^*, \tau_j^*) - \mathrm{spCl}(f(\mu)).\end{aligned}$$

(3) implies (4): Let ν be a fuzzy set on Y. Then

$$f((\tau_i, \tau_j) - \mathrm{spCl}(f^{-1}(\nu))) \leq (\tau_i^*, \tau_j^*) - \mathrm{spCl}(f(f^{-1}(\nu))) \leq (\tau_i^*, \tau_j^*) - \mathrm{spCl}\,\nu.$$

Hence

$$\begin{aligned}(\tau_i, \tau_j) - \mathrm{spCl}(f^{-1}(\nu)) &\leq f^{-1}(f((\tau_i, \tau_j) - \mathrm{spCl}(f^{-1}(\nu))))\\ &\leq f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spCl}\,\nu).\end{aligned}$$

(4) implies (5): Let ν be a fuzzy set on Y. Then

$$(\tau_i, \tau_j) - \mathrm{spCl}(f^{-1}(\nu^c)) \leq f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spCl}(\nu^c)).$$

Hence, by Lemma 2.6,

$$\begin{aligned} f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spInt}\,\nu) &= f^{-1}((((\tau_i^* \tau_j^*) - \mathrm{spCl}(\nu^c))^c) \\ &\leq ((\tau_i, \tau_j) - \mathrm{spCl}(f^{-1}(\nu^c)))^c \\ &= (\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(\nu)). \end{aligned}$$

(5) implies (1): Let ν be a $(\tau_i^*, \tau_j^*) - fspo$ set on Y. Then

$$f^{-1}(\nu) = f^{-1}((\tau_i^* \tau_j^*) - \mathrm{spInt}\,\nu) \leq (\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(\nu)).$$

Hence $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - fspo$ set on X and therefore, f is $fpsp$-irresolute continuous. □

Theorem 3.4. *Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a bijection. f is $fpsp$-irresolute continuous if and only if for each fuzzy set μ on X,*

$$(\tau_i^*, \tau_j^*) - spInt(f(\mu)) \leq f((\tau_i, \tau_j) - spInt\,\mu).$$

Proof. Let μ be a fuzzy set on X. Then, by Theorem 3.3,

$$f^{-1}((\tau_i^* \tau_j^*) - \mathrm{spInt}(f(\mu))) \leq (\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(f(\mu))).$$

Since f is a bijection,

$$(\tau_i^*, \tau_j^*) - \mathrm{spInt}(f(\mu)) = f(f^{-1}((\tau_i^* \tau_j^*) - \mathrm{spInt}(f(\mu)))) \leq f((\tau_i, \tau_j) - \mathrm{spInt}\,\mu).$$

Conversely, let ν be a fuzzy set on Y. Then

$$(\tau_i^*, \tau_j^*) - \mathrm{spInt}(f(f^{-1}(\nu))) \leq f((\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(\nu))).$$

Recall that f is a bijection. Hence

$$(\tau_i^*, \tau_j^*) - \mathrm{spInt}\,\nu = (\tau_i^*, \tau_j^*) - \mathrm{spInt}(f(f^{-1}(\nu))) \leq f((\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(\nu))).$$

and

$$\begin{aligned} f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spInt}\,\nu) &\leq f^{-1}(f((\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(\nu)))) \\ &= (\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(\nu)). \end{aligned}$$

Therefore, by Theorem 3.3, f is $fpsp$-irresolute contuous. □

Definition 3.5. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then f is called

(1) a fuzzy pairwise strong pre-irresolute open [$fpsp$-irresolute open] mapping if $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - fspo$ set on Y for each $(\tau_i, \tau_j) - fspo$ set μ on X and

(2) a fuzzy pairwise strong pre-irresolute closed [$fpsp$-irresolute closed] mapping if $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - fspc$ set on Y for each $(\tau_i, \tau_j) - fspc$ set μ on X.

It is clear that every $fpsp$-irresolute open mapping and every $fpsp$-irresolute closed mapping are $fpsp$ *open* and $fpsp$ *closed* respectively. But the converses are not true in general.

In fact, in Example 3.2, the identity mapping $i_X : (X, \tau_1^*, \tau_2^*) \to (X, \tau_1, \tau_2)$ is $fpsp$ *open*($fpsp$ *closed*) but not $fpsp$-irresolute open($fpsp$-irresolute closed).

Theorem 3.6. *Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then the following statements are equivalent:*

(1) f is $fpsp$-irresolute open.

(2) $f((\tau_i, \tau_j) - spInt\,\mu) \leq (\tau_i^*, \tau_j^*) - spInt(f(\mu))$ for each fuzzy set μ on X.

(3) $(\tau_i, \tau_j) - spInt(f^{-1}(\nu)) \leq f^{-1}((\tau_i^*, \tau_j^*) - spInt\,\nu)$ for each fuzzy set ν on Y.

Proof. (1) implies (2): Let μ be a fuzzy set on X. Then $f((\tau_i, \tau_j) - \text{spInt}\,\mu)$ is a $(\tau_i^*, \tau_j^*) - fspo$ set on Y and $f((\tau_i, \tau_j) - \text{spInt}\,\mu) \leq f(\mu)$. Hence

$$\begin{aligned} f((\tau_i, \tau_j) - \text{spInt}\,\mu) &= (\tau_i^*, \tau_j^*) - \text{spInt}(f((\tau_i, \tau_j) - \text{spInt}\,\mu)) \\ &\leq (\tau_i^*, \tau_j^*) - \text{spInt}(f(\mu)). \end{aligned}$$

(2) implies (3): Let ν be a fuzzy set on Y. Then

$$\begin{aligned} f((\tau_i, \tau_j) - \text{spInt}(f^{-1}(\nu))) &\leq (\tau_i^*, \tau_j^*) - \text{spInt}(f(f^{-1}(\nu))) \\ &\leq (\tau_i^*, \tau_j^*) - \text{spInt}\,\nu. \end{aligned}$$

Hence

$$\begin{aligned} (\tau_i, \tau_j) - \text{spInt}(f^{-1}(\nu)) &\leq f^{-1}(f((\tau_i, \tau_j) - \text{spInt}(f^{-1}(\nu)))) \\ &\leq f^{-1}((\tau_i^*, \tau_j^*) - \text{spInt}\,\nu). \end{aligned}$$

(3) implies (1): Let μ be a $(\tau_i, \tau_j) - fspo$ set on X. Then

$$\begin{aligned} \mu = (\tau_i, \tau_j) - \text{spInt}\,\mu &\leq (\tau_i, \tau_j) - \text{spInt}(f^{-1}(f(\mu))) \\ &\leq f^{-1}((\tau_i^*, \tau_j^*) - \text{spInt}(f(\mu))). \end{aligned}$$

We have

$$f(\mu) \leq f(f^{-1}((\tau_i^*, \tau_j^*) - \text{spInt}(f(\mu)))) \leq (\tau_i^*, \tau_j^*) - \text{spInt}(f(\mu)).$$

Hence $f(\mu) = (\tau_i^*, \tau_j^*) - \text{spInt}(f(\mu))$. Consequently, $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - fspo$ set on Y and therefore, f is $fpsp$-irresolute open. □

Theorem 3.7. *A mapping $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ is $fpsp$-irresolute closed if and only if $(\tau_i^*, \tau_j^*) - spCl(f(\mu)) \leq f((\tau_i, \tau_j) - spCl\,\mu)$ for each fuzzy set μ on X.*

Proof. Let μ be a fuzzy set on X. Then $f((\tau_i,\tau_j)-\text{spCl}\,\mu)$ is a $(\tau_i^*,\tau_j^*)-fspo$ set on Y and $f(\mu)\leq f((\tau_i,\tau_j)-\text{spCl}\,\mu)$. Hence

$$\begin{aligned}(\tau_i^*,\tau_j^*)-\text{spCl}(f(\mu)) &\leq (\tau_i^*,\tau_j^*)-\text{spCl}(f((\tau_i,\tau_j)-\text{spCl}\,\mu))\\ &= f((\tau_i,\tau_j)-\text{spCl}\,\mu).\end{aligned}$$

Conversely, let μ be a $(\tau_i,\tau_j)-fspc$ set on X. Then

$$\begin{aligned}(\tau_i^*,\tau_j^*)-\text{spCl}(f(\mu)) &\leq f((\tau_i,\tau_j)-\text{spCl}\,\mu)\\ &= f(\mu).\end{aligned}$$

Consequently, $f(\mu)$ is a $(\tau_i^*,\tau_j^*)-fspc$ set on Y and therefore f is a $fpsp$-irresolute closed mapping.

□

Theorem 3.8. *Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be a bijection. Then the following statements are equivalent:*
(1) f is $fpsp$-irresolute closed.
(2) $f^{-1}((\tau_i^*,\tau_j^*)-spCl\,\nu)\leq(\tau_i,\tau_j)-spCl(f^{-1}(\nu))$ for each fuzzy set ν on Y.
(3) f is $fpsp$-irresolute open.
(4) f^{-1} is $fpsp$-irresolute continuous.

Proof. (1) implies (2): Let ν be a fuzzy set on Y. Then, by Theorem 3.7,

$$(\tau_i^*,\tau_j^*)-\text{spCl}(f(f^{-1}(\nu)))\leq f((\tau_i,\tau_j)-\text{spCl}(f^{-1}(\nu))).$$

Hence

$$f^{-1}((\tau_i^*,\tau_j^*)-\text{spCl}(f(f^{-1}(\nu))))\leq f^{-1}(f((\tau_i,\tau_j)-\text{spCl}(f^{-1}(\nu)))).$$

Since f is a bijection,

$$f^{-1}((\tau_i^*,\tau_j^*)-\text{spCl}\,\nu)\leq(\tau_i,\tau_j)-\text{spCl}(f^{-1}(\nu)).$$

(2) implies (1): Let μ be a fuzzy set on X. Then

$$f^{-1}((\tau_i^*,\tau_j^*)-\text{spCl}(f(\mu)))\leq(\tau_i,\tau_j)-\text{spCl}(f^{-1}(f(\mu))).$$

Hence

$$f(f^{-1}((\tau_i^*,\tau_j^*)-\text{spCl}(f(\mu))))\leq f((\tau_i,\tau_j)-\text{spCl}(f^{-1}(f(\mu)))).$$

Since f is a bijection,

$$(\tau_i^*,\tau_j^*)-\text{spCl}(f(\mu))\leq f((\tau_i,\tau_j)-\text{spCl}\,\mu).$$

Therefore, by Theorem 3.7, f is $fpsp$-irresolute closed.

(2) implies (3): Let ν be a fuzzy set on Y. Then

$$f^{-1}((\tau_i^*,\tau_j^*)-\text{spCl}(\nu^c))\leq(\tau_i,\tau_j)-\text{spCl}(f^{-1}(\nu^c)).$$

By Lemma 2.6,

$$\begin{aligned}(\tau_i,\tau_j)-\mathrm{spInt}(f^{-1}(\nu)) &= ((\tau_i,\tau_j)-\mathrm{spCl}(f^{-1}(\nu^c)))^c\\ &\le f^{-1}(((\tau_i^*,\tau_j^*)-\mathrm{spCl}(\nu^c))^c)\\ &= f^{-1}((\tau_i^*,\tau_j^*)-\mathrm{spInt}\,\nu).\end{aligned}$$

Hence f is $fpsp$-irresolute open from Theorem 3.6.

(3) implies (4): Let ν be a fuzzy set on Y. Then

$$(\tau_i,\tau_j)-\mathrm{spInt}(f^{-1}(\nu))\le f^{-1}((\tau_i^*,\tau_j^*)-\mathrm{spInt}\,\nu).$$

Since f is a bijection, by Theorem 3.4, f^{-1} is $fpsp$-irresolute continuous.

(4) implies (2): It is clear from Theorem 3.3.

□

We have the following corollaries from Theorem 3.3, Theorem 3.7 and Theorem 3.6.

Corollary 3.9. *Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be a mapping. Then, f is a $fpsp$-irresolute closed and $fpsp$-irresolute continuous if and only if $f((\tau_i,\tau_j)-spCl\,\mu)=(\tau_i^*,\tau_j^*)-spCl(f(\mu))$ for each fuzzy set μ on X.*

Corollary 3.10. *Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be a mapping. Then, f is $fpsp$-irresolute open and $fpsp$-irresolute continuous if and only if $f^{-1}((\tau_i^*,\tau_j^*)-spCl\nu)=(\tau_i,\tau_j)-spCl(f^{-1}(\mu))$ for each fuzzy set ν on Y.*

A bijection $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ is called *a fuzzy pairwise strong pre-irresolute homeomorphism* if f and f^{-1} are $fpsp$-irresolute continuous mappings.

Theorem 3.11. *Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be a bijection. Then the following statements are equivalent:*

(1) f is a fuzzy pairwise strong pre-irresolute homeomorphism.

(2) f^{-1} is a fuzzy pairwise strong pre-irresolute homeomorphism.

(3) f and f^{-1} are $fpsp$-irresolute open ($fpsp$-irresolute closed).

(4) f is $fpsp$-irresolute continuous and $fpsp$-irresolute open ($fpsp$-irresolute closed).

(5) $f((\tau_i,\tau_j)-spCl\,\mu)=(\tau_i^*,\tau_j^*)-spCl(f(\mu))$ for each fuzzy set μ on X.

(6) $f((\tau_i,\tau_j)-spInt\,\mu)=(\tau_i^*,\tau_j^*)-spInt(f(\mu))$ for each fuzzy set μ on X.

(7) $f^{-1}((\tau_i^*,\tau_j^*)-spInt\nu)=(\tau_i,\tau_j)-spInt(f^{-1}(\nu))$ for each fuzzy set ν on Y.

(8) $(\tau_i,\tau_j)-spCl(f^{-1}(\nu))=f^{-1}((\tau_i^*,\tau_j^*)-spCl\nu)$ for each fuzzy set ν on Y.

Proof. (1) implies (2): It follows immediately from the definition of a fuzzy pairwise strong pre-irresolute homeomorphism.

(2) implies (3) and (3) implies (4): It follows from Theorem 3.8.

(4) implies (5): It follows from Theorem 3.8 and Corollary 3.9.

(5) implies (6): Let μ be a fuzzy set on X. Then, by Lemma 2.6,

$$\begin{aligned} f((\tau_i, \tau_j) - \mathrm{spInt}\,\mu) &= (f((\tau_i, \tau_j) - \mathrm{spCl}(\mu^c)))^c \\ &= ((\tau_i^*, \tau_j^*) - \mathrm{spCl}(f(\mu^c)))^c \\ &= (\tau_i^*, \tau_j^*) - \mathrm{spInt} f(\mu). \end{aligned}$$

(6) implies (7): Let ν be a fuzzy set on Y. Then

$$\begin{aligned} f((\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(\nu)) &= (\tau_i^*, \tau_j^*) - \mathrm{spInt}(f(f^{-1}(\nu))) \\ &= (\tau_i^*, \tau_j^*) - \mathrm{spInt}\,\nu. \end{aligned}$$

Hence

$$f^{-1}(f((\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(\nu))) = f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spInt}\,\nu).$$

Therefore,

$$(\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(\nu)) = f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spInt}\,\nu).$$

(7) implies (8): Let ν be a fuzzy set on Y. Then, by Lemma 2.6,

$$\begin{aligned} (\tau_i, \tau_j) - \mathrm{spCl}(f^{-1}(\nu)) &= (f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spInt}(\nu^c)))^c \\ &= ((\tau_i, \tau_j) - \mathrm{spInt}(f^{-1}(\nu^c)))^c \\ &= f^{-1}((\tau_i^*, \tau_j^*) - \mathrm{spCl}\,\nu). \end{aligned}$$

(8) implies (1): It follows from Theorem 3.8 and Corollary 3.10.

□

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