

UPRIGHT DRAWINGS OF GRAPHS ON THREE LAYERS

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ABSTRACT. An upright drawing of a planar graph G on k layers is a planar straight-line drawing of G , where the vertices of G are placed on a set of k horizontal lines, called layers and no two adjacent vertices are placed on the same layer. There is a previously known algorithm that decides in linear time whether a planar graph admits an upright drawing on k layers for a fixed value of k . However, the constant factor in the running time of the algorithm increases exponentially with k and makes it impractical even for $k = 3$. In this paper, we give a linear-time algorithm to examine whether a biconnected planar graph G admits an upright drawing on three layers and to obtain such a drawing if it exists. We also give a necessary and sufficient condition for a tree to have an upright drawing on three layers. Our algorithms in both the cases are much simpler and easier to implement than the previously known algorithms.

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1. Introduction

An *upright drawing of a planar graph* G is a planar straight-line drawing of G where the vertices of G are placed on a set of horizontal lines, called layers and no two adjacent vertices are placed on the same layer [11]. For example, Fig. 1(b) illustrates an upright drawing Γ_1 of the graph G_1 in Fig. 1(a) that occupies three layers. On the other hand, the graph G_2 of Fig. 1(c) does not admit any upright drawing on three layers although it admits an upright drawing Γ_2 on four layers as illustrated in Fig. 1(d). Thus the problem of determining whether a given graph admits an upright drawing on k layers for a given value of $k \geq 3$ is quite challenging although the problem is trivial for $k < 3$. In this paper, we

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give a linear-time algorithm to determine whether a biconnected planar graph G admits an upright drawing on three layers. We also address the problem of upright drawings of trees on three layers.

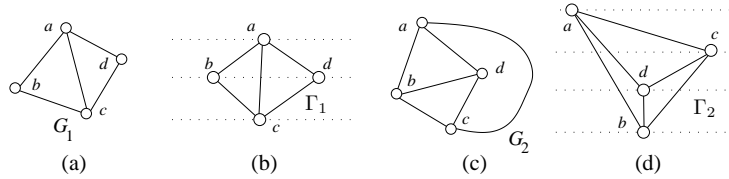


FIGURE 1. (a) The graph G_1 , (b) an upright drawing Γ_1 of G_1 on three layers, (c) the graph G_2 , and (d) an upright drawing Γ_2 of G_2 on four layers.

An upright drawing of a planar graph is a variant of the well studied graph drawing convention, named “layered drawings” [11]. A *layered drawing* of a planar graph G is a planar straight-line drawing of G such that the vertices of G are drawn on a set of layers. Thus an upright drawing of G is a layered drawing of G with the additional constraint that no two adjacent vertices of G are placed on the same layer. Layered drawings have important applications in VLSI layouts [8], DNA-mapping [12], information visualization [3], [7] etc. In some of these application areas, it is often desirable to obtain an upright drawing of a planar graph on a desired number of layers. For example in the “standard cell” technology employed during the VLSI layout design, the VLSI modules are placed on some constant number of previously fixed rows so that they can be lined up in rows on the integrated circuit. The placement of these modules thus gives a layered drawing of the graph obtained from the VLSI circuit where each vertex represents a module in the circuit and each edge represents an interconnection between two modules. Since the modules in a standard cell are designed so that the input and output lines are emitted from the top and the bottom of each module, this drawing is upright. Thus one can obtain a VLSI circuit layout on a standard cell with k rows from an upright drawing of the corresponding graph on k layers. There are some known algorithms to check whether a planar graph admits a layered drawing on two and three layers such as [1], [2], [6] etc. There are also some known results on “proper drawings” of planar graphs. A *proper drawing* of a planar graph G is an upright drawing of G with the additional constraint that adjacent vertices of G are placed on adjacent layers in the drawing. Dujmović *et al.* [4] employed a dynamic programming approach to give a linear-time algorithm that decides whether a planar graph admits a proper drawing on k layers for a fixed value of k . A slight modification of this algorithm yields a similar algorithm for upright drawings of planar graphs. Unfortunately, the constant factor ($= 2^{32k^3}$) [4] in the running time of the algorithm increases exponentially with k and is impractically large even for $k = 3$ [11]. Fößmeier, Kaufmann [5] and Suderman [11] addressed the

problem of proper drawings of planar graphs on three layers. However, there is no such algorithms for upright drawings of planar graphs that can be efficiently implemented to meet the requirements arising in many application areas. In this paper, we give a linear-time algorithm that determines whether a biconnected planar graph G admits an upright drawing on three layers. We also give a linear-time algorithm to obtain an upright drawing of G on three layers if it exists. Furthermore, we give a necessary and sufficient condition for a tree to have an upright drawing on three layers. These algorithms are much simpler in approach and much easier to implement than the previously known algorithms.

The rest of this paper is organized as follows. In Section 2, we present our algorithm to examine whether a biconnected planar graph G has an upright drawing on three layers and to obtain such a drawing if G has. Section 3 gives a characterization of a tree to admit an upright drawing on three layers. Finally, Section 4 concludes the paper with some suggestions on future works.

2. Upright Drawings of Biconnected Planar Graphs

In this section, we first give a linear-time algorithm to check whether a biconnected plane graph G admits an upright drawing on three layers and to obtain such a drawing of G if it exists. (Note that a plane graph is a planar graph with a fixed planar embedding.) Then we give a linear-time algorithm to examine whether a biconnected planar graph G has an upright drawing on three layers.

We first give the definition of the “simple weak dual graph” of a plane graph. Let G be a plane graph. G divides the plane into some regions, called the *faces* of G . The bounded regions are called the *inner faces* of G and the unbounded region is called the *outer face* of G . A vertex v of G is called an *outer vertex* if v is on the outer face of G ; otherwise it is called an *internal vertex*. The *simple weak dual graph* of a plane graph G is another graph H where the each vertex of H represents an inner face of G and there is an edge between two vertices u and v of H if the faces of G corresponding to u and v share one or more consecutive edges. We now have the following lemma that establishes a necessary condition for a biconnected plane graph to admit an upright drawing on three layers.

Lemma 1. *Let a biconnected plane graph G admit an upright drawing Γ on three layers. Then the simple weak dual graph G^* of G is a path.*

Proof. We first prove that G^* does not contain any cycle. Assume for a contradiction that G^* contains a cycle C . Then the faces of G corresponding to the vertices of G^* on C induces at least one internal vertex of G with degree three or more as illustrated in Fig. 2(a). Thus, to prove that G^* contains no cycle, it is sufficient to prove that G does not have any internal vertex with degree three or more.

Suppose there is an internal vertex v of G with degree three or more. Since the vertices on the top and bottom layers in Γ are on the outer face of G , v

is placed on the middle layer in Γ . Then none of the neighbors of v in G is placed on the middle layer since Γ is an upright drawing. Hence, either the top or the bottom layer contains at least two neighbors of v in G . Without loss of generality, let us assume that v has two neighbors u and w on the top layer in Γ . Since v is an internal vertex, there must be a path between u and w along the top layer as illustrated in Fig. 2(b). However, this would place adjacent vertices of G are on the same layer in Γ , a contradiction.

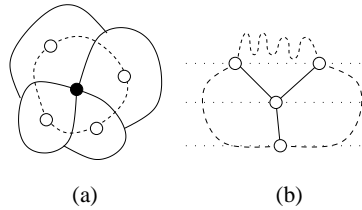


FIGURE 2. (a) A cycle in G^* induces an internal vertex with degree three or more in G , (b) G does not contain an internal vertex of degree three or more.

We thus assume that there is no cycle in G^* . Since each face of G occupies all three layers in Γ , each face shares edges with at most two other faces of G , one to its left and one to its right in Γ . Hence, every vertex of G^* has degree at most two and G^* is a path. \square

We call a biconnected plane graph (embedding) a *dual-path biconnected graph (embedding)* if its simple weak dual graph is a path. Lemma 1 implies that if a biconnected plane graph G admits an upright drawing on three layers, then G is a dual-path biconnected graph. However, this condition alone is not sufficient. Before we give a necessary and sufficient condition for a biconnected plane graph to have an upright drawing on three layers, we need some definitions. Let G be a dual-path biconnected graph with at least two faces and let G^* be its simple weak dual graph. Let F_l and F_r be the faces corresponding to the two end-vertices of G^* , and let F'_l and F'_r be the faces corresponding to the neighbors of F_l and F_r in G^* , respectively. If we delete those vertices of F_l (F_r) that are not on F'_l (F'_r), then the outer cycle of G is divided into two paths. Let us denote these two paths by P_t and P_b and call them the *top path* and the *bottom path*, respectively. Figure 4(a) illustrates the top path and the bottom path of a dual-path biconnected graph. Let the two end-vertices of P_t be t_l and t_r and the two end-vertices of P_b be b_l and b_r as illustrated in Fig. 4(a). We call t_l , b_l , t_r and b_r the *left-top vertex*, the *left-bottom vertex*, the *right-top vertex* and the *right-bottom vertex*, respectively. We denote by $odd(t_l)$ ($odd(t_r)$) the set of vertices that are at odd distance from t_l (t_r) along P_t . The set $\{b, d, f, h\}$ consists $odd(t_l)$ in Fig. 4(a). We also denote by $odd(b_l)$ ($odd(b_r)$) the set of vertices that are at odd distance from b_l (b_r) along P_b . Similarly we define the notations $even(t_l)$,

$even(t_r)$, $even(b_l)$ and $even(b_r)$. A set S of vertices in a graph G is said to be *independent* if G contains no edge between any pair of vertices in S . We now have the following theorem.

Theorem 1. *Let G be a dual path biconnected graph with at least two faces. Let t_l and b_l be the left-top and left-bottom vertices of G and let V_{in} be the set of internal vertices of G . Then G admits an upright drawing on three layers if and only if at least one of the three sets $S_{tb} = odd(t_l) \cup odd(b_l) \cup V_{in}$, $S_{tm} = odd(t_l) \cup even(b_l) \cup V_{in}$ and $S_{mb} = even(t_l) \cup odd(b_l) \cup V_{in}$ is independent in G and contains no vertices of degree greater than three.*

Proof. Suppose G admits an upright drawing Γ on three layers. Since the drawing of each face requires all three layers of Γ , the faces of G are placed in the order of the corresponding vertices along the simple weak dual graph of G . Let F_l and F_r be the faces corresponding to the two end-vertices of G^* . Then we may assume that F_l and F_r are drawn at the leftmost and rightmost position in Γ , respectively. Then the top path P_t and the bottom path P_b must be drawn between the drawings of F_l and F_r as illustrated in Fig. 3(a). One of these two paths (say P_t) must be drawn using the top layer and the middle layer only and the other (say P_b) using the middle layer and the bottom layer only; otherwise, there would have been some edge crossings between the two paths. Since t_l and b_l are on the common boundary of F_l and the face immediately to the right of F_l , either t_l and b_l are adjacent to each other or both of them are adjacent to some internal vertex. In either case, both t_l and b_l are not placed on the middle layer since no two adjacent vertices are placed on the same layer in Γ . Therefore, there are three possible cases regarding the placement of the two vertices t_l and b_l on these three layers:

- (i) t_l on the top layer and b_l on the bottom layer,
- (ii) t_l on the top layer and b_l on the middle layer, and
- (iii) t_l on the middle layer and b_l on the bottom layer.

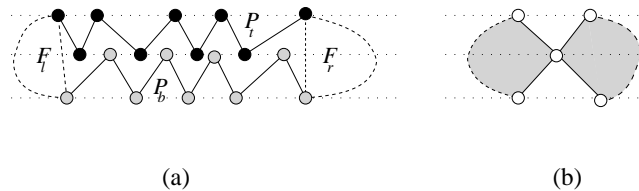


FIGURE 3. (a) P_t and P_b are drawn between the drawing of F_l and F_r , and (b) a vertex placed on the middle layer with degree greater than three is a cut vertex.

If t_l and b_l are placed on the top layer and the bottom layer, respectively, then all the vertices of the set $S_{tb} = odd(u) \cup odd(v) \cup V_{in}$ are placed on the middle layers in Γ . Similarly all the vertices of the set S_{tm} (S_{mb}) are placed on

the middle layer in Γ if u is placed on the top (middle) layer and v is placed on the middle (bottom) layer. Again the vertices of G placed on the same layer in an upright drawing gives an independent set in G . Furthermore Since G is biconnected, no vertex w of G with degree greater than three is placed on the middle layer in Γ ; otherwise, w would have been a cut vertex in G as illustrated in Fig. 3(b). Therefore, at least one of the three sets S_{tb} , S_{tm} and S_{mb} is independent in G and contains no vertices with degree greater than three in G .

Conversely if at least one of the three sets (say S_{tb}) is independent in G and contains no vertices of degree greater than three as illustrated in Fig. 4(a), then we can construct an upright drawing of G on three layers as follows. We first

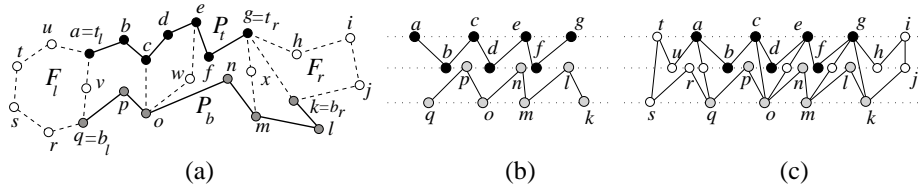


FIGURE 4. Illustration for the proof of Theorem 1.

place the vertices of P_t on the top layer and the middle layer with increasing x -coordinates from t_l so that all the vertices of $even(t_l)$ ($odd(t_l)$) are placed on the top (middle) layer. Similarly, we place the vertices of P_b on the bottom layer and the middle layer with increasing x -coordinates from b_l such that all the vertices of $even(b_l)$ ($odd(b_l)$) are placed on the bottom (middle) layer. While placing the vertices of P_b we take special care so that if a vertex v_t of P_t and a vertex v_b of P_b are adjacent to each other or have an internal vertex as their common neighbor, then these two vertices are placed in such positions that we can add an edge between them without creating any edge crossings. (See Fig. 4(b).) This is always possible because the degree of each vertex of $odd(t_l)$ and $odd(b_l)$ is at most three and hence each vertex of $odd(t_l)$ ($odd(b_l)$) has at most one neighbor not on P_t (P_b). We now place the internal vertices in the drawing. Note that each the internal vertex of G has degree two and has exactly one neighbor from P_t and exactly one neighbor from P_b ; otherwise G would not have been a dual-path biconnected graph. We place each of these internal vertices of G in such a position on the middle layer that we can add an edge between these vertices and their neighbors on the two paths P_t and P_b . Finally we place the vertices of F_l and F_r that have not been yet placed and add all the edges of G to complete the drawing. (See Fig. 4(c).) \square

Theorem 1 gives a necessary and sufficient condition for a biconnected plane graph to have an upright drawing on three layers. The proof of sufficiency also gives a linear-time algorithm to obtain an upright drawing of a biconnected plane graph G if G admits one. We now address the problem for a biconnected planar

graph. Let G be a biconnected planar graph. Since G may have an exponential number of embeddings, the naive approach of checking all these embeddings for an existence of an upright drawing on three layers would take exponential time. We now give a linear-time algorithm to check whether a biconnected planar graph G admits an upright drawing on three layers. We first have the following lemma.

Lemma 2. *Let $G = (V, E)$ be a biconnected planar graph that admits a dual-path biconnected embedding. Let H be a graph defined as follows: the vertex set of H is obtained by adding a vertex v to V and the edge set of H is obtained by adding an edge (u, v) to E for each vertex u of degree three or more in G . Then the following two conditions (a) and (b) hold:*

- (a) H is planar.
- (b) If Γ_H is a planar embedding of H where v is on the outerface and if Γ_G is an embedding of G obtained by deleting v from Γ_H , then Γ_G is a dual-path biconnected embedding.

Proof. (a) Let Γ be a dual-path biconnected embedding of G . Then all the vertices of G with degree three or more are on the outer cycle of Γ ; otherwise the dual graph of Γ would have contained a cycle as illustrated in Fig. 2(a), a contradiction. Thus one can obtain a plane embedding of H from Γ by placing v on the outer face and adding an edge from v to each vertex of degree three or more in G without edge crossings.

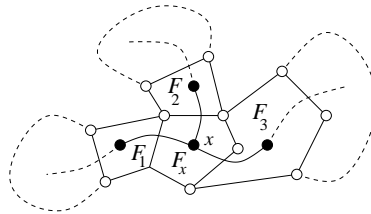


FIGURE 5. Illustration for the proof of Lemma 2(ii).

(b) Clearly the simple weak dual graph G^* of Γ_G contains no cycle since from the construction it is obvious that each vertex of degree three or more in G are on the outer face in Γ_G . If each vertex of G^* has degree at most two, then Γ_G is a dual-path biconnected embedding. We thus assume that the degree of a vertex x of G^* is at least three. Then the face F_x of Γ_G corresponding to x shares common boundary with at least three other faces F_1 , F_2 and F_3 other than the outer face as illustrated in Fig. 5. It is then trivial to see that for any embedding Γ' of G , the simple weak dual graph would contain either a cycle or a degree three vertex and hence Γ' is not a dual-path biconnected graph, a contradiction. \square

Let G be biconnected planar graph and let Γ be a dual-path biconnected embedding of G . Let t_l , t_r , b_l and b_r be the left-top vertex, the right-top vertex, the left-bottom vertex and the right-bottom vertex of Γ , respectively. We call a path of G a *left path* (*right path*) if its end-vertices are t_l (t_r) and b_l (b_r) and it contains no other vertices of the top or the bottom path. We call an embedding Γ^* of G a feasible embedding of G if the following conditions (a) and (b) hold: (a) Γ^* is a dual-path biconnected embedding, (b) the left path and the right path of Γ^* with the maximum length are both on the outer face on Γ^* . One can always obtain a feasible embedding from a dual-path biconnected graph by twisting or flipping the left paths across $\{t_l, b_l\}$ and the right paths across $\{t_r, b_r\}$ as illustrated in Fig. 6.

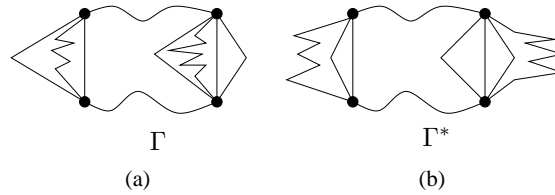


FIGURE 6. (a) a dual-path biconnected plane graph Γ , and (b) a feasible embedding Γ^* of Γ .

We now have the following lemma.

Lemma 3. *Let G be a biconnected planar graph that admits a dual-path biconnected embedding with at least two faces and let Γ^* be a feasible embedding of G . Then G admits an upright drawing on three layers if and only if Γ^* admits an upright drawing on three layers.*

Proof. It is sufficient to prove that if Γ^* does not admit an upright drawing on three layers, then no other embedding of G admits such a drawing. Let t_l , t_r , b_l and b_r be the left-top, the right-top, the left-bottom and the right-bottom vertices of Γ^* , respectively. One can observe that only the plane embeddings of G that can be obtained by flipping the left paths of Γ^* across $\{t_l, b_l\}$ and the right paths of Γ^* across $\{t_r, b_r\}$ have all the vertices with degree three or more on the outer face. Again in an embedding of G that admits an upright drawing on three layers, at most one left path and at most one right path have length greater than two and both are on the outer face. Since Γ^* has the left and the right paths of maximum length on the outer face, the result follows. \square

The following theorem follows from the results in Lemma 2, Lemma 3 and Theorem 1.

Theorem 2. *Let G be a biconnected planar graph. One can examine in linear time whether G admits an upright drawing on three layers. Furthermore, one can also obtain such a drawing of G in linear time if it exists.*

3. Upright Drawings of Trees

In this section, we give a necessary and sufficient condition for a tree T to have an upright drawing on three layers.

We first need some definitions. Let H be a subgraph of a graph G . Then $G - H$ denote the subgraph of G obtained by deleting all the vertices of H and the edges incident to these vertices. A tree T is called a *caterpillar* if T contains a path S , called the *spine* such that each component of $T - S$ is a single vertex. Similarly, T is called an *extended caterpillar* if T contains a path S , called the *spine* such that each component of $T - S$ is a caterpillar. A *strict caterpillar* is a caterpillar which is not a path and a *strict extended caterpillar* is an extended caterpillar which is not a caterpillar.

A path decomposition P of a graph $G = (V, E)$ is a sequence V_1, V_2, \dots, V_f of subsets of V such that the following conditions (a)–(c) hold: (a) $(\cup_{i=1}^f V_i) = V$, (b) for each edge $(u, v) \in E$, there is some index i ($1 \leq i \leq f$) such that $u, v \in V_i$, (c) for each $1 \leq i < j < k \leq f$, $V_i \cap V_k \subseteq V_j$. The width of P is $\max_{1 \leq i \leq f} (|V_i| - 1)$.

The pathwidth of a graph G is the minimum width of a path decomposition of G . Although finding the pathwidth of a graph G is NP-hard, Scheffler [9] gave a linear-time algorithm to find the pathwidth of a tree. It is trivial to see from the definition that a tree T has pathwidth zero (one) if and only if T is a path (a strict caterpillar). We now have the following lemma that defines trees with pathwidth two.

Lemma 4. *A tree T has pathwidth two if and only if T is a strict extended caterpillar.*

We need the following lemma from [9] to prove Lemma 4.

Lemma 5. *Let T be a tree and $p > 1$ be an integer. Then T has pathwidth at least p if and only if there exists a vertex u of T such that at least three components of $T - u$ has pathwidth at least $p - 1$.*

Proof of Lemma 4. Suppose the pathwidth of T is two. Then T has a path decomposition V_1, V_2, \dots, V_f of width two. Let v_1 be a vertex in V_1 , v_p a vertex in V_p . Then by the definition, the path S between v_1 and v_p contains at least one vertex from each V_i for $1 \leq i \leq f$. Therefore each component of $T - S$ has pathwidth at most one and is thus a caterpillar. Hence T is an extended caterpillar where S is the spine. Furthermore T is also a strict extended caterpillar since otherwise it would have pathwidth one.

Conversely if T is a strict extended caterpillar, then it surely has pathwidth at least two. We assume for a contradiction that T has pathwidth at least three. Then by Lemma 5, T has a vertex u such that $T - u$ has at least three components C_1, C_2 and C_3 with pathwidth at least two. Thus none of C_1, C_2 and C_3 is a caterpillar. Then for each path S of T , S contains no vertex from at least one of C_1, C_2 and C_3 and hence T is not an extended caterpillar, a contradiction. \square

Suderman gave a linear-time algorithm to obtain an upright drawing of a tree with pathwidth h on $\lceil 3h/2 \rceil$ layers [10]. Since an extended caterpillar has pathwidth at most two according to Lemma 4, the algorithm in [11] gives an upright drawing of an extended caterpillar on three layers. We now have the following theorem that shows that this sufficient condition is also necessary.

Theorem 3. *A tree T admits an upright drawing on three layers if and only if T is an extended caterpillar.*

Proof. Suppose T has an upright drawing Γ on three layers as illustrated in Fig. 7(a). Let u and v be the vertices of T with the minimum and the maximum x -coordinates in Γ , respectively and let S denote the unique path between u and v in T . T is an extended caterpillar if all the components C of $T - S$ are caterpillars. We thus assume that a component C of $T - S$ is not a caterpillar. Let Γ_C be the drawing of C contained in Γ . Then according to [5], Γ_C occupies all three layers in Γ . Thus it is not possible to draw the path S using straight-line segments without edge crossings with Γ_C , a contradiction, as illustrated in Fig. 7(b). Thus each component of $T - S$ is a caterpillar and T is an extended caterpillar where S is the spine.

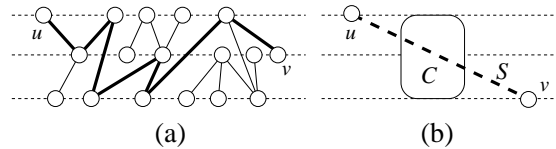


FIGURE 7. Illustration for the proof of Theorem 3.

Conversely if T is an extended caterpillar, then T has pathwidth at most two by Lemma 4 and we can obtain an upright drawing of T on three layers by the algorithm in [11]. \square

By Lemma 4 and Theorem 3, we can verify whether a tree T has an upright drawing on three layers in linear time as follows. By the linear-time algorithm of [9] we compute the pathwidth of T and verify whether the pathwidth is less than or equal to two or not. Furthermore if T admits a upright drawing on three layers, we can also obtain such a drawing by the linear-time algorithm of [11]. Thus the following corollary holds.

Corollary 1. *Let T be a tree. One can examine whether T admits an upright drawing on three layers in linear time. Furthermore, one can also obtain such a drawing of T in linear time if it exists.*

4. Conclusion

In this paper, we gave a linear-time algorithm to examine whether a biconnected planar graph G admits an upright drawing on three layers and to obtain such a drawing if it admits. We also give a characterization of a tree to have an upright drawing on three layers. Although there is a previously known algorithm in [4] that determines in linear time whether a planar graph admits an upright drawing on k layers for a fixed value of k , the constant factor in the running time is grows exponentially with k and is impractically large even for $k = 3$ [11]. Furthermore the difficulty of the problem seems to increase as the value of k increases. Thus although Suderman gave a linear-time algorithm in [11] that decides whether a planar graph admits a special variant of upright drawing, called a proper drawing on three layers, he acknowledges that his algorithm approach is hard to extend for four or more layers. In this context, the ideas developed in our algorithm can be seen as a base to be exploited to design practically feasible and implementable algorithms for upright or proper drawings of planar graphs on three or more layers. It is also an interesting open problem to give an algorithm that obtains an upright drawing of a planar graph G on the minimum number of layers.

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