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FUZZY INTERIOR Γ-IDEALS IN ORDERED Γ-SEMIGROUPS

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Abstract. In this paper we define fuzzy interior Γ-ideals in ordered Γ-semigroups. We prove that in regular(resp. intra-regular) ordered Γ-semigroups the concepts of fuzzy interior Γ-ideals and fuzzy Γ-ideals coincide. We prove that an ordered Γ-semigroup is fuzzy simple if and only if every fuzzy interior Γ-ideal is a constant function. We characterize intra-regular ordered Γ-semigroups in terms of interior (resp. fuzzy interior) Γ-ideals.

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1. Introduction

The notion of a fuzzy set in a set or (a fuzzy subset of a set) was introduced by Zadeh in [10], and since then this concept has been applied to various algebraic structures. Kuroki consider the fuzzification of interior ideals of semigroups in [5] and gave several properties of semigroups in terms of fuzzy interior ideals. Kehayopulu and Tsingelis first considered the fuzzy sets in ordered groupoids and ordered semigroups [3]. They discussed fuzzy analogous for several notions that have been proved to be useful in the theory of ordered groupoids/ordered semigroups. In [4], they have shown that the concepts of a fuzzy ideal and a fuzzy interior ideal coincide in case of regular and intra-regular ordered semigroups. They also shown that an ordered semigroup is simple if and only if it is fuzzy simple. Shabir and Khan extended the concept of interior ideals of ordered semigroups in intuitionistic fuzzy interior ideals and characterized several classes of ordered semigroups in terms of intuitionistic fuzzy interior ideals in [9]. Sen and Saha [7] defined the concept of a Γ-semigroup, and established a relation between regular Γ-semigroup and Γ-group (see also [7,8]). Kwon and

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Lee introduced the concept of Γ-ideals and weakly prime Γ-ideals in ordered Γ-semigroups in [6], and established basic properties of ordered Γ-semigroups in terms of weakly prime Γ-ideals.

In this paper we consider the fuzzification of the notion of interior Γ-ideals in ordered Γ-semigroups. We prove that in regular and intra-regular ordered Γ-semigroups the concepts of fuzzy Γ-ideals and fuzzy interior Γ-ideals coincide. Finally, we introduce the concept of a fuzzy simple ordered Γ-semigroup and prove that an ordered Γ-semigroup is simple if and only if it is fuzzy simple, and we characterize ordered Γ-semigroups in terms of interior Γ-ideals and in terms of fuzzy interior Γ-ideals.

2. Preliminaries

We conclude here some basic definitions of ordered $\Gamma-$ semigroups that are necessary for the subsequent results and for more details on ordered $\Gamma-$ semigroups we refer to [6] .

By an *ordered* $\Gamma-$*semigroup* we mean an *ordered set* M at the same time a $\Gamma-$ semigroup satisfying the following condition [6]:

$$a \leq b \Longrightarrow x\alpha a \leq x\alpha b \text{ and } a\beta x \leq b\beta x \text{ for all } x, a, b \in S \text{ and } \alpha, \beta \in \Gamma$$

If M is an ordered $\Gamma-$semigroup, and A a subset of M, we denote by $(A]$ the subset of M defined as follows;

$$(A] := \{t \in M \mid t \leq a \text{ for some } a \in A\}.$$

If $A = \{a\}$, then we write $(a]$ instead of $(\{a\}]$. If $A, B \subseteq M$,then $A \subseteq (A], B \subseteq (B], (A]\Gamma(B] \subseteq (A\Gamma B]$ and $((A]] = (A]$.

For $A, B \subseteq M$, we denote,

$$A\Gamma B := \{a\alpha b \mid a \in A, \alpha \in \Gamma \text{ and } b \in B\}.$$

An ordered $\Gamma-$semigroup M is called *regular* if for each $a \in M$ and for each $\alpha, \beta \in \Gamma$ there exists $x \in M$ such that $a \leq a\alpha x\beta a$.

Equivalent Definitions: (1) $A \subseteq (A\Gamma M\Gamma A]$ for each $A \subseteq M$. (2) $a \in (a\Gamma M\Gamma a]$ for each $a \in M$.

An ordered $\Gamma-$semigroup M is called *intra-regular* if for each $a \in M$ and for each $\alpha, \beta, \gamma \in \Gamma$ there exists $x, y \in M$ such that $a \leq x\alpha a\beta a\gamma y$. Equivalent Definitions: (1) $A \subseteq (M\Gamma A\Gamma A\Gamma M]$ for each $A \subseteq M$. (2) $a \in (M\Gamma a\Gamma a\Gamma M]$ for each $a \in M$.

Definition 1. *A non-empty subset A of an ordered $\Gamma-$semigroup M is called a left (resp. right) $\Gamma-$ideal of M if it satisfies* [6] :

(1) $M\Gamma A \subseteq A$ (*resp.* $A\Gamma M \subseteq A$),

(2) $a \leq b$ $M \ni b \leq a$ *implies* $b \in A$ *for all* $a, b \in M$.

Condition (2) is equivalent to the condition $(A] = A$.

Both a left $\Gamma-$ideal and a right $\Gamma-$ideal of an ordered $\Gamma-$semigroup M is called a $\Gamma-$*ideal* of S.

Definition 2. *A non-empty subset A of an ordered Γ-semigroup M is called an interior $\Gamma-$ideal of M if it satisfies:*
(1) $M\Gamma A\Gamma M \subseteq A$;
(2) $a \leq bM \ni b \leq a$ *implies* $b \in A$ *for all* $a, b \in M$.

Let M be an *ordered* Γ*-semigroup.* By *a fuzzy subset* f of M, we mean a mapping

$$f : M \longrightarrow [0, 1].$$

Definition 3. *A fuzzy subset f of an ordered Γ-semigroup M is called a fuzzy left (resp. right) $\Gamma-$ideal of M, if the following conditions are satisfied:*
(1) $f(x\alpha y) \geq f(y)$ *(resp.* $f(x\alpha y) \geq f(x)$*) for all* $x, y \in M$ *and for all* $\alpha \in \Gamma$;
(2) *If* $x \leq y$, *then* $f(x) \geq f(y)$ *for all* $x, y \in M$.

If f is both a fuzzy left $\Gamma-$*ideal* and right $\Gamma-$*ideal* then f is called a two-sided fuzzy $\Gamma-$*ideal* or simply fuzzy $\Gamma-$*ideal* of M.

3. Fuzzy interior $\Gamma-$ideals

Definition 4. *A fuzzy subset f of an ordered Γ-semigroup M is called a fuzzy interior $\Gamma-$ideal of M, if the following conditions are satisfied:*
(1) $f(x\alpha a\beta y) \geq f(a)$ *for all* $x, a, y \in M$ *and for all* $\alpha, \beta \in \Gamma$;
(2) *If* $x \leq y$, *then* $f(x) \geq f(y)$ *for all* $x, y \in M$.

In this section, we prove that in regular (resp. intra-regular) ordered Γ-semigroups the concepts of fuzzy $\Gamma-$ideals and fuzzy interior $\Gamma-$ideals coincide.

Let M be an ordered Γ-semigroup and $A \subseteq M$, the *characteristic function* χ_A of A is defined by

$$\chi_A : M \longrightarrow [0, 1] \mid x \longrightarrow \chi_A(x) := \begin{cases} 1 \text{ if } x \in A, \\ 0 \text{ if } x \notin A. \end{cases}$$

Proposition 1. *If $\{f_i : i \in I\}$ is a family of fuzzy interior $\Gamma-$ideals of an ordered Γ-semigroup M then $\bigcap_{i\in\Lambda} f_i$, if it is non-empty, is a fuzzy interior $\Gamma-$ideal of M, where $\left(\bigcap_{i\in\Lambda} f_i\right)(x) = \bigwedge_{i\in\Lambda} f_i(x)$ for all $x \in M$.*

Proof. For $x, y \in M$ if $x \leq y$, then we have

$$\begin{aligned} \left(\bigcap_{i\in\Lambda} f_i\right)(x) &= \left(\bigwedge_{i\in\Lambda} f_i\right)(x) = \bigwedge_{i\in\Lambda}(f_i(x)) \\ &\geq \bigwedge_{i\in\Lambda}(f_i(y)) \ \ (\text{since } x \leq y \Longrightarrow f_i(x) \geq f_i(y)) \\ &= \left(\bigwedge_{i\in\Lambda} f_i\right)(y) = \left(\bigcap_{i\in\Lambda} f_i\right)(y). \end{aligned}$$

Let $a, x, y \in M$ and $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned}\left(\bigcap_{i\in\Lambda} f_i\right)(x\alpha a\beta y) &= \left(\bigwedge_{i\in\Lambda} f_i\right)(x\alpha a\beta y) = \bigwedge_{i\in\Lambda}(f_i(x\alpha a\beta y)) \\ &\geq \bigwedge_{i\in\Lambda}(f_i(a)) \text{ (since } f_i(x\alpha a\beta y) \geq f_i(a)) \\ &= \left(\bigwedge_{i\in\Lambda} f_i\right)(a) = \left(\bigcap_{i\in\Lambda} f_i\right)(a).\end{aligned}$$

Thus $\bigcap_{i\in\Lambda} f_i$ is a fuzzy interior $\Gamma-$ideal of M. □

Proposition 2. *Every fuzzy $\Gamma-$ideal of an ordered Γ-semigroup M is a fuzzy interior $\Gamma-$ideal of M.*

Proof. Let f be a fuzzy $\Gamma-$ideal of an ordered Γ-semigroup M. Let $x, a, y \in M$ and $\alpha, \beta \in \Gamma$. Then

$f(x\alpha(a\beta y)) \geq f(a\beta y) \geq f(a)$ (because f is a fuzzy left and right $\Gamma -$ ideal of M).

Thus f is a fuzzy interior $\Gamma-$ideal of M. □

The converse of the above Proposition is not true in general, however, if M is regular then we have the following:

Proposition 3. *Every fuzzy interior $\Gamma-$ideal of an ordered Γ-semigroup M is a fuzzy $\Gamma-$ideal of M.*

Proof. Let f be a fuzzy interior $\Gamma-$ideal of an ordered Γ-semigroup M. Let $a, b \in M$ and $\alpha \in \Gamma$. Then

$$f(a\alpha b) \geq f(a).$$

Indeed: Since M is regular, there exists $x \in M$ and $\beta, \gamma \in \Gamma$ such that $a \leq a\beta x\gamma a$. Then

$$a\alpha b \leq (a\beta x\gamma a)\alpha b = (a\beta x)\gamma a\alpha b.$$

Since f is a fuzzy interior $\Gamma-$ideal of an ordered Γ-semigroup M we have

$$f(a\alpha b) \geq f((a\beta x)\gamma a\alpha b) \geq f(a).$$

Similarly, we can show that $f(a\alpha b) \geq f(b)$. Thus f is a fuzzy $\Gamma-$ideal of M. □

Combining Propositions 6 and 7, we have the following:

Proposition 4. *In regular ordered Γ-semigroups the concepts of fuzzy $\Gamma-$ideals and fuzzy interior $\Gamma-$ideals coincide.*

Proposition 5. *Let M be an intra-regular ordered Γ-semigroup. Then every fuzzt interior $\Gamma-$ideal of M is a fuzzy $\Gamma-$ideal of M.*

Proof. Let f be a fuzzy interior $\Gamma-$ideal of an intra-regular ordered Γ-semigroup M. Let $a, b \in M$ and $\alpha \in \Gamma$. Then

$$f(a\alpha b) \geq f(a).$$

Indeed: Since M is intra-regular, there exists $x, a, y \in M$ and $\beta, \gamma, \delta \in \Gamma$ such that $a \leq x\beta a\gamma a\delta y$. Then

$$a\alpha b \leq (x\beta a\gamma a\delta y)\alpha b = (x\beta a)\gamma a\delta(y\alpha b).$$

Since f is a fuzzy interior $\Gamma-$ideal of an ordered Γ-semigroup M we have

$$f(a\alpha b) \geq f((x\beta a)\gamma a\delta(y\alpha b)) \geq f(a).$$

Similarly, we can show that $f(a\alpha b) \geq f(b)$. Thus f is a fuzzy $\Gamma-$ideal of M. □

Combining Propositions 6 and 9, we have the following:

Proposition 6. *In intra-regular ordered* Γ*-semigroups the concepts of fuzzy* $\Gamma-$*ideals and fuzzy interior* $\Gamma-$*ideals coincide.*

Theorem 1. *Let M be an ordered* Γ*-semigroup, $\emptyset \neq I \subseteq M$. Then I is an interior $\Gamma-$ideal of M if and only if the characteristic function χ_I of I is a fuzzy interior $\Gamma-$ideal of M.*

Proof. $\Longrightarrow$. Suppose that I is an interior $\Gamma-$ideal of M and χ_I the *characteristic function* of I. Let $a, b \in M$, $a \leq b$ then $\chi_I(a) \geq \chi_I(b)$. Indeed: If $b \notin I$ then $\chi_I(b) = 0$. Since $\chi_I(a) \geq 0$, we have $\chi_I(a) \geq \chi_I(b)$. Let $b \in I$ then $\chi_I(b) = 1$. Since I is an interior $\Gamma-$ideal of M and $a \leq b$ we have $a \in I$. Then $\chi_I(a) = 1$. Again we have $\chi_I(a) \geq \chi_I(b)$.

Let $x, a, y \in M$ and $\alpha, \beta \in \Gamma$. If $a \in I$ then $\chi_I(a) = 1$. Since I is an interior $\Gamma-$ideal of M, we have $x\alpha a\beta y \in M\Gamma I\Gamma M \subseteq I$. Then we have $\chi_I(x\alpha a\beta y) = 1$ and hence $\chi_I(x\alpha a\beta y) \geq \chi_I(a)$.

$\Longleftarrow$. Assume that χ_I is a fuzzy interior $\Gamma-$ideal of M. Let $a, b \in M$, $a \leq b$. If $b \in I$ then $\chi_I(b) = 1$. Since $\chi_I(a) \geq \chi_I(b)$, we have $\chi_I(a) = 1$ and so $a \in I$.

Let $x, a, y \in M$ and $\alpha, \beta \in \Gamma$. If $a \in I$, then $\chi_I(a) = 1$. Since $\chi_I(x\alpha a\beta y) \geq \chi_I(a)$, we have $\chi_I(x\alpha a\beta y) = 1$ and so $x\alpha a\beta y \in I \Longrightarrow M\Gamma I\Gamma M \subseteq I$. □

4 Fuzzy Simple Ordered $\Gamma-$Semigroups

In this section we introduce the concept of fuzzy simple ordered $\Gamma-$semigroups, we prove that an ordered Γ-semigroup is simple if and only if it is simple, and we characterize this type of ordered $\Gamma-$semigroups in terms of fuzzy interior $\Gamma-$ideals.

An ordered $\Gamma-$semigroup M is called simple if it does not contain proper $\Gamma-$ideals, that is, for any $\Gamma-$ideal A of M, we have $A = M$.

Definition 5. *An ordered* Γ*-semigroup M is called fuzzy simple if every fuzzy $\Gamma-$ideal of M is a constant function, that is, for every fuzzy $\Gamma-$ideal f of M, we have $f(a) = f(b)$ for all $a, b \in M$.*

If M is an ordered Γ-semigroup and $a \in M$, we denote by I_a the subset of M defines as follows:

$$I_a := \{b \in M : f(b) \geq f(a)\}.$$

Proposition 7. *Let M be an ordered Γ-semigroup and f a fuzzy right $\Gamma-$ideal of M. Then I_a is a right $\Gamma-$ideal of M for every $a \in M$.*

Proof. Let M be an ordered Γ-semigroup and f a fuzzy right $\Gamma-$ideal of M. Since $a \in I_a$ for every $x \in M$, we have $I_a \neq \emptyset$. Let $b \in I_a, x \in M$ and $\alpha \in \Gamma$. We have to prove that $b\alpha x \in I_a$. Since f is a fuzzy right $\Gamma-$ideal of M, we have $f(b\alpha x) \geq f(b)$. Since $b \in I_a$ we have $f(b) \geq f(a)$. Thus $f(b\alpha x) \geq f(a)$, hence $b\alpha x \in I_a \Longrightarrow I_a \Gamma M \subseteq I_a$.

Let $b \in I_a$ and $M \ni x \leq b$. Then $x \in I_a$. Indeed: Since f a fuzzy right $\Gamma-$ideal of M and $x \leq b$ we have $f(x) \geq f(b)$. Since $b \in I_a$ we have $f(b) \geq f(a)$. Thus $f(x) \geq f(a)$, which implies that $x \in I_a$. □

In a similar way we can prove that:

Proposition 8. *Let M be an ordered Γ-semigroup and f a fuzzy left $\Gamma-$ideal of M. Then I_a is a left $\Gamma-$ideal of M for every $a \in M$.*

Combining Propositions 13 and 14, we have the following:

Proposition 9. *Let M be an ordered Γ-semigroup and f a fuzzy $\Gamma-$ideal of M. Then I_a is a $\Gamma-$ideal of M for every $a \in M$.*

Lemma 1. *Let M be an ordered Γ-semigroup, $\emptyset \neq A \subseteq M$. Then A is an ordered $\Gamma-$ideal of M if and only if the characteristic function χ_A of A is a fuzzy $\Gamma-$ideal of M.*

Proof. $\Longrightarrow$. Suppose that A is an ordered $\Gamma-$ideal of M and χ_A the *characteristic function* of A. Let $a, b \in M$ such that $a \leq b$ then $\chi_A(a) \geq \chi_A(b)$. I ndeed: If $b \notin A$, then $\chi_A(b) = 0$. Since $\chi_A(a) \geq 0$, we have $\chi_A(a) \geq \chi_A(b)$. Let $b \in A$ then $\chi_A(b) = 1$. Since A is an ordered $\Gamma-$ideal of M and $a \leq b$ we have $a \in A$. Thus $\chi_A(a) = 1$. Again we have $\chi_A(a) \geq \chi_A(b)$.

Let $x, y \in M$. If $x \in A$ then $\chi_A(x) = 1$.Since A is a right $\Gamma-$ideal of M, we have $x\alpha y \in A\Gamma M \subseteq A$. Then we have $\chi_A(x\alpha y) = 1$, hence $\chi_A(x\alpha y) \geq \chi_A(x)$.

$\Longleftarrow$. Assume that χ_A is a fuzzy $\Gamma-$ideal of M. Let $a, b \in M$ such that $a \leq b$. If $b \in A$, then $\chi_A(b) = 1$. Since $\chi_A(a) \geq \chi_A(b)$, we have $\chi_A(a) = 1$ and so $a \in A$.

Let $x, y \in M$ and $\alpha \in \Gamma$. If $x \in A$, then $\chi_A(x) = 1$. Since χ_A is a fuzzy right $\Gamma-$ideal of M, we have $\chi_A(x\alpha y) \geq \chi_A(x)$. Thus $\chi_A(x\alpha y) = 1$ and so $x\alpha y \in A \Rightarrow A\Gamma M \subseteq A$. □

Theorem 2. *An ordered Γ-semigroup M is simple if and only if it is fuzzy simple.*

Proof. Let M be a simple ordered Γ-semigroup, f a fuzzy $\Gamma-$ideal of M and $a, b \in M$. Since f is a fuzzy $\Gamma-$ideal of M and $a \in M$, so by Proposition 15, I_a is a $\Gamma-$ideal of M. Since M is simple we have $I_a = M$, and we have $b \in I_a$. Thus

$f(b) \geq f(a)$. By a similar way we can prove that $f(a) \geq f(b)$. Thus $f(b) = f(a)$ and so M is fuzzy simple.

$\Longleftarrow$. Suppose that M contains proper $\Gamma-$ideals and let A be a $\Gamma-$ideal of M such that $A \neq M$. Since A is proper $\Gamma-$ideal of M so by Lemma 16, χ_A is a fuzzy $\Gamma-$ideal of M. Let $x \in M$. Since χ_A is a constant fuzzy $\Gamma-$ideal of M. We have $f(x) = f(b)$ for every $b \in M$. Since $A \neq \emptyset$, let $a \in A$. Then $f(x) = f(a) = 1$, hence $x \in A$. Thus $M \subset A$, a contradiction. Thus $M = A$ and M is simple. □

Lemma 2. *An ordered* Γ*-semigroup* M *is simple if and only if for every* $a \in M$*, we have* $M = (M\Gamma a\Gamma M]$.

Proof. Let $a \in M$. Then $M = (M\Gamma a\Gamma M]$. Infact: Since $a \in M$ and M is simple, we have $a \in (a\Gamma M]$. Then

$$a \in (a\Gamma M] \subseteq ((M\Gamma a]\Gamma M] \subset (M\Gamma a\Gamma M],$$

and we have $M \subset (M\Gamma a\Gamma M]$. Hence $M = (M\Gamma a\Gamma M]$.

Conversely, suppose that M contains proper $\Gamma-$ideals of M and let A be a $\Gamma-$ideal of M such that $A \neq M$. Let $a \in A$. Then $b \leq b\alpha a\beta b$ for every $b \in M$ and every $\alpha, \beta \in \Gamma$ and we have $b\alpha a\beta b \in M\Gamma A\Gamma M \subseteq (M\Gamma A\Gamma M] \subseteq (A] = A$. Then $M \subset A$, a contradiction. Hence $A = M$. □

Theorem 3. *An ordered* Γ*-semigroup* M *is simple if and only if every fuzzy interior* $\Gamma-$*ideal of* M *is a constant function.*

Proof. $\Longrightarrow$. Let f be a fuzzy interior $\Gamma-$ideal of a simple ordered Γ-semigroup M and $a, b \in M$. Since M is simple and $b \in M$, by Lemma 18, we have $M = (M\Gamma b\Gamma M]$. Since $a \in M$, we have $a \in (M\Gamma b\Gamma M]$. Then there exist $x, y \in M$ and $\alpha, \beta \in \Gamma$ such that $a \leq x\alpha b\beta y$. Since f is a fuzzy interior $\Gamma-$ideal of M,we have $f(a) \geq f(x\alpha b\beta y) \geq f(b)$. In a similar way we can prove that $f(a) \leq f(b)$, hence $f(a) = f(b)$ and thus f is a constant function.

$\Longleftarrow$. Let f be a fuzzy $\Gamma-$ideal of M. By Proposition 6, f is a fuzzy interior $\Gamma-$ideal of M. By hypothesis, f is a constant function. Then M is fuzzy simple and, by Theorem 17, M is simple. □

Proposition 10. *Let* M *be an intra-regular ordered* Γ*-semigroup. Then for every interior* $\Gamma-$*ideals* A *and* B *of* M *we have,*

$$(A\Gamma A] = A \text{ and } (A\Gamma B] = (B\Gamma A].$$

Proof. (1) Let M be an intra-regular ordered Γ-semigroup and A, B be interior $\Gamma-$ideals of M. Let $a \in A$. Since M is intra-regular, there exist $x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$ such that

$$\begin{aligned} a &\leq x\alpha a\beta a\gamma y \leq x\alpha(x\alpha a\beta a\gamma y)\beta(x\alpha a\beta a\gamma y)\gamma y \\ &= ((x\alpha x)\alpha a\beta(a\gamma y)\beta(x\alpha a)\beta a\gamma(y\gamma y) \in (M\Gamma A\Gamma M)\Gamma(M\Gamma A\Gamma M) \subseteq A\Gamma A \\ &\Longrightarrow a \in (A\Gamma A] \Longrightarrow A \subseteq (A\Gamma A]. \end{aligned}$$

For the revesre inclusion, let $a \in (A\Gamma A]$, then $a \leq a_1 \alpha a_2$ for some $a_1, a_2 \in A$ and $\alpha \in \Gamma$. Then

$$a \leq x\alpha a\beta a\gamma y = a \leq (x\alpha a)\beta(a\gamma y) \leq x\alpha(a_1\alpha a_2)\beta(a_1\alpha a_2)\gamma y$$
$$= (x\alpha a_1\alpha a_2)\beta a_1\alpha(a_2\gamma y) \in M\Gamma A\Gamma M \subseteq A$$
$$\Longrightarrow a \in (A] = A \Longrightarrow (A\Gamma A] \subseteq A.$$

Thus $(A\Gamma A] = A$.

(2). Let A, B be interior $\Gamma-$ideals of M.Then $(A\Gamma B] = (B\Gamma A]$. Indeed: By (1), we have

$$(A\Gamma B] = ((A\Gamma B]\Gamma(A\Gamma B]] = ((A\Gamma B]\Gamma(A\Gamma B]\Gamma(A\Gamma B]\Gamma(A\Gamma B]]$$
$$\subseteq (((A\Gamma B)\Gamma(A\Gamma B)]\Gamma((A\Gamma B)\Gamma(A\Gamma B)]]$$
$$\subseteq (((M\Gamma B\Gamma M]\Gamma((M\Gamma A\Gamma M)]]$$
$$\subseteq ((B]\Gamma(A]]$$
$$= (B\Gamma A] \Longrightarrow (A\Gamma B] \subseteq (B\Gamma A].$$

By symmetry we have $(B\Gamma A] \subseteq (A\Gamma B]$. Thus $(A\Gamma B] = (B\Gamma A]$. □

Proposition 11. *Let M be an intra-regular ordered Γ-semigroup and f a fuzzy interior $\Gamma-$ideal of M.Then for every $a \in M$ and $\alpha \in \Gamma$ such that $a\alpha a \leq a$, we have*

$$f(a) = f(a\alpha a) \text{ and } f(a\alpha b) = f(b\alpha a).$$

Proof. (1). Let M be an intra-regular ordered Γ-semigroup, f a fuzzy interior $\Gamma-$ideal of M and $a \in M$, $\alpha \in \Gamma$. Then $f(a) = f(a\alpha a)$. Indeed: Since M is intra-regular and $a \in M$, there exist $x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$ such that $a \leq x\alpha a\beta a\gamma y$. Then

$$f(a) \geq f(x\alpha a\beta a\gamma y) \geq f(a\beta a).$$

Since $a\alpha a \leq a$ we have $f(a\alpha a) \geq f(a)$. Hence $f(a) = f(a\alpha a)$.

(2). Let $a, b \in M$ and $\alpha \in \Gamma$. Then $f(a\alpha b) = f(b\alpha a)$. Indeed: By (1), we have

$$f(a\alpha b) = f((a\alpha b)\alpha(a\alpha b)) = f(a\alpha(b\alpha a)\alpha b) \geq f(b\alpha a).$$

By symmetry, we have $f(b\alpha a) \geq f(a\alpha b)$. Hence $f(a\alpha b) = f(b\alpha a)$. □

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