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A NEW APPLICATION OF ADOMIAN DECOMPOSITION METHOD FOR THE SOLUTION OF FRACTIONAL FOKKER-PLANCK EQUATION WITH INSULATED ENDS

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Abstract. This paper presents the analytical solution of the fractional Fokker-Planck equation by Adomian decomposition method. By using initial conditions, the explicit solution of the equation has been presented in the closed form and then the numerical solution has been represented graphically. Two different approaches have been presented in order to show the application of the present technique. The present method performs extremely well in terms of efficiency and simplicity.

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1. Introduction

The fractional differential equations appear more and more frequently in different research areas and engineering applications. Nowadays, fractional diffusion equation plays important roles in modeling anomalous diffusion and subdiffusion systems, description of fractional random walk, unification of diffusion and wave propagation phenomenon, see, e.g. the reviews in [1-7], and references therein.

In this paper, we shall consider the following fractional Fokker-Planck equation for force free case [8, 9,10]

$$\frac{\partial u(x,t)}{\partial t} = K_0 D_t^{1-\gamma} \frac{\partial^2 u(x,t)}{\partial x^2}, \tag{1}$$

where ${}_0D_t^{1-\gamma}$ is the fractional derivative defined by the Riemann-Liouville operator [11, 12],

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$$D_t^{1-\gamma} f(t) = \frac{1}{\Gamma(\gamma)} \frac{d}{dt} \int_0^t \frac{f(\xi) d\xi}{(t-\xi)^{1-\gamma}}$$

K is the diffusion co-efficient having non-ordinary dimensions and $\gamma \in (0,1)$ is the anomalous diffusion exponent.

In this paper, we use the Adomian decomposition method (ADM) [13, 14] to obtain a solution of a fractional diffusion equation (1). Large classes of linear and nonlinear differential equations, both ordinary as well as partial, can be solved by the Adomian decomposition method [13-20]. A reliable modification of Adomian decomposition method has been done by Wazwaz [21]. The decomposition method provides an effective procedure for analytical solution of a wide and general class of dynamical systems representing real physical problems [13-29]. This method efficiently works for initial-value or boundary-value problems and for linear or nonlinear, ordinary or partial differential equations and even for stochastic systems. Moreover, we have the advantage of a single global method for solving ordinary or partial differential equations as well as many types of other equations. Recently, the solution of fractional differential equation has been obtained through Adomian decomposition method by the researchers [30-37]. The application of Adomian decomposition method for the solution of nonlinear fractional differential equations has also been established by Shawagfeh, Saha Ray and Bera [31, 34, 36].

2. The Fractional diffusion Equation model and Application of Adomain Decomposition Method for its Solution

We consider the following initial conditions:

$$u(x,0) = f(x), 0 < x < L \tag{2}$$

and boundary conditions:

$$u(0,t) = u(L,t) = 0, t \geq 0 \tag{3}$$

for fractional Fokker-Planck equation (1).

We adopt Adomian decomposition method for solving eq.(1). In the light of this method we assume that

$$u = \sum_{n=0}^{\infty} u_n \tag{4}$$

to be the solution of eq.(1).
Now, eq.(1) can be written as

$$L_t u(x,t) = K_0 D_t^{(1-\gamma)}(L_{xx}u(x,t)) \tag{5}$$

where $L_t \equiv \frac{\partial}{\partial t}$ which is an easily invertible linear operator, ${}_0D_t^{(1-\gamma)}(\bullet)$ is the Riemann-Liouville derivative of order $(1-\gamma)$, $L_{xx} = \frac{\partial^2}{\partial x^2}$.
We see that $f(x)$ is a periodic function with period L. The Fourier sine series of $f(x)$ in [0, L] can be obtained as

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{L} \int_0^L f(\xi) \sin\left(\frac{n\pi\xi}{L}\right) d\xi \sin\left(\frac{n\pi x}{L}\right) \tag{6}$$

Therefore, after considering $f(x)$ as Fourier sine series, we can take

$$u(x,0) = \sum_{n=1}^{\infty} \frac{2}{L} \int_0^L f(\xi) \sin\left(\frac{n\pi\xi}{L}\right) d\xi \sin\left(\frac{n\pi x}{L}\right) \tag{7}$$

because of the fact that Fourier sine series is well adapted to functions which are zero at the end points x=0 and x=L of the interval [0, L], since all the basis functions $\sin\left(\frac{n\pi x}{L}\right)$ have this property.
Therefore, by Adomian decomposition method, we can write,

$$u(x,t) = u(x,0) + L_t^{-1}(K_0 D_t^{(1-\gamma)}(L_{xx}u(x,t))) \tag{8}$$

where

$$u_0 = u(x,0)$$

$$= \sum_{n=1}^{\infty} \frac{2}{L} \int_0^L f(\xi) \sin\left(\frac{n\pi\xi}{L}\right) d\xi \sin\left(\frac{n\pi x}{L}\right)$$

$$u_1 = L_t^{-1}(K_0 D_t^{(1-\gamma)}(L_{xx}u_0))$$

$$u_2 = L_t^{-1}(K_0 D_t^{(1-\gamma)}(L_{xx}u_1))$$

$$u_3 = L_t^{-1}(K_0 D_t^{(1-\gamma)}(L_{xx}u_2))$$

and so on.
The decomposition series (4) solution is generally converges very rapidly in real physical problems [14]. The rapidity of this convergence means that few terms are required. Convergence of this method has been rigorously established by Cherruault [38], Abbaoui and Cherruault [39, 40] and Himoun, Abbaoui and Cherruault [41]. The practical solution will be the n-term approximation ϕ_n

$$\phi_n = \sum_{i=0}^{n-1} u_i(x,t)\ ,\ \ n \geq 1 \tag{9}$$

with

$$\lim_{n\to\infty} \phi_n = u(x,t)$$

(1) **Implementation of the present method**

Example 1. Let us consider initial conditions:

$$u(x,0) = x(1-x), 0 < x < 1 \tag{10}$$

and boundary conditions:

$$u(0,t) = u(1,t) = 0, t \geq 0 \tag{11}$$

for the eq.(1), as taken in[8, 9]. Physical $u(x,t)$ represents the temperature at any point x at any time t in a solid bounded by the planes $x = 0$ and $x = 1$. The Dirichlet's boundary conditions $u(0,t) = u(1,t) = 0$ express the fact that the ends $x = 0$ and $x = 1$ are kept at temperature zero. The initial distribution of temperature in the solid is described by the equation $u(x,0) = x(1-x)$, $0 < x < 1$.

We extend the domain of definition of $f(x)$ to (-1, 0) defining by $f(x) = -f(-x)$. Then $f(x)$ becomes odd function in the interval (-1, 1). The Fourier sine series of $f(x)$ in [0, 1] can be obtained as

$$f(x) = \sum_{n=1}^{\infty} \frac{4(1-\cos(n\pi))}{n^3\pi^3} \sin(n\pi x) \tag{12}$$

Therefore, after considering $f(x)$ as Fourier sine series, we can take

$$u(x,0) = \sum_{n=1}^{\infty} \frac{4(1-\cos(n\pi))}{n^3\pi^3} \sin(n\pi x) \tag{13}$$

because of the fact that Fourier sine series is well adapted to functions which are zero at the end points x=0 and x=1 of the interval [0, 1], since all the basis functions $\sin(n\pi x)$ have this property.

We will then obtain from recursive scheme for Adomian decomposition method

$$u_0 = u(x,0)$$

$$= \sum_{n=1}^{\infty} \frac{4(1-\cos(n\pi))}{n^3\pi^3} \sin(n\pi x)$$

$$u_1 = L_t^{-1}(K_0 D_t^{(1-\gamma)}(L_{xx}u_0))$$

$$= \frac{-t^{\gamma}}{\Gamma(\gamma+1)} \sum_{n=1}^{\infty} \frac{4(1-\cos(n\pi))}{n^3\pi^3} K n^2 \pi^2 \sin(n\pi x)$$

$$u_2 = L_t^{-1}(K_0 D_t^{(1-\gamma)}(L_{xx}u_1))$$

$$= \frac{t^{2\gamma}}{\Gamma(2\gamma+1)} \sum_{n=1}^{\infty} \frac{4(1-\cos(n\pi))}{n^3\pi^3} K^2 n^4 \pi^4 \sin(n\pi x)$$

$$u_3 = L_t^{-1}(K_0 D_t^{(1-\gamma)}(L_{xx}u_2))$$

$$= \frac{-t^{3\gamma}}{\Gamma(3\gamma+1)} \sum_{n=1}^{\infty} \frac{4(1-\cos(n\pi))}{n^3\pi^3} K^3 n^6 \pi^6 \sin(n\pi x)$$

$$u_4 = L_t^{-1}(K_0 D_t^{(1-\gamma)}(L_{xx}u_3))$$

$$= \frac{t^{4\gamma}}{\Gamma(4\gamma+1)} \sum_{n=1}^{\infty} \frac{4(1-\cos(n\pi))}{n^3\pi^3} K^4 n^8 \pi^8 \sin(n\pi x)$$

and so on.
Therefore, the solution is

$$u(x,t) = \sum_{n=1}^{\infty} \frac{4(1-\cos(n\pi))}{n^3\pi^3} \sin(n\pi x) \sum_{k=0}^{\infty} \frac{(-Kn^2\pi^2 t^{\gamma})^k}{\Gamma(k\gamma+1)}$$

$$= \sum_{n=1}^{\infty} \frac{4(1-(-1)^n)}{n^3\pi^3} \sin(n\pi x) E_{\gamma}(-Kn^2\pi^2 t^{\gamma})$$

$$= \frac{8}{\pi^3} \sum_{n=0}^{\infty} \frac{\sin[(2n+1)\pi x] E_{\gamma}[-K(2n+1)^2\pi^2 t^{\gamma}]}{(2n+1)^3} \tag{14}$$

where $E_{\lambda}(z)$ is the Mittag-Leffler function in one parameter.
The solution (14) can be verified through substitution in eq. (1).
Example 2. Let us consider the problem of anomalous subdiffusion of particle in a finite medium with initial condition:

$$u(x,0) = f(x) = \delta(x - L/2), \tag{15}$$

and absorbing boundary conditions:

$$u(0,t) = u(L,t) = 0, t \geq 0 \tag{16}$$

for the eq.(1), as taken in [10].

We extend the domain of definition of $f(x)$ to (-L, 0) defining by $f(x) = -f(-x)$. Then $f(x)$ becomes odd function in the interval (-L, L). The Fourier sine series of $f(x)$ in [0, L] can be obtained as

$$f(x) = \frac{2}{L}\sum_{n=0}^{\infty}(-1)^n \sin\frac{(2n+1)\pi x}{L} \tag{17}$$

Therefore, after considering $f(x)$ as Fourier sine series, we can take

$$u(x,0) = \frac{2}{L}\sum_{n=0}^{\infty}(-1)^n \sin\frac{(2n+1)\pi x}{L} \tag{18}$$

because of the fact that Fourier sine series is well adapted to functions which are zero at the end points x=0 and x=L of the interval [0, L], since all the basis functions sin $\frac{(2n+1)\pi x}{L}$ have this property.
We will then obtain from recursive scheme for Adomian decomposition method

$$\begin{aligned}
u_0 &= u(x,0)\\
&= \frac{2}{L}\sum_{n=0}^{\infty}(-1)^n \sin\frac{(2n+1)\pi x}{L}\\
u_1 &= L_t^{-1}(K_0 D_t^{(1-\gamma)}(L_{xx}u_0))\\
&= \frac{2}{L}\sum_{n=0}^{\infty}(-1)^n \sin\frac{(2n+1)\pi x}{L}\left\{\frac{(2n+1)\pi}{L}\right\}^2\left\{\frac{-Kt^\gamma}{\Gamma(\gamma+1)}\right\}\\
u_2 &= L_t^{-1}(K_0 D_t^{(1-\gamma)}(L_{xx}u_1))\\
&= \frac{2}{L}\sum_{n=0}^{\infty}(-1)^n \sin\frac{(2n+1)\pi x}{L}\left\{\frac{(2n+1)\pi}{L}\right\}^4\left\{\frac{K^2t^{2\gamma}}{\Gamma(2\gamma+1)}\right\}
\end{aligned}$$

and so on.
Therefore, the solution is

$$\begin{aligned}
u(x,t) &= \frac{2}{L}\sum_{n=0}^{\infty}(-1)^n \sin\frac{(2n+1)\pi x}{L}\sum_{r=0}^{\infty}\frac{\left\{-K\left\{\frac{(2n+1)\pi}{L}\right\}^2 t^\gamma\right\}^r}{\Gamma(r\gamma+1)}\\
&= \frac{2}{L}\sum_{n=0}^{\infty}(-1)^n \sin\frac{(2n+1)\pi x}{L}E_\gamma(-K\frac{(2n+1)^2\pi^2}{L^2}t^\gamma)
\end{aligned} \tag{19}$$

where $E_\lambda(z)$ is the Mittag-Leffler function in one parameter.

3. Alternative approach

We can obtain the same solutions (14) and (19) in another way. First we take finite fourier sine transform of eq. (1), integrating the second term of the resulting equation by parts, and then applying the boundary conditions, we obtain

$$\frac{d\overline{u}(n,t)}{dt} + \frac{n^2\pi^2}{L^2} K_0 D_t^{1-\gamma} \overline{u}(n,t) = 0 \tag{20}$$

where n is a wave number, and

$$\overline{u}(n,t) = \int_0^L u(x,t) \sin\left(\frac{n\pi x}{L}\right) dx \tag{21}$$

is the finite sine transform of $u(x,t)$.
Taking finite sine transform of eq. (2), we obtain

$$\overline{u}(n,0) = \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx \tag{22}$$

Let us rewrite the eq. (20) in an operator form

$$L_t \overline{u} = -\frac{n^2\pi^2}{L^2} K_0 D_t^{1-\gamma} \overline{u}(n,t) \tag{23}$$

where $L_t \equiv \frac{d}{dt}$ which is an easily invertible linear operator, ${}_0D_t^{1-\gamma}(\bullet)$ is the Riemann-Liouville derivative of order $(1-\gamma)$. Applying the inverse operator L_t^{-1} to the eq. (23) yields

$$\overline{u}(n,t) = \overline{u}(n,0) - \frac{n^2\pi^2}{L^2} K_0 D_t^{1-\gamma} \overline{u}(n,t)$$

The Adomian decomposition method [13, 14] assumes an infinite series solution for unknown function $\overline{u}(n,t)$ in the form

$$\overline{u}(n,t) = \sum_{k=0}^{\infty} \overline{u}_k(n,t), \tag{24}$$

Therefore, by Adomian decomposition method, we can write,

$$\overline{u}_0 = \overline{u}(n,0)$$

$$= \int_0^L f(x) \sin(n\pi x)\, dx$$

$$\overline{u}_1 = -\frac{n^2\pi^2}{L^2} K_0 D_t^{1-\gamma}(\overline{u}_0)$$

$$\overline{u}_2 = -\frac{n^2\pi^2}{L^2} K_0 D_t^{1-\gamma}(\overline{u}_1)$$

$$\overline{u}_3 = -\frac{n^2\pi^2}{L^2} K_0 D_t^{1-\gamma}(\overline{u}_2)$$

and so on.
Therefore, the entire components $\overline{u}_0$, $\overline{u}_1$, $\overline{u}_2$, ... are identified and the series solution thus entirely determined. In this case the exact solution in a closed form may be obtained. The practical solution will be the n-term approximation ϕ_n

$$\phi_n = \sum_{i=0}^{n-1} \overline{u}_i(n,t) \ , \quad n \geq 1 \tag{25}$$

with

$$\lim_{n\to\infty} \phi_n = \overline{u}(n,t)$$

(1) **Alternative approach for 1st Example**

let us consider the initial condition (10) and boundary conditions (11) for eq. (1). Then from recursive relations of Adomian decomposition method with initial conditions eq. (22) gives

$$\overline{u}_0 = \overline{u}(n,0)$$

$$= \int_0^1 f(x)\sin(n\pi x)dx$$

$$= \frac{2(1-\cos(n\pi))}{n^3\pi^3}$$

$$\begin{aligned}\overline{u}_1 &= -n^2\pi^2 L_t^{-1}(K_0 D_t^{1-\gamma}(\overline{u}_0)) \\ &= \tfrac{2(1-\cos(n\pi))}{n^3\pi^3}\left[-Kn^2\pi^2 \tfrac{t^\gamma}{\Gamma(\gamma+1)}\right]\end{aligned}$$

$$\begin{aligned}\overline{u}_2 &= -n^2\pi^2 L_t^{-1}(K_0 D_t^{1-\gamma}(\overline{u}_1)) \\ &= \tfrac{2(1-\cos(n\pi))}{n^3\pi^3}\left[K^2n^4\pi^4 \tfrac{t^{2\gamma}}{\Gamma(2\gamma+1)}\right]\end{aligned}$$

$$\begin{aligned}\overline{u}_3 &= -n^2\pi^2 L_t^{-1}(K_0 D_t^{1-\gamma}(\overline{u}_2)) \\ &= \tfrac{2(1-\cos(n\pi))}{n^3\pi^3}\left[-K^3n^6\pi^6 \tfrac{t^{3\gamma}}{\Gamma(3\gamma+1)}\right]\end{aligned}$$

$$\begin{aligned}\overline{u}_4 &= -\tfrac{n^2\pi^2}{4} L_t^{-1}({}_0D_t^{1-\gamma}(\overline{u}_3)) \\ &= \tfrac{2(1-\cos(n\pi))}{n^3\pi^3}\left[K^4n^8\pi^8 \tfrac{t^{4\gamma}}{\Gamma(4\gamma+1)}\right]\end{aligned}$$

and so on.
Therefore, the series $\overline{u}(n,t)$ becomes

$$\overline{u}(n,t) = \frac{2(1-\cos(n\pi))}{n^3\pi^3}\sum_{k=0}^{\infty}\frac{\left(-Kn^2\pi^2t^{\gamma}\right)^k}{\Gamma\left(k\gamma+1\right)} \tag{26}$$

$$= \frac{2(1-\cos(n\pi))}{n^3\pi^3}E_{\gamma}(-Kn^2\pi^2t^{\gamma})$$

Taking the inverse finite sine transform of eq. (**??**) we obtain the solution

$$u(x,t) = \frac{4}{\pi^3}\sum_{n=1}^{\infty}\frac{(1-\cos(n\pi))}{n^3}E_{\gamma}(-Kn^2\pi^2t^{\gamma})\sin\left(n\pi x\right)$$

$$= \sum_{n=1}^{\infty}\frac{4(1-(-1)^n)}{n^3\pi^3}\sin(n\pi x)E_{\gamma}(-Kn^2\pi^2t^{\gamma})$$

$$= \frac{8}{\pi^3}\sum_{n=0}^{\infty}\frac{\sin[(2n+1)\pi x]E_{\gamma}[-K(2n+1)^2\pi^2t^{\gamma}]}{(2n+1)^3} \tag{27}$$

where $E_{\lambda}\left(z\right)$ is the Mittag-Leffler function in one parameter.
The solution (27) can be verified through substitution in eq. (1). The two solutions (14) and (27) are same.

(1) (a) **Alternative approach for 2nd Example**

We can obtain the same solution (19) in another way. Let us consider the initial condition (15) and boundary conditions (16) for eq. (1). Then from recursive relations of Adomian decomposition method with initial conditions eq. (22) gives

$$\overline{u}_0 = \overline{u}(n,0)$$

$$= \int_0^1 \delta(x-L/2)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \sin\left(\frac{n\pi}{2}\right)$$

$$\begin{aligned}\overline{u}_1 &= -\tfrac{Kn^2\pi^2}{L^2}L_t^{-1}({}_0D_t^{1-\gamma}(\overline{u}_0))\\ &= -\tfrac{Kn^2\pi^2}{L^2}\tfrac{t^{\gamma}}{\Gamma(\gamma+1)}\sin\left(\tfrac{n\pi}{2}\right)\end{aligned}$$

$$\begin{aligned}\overline{u}_2 &= -\tfrac{Kn^2\pi^2}{L^2}L_t^{-1}({}_0D_t^{1-\gamma}(\overline{u}_1))\\ &= \tfrac{K^2n^4\pi^4}{L^4}\tfrac{t^{2\gamma}}{\Gamma(2\gamma+1)}\sin\left(\tfrac{n\pi}{2}\right)\end{aligned}$$

and so on.
Therefore, the series $\overline{u}(n,t)$ becomes

$$\overline{u}(n,t) = \sum_{r=0}^{\infty} \frac{\left\{-\frac{Kn^2\pi^2}{L^2}t^\gamma\right\}^r \sin\left(\frac{n\pi}{2}\right)}{\Gamma(r\gamma+1)} \tag{28}$$

$$= \frac{2}{L}\sum_{n=1}^{\infty} E_\gamma(-K\frac{n^2\pi^2}{L^2}t^\gamma)\sin\frac{n\pi}{2}\sin\frac{n\pi x}{L}$$

$$= \frac{2}{L}\sum_{n=0}^{\infty}(-1)^n \sin\frac{(2n+1)\pi x}{L} E_\gamma(-K\frac{(2n+1)^2\pi^2}{L^2}t^\gamma)$$

Taking the inverse finite sine transform of eq. (**??**) we obtain the solution

$$u(x,t) = \frac{4}{\pi^3}\sum_{n=1}^{\infty}\frac{(1-\cos(n\pi))}{n^3}E_\gamma(-Kn^2\pi^2t^\gamma)\sin(n\pi x)$$

$$= \sum_{n=1}^{\infty}\frac{4(1-(-1)^n)}{n^3\pi^3}\sin(n\pi x)E_\gamma(-Kn^2\pi^2t^\gamma)$$

$$= \frac{8}{\pi^3}\sum_{n=0}^{\infty}\frac{\sin[(2n+1)\pi x]E_\gamma[-K(2n+1)^2\pi^2t^\gamma]}{(2n+1)^3} \tag{29}$$

where $E_\lambda(z)$ is the Mittag-Leffler function in one parameter.
5. Numerical Results and Discussions.
Fig. 1 presenting the decomposition method solution at t=0.25 for $\gamma = 0.5$.
Fig. 2 presenting the decomposition method solution at t=0.5 for $\gamma = 0.75$.
Fig. 3 presenting the decomposition method solution at t=0.5 for $\gamma = 1$.
Fig. 4 presenting the decomposition method solution at t=0.005, 0.05 and 0.5 for $\gamma = 0.75$.
In the present numerical analysis we assume $K = 1$ for the eq. (1). Equation (14) has been used to draw the figures. Figures 1 and 2 cited fast diffusion behaviour. Figure 3 shows very slow diffusion behaviour. Figure 4 shows fast diffusion when t is small and exhibits slow diffusion as t increases. Figures 1-4 have been drawn using the *Mathematica* software [42].

4. Conclusion

This paper presents an analytical scheme to obtain the solution of a fractional Fokker-Planck equation. In the present analysis, Admian decomposition method has been successfully applied in two different new ways. In both approaches same result has been achieved. Physical significance of the solution has been graphically presented in this paper. In our previous papers [32-37] we have already as well as successfully exhibit the applicability of Adomian decomposition method to obtain a solution for dynamic system containing factional derivative. In this work we demonstrate that this method is also well suited to solve fractional Fokker-Planck equation. The decomposition method is straightforward, without restrictive assumptions and the components of the series solution can be easily computed using any mathematical symbolic package. Moreover, this method does not change the problem into a convenient one for the use of linear theory. It, therefore, provides more realistic series solutions that generally converge very rapidly in real physical problems. When solutions are computed numerically, the rapid convergence is obvious. Moreover, no linearization or perturbation is required. It can avoid the difficulty of finding the inverse of Laplace Transform and can reduce the labour of perturbation method. Furthermore, as the decomposition method does not require discretization of the variables, i. e., time and space, it is not affected by computational round off errors and one is not faced with necessity of large computer memory and time. Consequently, the computational size will be reduced.

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