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ON THE C-NETS

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Abstract. In this paper, we define the concept of a c-net and study the convergence of c-nets. Also we show that a c-net in a topological space X has a convergent sub-c-net if and only if X is a Lindelöf space, if every G_δ set is open in X.

1. Introduction

It is well known [1, 3, 9] that the order structure plays the important role in the study of various mathematical structures.

The concept of a net first introduced by E. H. Moore and H. L. Smith in 1922([8]), is to generalize the notion of a sequence. They proved that a topological space is Hausdorff if and only if each net in X converges to at most one point. In this paper, we define the concept of a c-net which is a special type of a net, and then study the relation between topological spaces and the convergence of c-nets. We show that $p \in S^{'}$ if and only if there is a c-net $(x_\alpha) \longrightarrow p \in X$, where X is a topological space, every G_δ set is open and $S^{'}$ is the derived set of $S \subseteq X$. Using this, we show that if a c-net in a topological space X has a convergent sub-c-net, then X is a Lindelöf space. Using c-nets, we show that if X is a Lindelöf space and every G_δ set is open in X, then each c-net in X has a convergent sub-c-net.

For a further development in this field we refer to [4, 6, 10]. For terminologies not introduced in this paper, we refer to [2, 5, 7].

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2. Preliminaries

In this section, we introduce the concepts of preordered sets, partially ordered sets and nets.

Definition 2.1. Let P be a nonempty set and $\prec$ be a relation in P. Then $(P, \prec)$ is called a partially ordered set if $\prec$ satisfies the following properties:
(1) $\prec$ is transitive in P.
(2) $\prec$ is reflexive in P.
(3) $\prec$ is antisymmetric in P.

If $\prec$ satisfies (1) and (2), then $(P, \prec)$ is called a preordered set.

In this thesis, a partially ordered set is called a poset, and a preordered set is called a preset.

Definition 2.2. Let D be a nonempty subset of a preset $(P, \prec)$. Then D is called a directed set if $\prec$ satisfies the following property:

If $a \in D$ and $b \in D$, then there exists $c \in D$ such that $a \prec c$ and $b \prec c$.

Definition 2.3. Let X be a set. A net in X is a map $c : (D, \prec) \longrightarrow X$, where $(D, \prec)$ is a directed set.

Definition 2.4. Let $c : (D, \prec) \longrightarrow X$ be a net in X.

If $k : (D^*, \prec^*) \longrightarrow D$ is a net and for each $p \in D$, there exists $p^* \in D^*$ with $p^* \prec^* d^*$ implies $p \prec k(d^*)$ $(d^* \in D^*)$, then the net $c \circ k : (D^*, \prec^*) \longrightarrow X$ is called a subnet of c.

Definition 2.5. Let X be a topological space and $x : (D, \prec) \longrightarrow X$ be a net in X and $p \in X$. Then (x_α) converges to p, denoted by $(x_\alpha) \longrightarrow p$, if for each open neighborhood U of p, there exists an $\alpha_0 \in D$ such that for $\alpha_0 \prec \alpha$, $x_\alpha = x(\alpha) \in U$.

Definition 2.6. Let $(D, \prec)$ be a directed set. A subset D^* of D is said to be a cofinal subset of D if for each $d \in D$, there exists $d^* \in D^*$ with $d \prec d^*$.

Theorem 2.7. *Let X be a topological space and $x : (D, \prec) \longrightarrow X$ be a net in X. Then*
(a) *If $x_\alpha = x$ for each $\alpha \in D$, then the net $(x_\alpha) \longrightarrow x$.*
(b) *If D^* is a cofinal subset of D, then $x : (D^*, \prec) \longrightarrow X$ is a subnet of $x : (D, \prec) \longrightarrow X$.*
(c) *If the net $(x_\alpha) \longrightarrow x$, then every subnet of (x_α) also converges to p.*

(d) *Let* $\beta \in D$ *and* $D_\beta = \{\alpha \in D : \beta \prec \alpha\}$, *then* $x : (D_\beta, \prec) \longrightarrow X$ *is a subnet of* $x : (D, \prec) \longrightarrow X$.

THEOREM 2.8. *Let* X *be a topological space. Then* p *is a limit point of a subset* S *of* X *if and only if there exists a net* (x_α) *in* $S - \{p\}$ *that converges to* p.

THEOREM 2.9. *A topological space is Hausdorff if and only if each net converges to at most one point.*

THEOREM 2.10. *Let* X *be a topological space. Then* X *is compact if and only if each net* (x_α) *in* X *has a convergent subnet.*

3. The convergence of c-nets

In this section, we introduce the concept of c-nets, and then we compare the properties of sequences, nets and c-nets.

DEFINITION 3.1. Let D be a nonempty subset of a preset $(P, \prec)$. Then $(D, \prec)$ is called a countably directed set if $\prec$ satisfies the following property :
If $a_n \in D$ for each $n \in K$, where K is a countable set, then there exists $b \in D$ with $a_n \prec b$. That is, every countable subset of D has an upper bound in D.

EXAMPLE 3.2. (1) Let $(X, \mathcal{T})$ be a topological space and $\mathcal{F}$ be a collection of all closed subsets of X. Then $(\mathcal{T}, \subseteq)$ and $(\mathcal{F}, \subseteq)$ are countably directed sets since X is an upper bound of every countable subset $\mathcal{D}$ of $\mathcal{T}(\mathcal{F}$, resp.).

(2) Let $(X, \mathcal{T})$ be a topological space, $x \in X$ and $\mathcal{N}_x$ be the neighborhood system of x. Then $(\mathcal{N}_x, \subseteq)$ is a countably directed set since X is an upper bound of every countable subset $\mathcal{D}$ of $\mathcal{N}_x$.

(3) Suppose that X is a nonempty set and $\mathcal{F}$ is a nonempty collection of nonempty subsets of X and
  (a) if $A \in \mathcal{F}$ and $B \in \mathcal{F}$, then $A \cap B \in \mathcal{F}$ and
  (b) if $A \in \mathcal{F}$, then each subset of X that contains A is also an element of $\mathcal{F}$.

  Then $\mathcal{F}$ is called a filter in X. If $\mathcal{F}$ is a filter in X, then $(\mathcal{F}, \subseteq)$ is a countably directed set since X is an upper bound of every countable subset $\mathcal{D}$ of $\mathcal{F}$.

REMARK 3.3. (1) If $(D, \prec)$ is a countably directed set, then $(D, \prec)$ is a directed set.

(2) The set of all natural numbers $(\mathbb{N}, \leq)$ is directed but not countably directed. But $(\mathbb{N}, \geq)$ is countably directed.
(3) Let X be an infinite set. Then $(Fin(X), \subseteq)$ is directed but not countably directed, where $Fin(X) = \{A \subseteq X \mid A \ is \ finite\}$.
But $(Count(X), \subseteq)$ is countably directed,
where $Count(X) = \{B \subseteq X \mid B \ is \ countable\}$.
(4) Let $[a, b]$ be a closed interval in $\mathbb{R}$. By a partition of $[a, b]$, we mean a finite set of points $\{x_0, x_1, \cdots, x_n\}$ and
$a = x_0 < x_1 < x_2 < \cdots < x_n = b$. Let $\mathcal{P}[a, b]$ be the collection of all partitions of $[a, b]$. Then $(\mathcal{P}[a, b], \subseteq)$ is directed but not countably directed.

Definition 3.4. Let X be a set. A c-net in X is a map
$c : (D, \prec) \longrightarrow X$,where $(D, \prec)$ is a countably directed set.

Analogous to the notation used for nets, if $s : (D, \prec) \longrightarrow X$ is a c-net, then we shall write (s_α) for $s : (D, \prec) \longrightarrow X$.

Example 3.5. (1) Let f be a sequence in X, then f is not a c-net by Remark 3.3- (2).
(2) Let $c : (D, \prec) \longrightarrow X$ be a c-net, then c is a net by Remark 3.3 - (1).
(3) In general, a net need not be a sequence.
For example, $c : (Fin(\mathbb{N}), \subseteq) \longrightarrow \mathbb{N}$ defined by,
$$c(K) = \begin{cases} \text{the least element of K} & (\text{if } K \neq \emptyset) \\ 1 & (\text{if } K = \emptyset) \end{cases}$$
is a net but not a sequence.
(4) A net need not be a c-net by Remark 3.3-(2).
(5) A c-net is not a sequence.
For example, $c : (Count(\mathbb{N}), \subseteq) \longrightarrow \mathbb{N}$ defined by,
$$c(K) = \begin{cases} \text{the least element of K} & (\text{if } K \neq \emptyset) \\ 1 & (\text{if } K = \emptyset) \end{cases}$$
is a c-net but not a sequence.

Definition 3.6. Let $c : (D, \prec) \longrightarrow X$ be a c-net in X.
If $k : (D^*, \prec^*) \longrightarrow D$ is a c-net and $p \in D$, there exists $p^* \in D^*$ with $p^* \prec^* d^*$ implies $p \prec k(d^*)$ $(d^* \in D^*)$.
Then the c-net $c \circ k : (D^*, \prec^*) \longrightarrow X$ is called a sub-c-net of c.

Example 3.7. (1) Let $D^* = \{a, b, c, \}$ and $D = \{1, 2\}$.
Define a relation $a \prec^* b \prec^* c$ on D^* and $1 \prec 2$ on D. Define $s^* : (D^*, \prec^*) \longrightarrow D$ by $s^*(a) = 1$, $s^*(b) = 2$, $s^*(c) = 2$ and

$s : (D, \prec) \longrightarrow \mathbb{N}$ by $s(1) = 1,\ s(2) = 2$, then $s \circ s^* : D^* \longrightarrow \mathbb{N}$ is a sub-c-net of s. But if we define $s^*(a) = 1,\ s^*(b) = 2,\ s^*(c) = 1$ and $s(1) = 1,\ s(2) = 2$, then $s \circ s^*$ is not a sub-c-net of s, because if $s \circ s^* : D^* \longrightarrow \mathbb{N}$ is a sub-c-net of s, then for $2 \in D$ there exists $e \in D^*$ and $e \prec^* d$ implies $2 \prec s(d)$. But $e \in D^*$ does not exist.

(2) Let s be the c-net from $(Count(\mathbb{N}), \subseteq)$ to $\mathbb{N}$ defined by the example 3.5-(5), and s^* be a c-net from $(Count(\mathbb{N}), \subseteq)$ to $(Count(\mathbb{N}), \subseteq)$ by $s^*(K) = K \cup \{p\}$ for any $p \in \mathbb{N}$. Then $s \circ s^* : (Count(\mathbb{N}), \subseteq) \longrightarrow \mathbb{N}$ is a sub-c-net of s. Let $A \in Count(\mathbb{N})$ and $\{p\} \subseteq A$, then for $A \subseteq B$, $A \subseteq S^*(B) = B$.

If $A = \emptyset$, then $\emptyset \subseteq s^*(B) = B \cup \{p\}$. If $p \notin A \neq \emptyset$ and $A \subseteq B$, then $A \subseteq s^*(B) = B \cup \{p\}$.

In all, $s \circ s^* : (Count(\mathbb{N}), \subseteq) \longrightarrow \mathbb{N}$ is a sub-c-net of s.

(3) Let s be the c-net from $(Count(\mathbb{N}), \subseteq)$ to $\mathbb{N}$ defined by the example 3.5-(5) and s^* be a c-net from $(Count(\mathbb{N}), \subseteq)$ to $(Count(\mathbb{N}), \subseteq)$ by $s^*(K) = \mathbb{N} - K$. Then $s \circ s^* : (Count(\mathbb{N}), \subseteq) \longrightarrow \mathbb{N}$ is not a sub-c-net of s. If $s \circ s^* : (Count(\mathbb{N}), \subseteq) \longrightarrow \mathbb{N}$ is a sub-c-net of s, then for any $K \in Count(\mathbb{N})$, there exists $P^* \in Count(\mathbb{N})$ and for $P^* \subseteq T$, $K \subseteq s^*(T) = \mathbb{N} - T$. It is a contradiction, for $T = \mathbb{N}$ implies $K = \emptyset$.

Example 3.8. Let $c : (D, \prec) \longrightarrow X$ be a c-net in X, then $c^* : (D^*, \prec^*) \longrightarrow X$ is a sub-c-net of c and $c^{**} : (D^{**}, \prec^{**}) \longrightarrow X$ be a sub-c-net of c^*. Then $c^{**} : (D^{**}, \prec^{**}) \longrightarrow X$ is a sub-c-net of c.

Definition 3.9. Let X be a topological space, $x : (D, \prec) \longrightarrow X$ be a c-net in X and $p \in X$. Then (x_α) is said to converge to p if for each open neighborhood U of p, there exists $\alpha_0 \in D$ such that for $\alpha_0 \prec \alpha \in D$, $x_\alpha = x(\alpha) \in U$.

Example 3.10. (1) Let $(D, \prec)$ be a countably directed poset with the largest element e and X be a topological space. Then every c-net $s : (D, \prec) \longrightarrow X$ converges to $s(e)$. If D is a preset, then largest elements are not unique in general.

(2) Let $\mathcal{T}$ be the set of all triangles and $\rhd$ be a relation in $\mathcal{T}$ defined by $T_1 \rhd T_2$ if T_1 and T_2 are similar figures for $T_1,\ T_2 \in \mathcal{T}$.
Then $s : (\mathcal{T}, \rhd) \longrightarrow (\mathbb{R}, \mathcal{U})$, defined by $s(T) =$ area of T, is a c-net in S which is not convergent to $s(T^*)$, where T^* is a largest element of $\mathcal{T}$ and $(\mathbb{R}, \mathcal{U})$ is the real line with the usual topology.

Example 3.11. Let X be a co-countable space , i.e. , X is a space endowed with the topology $C_c = \{X - G \mid G \text{ is countable }\} \cup \{\emptyset\}$, $x \in X$. Let $\mathcal{N}_x$ be the neighborhood system of x. Then $(\mathcal{N}_x, \supseteq)$ is a countably directed set. For each $U \in \mathcal{N}_x$, let $x(U) \in U$, then the c-net $x : (\mathcal{N}_x, \supseteq) \longrightarrow X$ converges to x.

Definition 3.12. Suppose that $(D, \prec)$ is a countably directed set. A subset D^* of D is said to be a cofinal subset of D if for each $d \in D$, there exists $d^* \in D^*$ with $d \prec d^*$.

Remark 3.13. Let $(D, \prec)$ be a countably directed set. If a subset D^* of D is a cofinal subset of D, then $(D^*, \prec)$ is a countably directed set.

Theorem 3.14. *Let X be a topological space and $x : (D, \prec) \longrightarrow X$ be a c-net in X. Then we have the following :*

(a) *If $x_\alpha = x$ for each $\alpha \in D$, then the c-net (x_α) converges to x.*
(b) *If D^* is a cofinal subset of D, then $x^* : (D^*, \prec) \longrightarrow X$ is a sub-c-net of x.*
(c) *If the c-net (x_α) converges to a point p, then every sub-c-net of (x_α) also converges to p.*
(d) *Let $\beta \in D$ and $D_\beta = \{\alpha \in D : \beta \prec \alpha\}$. Then $x_\beta : (D_\beta, \prec) \longrightarrow X$ is a sub-c-net of x.*

Proof. (a) Take any open neighborhood U of x, then $x_\alpha = x \in U$ for all $\alpha \in D$. Hence (x_α) converges to x.
(b) The identity map $i : (D^*, \prec) \longrightarrow (D, \prec)$ is a c-net by Remark 3.13. Since D^* is a cofinal subset of D, for each $p \in D$, there exists $p^* \in D^*$ with $p \prec p^*$. So $p^* \prec d^*$ implies $p \prec i(d^*)$. Hence $x^* = x \circ i : D^* \longrightarrow X$ is a sub-c-net of x.
(c) Let $k : (E, \prec^*) \longrightarrow X$ be a sub-c-net of a c-net x, then there is a c-net $l : (E, \prec^*) \longrightarrow D$ with $k = x \circ l$ and for each $p \in E$, there exists $p^* \in E$ with $p^* \prec^* d^*$ implies $p \prec l(d^*)$. Take any open neighborhood G of p. Then there exists $\alpha_0 \in D$ such that for $\alpha_0 \prec \alpha$, $x(\alpha) \in G$. Then for the α_0, there exists $p_0 \in E$ and $p_0 \prec^* d_0$ implies $\alpha_0 \prec l(d_0)$. Hence there exists $p_0 \in E$ and $k_p \in G$ for all $p_0 \prec^* p$.
(d) Take any $\gamma \in D$. Since D is countably directed, for $\gamma, \beta \in D$, there exists $\delta \in D$ with $\gamma,\ \beta \prec \delta$. Thus $\delta \in D_\beta$ and $\gamma \prec \delta$. Hence D_β is a cofinal subset of D. Thus $x_\beta : (D_\beta, \prec) \longrightarrow X$ is a sub-c-net of x.

□

Lemma 3.15. *Let X be a topological space, then the followings are equivalent:*

(1) *Every G_δ set in X is open, where G_δ set is a countable intersection of open subsets of X.*
(2) *For any $x \in X$, $(\mathcal{N}_x, \supseteq)$ is a countably directed set, where $\mathcal{N}_x$ is the neighborhood system of x.*

THEOREM 3.16. *Let X be a topological space, $p \in X$ and every G_δ set is open. Then p is a limit point of a subset S of X if and only if there is a c-net (x_α) in $S - \{p\}$ that converges to p.*

Proof. Let $\mathcal{N}_p$ be a neighborhood system of p. Then $(\mathcal{N}_p, \supseteq)$ is a countably directed set. Since p is a limit point of S, for any $U \in \mathcal{N}_p$, $U \cap (S - \{p\}) \neq \emptyset$. Let $x : (\mathcal{N}_p, \supseteq) \longrightarrow S - \{p\}$ be a choice function. Then (x_U) is a c-net in $S - \{p\}$. Take any $V \in \mathcal{N}_p$, then for $V \supseteq U$, $x_U = x(U) \in U \subseteq V$. Hence (x_U) converges to p. Conversely, since there is a c-net $x : (D, \prec) \longrightarrow S - \{p\}$ which converges to p, for any $U \in \mathcal{N}_p$, there exists $\alpha_0 \in D$ such that $x_\alpha \in U$ for $\alpha_0 \prec \alpha$. So $U \cap (S - \{p\}) \neq \emptyset$. Hence p is a limit point of S. □

EXAMPLE 3.17. Let X be a co-countable spaces. Then p is a limit point of a subset S of X if and only if there exists a c-net (x_α) in $S - \{p\}$ that converges to p by the above Theorem 3.16.

COROLLARY 3.18. *Let X be a topological space, $p \in X$ and every G_δ set is open. Then p is in the closure of S if and only if there is a c-net (x_α) in S that converges to p.*

Every convergent sequence in a Hausdorff space has a unique limit. Among first countable spaces those that are Hausdorff can be characterized by this property, although general Hausdorff space can not be.

As we know, a topological space is Hausdorff if and only if each net converges to at most one point ([5]). Since every c-net is a net, we have the following :

COROLLARY 3.19. *If a topological space X is Hausdorff then each c-net in X converges to at most one point.*

THEOREM 3.20. *Let X be a topological space. If a c-net (x_α) in X has a convergent sub-c-net, then X is a Lindelöf space.*

Proof. Let $\mathcal{F} = \{F : F$ is closed subset of $X\}$ has countable intersection property and $\mathcal{F}^* = \{\bigcap Count(\mathcal{F}) : Count(\mathcal{F}) \subseteq \mathcal{F}\}$, then $(\mathcal{F}^*, \supseteq)$ is a countably directed set. Define a c-net $x : (\mathcal{F}^*, \prec) \longrightarrow X$ by $x(F) \in F$, then there is a convergent sub-c-net $x^* = x \circ k : (D, \prec) \longrightarrow X$. Suppose that (x^*_α) converges to $p \in X$. We complete the proof by showing that $p \in \bigcap \mathcal{F}$.

Suppose that $p \notin \bigcap \mathcal{F}$, then there exists $F_0 \in \mathcal{F}$ with $p \notin F_0$. Since x^* is a sub-c-net of x, there exists $\beta_0 \in D$ and for $\beta_0 \prec \beta$, $F_0 \supseteq k(\beta)$. Note that $X - F_0$ is an open neighborhood of p. Since (x^*_α) converges to p, there exists $\beta_1 \in D$ such that for $\beta_1 \prec \beta$, $x^*(\beta) \in X - F_0$. Let $\beta_2 = \max\{\beta_0,\ \beta_1\}$, then for $\beta_2 \prec \beta$, $F_0 \supseteq k(\beta)$ and $x^*(\beta) = x(k(\beta)) \in X - F_0 \subseteq X - k(\beta)$. It is a contradiction to $x(k(\beta)) \notin k(\beta)$. Thus, $\bigcap \mathcal{F} \neq \emptyset$, and hence X is a Lindelöf space. □

Theorem 3.21. *Let X be a topological space and every G_δ set is open. If X is a Lindelöf space, then each c-net (x_α) in X has a convergent sub-c-net.*

Proof. Suppose that X is a Lindelöf space. Let $x : (D, \prec) \longrightarrow X$ be a c-net in X, $D_\beta = \{x(\alpha) : \alpha \in D,\ \beta \prec \alpha\}$ and $\mathcal{F} = \{\overline{D}_\alpha : \alpha \in D\}$. Then $\mathcal{F}$ has countable intersection property, because for any $\overline{D}_{\alpha_i} \in D$, $i \in K$ and K is countable, there exists $\alpha_0 \in D$ such that $\alpha_i \prec \alpha_0$ for all $i \in K$ since D is countably directed,. Then $x(\alpha_0) \in \overline{D}_{\alpha_i}$ for all $i \in K$. So $x(\alpha_0) \in \bigcap_{i\in K} \overline{D}_{\alpha_i}$ and $\bigcap_{i \in K} \overline{D}_{\alpha_i} \neq \emptyset$. Since X is Lindelöf, $\bigcap \mathcal{F} \neq \emptyset$.

Let $p \in \bigcap \mathcal{F}$ and $\mathcal{N}_p$ be the neighborhood system of p. For every (U_1, α_1) and (U_2, α_2) in $\mathcal{N}_p \times D$, let $(U_1, \alpha_1) \preceq (U_2, \alpha_2)$ if and only if $U_1 \supseteq U_2$ and $\alpha_1 \prec \alpha_2$. Then $(\mathcal{N}_p \times D, \preceq)$ is a countably directed set. Note that since $p \in \bigcap \mathcal{F}$, for any $U \in \mathcal{N}_p$ and any $\alpha \in D$, $U \cap D_\alpha \neq \emptyset$. So there exists $x(\gamma)$ with $\alpha \prec \gamma$ with $x(\gamma) \in U \cap D_\alpha$, because $D_\alpha = \{x(\gamma) : \gamma \in D,\ \alpha \prec \gamma\}$. Let $k : (\mathcal{N}_p \times D, \preceq) \longrightarrow D$ by $k(U, \alpha) = \gamma$ with $x(\gamma) \in U \cap D(\alpha)$. Since $p \in \bigcap \mathcal{F}$, we can choose a γ with $x(\gamma) \in U \cap D(\alpha)$ for every $\alpha \in D$ and every $U \in \mathcal{N}_p$. Thus, $x^* = x \circ k : (\mathcal{N}_p \times D, \preceq) \longrightarrow X$ is a sub-c-net of x, because for each $\alpha \in D$, there exists $(U, \alpha) \in \mathcal{N}_p \times D$ such that for $(U, \alpha) \preceq (V, \beta)$, $\alpha \prec k(V, \beta)$. We complete the proof by showing that $(x^*_{(U,\alpha)})$ converges to p. Take any $U_p \in \mathcal{N}_p$, then there exists $\alpha \in D$ such that $x(\alpha) \in U_p \cap D_{\alpha_0}$ for some $\alpha_0 \in D$ and $\alpha_0 \prec \alpha$. Then for $(U_p, \alpha) \preceq (V, \beta)$, $x^*(V, \beta) \in V \cap D_\beta \subseteq U \cap D_\beta \subseteq U_p$. □

References

[1] D. W. Barnes and J. M. Mack, *An algebraic Introduction to Mathematical Logic,* Springer Verlag, New York, 1975.

[2] N. Bourbaki, *General Topology,* Part I, Hermann, Paris, 1966.

[3] B. A. Davey and H. A. Priestly, *Introduction to lattices and order,* Cambridge University Press, Cambridge, 1990.

[4] J. Dugundji, *Topology,* Allyn and Bacon., Boston, 1966.

[5] R. H. Kasriel, *Undergraduate topology,* W. B. Saunders., London, 1971.

[6] J. L. Kelley, *General topology,* Springer Verlag, New York, 1955.

[7] S. O. Lee, *On countably approximating lattices,* J. of Korean Math. Soc. **25** (1988), 3-11.
[8] E. H. Moore and H. L. Smith, *A General Theory of Limits,* American Journal of Mathematics. **44** (1922), 102-121.
[9] L. Nachbin, *Topology and Order,* D. Van Nostrand Co., Princeton, 1965.
[10] H. L. Royden, *Real Analysis,* Third edition, Macmillan, 1988.

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