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INFINITESIMAL GENERATORS OF THE GENERALIZED FOURIER–GAUSS TRANSFORMS

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Abstract. In this note, we investigate the infinitesimal generators of the transformation groups induced by the Fourier–Gauss and Fourier–Mehler transforms on white noise functionals.

1. Introduction

As a natural extension of the finite dimensional Laplacian, Gross [3] introduced an infinite dimensional Laplacian which is called the *Gross Laplacian*. Since then the Gross Laplacian has been studied by many mathematicians, among others, Kuo studied the Gross Laplacian and number operator as continuous linear operators acting on the space of test white noise functionals.

On the other hand, in [2], the authors studied all one–parameter transformation groups induced by the Fourier–Gauss transform and Fourier–Mehler transform based on the white noise theory, and their infinitesimal generators which are linear combinations of the Gross Laplacian and the number operator. For more study of the transforms, we also refer to [5, 8, 9]. In [1], the Fourier–Gauss and Fourier–Mehler transforms have been generalized to transforms of operator parameters and so the generalized transforms are called the *generalized Fourier–Gauss* and *generalized Fourier–Mehler transforms* of which the unitarities are studied in [6]. Recently, in [7], the authors studied relations between the Bogoliubov transformations and the generalized Fourier–Gauss transforms.

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Main purpose of this paper is to study the one–parameter transformation groups induced by the generalized Fourier–Gauss and generalized Fourier–Mehler transforms, and their infinitesimal generators.

This paper is organized as follows: In Section 2 we recall basic notions in white noise theory with analytic characterization for operator symbols [10]. In Section 3 we study basic properties of the generalized Fourier–Gauss and generalized Fourier–Mehler transforms. In Section 4 we investigate one-parameter transformation groups induced by the generalized Fourier–Gauss and generalized Fourier–Mehler transforms, and their infinitesimal generators which are linear combinations of the generalized Gross Laplacian and the generalized Beltrami Laplacian.

2. Preliminaries

Let (T,ν) be a measure space, where T is a topological space. Let $H_{\mathbb{R}} = L^2(T,\nu)$ be the real Hilbert space of all square integrable functions on T with respect to ν with the norm $|\cdot|_0$ generated by the inner product $\langle\cdot,\,\cdot\rangle$. Let A be a positive self-adjoint operator in $H_{\mathbb{R}}$ satisfying that there exists a sequence $\{\lambda_j\}_{j=0}^{\infty}$ with

$$1 < \lambda_0 \le \lambda_1 \le \lambda_2 \le \dots, \qquad \sum_{j=0}^{\infty} \lambda_j^{-2} < \infty$$

and a complete orthonormal basis $\{e_j\}_{j=0}^{\infty}$ of $H_{\mathbb{R}}$ such that $Ae_j = \lambda_j e_j$. Then we have

$$\rho \equiv \| A^{-1} \|_{\mathrm{OP}} = \lambda_0^{-1}, \qquad \| A^{-q} \|_{\mathrm{HS}}^2 = \sum_{j=0}^{\infty} \lambda_j^{-2q}.$$

For each $p \ge 0$, let

$$E_{\mathbb{R},p} = \{\xi \in H_{\mathbb{R}}\,;\, |\,\xi\,|_p \equiv |\,A^p\xi\,|_0 < \infty\}$$

and $E_{\mathbb{R},-p}$ be the completion of $H_{\mathbb{R}}$ with respect to $|\cdot|_{-p} \equiv |\,A^{-p}\cdot|_0$. Then by the chain of Hilbert spaces $\{E_{\mathbb{R},p}\,;\, p\in\mathbb{R}\}$ with

$$E_{\mathbb{R}} = \operatorname*{proj\,lim}_{p\to\infty} E_{\mathbb{R},p}, \qquad E_{\mathbb{R}}^* \cong \operatorname*{ind\,lim}_{p\to\infty} E_{\mathbb{R},-p},$$

we get a real Gelfand triple

$$E_{\mathbb{R}} \subset H_{\mathbb{R}} \subset E_{\mathbb{R}}^*, \tag{2.1}$$

where $E_{\mathbb{R}}$ and $E_{\mathbb{R}}^*$ are mutually dual spaces. Finally, by taking complexification of (2.1), we have a complex Gelfand triple:

$$E \subset H \subset E^*.$$

The canonical $\mathbb{C}$-bilinear form on $E^* \times E$ which is compatible with the inner product of H is denoted by $\langle \cdot, \cdot \rangle$ again.

For each $p \in \mathbb{R}$, let E_p be complexification of $E_{\mathbb{R},p}$. The (Boson) Fock space over E_p is defined by

$$\Gamma(E_p) = \left\{ \phi = (f_n)_{n=0}^{\infty};\, f_n \in E_p^{\widehat{\otimes} n}, \| \phi \|_p^2 = \sum_{n=0}^{\infty} n! \, | f_n |_p^2 < \infty \right\},$$

where $E_p^{\widehat{\otimes} n}$ is the symmetric n-fold tensor product. From the chain of Fock spaces $\{\Gamma(E_p)\,;\, p \in \mathbb{R}\}$, by setting

$$(E) = \operatorname*{proj\,lim}_{p \to \infty} \Gamma(E_p), \qquad (E)^* = \operatorname*{ind\,lim}_{p \to \infty} \Gamma(E_{-p}),$$

we construct the Gelfand triple:

$$(E) \subset \Gamma(H) \subset (E)^*$$

which is referred to as the *Hida–Kubo–Takenaka space* in the white noise theory [4, 8, 11]. The topology on (E) is given by the norms

$$\| \phi \|_p^2 = \sum_{n=0}^{\infty} n! \, | f_n |_p^2 , \quad \phi = (f_n), \quad p \in \mathbb{R}.$$

On the other hand, for each $\Phi = (F_n) \in (E)^*$ there exists $p \geq 0$ such that $\Phi \in \Gamma(E_{-p})$ and

$$\| \Phi \|_{-p}^2 \equiv \sum_{n=0}^{\infty} n! \, | F_n |_{-p}^2 < \infty.$$

The canonical $\mathbb{C}$-bilinear form on $(E)^* \times (E)$ is denoted by $\langle\!\langle \cdot, \cdot \rangle\!\rangle$ and we have

$$\langle\!\langle \Phi, \phi \rangle\!\rangle = \sum_{n=0}^{\infty} n! \, \langle F_n, f_n \rangle , \quad \Phi = (F_n) \in (E)^*,$$

for any $\phi = (f_n) \in (E)$. For each $\xi \in E$,

$$\phi_\xi = \left(1, \xi, \frac{\xi^{\otimes 2}}{2!}, \dots, \frac{\xi^{\otimes n}}{n!}, \dots \right)$$

is called an *exponential vector* (or *coherent vector*). In particular, ϕ_0 is called the *vacuum vector*.

A continuous linear operator Ξ from (E) into $(E)^*$ is called a *white noise operator*. Let $\mathcal{L}((E),(E)^*)$ denote the space of all white noise operators equipped with the topology of bounded convergence. The *symbol* of $\Xi \in \mathcal{L}((E),(E)^*)$ is defined by

$$\widehat{\Xi}(\xi,\eta) = \langle\langle \Xi\phi_\xi,\, \phi_\eta \rangle\rangle\,, \qquad \xi,\eta \in E.$$

Then we have the following analytic characterization of operator symbols.

Theorem 2.1 ([10]). *Let Θ be a $\mathbb{C}$-valued function on $E\times E$. Then Θ is the symbol of an operator in $\mathcal{L}((E),(E))$ if and only if*

(i) *for any $\xi,\xi_1,\eta,\eta_1 \in E$, the function*

$$z, w \longmapsto \Theta(z\xi+\xi_1, w\eta+\eta_1)$$

is an entire holomorphic function on $\mathbb{C}\times\mathbb{C}$.

(ii) *for any $p \geq 0$ and $\epsilon > 0$, there exist two positive constants C and q such that*

$$|\Theta(\xi,\eta)| \leq C e^{\epsilon(|\xi|^2_{p+q}+|\eta|^2_{-p})}, \qquad \xi,\eta \in E.$$

In this case,

$$\|\, \Xi\phi \,\|_{p-1} \leq CM(\epsilon,q,r)\, \|\, \phi \,\|_{p+q+r+1}\,, \qquad \phi \in (E),$$

where $M(\epsilon,q,r)$ is a (finite) constant for $\epsilon < (2e^3\delta^2)^{-1}$, $r \geq r_0(q) \geq 0$.

For each $U \in \mathcal{L}(E,E^*)$ and $V \in \mathcal{L}(E,E)$, the *generalized Gross Laplacian* or *U–Gross Laplacian* $\Delta_{\mathrm{G}}(U)$ and the *generalized Beltrami Laplacian* $N(V)$ is defined by

$$\widehat{\Delta_{\mathrm{G}}(U)}(\xi,\eta) = \langle U\xi,\, \xi\rangle\, e^{\langle \xi,\,\eta\rangle}, \qquad \widehat{N(V)}(\xi,\eta) = \langle V\xi,\, \eta\rangle\, e^{\langle \xi,\,\eta\rangle}$$

for $\xi,\,\eta \in E$. Then by applying Theorem 2.1, we can easily see that $\Delta_{\mathrm{G}}(U) \in \mathcal{L}((E),(E))$ and $N(V) \in \mathcal{L}((E),(E))$.

3. Generalized Fourier–Gauss transforms

For a nuclear Fréchet space $\mathfrak{X}$ with defining Hilbertian norms $\{\|\cdot\|_\alpha\}$, we denote by $GL(\mathfrak{X})$ the group of all linear homeomorphisms from $\mathfrak{X}$ onto itself.

For each $U \in \mathcal{L}(E,E^*)$ and $V \in \mathcal{L}(E,E)$, by Theorem 2.1 there exists an operator $\mathcal{G}_{U,V} \in \mathcal{L}((E),(E))$ such that

$$\mathcal{G}_{U,V}\phi_\xi = \phi_{V\xi} \exp\langle U\xi,\, \xi\rangle, \qquad \xi \in E.$$

The operator $\mathcal{G}_{U,V} \in \mathcal{L}((E),(E))$ is called the *generalized Fourier–Gauss transform* or $\mathcal{G}_{U,V}$*-transform*. The adjoint operator $\mathcal{G}^*_{U,V}$ of $\mathcal{G}_{U,V}$ is denoted by $\mathcal{F}_{U,V}$ and called the *generalized Fourier–Mehler transform* or $\mathcal{F}_{U,V}$*-transform*. For the more study of the generalized Fourier–Gauss and generalized Fourier–Mehler transforms, we refer to [1].

THEOREM 3.1 ([1]). *Let $U, U' \in \mathcal{L}(E, E^*)$ and $V, V' \in \mathcal{L}(E, E)$. Then in order that $\mathcal{G}_{U',V'}\mathcal{G}_{U,V}\phi = \phi$ for any $\phi \in (E)$, it is necessary and sufficient that*

$$V'V = I \quad \text{and} \quad V^*U'V + U = 0 \quad \text{(the zero operator)}.$$

Therefore, $G = \{\mathcal{G}_{U,V} | U \in \mathcal{L}(E, E^*),\ V \in GL(E)\}$ (resp. $\{\mathcal{F}_{U,V} | U \in \mathcal{L}(E, E^*),\ V \in GL(E)\}$) is a subgroup of $GL((E))$ (resp. $GL((E)^*)$). Moreover, we have $\mathcal{G}_{0,I} = I$ and

$$\mathcal{G}_{U',V'}\mathcal{G}_{U,V} = \mathcal{G}_{V^*U'V+U,V'V}.$$

4. One-parameter transformation groups

Let $\mathfrak{X}$ be a nuclear Fréchet space with defining Hilbertian norms $\{\|\cdot\|_\alpha\}_{\alpha\in\mathfrak{I}}$. A one-parameter subgroup $\{g_\theta\}_{\theta\in\mathbb{R}}$ of $GL(\mathfrak{X})$ is said to be *differentiable* if, for every $\xi \in \mathfrak{X}, \lim_{\theta\to 0}((g_\theta\xi - \xi)/\theta)$ converges in the topology of $\mathfrak{X}$. In this case, a linear operator X from $\mathfrak{X}$ into itself defined by

$$X\xi = \lim_{\theta\to 0} \frac{g_\theta\xi - \xi}{\theta}, \qquad \xi \in \mathfrak{X},$$

is called the *infinitesimal generator* of $\{g_\theta\}_{\theta\in\mathbb{R}}$. It is shown that $X \in \mathcal{L}(\mathfrak{X}, \mathfrak{X})$ and that, for any $\theta \in \mathbb{R}$, we have $g_\theta X\phi = Xg_\theta\phi$ for $\phi \in \mathfrak{X}$. For the proof, see Section 5.2 in [11].

A differentiable one-parameter subgroup $\{g_\theta\}_{\theta\in\mathbb{R}} \subset GL(\mathfrak{X})$ with the infinitesimal generator X is called *regular* if for any $\alpha \in \mathfrak{I}$, there exists $\beta \in \mathfrak{I}$ such that

$$\lim_{\theta\to 0} \sup_{\|\xi\|_\beta \le 1} \left\| \frac{g_\theta\xi - \xi}{\theta} - X\xi \right\|_\alpha = 0.$$

LEMMA 4.1. *Let U and V be two differentiable maps on $\mathbb{R}$ such that $U(\theta) \in \mathcal{L}(E, E^*)$ and $V(\theta) \in \mathcal{L}(E, E)$ for any $\theta \in \mathbb{R}$. We assume that V is regular. Then $\{\mathcal{G}_{U(\theta),V(\theta)}\}_{\theta\in\mathbb{R}}$ is a one-parameter subgroup of $GL((E))$ if and only if $\{V(\theta)\}_{\theta\in\mathbb{R}}$ is a one–parameter subgroup of $GL(E)$ and $U(\theta)$*

is given by

$$U(\theta) = \int_0^\theta V(s)^* U V(s) ds, \tag{4.1}$$

for $U = U'(0) \in \mathcal{L}(E, E^*)$.

Remark 4.2. In (4.1), the integral is in the Pettis sense and the integrability is justified from the equicontinuity of $V(\theta)$ on every finite interval which is implied by the assumption that V is regular.

Proof. Let $\{\mathcal{G}_{U(\theta),V(\theta)}\}_{\theta\in\mathbb{R}}$ be a one-parameter subgroup of $GL((E))$. Then it holds that, for any $\theta_1, \theta_2 \in \mathbb{R}$ and $\phi \in (E)$,

$$\begin{aligned}\mathcal{G}_{U(\theta_2),V(\theta_2)}\mathcal{G}_{U(\theta_1),V(\theta_1)}\phi &= \mathcal{G}_{U(\theta_1)+V(\theta_1)^*U(\theta_2)V(\theta_1),V(\theta_2)V(\theta_1)}\phi \\ &= \mathcal{G}_{U(\theta_1+\theta_2),V(\theta_1+\theta_2)}\phi.\end{aligned}$$

So, $U(\theta)$ and $V(\theta)$ satisfy the equations:

$$U(\theta_1+\theta_2) = U(\theta_1) + V(\theta_1)^* U(\theta_2) V(\theta_1) \tag{4.2}$$

$$V(\theta_1+\theta_2) = V(\theta_2)V(\theta_1) \tag{4.3}$$

for $\theta_1, \theta_2 \in \mathbb{R}$. From the group property, $V(0) = I$ and $U(0) = 0$. Therefore, by (4.3), $\{V(\theta)\}_{\theta\in\mathbb{R}}$ is a one–parameter subgroup of $GL(E)$. To find the solution of (4.2), we compute $U'(\theta)$ as follows:

$$\begin{aligned}U'(\theta) &= \lim_{h\to 0} \frac{U(\theta+h) - U(\theta)}{h} \\ &= \lim_{h\to 0} \frac{V(\theta)^*(U(h) - U(0))V(\theta)}{h} \\ &= V(\theta)^* U'(0) V(\theta).\end{aligned}$$

Therefore, we have the equality (4.1). Now, we can see that (4.1) is the solution of (4.2). In fact, for any $\theta_1, \theta_2 \in \mathbb{R}$, we obtain that

$$\begin{aligned}U(\theta_1+\theta_2) &= \int_0^{\theta_1+\theta_2} V(s)^* U V(s)\, ds \\ &= \int_0^{\theta_1} V(s)^* U V(s)\, ds + \int_{\theta_1}^{\theta_1+\theta_2} V(s)^* U V(s)\, ds \\ &= U(\theta_1) + \int_0^{\theta_2} V(t+\theta_1)^* U V(t+\theta_1)\, dt \\ &= U(\theta_1) + V(\theta_1)^* U(\theta_2) V(\theta_1),\end{aligned}$$

as desired. The converse is straightforward. □

REMARK 4.3 ([11]). An infinitesimal generator X of a differentiable one–parameter subgroup $\{g_\theta\}_{\theta\in\mathbb{R}}$ of $GL(\mathfrak{X})$ is continuous, i.e., $X \in \mathcal{L}(\mathfrak{X},\mathfrak{X})$. Let $\{V(\theta)\}_{\theta\in\mathbb{R}}$ be a differentiable one–parameter subgroup of $GL(E)$. Then there exists an infinitesimal generator $V \in \mathcal{L}(E,E)$ and so we write $V(\theta) = e^{\theta V}$ (see Proposition 5.2.2 in [11]).

Let $\mathcal{L}_{\mathcal{E}}(\mathfrak{X},\mathfrak{X})$ be the class of operators $X \in \mathcal{L}(\mathfrak{X},\mathfrak{X})$ satisfying that there exists $r > 0$ such that $\{(rX)^n/n!\}_{n=0}^{\infty}$ is equicontinuous, i.e., for any $\alpha \in \mathfrak{I}$ there exist $C = C(\alpha) \geq 0$ and $\beta = \beta(\alpha) \in \mathfrak{I}$ such that

$$\sup_{n\geq 0} \frac{1}{n!} \| (rX)^n \xi \|_{\alpha} \leq C \| \xi \|_{\beta}, \qquad \xi \in \mathfrak{X}.$$

For any $U \in \mathcal{L}(E,E^*)$ and $V \in \mathcal{L}_{\mathcal{E}}(E,E)$, we define a one–parameter subgroup $\{\mathcal{H}_{U,V;\theta}\}_{\theta\in\mathbb{R}}$ of $GL((E))$ by

$$\mathcal{H}_{U,V;\theta} = \mathcal{G}_{U(\theta),\, e^{\theta V}} \quad \text{and} \quad U(\theta) = \int_0^\theta e^{sV^*} U e^{sV}\, ds.$$

We can show that the one-parameter group $\{\mathcal{H}_{U,V;\theta}\}_{\theta\in\mathbb{R}}$ is differentiable and regular as follows.

THEOREM 4.4. *Let $U \in \mathcal{L}(E,E^*)$ and $V \in \mathcal{L}_{\mathcal{E}}(E,E)$. Then $\{\mathcal{H}_{U,V;\theta}\}$ is a regular one–parameter subgroup of $GL((E))$ with infinitesimal generator $\Delta_{\mathrm{G}}(U) + N(V)$.*

Proof. By Lemma 4.1, $\{\mathcal{H}_{U,V;\theta}\}_{\theta\in\mathbb{R}}$ is a one-parameter subgroup of $GL((E))$. For any $\xi, \eta \in E$, we put

$$f(\theta) = \widehat{\mathcal{H}}_{U,V;\theta}(\xi,\eta) = \exp\left\{\left\langle \int_0^\theta e^{sV^*} U e^{sV} ds \xi,\, \xi \right\rangle + \left\langle e^{\theta V}\xi,\, \eta\right\rangle\right\}.$$

Then we have

$$\begin{aligned} f'(\theta) &= \left\{\left\langle e^{\theta V^*} U e^{\theta V}\xi,\, \xi\right\rangle + \left\langle V e^{\theta V}\xi,\, \eta\right\rangle\right\} f(\theta), \\ f''(\theta) &= \left\{\left\langle (V^* e^{\theta V^*} U e^{\theta V} + e^{\theta V^*} U V e^{\theta V})\xi,\, \xi\right\rangle + \left\langle V^2 e^{\theta V}\xi,\, \eta\right\rangle\right\} f(\theta) \\ &\quad + \left\{\left\langle e^{\theta V^*} U e^{\theta V}\xi,\, \xi\right\rangle + \left\langle V e^{\theta V}\xi,\, \eta\right\rangle\right\}^2 f(\theta). \end{aligned}$$

Let $\theta_0 > 0$ be fixed. Then we note that, whenever $|\theta| \le \theta_0$, for some p, $q \ge 0$ and $\gamma > 0$,

$$\begin{aligned} \left|f''(\theta)\right| &\le C\exp\{K_1\,|\,\xi\,|_{p+q}\,|\,\xi\,|_{-p} + K_2\,|\,\xi\,|_{p+q}\,|\,\eta\,|_{-p}\} \\ &\le C\exp\left\{K_1\rho^{(2p+q)}\,|\,\xi\,|_{p+q}^2 + \frac{K_2}{2}\left(\gamma^2\,|\,\xi\,|_{p+q}^2 + \frac{1}{\gamma^2}\,|\,\eta\,|_{-p}^2\right)\right\} \\ &= C\exp\left\{\left(K_1\rho^{(2p+q)} + \frac{K_2\gamma^2}{2}\right)|\,\xi\,|_{p+q}^2 + \frac{K_2}{2\gamma^2}\,|\,\eta\,|_{-p}^2\right\}, \end{aligned}$$

with some constants $C = C(U, V; \theta_0)$, $K_1 = K_1(U, V; \theta_0)$ and $K_2 = K_2(U, V; \theta_0)$. Now, we put

$$g_\theta(\xi, \eta) = f(\theta) - f(0) - f'(0)\theta.$$

Then, by Taylor theorem, whenever $|\theta| \le \theta_0$ it follows that

$$\begin{aligned} |g_\theta(\xi, \eta)| &\le \frac{|\theta|^2}{2}\max_{|\theta|\le\theta_0}\left|f''(\theta)\right| \\ &\le \frac{|\theta|^2}{2}C\exp\left\{\left(K_1\rho^{(2p+q)} + \frac{K_2\gamma^2}{2}\right)\rho^{2r}\,|\,\xi\,|_{p+q+r}^2 + \frac{K_2}{2\gamma^2}\,|\,\eta\,|_{-p}^2\right\} \end{aligned}$$

for $\xi, \eta \in E$. Now let $\varepsilon \ge 0$ be given. We can find $q \ge 0$, $r \ge 0$ and $\gamma > 0$ such that

$$\left(K_1\rho^{(2p+q)} + \frac{K_2\gamma^2}{2}\right)\rho^{2r} < \varepsilon \quad \text{and} \quad \frac{K_2}{2\gamma^2} < \varepsilon.$$

So, there exists $q \ge 0$ such that

$$|g_\theta(\xi, \eta)| \le \frac{|\theta|^2}{2}C\exp\{\varepsilon(|\,\xi\,|_{p+q}^2 + |\,\eta\,|_{-p}^2)\}, \ |\theta| \le \theta_0, \ \xi, \eta \in E.$$

It then follows from Theorem 2.1, that there exists $\Xi_\theta \in \mathcal{L}((E), (E))$ such that $\widehat{\Xi}_\theta = g_\theta$ and

$$\|\,\Xi_\theta\phi\,\|_{p-1} \le \frac{|\theta|^2}{2}CM(\varepsilon, q, r)\,\|\,\phi\,\|_{p+q+r+1}\,, \ \phi \in (E), \tag{4.4}$$

for some $r \ge 0$. On the other hand, we see that

$$\begin{aligned} f(0) &= e^{\langle \xi,\, \eta\rangle} = \widehat{I}(\xi, \eta), \\ f'(0) &= \{\langle U\xi,\, \xi\rangle + \langle V\xi,\, \eta\rangle\}\, e^{\langle \xi,\, \eta\rangle} = \widehat{\Delta_{\mathrm{G}}(U)}(\xi, \eta) + \widehat{N(V)}(\xi, \eta). \end{aligned}$$

Hence we have

$$\Xi_\theta = \mathcal{H}_{U,V;\theta} - I - \theta\left(\Delta_{\mathrm{G}}(U) + N(V)\right).$$

From (4.4), it follows that

$$\sup_{\|\phi\|_{p+q+r+1}} \left\| \frac{\mathcal{H}_{U,V;\theta}\phi - \phi}{\theta} - (\Delta_{\mathrm{G}}(U) + N(V))\,\phi \right\|_{p-1} \le \frac{|\theta|}{2} CM(\varepsilon, q, r) \to 0$$

as $\theta \to 0$. Therefore, the proof is completed. □

By the duality of Theorem 4.4, we have the following theorem.

THEOREM 4.5. *Let $U \in \mathcal{L}(E, E^*)$ and $V \in \mathcal{L}_{\mathcal{E}}(E, E)$. Then $\{\mathcal{H}^*_{U,V;\theta}\}$ is a differentiable one-parameter subgroup of $GL((E)^*)$ with infinitesimal generator $\Delta^*_{\mathrm{G}}(U) + N(V^*)$.*

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