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NUMBER OF THE NONTRIVIAL SOLUTIONS OF THE NONLINEAR BIHARMONIC PROBLEM

Tacksun Jung and Q-Heung Choi*

Abstract. We investigate the number of the nontrivial solutions of the nonlinear biharmonic equation with Dirichlet boundary condition. We give a theorem that there exist at least three nontrivial solutions for the nonlinear biharmonic problem. We prove this result by the finite dimensional reduction method and the shape of the graph of the corresponding functional on the finite reduction subspace.

1. Introduction

Let Ω be a smooth bounded region in R^n with smooth boundary $\partial\Omega$, Δ^2 denote the biharmonic operator and $c \in R$. Let $g : R \to R$ be a differentiable function such that $g(0) = 0$, and $c \in R$. In this paper we investigate the number of the weak solutions of the following nonlinear biharmonic equation with Dirichlet boundary condition

$$\begin{aligned} \Delta^2 u + c\Delta u &= g(u) \qquad \text{in } \Omega, \\ u = 0, \qquad \Delta u &= 0 \qquad \text{on } \partial\Omega. \end{aligned} \tag{1.1}$$

The eigenvalue problem

$$\begin{aligned} \Delta^2 u + c\Delta u &= \Lambda u \qquad \text{in } \Omega, \\ u = 0, \qquad \Delta u &= 0 \qquad \text{on } \partial\Omega, \end{aligned}$$

has infinitely many eigenvalues $\Lambda_k = \lambda_k(\lambda_{k-c})$, $k \geq 1$ and corresponding eigenfunctions ϕ_k, $k \geq 1$, the suitably normalized with respect to $L^2(\Omega)$ inner product, of where each eigenvalue λ_k is repeated as often as its

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*Corresponding author.

multiplicity, where λ_k, $k \geq 1$, are the infinitely many eigenvalues and ϕ_k, $k \geq 1$, are the corresponding eigenfunctions, suitably normalized with respect to $L^2(\Omega)$ inner product of the eigenvalue problem

$$\Delta u + \lambda u = 0 \qquad \text{in } \Omega,$$

$$u = 0 \qquad \text{on } \partial\Omega.$$

We recall that $\Lambda_1 \leq \Lambda_2 \leq \ldots \to +\infty$, and that $\phi_1(x) > 0$ for $x \in \Omega$. We assume that $\lambda_k < c < \lambda_{k+1}$. We also assume that $g \in C^1(R, R)$ satisfies the following conditions:
(g1) There exist $\alpha < \beta$ such that

$$\alpha \leq g'(u) \leq \beta.$$

(g2) Let $\Lambda_{j+1}, \Lambda_{j+2}, \Lambda_{j+m}$, $m \geq 1$, be all eigenvalues with in $[\alpha, \beta]$(without loss of generality, we may assume that α, β are not the eigenvalues Λ_i, $i \geq 1$). Suppose that there exist γ and C such that $\Lambda_{j+m} < \gamma < \beta$ and

$$G(u) \geq \frac{1}{2}\gamma\|u\|^2 - C, \qquad \forall u \in R,$$

where $G(\xi) = \int_0^\xi g(t)dt$.
(g3) $g(0) = 0$.
(g4) There exists eigenvalue $\Lambda_l \in [\Lambda_{j+1}, \Lambda_{j+m})$ such that

$$\Lambda_l < g'(0) < \Lambda_{l+1}.$$

Choi and Jung [2] show that the problem

$$\begin{aligned} \Delta^2 u + c\Delta u &= bu^+ + s \qquad \text{in } \Omega, \\ u = 0, \qquad \Delta u &= 0 \qquad \text{on } \partial\Omega, \end{aligned} \tag{1.2}$$

has at least two nontrivial solutions when ($c < \lambda_1$, $\Lambda_1 < b < \Lambda_2$ and $s < 0$) or ($\lambda_1 < c < \lambda_2$, $b < \Lambda_1$ and $s > 0$). They obtained these results by use of the variational reduction method. They [3] also proved that when $c < \lambda_1$, $\Lambda_1 < b < \Lambda_2$ and $s < 0$, (1.2) has at least three nontrivial solutions by use of the degree theory. Tarantello [5] also studied (1.1). She show that if $c < \lambda_1$ and $b \geq \Lambda_1$, then (1.1) has a negative solution. She obtained this result by the degree theory. Micheletti and Pistoia [4] also proved that if $c < \lambda_1$ and $b \geq \Lambda_2$, then (1.1) has at least four solutions by the variational linking theorem and Leray-S chauder degree theory. In this paper we are looking for the weak solutions of (1.1), that is,

$$\int_\Omega \Delta^2 u \cdot v + c\Delta u \cdot v - g(u)v = 0, \qquad \forall v \in H,$$

where H is introduced in section 2. Our main result is the following.

THEOREM 1.1. *Assume that g satisfies the conditions $(g1)$-$(g4)$. Then (1.1) has at least three nontrivial solutions.*

The outline of the proof is as follows: In section 2 we introduce the Hilbert space H and show that the corresponding functional $I(u)$ of (1.1) is in $C^1(H, R)$, *Fréchet* differentiable and satisfies the Palais-Smale condition. In section 3, we prove Theorem 1.1. For the proof of Theorem 1.1 we use the finite dimensional reduction method to reduce the theory on the infinite dimensional space to the one on the finite dimensional subspace. So we obtain the critical points results of the functional on the infinite space H from the critical points results of the corresponding functional $\tilde{I}(v)$ on the finite dimensional reduction subspace.

2. Finite dimensional reduction method

We assume that $\lambda_k < c < \lambda_{k+1}$, $c \in R$. Any element u in $L^2(\Omega)$ can be written as

$$u = \sum h_k \phi_k \quad \text{with} \quad \sum h_k^2 < \infty.$$

We define a subspace H of $L^2(\Omega)$ as follows

$$H = \{u \in L^2(\Omega) | \ \sum |\Lambda_k| < \infty\}.$$

Then this is a complete normed space with a norm

$$\|u\| = [\sum |\Lambda_k| h_k^2]^{\frac{1}{2}}.$$

Since $\lambda_k \to +\infty$ and c is fixed, we have
(i) $\Delta^2 u + c\Delta u \in H$ implies $u \in H$.
(ii) $\|u\| \geq C\|u\|_{L^2(\Omega)}$, for some $C > 0$.
(iii) $\|u\|_{L^2(\Omega)} = 0$ if and only if $\|u\| = 0$,
which is proved in [1].

From the conditions on g, we have the following lemma:

Lemma 2.1. *Assume that g satisfies the conditions $(g1)$-$(g4)$. Then the solutions in $L^2(\Omega)$ of*

$$\Delta^2 u + c\Delta u = g(u) \qquad \text{in } L^2(\Omega)$$

belong to H.

Proof. Let $g(u) = \sum h_k \phi_k \in L^2(\Omega)$. Then

$$(\Delta^2 + c\Delta)^{-1}(g(u)) = \sum \frac{1}{\lambda_k(\lambda_k - c)} h_k \phi_k.$$

Hence we have

$$\|(\Delta^2 + c\Delta)^{-1} g(u)\|^2 = \sum |\lambda_k(\lambda_k - c)| \frac{1}{\lambda_k(\lambda_k - c))^2} h_k^2 \leq C \sum h_k^2$$

for some $C > 0$, which means that

$$\|(\Delta^2 + c\Delta)^{-1} g(u)\| \leq C_1 \|u\|_{L^2(\Omega)}.$$

□

With the aid of Lemma 2.1 it is enough that we investigate the existence of solutions of (1.1) in the subspace H of $L^2(\Omega)$. Let us define the functional in $H \times R$,

$$I(u) = \int_\Omega \frac{1}{2}[|\Delta u|^2 - \frac{c}{2}|\nabla u|^2 - G(u)]dx,$$

where $G(\xi) = \int_0^\xi g(t)dt$. Then $I(u)$ is well defined. By the following Lemma 2.2, $I(u) \in C(H, R)$, *Fréchet* differentiable in H, so the solutions of (1.1) coincide with the critical points of $I(u)$.

Lemma 2.2. *Assume that $g(u)$ satisfies the conditions $g(1)$-$g(4)$. Then $I(u)$ is continuous and Fréchet differentiable in H and*

$$DI(u)(h) = \int_\Omega \Delta u \cdot \Delta h - c\nabla u \cdot \nabla h - g(u)h \tag{2.1}$$

for $h \in H$

Proof. Let $u \in H$. First we will prove that $I(u)$ is continuous. We consider

$$\begin{aligned} &|I(u+v) - I(u)| \\ = &\int_\Omega [\frac{1}{2}|\Delta(u+v)|^2 - \frac{c}{2}|\nabla(u+v)|^2 - G(u+v)] \\ &\qquad\qquad - \int_\Omega [\frac{1}{2}|\Delta u|^2 - \frac{c}{2}|\nabla u|^2 G(u)] \\ = &\int_\Omega [u \cdot (\Delta^2 v + c\Delta v) + \frac{1}{2} v \cdot (\Delta^2 v + c\Delta v) - G(u+v) - G(u)]. \end{aligned}$$

Let $u = \sum h_k \phi_k$, $v = \sum \tilde{h}_k \phi_k$. Then we have

$$|\int_\Omega u \cdot (\Delta^2 v + c\Delta v)dx| = |\sum \int_\Omega \Lambda_k h_k \tilde{h}_k| \le \|u\|\|v\|,$$

$$|\int_\Omega v \cdot (\Delta^2 v + c\delta v)dx| = |\sum \Lambda_k \tilde{h}_k^2| \le \|v\|^2.$$

On the other hand, by Mean Value Theorem and $(g1)$, we have

$$\begin{aligned} G(u+v) - G(u) &= \int_0^{u+v} g(s)ds - \int_0^u g(s)ds \\ &= \frac{1}{2} g'(t)(u+v)^2 - \frac{1}{2} g'(t')u^2 \\ &\le \max\{|\alpha|, |\beta|\}|v|(|u| + |v|) \\ &\le C \max\{|\alpha|, |\beta|\}\|v\|(\|u\| + \|v\|). \end{aligned}$$

With the above results, we see that $I(u)$ is continuous at u. To prove $I(u)$ is *Fréchet* differentiable at $u \in H$, we consider

$$\begin{aligned} |I(u+v) &- I(u) - DI(u)v| \\ &= |\int_\Omega \frac{1}{2} v(\Delta^2 v + c\Delta v) - G(u+v) + G(u) - g(u)v| \\ &\le \frac{1}{2}\|v\|^2 + C\gamma\|v\|(\|u\| + \|v\|) + M\|v\| \\ &\le C'\|v\|(\|v\| + \|u\| + \|v\| + 1). \end{aligned}$$

□

By the following Lemma 2.3 (finite dimensional reduction method), we can get the critical results of the functional on the infinite dimensional space H from that of the functional on the finite dimensional one.

Let V be m dimensional subspace of H spanned by $\phi_{j+1},\dots,\phi_{j+m}$ whose eigenvalues are $\Lambda_j,\dots,\Lambda_{j+m}$. Let W be the orthogonal complement of V in H. Let $P : H \to V$ be the orthogonal projection of H onto V and $I - P : H \to W$ denote that of H onto W. Then every element $u \in L^2(\Omega)$ is expressed by $u = v + z$, $v \in Pu$, $z = (I - P)u$. Then (1.1) is equivalent to the two systems in the two unknowns v and z:

$$\Delta^2 v + c\Delta v = P(g(v + z)) \qquad \text{in } \Omega,$$

$$\Delta^2 z + c\Delta z = (I - P)(g(v + z)) \qquad \text{in } \Omega,$$

$$v = 0, \qquad \Delta v = 0 \qquad \text{on } \partial\Omega,$$

$$z = 0, \qquad \Delta z = 0 \qquad \text{on } \partial\Omega.$$

Let W_1 be a subspace of W spanned by eigenvalues $\Lambda_1, \dots, \Lambda_j$ and W_2 be a subspace of W spanned by eigenvalues Λ_i, $i \geq j+m+1$. Let $v \in V$ be fixed and consider the function $h : W_1 \times W_2 \to R$ defined by

$$h(w_1, w_2) = I(v + w_1 + w_2).$$

The function h has continuous partial *Fréchet* derivatives $D_1 h$ and $D_2 h$ with respect to its first and second variables given by

$$D_i h(w_1, w_2)(y_i) = DI(v + w_1 + w_2)(y_i) \tag{2.2}$$

for $y_i \in W_i$, $i = 1, 2$. We recall that if I is a function of class C^1 and u_0 is a critical point of I, then u_0 is called of mountain pass type if for every open neighborhood U of $I^{-1}(-\infty, I(u_0)) \cap U \neq \emptyset$ and $I^{-1}(-\infty, I(u_0)) \cap U$ is not pass connected.

LEMMA 2.3. *Assume that g satisfies the conditions $(g1)$-$(g4)$. Then (i) there exists $m_1 < 0$ such that if w_1 and y_1 are in W_1 and $w_2 \in W_2$, then*

$$(D_1 h(w_1, w_2) - D_1 h(y_1, w_2))(w_1 - y_1) \leq m_1 \|w_1 - y_1\|^2,$$

(ii) there exists $m_2 > 0$ such that if w_2 and y_2 are in W_2 and $w_1 \in W_1$, then

$$(D_2 h(w_1, w_2) - D_2 h(w_1, y_2))(w_2 - y_2) \geq m_2 \|w_2 - y_2\|^2$$

(iii) there exists a unique solution $z \in W$ of the equation

$$\Delta^2 z + c\Delta z = (I - P)(g(v + z)) \qquad \text{in } W. \tag{2.3}$$

If we put $z = \theta(v)$, then θ is continuous on V and satisfies a uniform Lipschitz condition in v with respect to the L^2 norm(also norm $\|\cdot\|$). Moreover

$$DI(v + \theta(v))(w) = 0 \qquad \text{for all } w \in W.$$

(iv) If $\tilde{I} : V \to R$ is defined by $\tilde{I}(v) = I(v+\theta(v))$, then $\tilde{I}$ has a continuous Fréchet derivative $D\tilde{I}$ with respect to v, and

$$D\tilde{I}(v)(h) = DI(v + \theta(v))(h) \qquad \text{for all } v, h \in V. \tag{2.4}$$

(v) If $v_0 \in V$ is a critical point of $\tilde{I}$ if and only if $v_0 + \theta(v_0)$ is a critical point of I.
(vi) Let $S \subset V$ and $\Sigma \subset H$ be open bounded regions such that

$$\{v + \theta(v); v \in S\} = \Sigma \cap \{v + \theta(v); v \in V\}.$$

If $D\tilde{I}(v) \neq 0$ for $v \in \partial S$, then

$$d(D\tilde{I}, S, 0) = d(DI, \Sigma, 0),$$

where d denote the Leray-Schauder degree.
(vii) If $u_0 = v_0 + \theta(v_0)$ is a critical point of mountain pass type of I, then v_0 is a critical point of mountain pass type of $\tilde{I}$.

Proof. (i) According to the variation all characterization of the eigenvalues $\{\Lambda_j\}_{j=1}^{\infty}$ we have

$$\|w_1\|^2 \leq \Lambda_j \|w_1\|^2_{L^2(\Omega)} \tag{2.4.a}$$

for all $w_1 \in W_1$ and

$$\|w_2\|^2 \geq \Lambda_{j+m+1} \|w_1\|^2_{L^2(\Omega)} \tag{2.5}$$

for all $w_2 \in W_2$. If w_1 and y_1 are in W_1 and $w_2 \in W_2$, then

$$\begin{aligned}
&(D_1 h(w_1, w_2) - D_1 h(y_1, w_2))(w_1 - y_1) \\
&= \int_\Omega |\Delta(w_1 - y_1)|^2 - c|\nabla(w_1 - y_1)|^2 - (g(v + w_1 + w_2) \\
&\qquad\qquad -g(v + y_1 + w_2))(w_1 - y_1)dx.
\end{aligned}$$

Since $(g(\xi_2) - g(\xi_1))(\xi_2 - \xi_1) > \alpha(\xi_2 - \xi_1)$ and (2.4.a), we see that if w_1 and y_1 are in W_1 and $w_2 \in W_2$, then

$$(D_1 h(w_1, w_2) - D_1 h(y_1, w_2))(w_1 - y_1) \leq m_1 \|w_1 - y_1\|^2$$

where $m_1 = 1 - \frac{\alpha}{\Lambda_j} < 0$.
(ii) Similarly, using the fact that $(g(\xi_2) - g(\xi_1))(\xi_2 - \xi_1) \le \beta(\xi_2 - \xi_1)^2$ and (2.5) we see that if w_2 and y_2 are in W_2 and $w_1 \in W_1$, then

$$(D_2h(w_1, w_2) - D_2h(w_1, y_2))(w_2 - y_2) \ge m_2\|w_2 - y_2\|^2$$

where $m_2 = 1 - \frac{\beta}{\Lambda_{j+m+1}} > 0$.
(iii) Let $\delta = \frac{\alpha+\beta}{2}$. If $g_1(\xi) = g(\xi) - \delta\xi$, the equation (2.3) is equivalent to

$$z = (\Delta^2 + c\Delta - \delta)^{-1}(I - P)(g_1(v + z)) \tag{2.6}$$

Since $(\Delta^2 + c\Delta - \delta)^{-1}(I - P)$ is self adjoint, compact and linear map from $(I-P)L^2(\Omega)$ into itself, the eigenvalues of $(\Delta^2 + c\Delta - \delta)^{-1}(I - P)$ are $(\Lambda_l - \delta)^{-1}$, $l \le j$ or $l \ge j + m + 1$. Therefore its L_2 norm is $(\min\{|\Lambda_j - \delta|, |\Lambda_{j+m+1} - \delta|\}^{-1}$. Since $|g_1(\xi_2) - g_1(\xi_1)| \le \max\{|\alpha - \delta|, |\beta - \delta|\}|\xi_2 - \xi_1| = \frac{|\alpha+\beta|}{2}|\xi_2 - \xi_1|$, it follows that the right-hand side of (2.6) defines, for fixed $v \in V$, a Lipschitz mapping of $(I - P)L^2(\Omega)$ into itself with Lipschitz constant $r < 1$. Therefore, by the contraction mapping principle, for given $v \in V$, there exists a unique $z = (I - P)L^2(\Omega)$ which satisfies (2.6). If $\theta(v)$ denote the unique $z \in (I - P)L^2(\Omega)$ which solves (2.3), then θ is continuous and satisfies a uniform Lipschitz condition in v with respect to the L^2 norm(also norm $\|\|$). In fact, if $z_1 = \theta(v_1)$ and $z_2 = \theta(v_2)$, then

$$\begin{aligned}
\|z_1 \;\; - \;\; & z_2\|_{L^2(\Omega)} \\
&= \|(\Delta^2 + c\Delta - \delta)^{-1}(I - P)(g_1(v_1 + z_1) - g_1(v_2 + z_2))\|_{L^2(\Omega)} \\
&\le r\|(v_1 + z_1) - (v_2 + z_2)\|_{L^2(\Omega)} \\
&\le r(\|v_1 - v_2\|_{L^2(\Omega)} + \|z_1 - z_2\|_{L^2(\Omega)}) \le r\|v_1 - v_2\| + r\|z_1 - z_2\|.
\end{aligned}$$

Hence

$$\|z_1 - z_2\| \le C\|v_1 - v_2\|, \qquad C = \frac{r}{1 - r}. \tag{2.7}$$

Let $u = v + z$, $v \in V$ and $z = \theta(v)$. If $w \in (I - P)L^2(\Omega) \cap H$, then from (2.3) we see that

$$\int_\Omega [\Delta z \cdot \Delta w - c\nabla z \cdot \nabla w - (I - P)(g(v + z)w)]dx = 0.$$

Since

$$\int_\Omega \Delta z \cdot \Delta w = 0 \qquad \text{and} \int_\Omega \nabla v \cdot \nabla w = 0,$$

we have

$$DI(v + \theta(v))(w) = 0. \tag{2.8}$$

(iv) Since the functional I has a continuous *Fréchet* derivative DI, $\tilde{I}$ has a continuous *Fréchet* derivative $D\tilde{I}$ with respect to v.
(v) Suppose that there exists $v_0 \in V$ such that $D\tilde{I}(v_0) = 0$. From $D\tilde{I}(v)(h) = DI(v + \theta(v))(h)$ for all $v, h \in V$, $DI(v_0 + \theta(v_0))(h) = 0$ for all $h \in V$. Since $DI(v+\theta(v))(w)$ for all $w \in W$ and H is the direct sum of V and W, it follows that $DI(v_0 + \theta(v_0)) = 0$. Thus $v_0 + \theta(v_0)$ is a solution of (1.1). Conversely if u is a solution of (1.1) and $v = Pu$, then $D\tilde{I}(v) = 0$.
(vi) The proof of part (vi) follows by arguing as in Lemma 2.6 of [11].
(vii) Suppose v_0 is not of mountain pass type of $\tilde{I}$. Let S be an open neighborhood of v_0 in V such that $\tilde{I}^{-1}(-\infty, \tilde{I}(v_0)) \cap S$ is empty or path connected. If $\tilde{I}^{-1}(-\infty, \tilde{I}(v_0)) \cap S$ is empty, by part (i) we see that $\{v + w : v \in V, w \in W\} \cap I^{-1}(-\infty, I(u_0))$ is also empty. Thus u_0 is not of mountain pass type for I. If $\tilde{I}^{-1}(-\infty, \tilde{I}(v_0)) \cap S$ is path connected, Letting $T = \{v + w : v \in V, \|w - \theta(v)\| < 1\}$ and using again (i) it is seen that $T \cap I^{-1}(-\infty, I(u_0))$ is also path connected. □

3. Proof of Theorem 1.1

We shall show that $\tilde{I}(v)$ satisfies the (P.S.) condition.

LEMMA 3.1. *Assume that g satisfies the conditions $(g1)$-$(g4)$. Then $\tilde{I}(v)$ satisfies the Palais-Smale condition.*

Proof. Let us set $u(v) = v + w(v)$, $v \in V$, $w(v) \in W$. Then we have

$$\tilde{I}(v) = \int_\Omega [\frac{1}{2}|\Delta v + \Delta w(v)|^2 - \frac{c}{2}|\nabla v + \nabla w(v)|^2]dx$$

$$- \int_\Omega G(v + w(v))dx.$$

Moreover we have

$$\begin{aligned}\tilde{I}(v) &= I(v+w(v)) = I(u(v)) \\ &= \int_\Omega [\frac{1}{2}|\Delta u(v)|^2 - \frac{c}{2}|\nabla u(v)|^2 - \int_\Omega G(u(v))dx \\ &= \int_\Omega [\frac{1}{2}|\Delta v|^2 - \frac{c}{2}|\nabla v|^2]dx - \int_\Omega G(u(v))dx \\ &\quad +\{\int_\Omega \frac{1}{2}|\Delta u(v)|^2 - \frac{c}{2}|\nabla u(v)|^2 - \frac{1}{2}|\Delta v|^2 + \frac{c}{2}|\nabla v|^2 \\ &\quad - \int_\Omega [G(u(v)) - G(v)]dx\}.\end{aligned}$$

The terms in the bracket are equal to

$$\begin{aligned}&- \int_\Omega [G'(sw(v) - v)w(v)dx]ds + \frac{1}{2}\int_\Omega (\Delta^2 u(v) + c\Delta u(v))w(v)dx \\ &= \int_\Omega \int_0^1 G''(sw(v)+v)w(v)w(v)sdsdx \\ &\qquad\qquad - \frac{1}{2}\int_\Omega (\Delta^2 w(v) + c\Delta w(v))w(v)dx\end{aligned}$$

Thus we have

$$\begin{aligned}\tilde{I}(v) &\leq \int_\Omega [\frac{1}{2}|\Delta v|^2 - \frac{c}{2}|\nabla v|^2]dx \\ &\quad - \int_\Omega G(v)dx \\ &\leq \frac{1}{2}\{\Lambda_{j+m} - \gamma\}\|v\|^2 + C|\Omega| \longrightarrow -\infty \text{ as } \|v\| \to \infty.\end{aligned}$$

Thus $-\tilde{I}(v)$ is bounded from below and, so satisfies the (P.S.) condition. □

PROOF OF THEOREM 1.1

By Lemma 3.1, $I(v)$ is bounded above, satisfies the (P.S.) condition and $\tilde{I}(v) \to -\infty$ as $\|v\| \to \infty$. We claim that 0 is neither a minimum nor degenerate. In fact, we note that $0 = 0 + \theta(0)$, $\theta(0) = 0$. Since $I + \theta$ is continuous, I is identity map, there exists a small neighborhood B of 0 such that if $v \in B$, then, by $(g4)$,

$$\frac{1}{2}\int_\Omega (\Delta^2 v + c\Delta v)vdx - \frac{\bar{\Lambda}}{2}\int_\Omega G(v)dx + o(\|v\|^2) \leq \tilde{I}(v)$$

$$\leq \frac{1}{2}\int_\Omega (\Delta^2 v + c\Delta v)v dx - \frac{\Lambda}{2}\int_\Omega G(v)dx + o(\|v\|^2),$$

where $(\Lambda, \bar{\Lambda}) \subset (\Lambda_l, \Lambda_{l+1})$. Thus $\tilde{I}(v)$ has at least three nontrivial weak solutions.

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Department of Mathematics
Kunsan National University
Kunsan 573-701, Korea
E-mail: tsjung@kunsan.ac.kr

Department of Mathematics Education
Inha University
Incheon 402-751, Korea
E-mail: qheung@inha.ac.kr