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이동통신 네트워크에서 주파수간 간섭과 서비스 장애를 최소화하는 주파수 재할당 방법

한 정 희*

Minimizing Frequency Drop Cost and Interference Cost in Reconfiguring Radio Networks

Junghee Han*

■ Abstract ■

In this paper, we present a frequency reassignment problem (FRP) that arises when we install new base stations or reconfigure radio networks to increase the capacity or to expand service area. For this problem, we develop an integer programming (IP) model, and develop cutting planes to enhance the mathematical representation of the model. Also, we devise an effective tabu search algorithm to obtain tight upper bounds within reasonable time bounds. Computational results exhibit that the developed cutting planes are effective for reducing the computing time as well as for increasing lower bounds. Also, the proposed tabu search algorithm finds a feasible solution of good quality within reasonable time bound.

Keyword : Radio Network, Frequency Reassignment, Integer Programming, Heuristic

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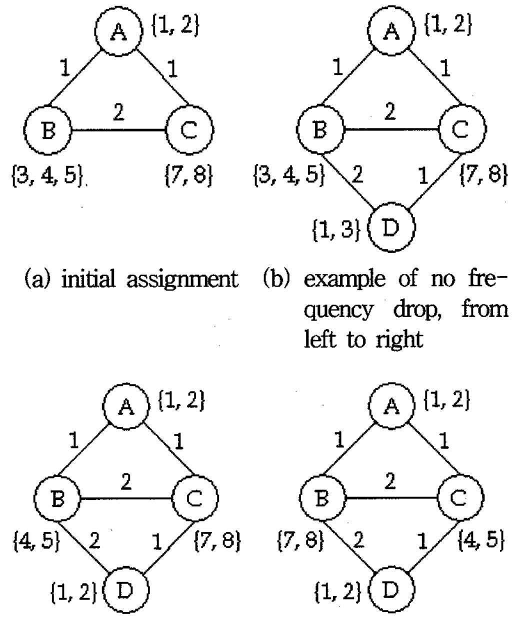
* Department of Business Administration, Kangwon National University

1. Introduction

This paper deals with a radio frequency reassignment problem arising from the reconfiguration of a radio access network (RAN). We may install new base stations (BSs) in order to expand service area or to increase the capacity of a RAN. Also, we may reassign frequencies to BSs due to the change of geographical environment of a RAN, for example, construction of large buildings, roads and so forth. In this case, we need to minimize the change of frequency reassignment. In particular, dropping a frequency from a BS may force a mobile station (MS) using this frequency to be handed over to other frequency assigned to adjacent BSs. If a number of MSs are handed over to a BS simultaneously and the BS does not have enough empty frequencies to accommodate them all, some MSs may lose their connections to BSs. When we reconfigure a RAN, we also need to minimize interference among frequencies. Thus, in this paper, we consider a frequency reassignment problem (FRP) that minimizes the sum of interference cost and frequency drop cost, while providing the minimum number of frequencies required for each BS. [Figure 1] illustrates the nature of the problem FRP.

Suppose that there are three BSs A, B and C, and that a number of frequencies are assigned to them. They are {1, 2}, {3, 4, 5} and {7, 8} for BSs A, B and C, respectively, where numbers denote the frequency indices. Also, we assume that the minimum distances between frequencies to avoid interference are 1, 1 and 2 for pairs of BSs (A, B), (A, C) and (B, C), respectively. This is illustrated in Part (a) of [Figure 1]. Now, we add a new BS labeled as D requiring at least two frequencies. And, suppose that the minimum distances between frequencies for pairs of BSs (B, D) and (C, D) are



(a) initial assignment (b) example of no frequency drop, from left to right

(c) example of one frequency drop (d) example of five frequencies drop, from left to right

[Figure 1] Illustration of frequency reassignment

2 and 1, respectively. Here, we assume that the minimum number of frequencies for BS B changed from three to two, and that the available frequency set for this network is {1, ⋯, 8}. Then, we may assign frequencies 1 and 3 to a new BS D without dropping any frequency from the existing BSs. See Part (b). However, in this case, frequency 3 appears in both BSs B and D. Thus, MSs using this frequency may experience significant interference. While, if we assign frequencies 1 and 2 to BS D at the expense of dropping a frequency 3 from the BS B, we obtain an interference-free frequency assignment (Part (c)). Part (d) displays another interference-free frequency assignment, where frequencies {1, 2}, {7, 8}, {4, 5} and {1, 2} are assigned to BSs A, B, C and D, respectively. However, in this case, we should drop all the frequencies {3, 4, 5} and {7, 8} from the BSs B and C, respectively,

which may cause significant disorder of mobile communication services. This implies that reassigning frequencies in consideration of frequency drop cost as well as interference cost is important to prevent service failures that may occur during the reconfiguration of a RAN. Numerous reports on service failure during the frequency reassignment process in global system for mobile communication (GSM) based networks motivate this study ([16] and [17]).

There exist numerous studies on generic frequency assignment problem (FAP). However, only a few researches in the literature have dealt frequency reassignment problem. For example, Han (2005) proposed a PN(Pseudo Noise)-code reassignment problem in code division multiple access network (CDMA) that seeks to minimize the sum of PN-code reassignment cost and interference cost that may incur during the PN-code reassignment process, where he assumes that only one PN code is assigned to each BS. The frequency reassignment problem addressed in this paper can be conceptualized as an extension of Han (2005) by assuming that multiple frequencies are assigned to each BS, which is the case in GSM based networks. Due to the relaxation on the number of frequencies assigned to each BS, in this paper, we need to take into account the number of frequencies dropped from the BSs as a new cost component. Below some distinguished research results on generic FAPs are summarized. The first model of FAP, referred to as Min-FAP, is to minimize the total number of frequencies needed to satisfy the minimum distance requirements for all pairs of adjacent BSs. This type of FAP was dealt by Hale (1980), Gamst and Rave (1982), Hao et al. (1990) and Sung and Wong (1997). Hale (1980) showed that this problem can be expressed as a graph coloring problem. Another type of FAP, referred to as Max-FAP, is to maximize the total number of frequencies assigned to the BSs, while satisfying the minimum distance requirements among frequencies for all pairs of adjacent BSs. Gamst and Rave (1982), Marthar and Mattfeldt (1993), Chang and Kim (1997), Sung and Wong (1997) and Tiourine et al. (2000) dealt with the Max-FAP. Hao et al. (1990) also developed a tabu search algorithm to solve a FAP that minimizes the total interference among frequencies, referred to as MI(Minimum Interference)-FAP. For the MI-FAP, Tiourine et al. (2000) developed a heuristic algorithm and compared the performance with tabu search algorithm developed by Hao et al. (1990). Also, Tiourine et al. (2000) obtained a tight lower bound by reformulation, and found an optimal solution using the branch-and-bound combined with some preprocessing routines. Besides, quite many research papers on FAP are well summarized in the work by Aardal et al. (2001).

The remainder of this paper is organized as follows. In Section 2, we develop a IP formulation. In Section 3, we develop some cutting planes in order to obtain tight lower bounds. In Section 4, we devise an effective tabu search algorithm to find good quality feasible solutions within reasonable time bound. Computational results are provided in Section 5, and Section 6 concludes this paper.

2. Formulation

Let N be the set of BSs, and let F be the set of frequencies. Also let $F(i) \subseteq F$ be the set of frequencies that are currently assigned to BS $i \in N$, and let $b(i)$ be the number of frequencies required at BS $i \in N$. Define $E = \{i, j\ (> i) \in N : r(i, j) > 0\}$, where $r(i, j)$ denotes the minimum distance between two frequencies assigned to BSs i and j ($> i$) $\in N$, respectively, to avoid interference. Let $x_{if} = 1$ if frequency $f \in F$ is assigned to BS $i \in N$ and 0 otherwise, and let $y_{ik} = 1$ if the number of frequencies dropped from $F(i)$ is equal to k ($= 1, \cdots, |F(i)|$) and 0 otherwise. Setting $y_{ik} = 1$ entails frequency drop cost p_{ik} (> 0). Note that we need not consider frequency drop cost for a new BS. Also, let $u_{ijfg} = 1$ if $x_{if} = x_{jg} = 1$ for $(i, j) \in E$, f and $g \in F$ such that $|f - g| < r(i, j)$ and 0 otherwise. If $u_{ijfg} = 1$, interference cost $q_{ij|f-g|}$ arises between frequencies f and $g \in F$ that are assigned to BSs i and j ($> i$) $\in N$, respectively. We set both interference cost and frequency drop cost based on some proper assumptions. We assume that interference cost $q_{ij|f-g|}$ increases as $|f-g|$ decreases. Also, we assume that it is realistic to set interference cost greater than frequency drop cost since interference influences on service quality permanently unless the current frequency assignment is not re-optimized, while the service quality degrade resulting from the frequency drop is temporal. Obviously, frequency drop cost should increase as the number of frequencies dropped from a BS increases. Then, we should find an optimal trade-off between interference cost and frequency drop cost. Also, if a number of frequencies should be dropped (changed), we need to determine whether to conduct major change of frequency assignment for a few BSs or to conduct minor change of frequency assignment for a wide range of BSs. Using the notations defined above, we can formulate the FRP as a linear IP model as follows.

FRP :

Min $\sum_{i \in N'} \sum_{k \leq |F(i)|} p_{ik} y_{ik} + \sum_{(i, j) \in E} \sum_{f, g \in F : |f-g| < r(i, j)} q_{ij|f-g|} u_{ijfg}$

Subject to

$$\sum_{f \in F} x_{if} \geq b(i) \qquad i \in N, \qquad (1)$$

$$\sum_{f \in F(i)} x_{if} + \sum_{k \leq |F(i)|} k\, y_{ik} = |F(i)| \qquad i \in N, \qquad (2)$$

$$\sum_{k \leq |F(i)|} y_{ik} \leq 1 \qquad i \in N, \qquad (3)$$

$$x_{if} + x_{jg} \leq 1 + u_{ijfg} \qquad (i, j) \in E,\ f, g \in F : |f - g| < r(i, j), \qquad (4)$$

all the variables are binary, (5)

where $N' = \{i \in N : |F(i)| > 1\}$.

Constraint (1) forces that at least $b(i)$ frequencies should be assigned to BS $i \in N$. Constraints (2) and (3) express the number of frequencies dropped from the set of original frequencies assigned to each BS. Constraint (4) expresses the interference between two frequencies assigned to adjacent BSs.

If $|F(i)| \geq b(i)$ for all $i \in N$ and the current frequency assignment is interference free, we need not change any frequency. Thus, we assume that $F(i) < b(i)$ for some $i \in N$ or that there exists interference among the current frequency assignments to BSs. In the following section, we develop some cutting planes that improve the lower bound of formulation FRP.

3. Cutting Planes

Remark 1 : Note that any optimal solution satisfies that

$$\sum_{f \in F} x_{if} \leq \max\{|F(i)|, b(i)\} \quad i \in N. \qquad (6)$$

Thus, if $|F(i)| \leq b(i)$ for some $i \in N$, we see that any optimal solution satisfies the constraint (1) at equality. □

Proposition 1 : Suppose that $|F(i)| > b(i)$ for some $i \in N$. Then, any optimal solution satisfies that

$$\sum_{f \in F} x_{if} = |F(i)| - \sum_{k=1,\cdots,|F(i)|-b(i)} k\, y_{ik} - \sum_{k=1,\cdots,b(i)} (|F(i)|-b(i))\, y_{i,(|F(i)|-b(i)+k)}. \qquad (7)$$

Proof : Let (x^*, y^*) be an optimal solution. If $\sum_{k \le |F(i)|} y^*_{ik} = 0$, equality (7) becomes $\sum_{f \in F} x^*_{if} = |F(i)|$. In this case, we need not add any frequency in $F \backslash F(i)$ to BS $i \in N$ since $|F(i)| > b(i)$. Note that addition of any frequency in $F \backslash F(i)$ to the BS i does not decrease the total cost. If $\sum_{k=1,\cdots,|F(i)|-b(i)} y^*_{ik} = 1$, equality (7) becomes $\sum_{f \in F} x^*_{if} = |F(i)| - k^\circ$, where $k^\circ = arg\{k = 1,\cdots, |F(i)| - b(i) : y^*_{ik} = 1\}$, which is greater than or equal to $b(i)$. Thus, we need not add any frequency in $F \backslash F(i)$ to the BS i. Finally, if $\sum_{k=1,\cdots,b(i)} y^*_{i,(|F(i)|-b(i)+k)} = 1$, equality (7) becomes $\sum_{f \in F} x^*_{if} = b(i)$. Even in this case, we need not force that $\sum_{f \in F} x^*_{if} > b(i)$ by adding additional frequencies to the BS i. This completes the proof. □

Now, we consider a valid inequality based on a clique subgraph of $G(N, E)$. For a clique inducing node set $C \subseteq N$, Padberg (1973) considered the following inequality

$$\sum_{i \in C} x_{if(i)} \le 1, \qquad i, j\ (> i) \in C : |f(i)-f(j)| < r(i, j), \qquad (8)$$

where $f(i)$ denotes the frequency assigned to node $i \in C$, and inequality of type (8) was used by Fischetti et al. (2000) for solving the MaxFAP in a branch-and-cut framework. However, due to the u variables representing the interference level between two frequencies, we modify inequality (8) to deal with u variables. □

Proposition 2 : Consider a triple of nodes $\{i, j, j'\} \subseteq N$. If $|f - g| < r(i, j)$, $|f - g'| < r(i, j')$ and $|g - g'| < r(j, j')$ for f, g and $g' \in F$, any optimal solution satisfies that

$$x_{if}+x_{jg}+x_{j'g'} \le 1+u_{ijfg}+u_{ij'fg'}+u_{jj'gg'}. \qquad (9)$$

Proof : Let (x^*, u^*) be an optimal solution. If $x^*_{if} = x^*_{jg} = x^*_{j'g'} = 1$, we see that $u^*_{ijfg} = u^*_{ij'fg'} = u^*_{jj'gg'} = 1$ from the constraint (4), which satisfies the inequality (9). Now, we assume that, for example, $x^*_{if} = x^*_{jg} = 1$ and $x^*_{j'g'} = 0$. Then, we see that $u^*_{ijfg} = 1$ from the constraint (9) and that $u^*_{ij'fg'} = u^*_{jj'gg'} = 0$ from the direction of the objective function, which also satisfies the inequality (9). If $u^*_{ij'fg'} = 1$ (or $u^*_{jj'gg'} = 1$) when $x^*_{if} = x^*_{jg} = 1$ and $x^*_{j'g'} = 0$, this solution cannot be optimal since $u^*_{ijfg} = 1$. Thus, in this case, we see that $u^*_{ij'fg'} = u^*_{jj'gg'} = 0$ if $x^*_{if} = x^*_{jg} = 1$ and $x^*_{j'g'} = 0$, which also satisfies the inequality (9). This completes the proof. □

Suppose that we have an optimal solution to the LP-relaxation of FRP such that $x_{if} = x_{jg} = x_{j'g'} = 0.5$. In this case, constraint (4) is satisfied by $u_{ijfg} = u_{ij'fg'} = u_{jj'gg'} = 0$, while inequality (9) becomes $0.5 \le u_{ijfg}+u_{ij'fg'}+u_{jj'gg'}$ Here, we see that $u_{ijfg} = 0.5$ and $u_{ij'fg'} = u_{jj'gg'} = 0$ if $q_{ij|f-g|} \le \min\{q_{ij'|f-g'|}, q_{jj'|g-g'|}\}$. Although we can define inequality (9) over a clique inducing node set rather than just a triangle, we do not consider a clique inducing node set $C \subseteq N$ with $|C| \ge 4$ to separate the inequality of type (9) since the inequality (9) is not rarely violated by the fractional optimal solution of FRP if $|C| \ge 4$.

4. Tabu Search

Motivated by the successful application of tabu search to MI-FAP (Tiourine et al., 2000) that is closely related with the problem FRP, we develop an effective tabu search algorithm tailored to the problem FRP considering both frequency drop cost and interference cost. The proposed tabu search algorithm has some characteristics compared with that developed for MI-FAP (Tiourine et al., 2000). First, we allow solution infeasibility temporarily in terms of constraint (1), which turned out to be effective when dealing with frequency drop cost. Second, we employ a list consisting of elite solutions, from which we select one and resume the search process. Also, we allow the implementation of tabu moves probabilistically. To describe the proposed tabu search algorithm precisely, we define some notations.

Let us denote the current solution by (x^o, y^o, u^o). We define $F^o(i) = \{f \in F : x^o_{if} = 1\}$ as a set of frequencies currently assigned to BS $i \in N$. Also, we define $\{F^o(i) : \forall i \in N\}$ and Z^o as the current solution and its objective value, respectively.

4.1 Initial Procedure

Initialize : Let $F^o(i) = F(i)$ for all $i \in N$.

Step 1 : Pick an arbitrary BS $i \in N$ such that $F^o(i) < b(i)$, and add a frequency $f \in F \backslash F^o(i)$ with minimum interference cost to $F^o(i)$. If $F^o(i) \geq b(i)$ for all $i \in N$, go to Step 2. Otherwise, repeat this step.

Step 2 : Find a minimum D[i, f], where D[i, f] denotes the change of total cost by dropping frequency $f \in F^o(i)$ from BS $i \in N$, for BSs $i \in N$ such that $F^o(i) > b(i)$, and go to Step 3.

Step 3 : If D[i, f] < 0, delete frequency f from $F^o(i)$, and go to Step2. Otherwise, stop.

Next, we describe moves and tabu list. Also, we consider elite solution list (ESL) that would effectively guide moves to an unexplored search space.

4.2 Moves

We consider two types of move, *improving move* and *random move*. We define improving move to change the frequency $f \in F^o(i)$ to a new frequency $g \in F \backslash F^o(i)$ for some $i \in N$ such that the resulting solution reduces the total cost by at least *MinDelta* (%), where such a triple $\langle i, f, g \rangle$ is chosen at random. Also, we define random move that changes a frequency $f \in F^o(i)$ to a new frequency $g \in F \backslash F^o(i)$ for some $i \in N$. If the resulting solution by a random move increases the total cost of the previous solution by more than *MaxDelta* (%), we restore to the previous solution. Note that two moves defined above are very simple. However, we attempt to find good quality feasible solutions by calibrating parameters adaptively under the guide of tabu list and by employing restarting mechanism that seeks to find a high quality solution from an elite solution at different configuration of neighborhood.

4.3 Tabu List

We consider a tabu list consisting of recently executed *MaxTL* moves. That is, tabu list is augmented at every implementation of improving and random moves. However, we examine tabu list only when we evaluate random moves. An interesting strategy for evaluating random moves with reference to tabu list is that we prohibit a random move in the tabu list probabilistically from execution. That is, even if a random move matches with an element in the tabu list, we allow the execution of this random move if the value generated from a uniform distribution in the range [0, 1] is less than *ProbTL*. The reason we employ probabilistic tabu list is that we need not prohibit all the moves in the tabu list to escape from the loop of local optima. For example, suppose that we have just performed a series of moves $a \rightarrow b \rightarrow c \rightarrow d$ ($\rightarrow a$). In most cases, we can escape from this loop of moves by prohibiting some of these moves. If we prohibit only two moves, for example, b and d, which means that the random values associated with theses two moves are greater than *ProbTL*, we may find a new direction of moves by allowing a tabu move a temporarily, for example, $a \rightarrow e$, from which we may reach unexplored search space.

4.4 Elite Solution List

In our tabu search algorithm, we employ an elite solution list (ESL) consisting of up to *MaxESL* best solutions. More than one solution in ESL can have the same objective value if they are different each other in terms of $\{F^{\circ}(i) : \forall i \in N\}$. The *ESL* is updated at every execution of improving move. That is, if the current solution is better than the worst solution in ESL and if the cardinality of ESL equals *MaxESL*, we replace the worst solution in ESL with the current solution. The management strategy of ESL enables us to improve the average quality of feasible solutions in ESL as the computing time allowed for our heuristic algorithm increases.

4.5 Overall Procedure

Define $\{F^{*}(i) : \forall i \in N\}$ and Z^{*} as the best solution and its objective value, respectively, among the solutions in ESL. Below, we describe overall procedure for improving the current solution.

Step 0 (Initialize) : Obtain a feasible solution $\{F^{\circ}(i) : \forall i \in N\}$ and its objective value Z° by performing the InitialProcedure. Set *MinDelta* = 5% and $ESL = \{\{F^{\circ}(i) : \forall i \in N\}\}$. And, let *IterCnt* = 0.

Step 1 : Perform improving moves with *MinDelta* until Z° is not reduced, and go to Step 2.

Step 2 : Set *MinDelta* = *MinDelta* × 0.8. If *MinDelta* × Z° is less than the minimum unit cost (for example, 1), go to Step 3, else, go to Step 1.

Step 3. : If Z° is updated in Steps 1 and 2, set *IterCnt* = 0, else, set *IterCnt* = *IterCnt*+1. If *IterCnt* = *MaxIterCnt*, go to Step 6, else, calculate Z°_{Q} denoting the interference cost of the current solution and go to Step 4.

Step 4 : Pick an arbitrary pair of BS $i \in N$ and frequency $f \in F^{\circ}(i)$ incurring interference, and delete the frequency f from $F^{\circ}(i)$. Repeat this frequency deletion until the total interference cost of the resulting (infeasible) solution is less than or equal to $Z^{\circ}_{Q} \times$*MinInter* (%), and go to Step 5.

Step 5 : For an arbitrary BS $i \in N$ such that $|F^{\circ}(i)| < b(i)$, add at least $b(i)-|F^{\circ}(i)|$ frequencies considering both cost factors p and q (by optimally solving the problem Add(i) defined below), and update $F^{\circ}(i)$. Repeat this frequency addition until the feasibility of the solution is satisfied ($|F^{\circ}(i)| \geq b(i)$ for all $i \in N$). Update *ESL* with this solution. Set *MinDelta* = 5%, and go to Step 1.

$$\text{Add}(i) : \text{Minimize } \sum_{k \leq |F(i)|} p_{ik}\, y_{ik} + \sum_{j \in N : (i,\, j) \text{ or } (j,\, \text{I}) \in N} \sum_{g \in F^{\circ}(j)} \sum_{f \in F : |f-g| < r(i,\, j)} q_{ij|f-g|}\, x_{if}$$

Subject to (1)~(3) and (5).

Step 6 : If time limit expired, stop, else, pick an arbitrary solution from *ESL*. Denote this solution and its objective value by $\{F^{o}(i) : \forall i \in N\}$ and Z^{o}, respectively, and perform random moves for *MaxRM* times. Set *MinDelta* = 2% and *IterCnt* = 0, and go to Step1.

Note that random moves are executed only in Step 6 for a solution randomly selected from ESL. Although we could perform improving move → random move → improving move to diversify the neighborhood of the current solution, preliminary test exhibits that this type of procedure is not effective for improving the solution. Instead, by performing feasibility recovering procedure (Step 5) after partially destroying the feasibility (Step 4) between successive improving move procedures (Step 1), we could effectively enhance the solution. Also, note that we resort to restarting scheme (Step 6) only when we cannot improve the current solution for a number of move iterations. That is, we perform random moves for a solution in the ESL to guide the improving move (Step 1) to a new direction.

4.6 Parameter Tuning

There are eight parameters : *MinDelta* for improving move, *MaxDelta* and *MaxRM* for random move, *MinInter* for frequency deletion, *MaxTL* and *ProbTL* for tabu list and *MaxESL* for ESL, *MaxIterCnt* for feasibility destroy and recover procedure. They are tuned as follows.

- *MinDelta* : Note that *MinDelta* is determined adaptively as described in Steps 1 and 2 of the Improving Procedure : If *MinDelta* is set to a large value (for example, 10%) initially, we may waste computing time without improving the current solution until *MinDelta* is adjusted to a smaller value. On the other hand, if we set initial *MinDelta* to a very small value (for example, 0.1%), we may consume long computing time for improving moves with marginal improvement. We found that setting *MinDelta* = 5% at Step 0 and to 2% at Step7, respectively, can be a reasonable compromise in terms of computing time and solution quality obtained by improving moves. The reason we set *MinDelta* = 2% at Step 6 (smaller value than that being set at Step 0) is due to the observation that any solution in *ESL* is rarely improved by the same amount with the improvement obtained by a single improving move when starting from the initial solution.
- *MaxDelta* and *MaxRM* : The reason we perform random moves is to diversify the solution. However, we need to prevent the current solution from becoming too aggravated. In this context, we have set *MaxDelta* = 1% and *Max RM* = 5 times. Setting *MaxDelta* and *MaxRM* to large values is not recommended since, in that case, we observed that the solution quality highly fluctuates and Z^{*} is slowly updated.
- *MinInter* : We have set *MinInter* = 50% since we have generated test problems such that there is no initial interference among $F(i)$'s for $i \in N$ and that average value of $b(i)$'s is slightly increased compared with that of $|F(i)|$'s. However, note that as we decrease *MinInter* the range of solution quality of the recovered solution increases. Thus, we should be careful when setting *MinInter*, in particular, if we deal with a network having large demands of $b(i)$.
- *MaxTL* and *ProbTL* : The size of *MaxTL* affects the memory consumption, solution quality and computing time. However, we focused primarily on the solution quality and computing

time. In this paper, we have set $MaxTL = |N| \times |F|$. And, we have set $ProbTL = 0.5$. That is, the probability we perform a random move belonging to the tabu list is 50%.

- *MaxESL* : If *MaxESL* is set to a large value or to a very small value, Z^* is slowly updated. In consideration of computing time and solution quality, we have set $MaxESL = \lfloor |N|/2 \rfloor$.
- *MaxIterCnt* : We set $MaxIterCnt = 2$. There is no significant difference in terms of solution quality for a given time limit whether we set *MaxIterCnt* to a large value or not.

5. Computational Results

In this section, we report computational results of the proposed solution procedure. Test problems are generated as follows.

Step 1 : (Generate a network). Generate $|N|$ BSs on a square with dimension 1000 by 1000 at random, and calculate the distance $D(i, j)$ for all pairs of BSs i and j $(> i) \in N$. If $D(i, j) > R\times 1000$, set $r(i, j) = 0$ and otherwise, set $r(i, j) = \lceil 3\times(400-D(i, j))/400 \rceil$ for all i and j $(> i) \in N$, where R (< 1) is the minimum value that guarantees the connectivity of the resulting network. The R can be found by trial-and-error.

Step 2 : (Generate $F(i)$). Pick a BS at random, and assign the lowest index frequency in F not incurring any interference with other frequencies assigned to adjacent BSs. Repeat the above procedure until further assignment of frequency to any BS is not possible. Let $F(i)$ denote the set of frequencies assigned to BS $i \in N$.

Step 3 : (Set $b(i)$). Set $b(i) = |F(i)|/2\times(1+Uniform[0, 1])$ if $|F(i)| > 0.9\times AvgF$, where $AvgF$ is the average of $|F(i)|$ over $i \in N$. Otherwise, set $b(i) = |F(i)|+AvgF\times Uniform[0, 1]$.

Step 4 : (Set cost factors). Set $p_{i1} = \lceil |F|/|F(i)| \rceil$ and $p_{i|F(i)|} = |F|$ for all $i \in N$. And, set $p_{ik} = p_{i1}+(p_{i|F(i)|}-p_{i1})/\log(|F(i)|)\times k \rceil$ for $k = 2, \cdots, |F(i)|-1$ and $i \in N$. Also, set $q_{ij|f-g|} = 10+50\times r(i, j)/(|f-g|+1) \rceil$ for all $(i, j) \in E$, f and $g \in F$.

The coding was done in C and all runs were made on a Pentium IV 3.2 GHz PC, 2GB RAM with CPLEX version 10.0 as a LP/MIP solver. We report computational results of 60 test problems in <Table 1>~<Table 3>. The following notations are used in <Table 1>~<Table 3>.

- P : model FRP,
- Pe : model FRP enhanced by *optimality* cuts (7) and (9) and with inequality constraint (1) replaced by equality constraint (see Remark 1).

For all test problems displayed in <Table 1>~<Table 3>, we have interrupted the CPLEX optimization procedure in 7,200 CPU seconds. Also, we have run the tabu search algorithm for only 300 seconds. The "Ratio" of the proposed tabu search solution is calculated as follows.

Ratio = 100%×(Tabu upper bound−Min{Upper bound P, Upper bound Pe})/Min{Upper bound P, Upper bound Pe}).

From <Table 1>, we see that the LP-relaxation lower bound of Pe is far better than that of P. Also, we find better than (or equally good) and worse solutions using the enhanced model Pe for six and thirteen problems, respectively, out of twenty, 〈Table 1〉 Computational results : N = 20, F = 20

No	LP relaxation lower bound			Upper bound			Ratio (%)	Elapsed time (seconds)		
	P	Pe	Best	P	Pe	Tabu		P	Pe	Tabu
1	889	1170	1215	1215	1215	1230	1.23	2740	18	300
2	674	1222	1547	1600	1565	1585	1.28	7200	7200	300
3	1099	1357	1470	1470	1470	1470	0	1601	32	300
4	774	1256	1480	1520	1480	1490	0.68	7200	3900	300
5	680	1039	1350	1415	1350	1370	1.48	7200	7200	300
6	1495	1793	1840	1840	1840	1840	0	164	4	300
7	544	799	920	920	920	935	1.63	622	1006	300
8	673	1029	1120	1120	1120	1125	0.45	5234	247	300
9	655	984	1035	1035	1035	1040	0.48	7200	84	300
10	285	451	555	555	555	570	2.7	98	888	300
11	1431	1821	1895	1895	1895	1895	0	6314	119	300
12	805	1165	1355	1355	1355	1370	1.11	6101	454	300
13	909	1427	1650	1670	1650	1655	0.3	7200	7200	300
14	500	762	1020	1025	1025	1050	2.44	7200	7200	300
15	979	1415	1631	1800	1760	1810	2.84	7200	7200	300
16	763	1024	1100	1100	1100	1100	0	245	54	300
17	590	816	955	955	955	965	1.05	1337	339	300
18	680	960	1280	1495	1505	1510	1.0	7200	7200	300
19	1210	1497	1560	1560	1560	1565	0.32	17	1	300
20	684	1097	1210	1255	1210	1210	0	7200	7200	300

when compared with the solutions based on the original model P. This implies that applying the optimality cuts (7) and (9) along with replacing the inequality constraint (1) by equality constraint can be a viable approach to find a better feasible solution within limited computing time. Note that the computing time elapsed to find an optimal solution is significantly reduced for eleven problems by applying the optimality cuts (7) and (9) although there are some exceptions, i.e., problems #7 and #19. Also, the proposed tabu search algorithm finds good quality feasible solutions consuming smaller computing times. Note that the tabu search solution is better than or equal to the best CPLEX solution for five test problems out of twenty (Ratio $\leq$ 0), and the worst Ratio does not exceed 2.84%.

We conduct additional experiments for larger test problems. They are displayed in <Table 2> and <Table 3>. Similar to the results of <Table 1>, we see that the LP-relaxation lower bounds of Pe are also quite tight. And, we find better or equally good solutions using the enhanced model Pe for all test problems in <Table 2> and for seventeen test problems out of twenty in <Table 3>, when compared with the solutions based on the original model P. Also, the proposed tabu search algorithm outperforms the CPLEX optimization procedure based on the models P and Pe in terms of upper bound and computing time. Note that the proposed tabu search algorithm finds better or

〈Table 2〉 Computational results : N = 30, F = 20

No	LP relaxation lower bound			Upper bound			Ratio	Elapsed time (seconds)		
	P	Pe	Best	P	Pe	Tabu		P	Pe	Tabu
21	942	1498	1695	1745	1695	1710	0.88	7200	6333	300
22	1187	1924	2160	2310	2160	2160	0	7200	7200	300
23	928	1339	1684	1920	1865	1815	2.68	7200	7200	300
24	752	1009	1190	1195	1190	1190	0	7200	7200	300
25	993	1416	1726	1910	1860	1850	0.54	7200	7200	300
26	803	1378	1670	2090	1890	1850	2.12	7200	7200	300
27	1067	1647	1820	1895	1850	1830	1.08	7200	7200	300
28	1088	1676	1765	1775	1765	1780	0.85	7200	179	300
29	546	890	1135	1165	1145	1190	3.93	7200	7200	300
30	1307	1805	1975	2000	1975	2035	3.04	7200	7200	300
31	1421	2195	2870	2960	2890	2910	0.69	7200	7200	300
32	623	887	1005	1005	1005	1015	1.0	5905	365	300
33	640	1015	1120	1125	1120	1135	1.34	7200	492	300
34	748	1129	1220	1335	1335	1270	4.87	7200	7200	300
35	989	1344	1510	1635	1530	1530	0	7200	7200	300
36	936	1378	1470	1575	1505	1485	1.33	7200	7200	300
37	1577	2165	2290	2355	2290	2295	0.22	7200	63	300
38	971	1337	1880	1960	1905	1885	1.05	7200	7200	300
39	1019	1590	1810	1870	1840	1810	1.63	7200	7200	300
40	927	1448	1590	1605	1590	1590	0	7200	3766	300

equivalently good feasible solutions for twelve test problems out of twenty in <Table 2>, and for all test problems in <Table 3>, when compared with those of P and Pe. The maximum Ratio does not exceed 3.93% for test problem in <Table 2>. While note that Ratio ranges from -9.54% up to -60.5% for the largest test problems considered in this paper as displayed in <Table 3>.

From <Table 1>~<Table 3>, we see that

(a) the model Pe enhanced by optimality cuts (7) and (9) and by the equality constraint (6) outperforms the original model P in terms of solution quality,

(b) the proposed tabu search algorithm performs similar to the model Pe in terms of solution quality for small size problems, while consuming far smaller computing times,

(c) the proposed tabu search algorithm finds far better feasible solutions for larger test problems, when compared with the enhanced model Pe, reducing the total cost at least 9.54% up to 60.5%.

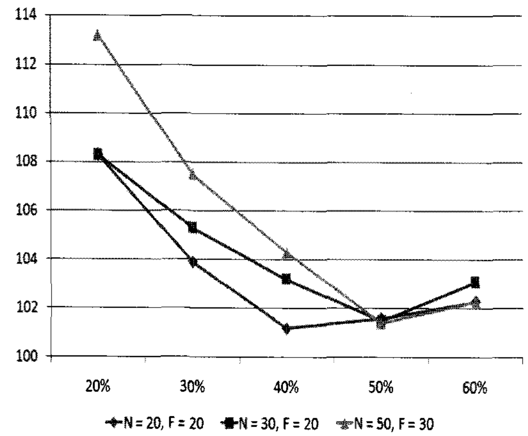
Now, we focus on the behavior of the tabu search algorithm for some combinations of parameters. Although there are 8 parameters affecting the performance of the tabu search algorithm, we investigate the influence on the performance of the tabu search algorithm for only four parameters, *MinIn*

<Table 3> Computational results : N = 50, F = 30

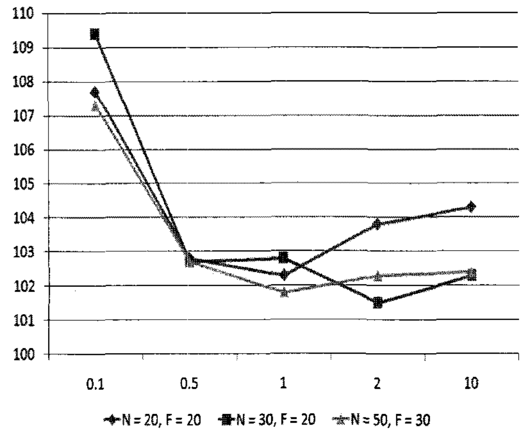
No	LP relaxation lower bound			Upper bound			Ratio	Elapsed time (seconds)		
	P	Pe	Best	P	Pe	Tabu		P	Pe	Tabu
41	264	580	590	6429	7011	2634	59.1	7200	7200	300
42	256	548	553	4190	3480	2832	18.6	7200	7200	300
43	404	1087	1141	3090	3090	2194	28.9	7200	7200	300
44	669	1692	1794	5536	4079	2204	45.9	7200	7200	300
45	363	856	898	3223	2716	1924	29.1	7200	7200	300
46	194	386	391	2905	2905	2042	29.7	7200	7200	300
47	277	610	616	5611	5463	2743	49.8	7200	7200	300
48	287	682	764	4166	4034	2364	41.4	7200	7200	300
49	349	894	942	3127	3004	1937	35.5	7200	7200	300
50	347	859	928	5314	4182	1829	56.3	7200	7200	300
51	355	910	1003	2494	2302	1956	15.1	7200	7200	300
52	306	759	802	3433	3321	2012	39.4	7200	7200	300
53	260	549	552	5887	5825	2956	49.2	7200	7200	300
54	263	553	559	6450	6450	2743	57.4	7200	7200	300
55	178	372	382	5078	5023	2258	55.1	7200	7200	300
56	416	929	996	4242	3145	2845	9.54	7200	7200	300
57	510	1568	1607	8037	3028	1933	36.2	7200	7200	300
58	256	546	554	5522	5590	2351	57.4	7200	7200	300
59	244	504	506	4708	4700	1857	60.5	7200	7200	300
60	342	816	842	2868	2892	2201	23.2	7200	7200	300

ter, *MaxTL*, *ProbTL* and *MaxESL*, since the performance of the tabu search algorithm is not sensitive to the other 4 parameters, or they are determined adaptively. In [Figure 2]~[Figure 5], we compare the tabu search solutions obtained by running 300 seconds for varying parameters of *Min Inter*, *MaxTL*, *ProbTL* and *MaxESL*, respectively, where x-axis represents the parameter, and y-axis represents the percentage ratio of the solution associated with the parameter value to the best tabu search solution. Since we consider test problems of three different classes in size as displayed in <Table 1>~<Table 3>, we present average percentage ratio for each problem size.

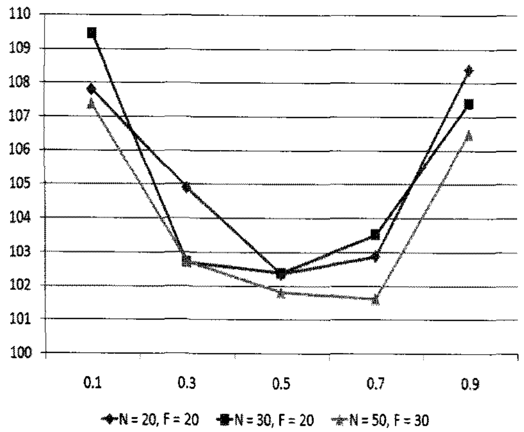
Note that average percentage ratio against varying *MinInter* exhibits similar pattern for all problem size as displayed in [Figure 2]. Although the average percentage ratio is minimum when *Min Inter* = 40% for N = 20 and F = 20, the overall performance is best when *MinInter* is around 50%. Also, in [Figure 3], note that the overall performance of average performance is best when *MaxTL* = $|N| \times |F|$, although there is an exception when N = 30 and F = 20. In [Figure 4], we see that *ProbTL* = 0.5 seems the best parameter, although the average percentage ratio is best for other value *ProbTL* = 0.7 for problem size of N = 50 and F = 30. While, in [Figure 5], we see that *MaxESL* = $\lfloor |N|/2 \rfloor$ always outperforms other values of *MaxESL* regardless of problem size.



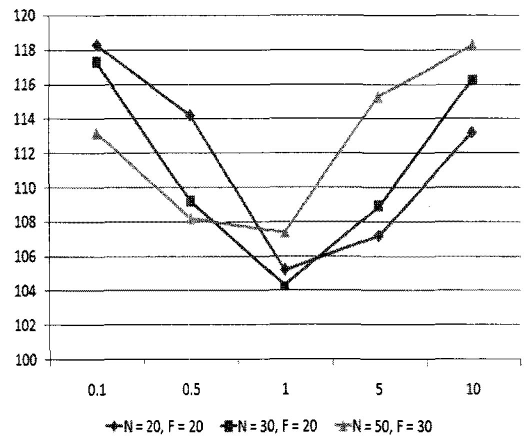
[Figure 2] Percentage ratio of the solution to the best solution for *MinInter* = {20%, 30%, 40%, 50%, 60%}, where *MaxTL* = |N|×|F|, *ProbTL* = 0.5 and *MaxESL* = ⌊|N|/2⌋



[Figure 3] Percentage ratio of the solution to the best solution for *MaxTL* = |M×|F|× {0.1, 0.5, 1, 2, 10}, where *MinInter* = 50%, *ProbTL* = 0.5 and *MaxESL* = ⌊|N|/2⌋



[Figure 4] Percentage ratio of the solution to the best solution for *ProbTL* = {0.1, 0.3, 0.5, 0.7, 0.9}, where *MinInter* = 50%, *MaxTL* = |M×|F| and *MaxESL* = ⌊|N|/2⌋



[Figure 5] Percentage ratio of the solution to the best solution for *MaxESL* = ⌊|N|/2⌋ × {0.1, 0.5, 1, 5, 10}, where *MinInter* = 50%, *MaxTL* = |M×|F| and *ProbTL* = 0.5.

6. Conclusions

In this paper, we considered a frequency reassignment problem arising from the reconfiguration of radio networks. For this problem, we developed an IP model. Also, we developed two optimality cuts in order to derive tight lower bounds. For solving large problem instances, we developed an effective tabu search algorithm. Computational results show that the developed cutting planes improve the lower bound significantly and are effective for reducing the computing time to find an optimal solution. Also, we observed that the proposed tabu search algorithm finds better or equally good feasible solutions using far less computing time when compared with the CPLEX optimization procedure.

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◈ 저 자 소 개 ◈

Junghee Han (jhhan@kangwon.ac.kr)

He received B.A, M.A. and Ph.D. degrees from Korea University, Dept. of Industrial Engineering. Then, he worked at LG Electronics, WCDMA R&D Center as a senior researcher. He is currently a associate professor at Kangwon National University, Dept. of Business Administration. His research interests include optimization of telecommunication networks, scheduling, forecasting, and so forth.