대한수학교육학회지 수학교육학연구 제 20 권 제 4 호
Journal of Educational Research in Mathematics
Vol. 20, No. 4, 477-491. Nov 2010

Mathematics as Engaged Practice: Professional Mathematicians' Conceptions of Mathematics

Ju, Mi Kyung*

This research took an interpretive approach to investigate professional mathematicians' conception of mathematics, particularly focusing on their beliefs about the nature of mathematics as a discipline, and the relation between the discipline and themselves as knowers. The analysis shows that the professional mathematicians consider mathematics as human practice. For mathematicians, mathematics as a product is considered as a crystalization of practice that emerges in the dialogical relation between the discipline and its practitioners. This dialogical nature of mathematics suggests that professional mathematicians consider mathematics not as isolated fixed knowledge but as something they are playfully engaged with. The results of this research extend our understanding of what mathematics is and provide an alternative perspective on mathematics to make the learning of mathematics more accessible by dismantling the myth of the rationalist pure objectivity in mathematics.

I. Introduction

The results of international achievement assessments such as TIMSS and PISA have shown that our students' affective attitudes toward mathematics are disproportionally low compared to their academic achievement (Kim, et al., 2008; Mullis, et al., 2008; OECD, 2007). This discrepancy should be taken seriously due to its possible harmful impacts. Negative attitudes to mathematics could lead to avoidance of mathematics and create low self-esteem or low confidence in mathematics, which in turn would consolidate their negative attitudes. This vicious circle would consequently trigger a shortage in the supply of qualified mathematics specialists for the academic and economic development of the nation. Furthermore, the tendency to avoid mathematics would contribute to keeping the majority of society mathematically illiterate and as a result, to having them marginalized in the mathematised culture of modern society. In this regard, educational efforts to improve students' attitudes to mathematics is critical to support every student to develop as a whole person with positive self-esteem and to participate in society as a mathematically literate citizen. Therefore it is one of the pending issues for mathematics educators to seek for remedies to improve our students' attitudes toward mathematics.

This research is based on the assumption that one of cures for students' negative attitudes to mathematics is to reconstruct their beliefs to view mathematics as more accessible knowledge by providing alternative conceptions of the discipline. Researchers note that those negative public images of mathematics have been developed largely through mass media without reference to adequate role models of mathematicians (Brown & Porter, 2001). Therefore, it is of essence

* Hanyang University, mkju11@hanyang.ac.kr

to provide role models to extend our understanding of what is mathematics (Brown & Porter, 2001; Ernest, 1996; Lim, 2002; Renssa, 2006). For the purpose, this research has been conducted as an inquiry into conceptions of mathematics held by professional mathematics working at a university.

However, it is necessary to note that this selection does not suggest that the academic mathematics are given the priority over mathematics produced by other groups of people. Anthropological studies have shown that mathematics is sociocultural knowledge constructed in the cultural context of communal practce (D'Ambrosio, 1985; Lave, 1988; Nunes, 1992; Powell & Frankenstein, 1997; Scribner, 1977). In this perspective, academic mathematics is one of the cultural mathematics system, that is, an ethnomathematics produced by a group of professional mathematicians (Restivo, 1994). Professional mathematicians are considered as a group of interest in the regard that mathematics is the object of their profession and as a consequence, professional mathematicians are intensely involved with mathematics. Furthermore, since the academic mathematics provides the disciplinary foundation to the school mathematics curricula, it would be of importance to investigate the professional mathematicians' conceptions of mathematics.

Therefore, this research took an interpretive approach to professional mathematicians' conceptions of mathematics. Specifically, this research sought for an understanding of how professional mathematicians think of mathematics by listening to their own narratives about the nature of mathematics and their relationship with the discipline.

II. Conceptions about Mathematics

Lloyd & Wilson (1998) defined conceptions as a person's general mental structures that encompasses knowledge, beliefs, understandings, preferences, and views. Thom- pson (1992) defined conception of mathematics as a set of certain beliefs about mathematics. While these scholars treated beliefs as a subset of conceptions, some scholars characterize "beliefs" as a broader notion. For instance, Richardson (1996) defined "beliefs" as "psychologically held understandings, premises, or propositions about the world that are felt to be true" (p. 103). Literature shows that scholars identified different constituents of a belief system so that beliefs includes broad categories of related characteristics. For instance, Furinghetti & Pehkonen (2002) argued that beliefs about mathematics consists of beliefs on the nature of mathematics as such, beliefs on the subject of mathematics, beliefs on the origins of mathematical tasks. McLeod (1992) said that beliefs about mathematics include beliefs about mathematics as a discipline and beliefs about mathematics learning, which Underhill (1988) distinguished. As these cases suggest, the notion of beliefs about mathematics is not so strictly defined. Also it is a rather broad category of characteristics like conception. Thus, in this research, the terms such as "beliefs" and "images" are used interchangeably with the term "conception".

Beliefs about mathematics have been one of important issues in mathematics education as researchers have revealed how beliefs function as an influential factor in mathematics learning and problem solving (Leder, Pehkonen, & Torner, 2002; McLeod & Adams, 1989). For instance, Schoenfeld (1985) identified beliefs as one of the factors for successful problem solving. In their research of gender difference in mathematics, Fennema and Sherman (1977) argued that a student's beliefs about mathematics is a significant indicator for successful mathematics learning. In general, it is argued that Students' beliefs about mathematics create a context within which emotional responses to mathematics develop and kind of knowledge that students eventually learn about mathematics (McLeod, 1992).

In addition to students' learning mathematics, researchers have reported that there is a higher degree of consistency between a teacher's beliefs about mathematics and one's teaching practice (Leder, Pehkonen, & Torner, 2002; McLeod & Adams, 1989). Thompson (1992) noted that mathematics teachers' beliefs shape their practice of teaching at a fundamental level and as a consequence, students' beliefs about mathematics. For instance, if a teacher assumes that mathematics is a discipline full of accurate results and infallible procedures, then his/her teaching would focus on clear presentation of procedures and skills and place undue emphasis on the manipulation of symbols. Ernest (2009) contrasted two popular images of mathematics: "traditional image" and "humanistic image". By "traditional image", Ernest meant the conception of mathematics as a system of objective and abstract knowledge, in other words, being isolated from human-being, in the sense, being super-human, asocial, acultural, apolitical and absolute. On the contrary, "humanistic image" depicts mathematics as corrigible and revisable knowledge that is originated from human inquiry. Based on this distinction, Ernest (ibid.) explained the link between the conception of mathematics and the teaching and learning of mathematics by discussing how different kind of teaching practice each conception of mathematics would lead to.

Therefore, beliefs about mathematics held either by students or by teachers play critical influence on their experience with mathematics in terms of learning or teaching of mathematics, respectively. Moreover, the interface between students' beliefs about mathematics and teachers' beliefs about mathematics suggests that beliefs are shaped in social context (Op't Eynde, de Corte, & Verschafel, 2002; Yackel & Rasmussen, 2002). Specifically, while each student is committed to a certain belief system about mathematics depending on one's interests in and experience with mathematics, a student's belief system of mathematics is also influenced by teachers, parents, peers, and mass media. In this respect, it is of importance to reflect upon the question of what kind of public images communicated in society. Unfortunately, in our society, negative images and myths of mathematics are widespread. Mathematics is often considered as a subject matter that is difficult. It is considered as a well-stocked warehouse of ready-to-use formula and skills. These formula and skills are fixed and universal. Most students believe that there always is only one right way to solve a mathematical problem, which is most often given by a teacher. It is only the intelligent few who can access that right way without teachers' help. Also, mathematics is computation, which is considered as uncreative since it is simply about reproducing something already known (Ernest, 1996; Lim, 2002; Renssa, 2006).

While the public images of mathematics depict the discipline as difficult, cold, abstract, masculine, immutable, external, intractable, or uncreative, some research adapted professional mathematicians's point of view to challenge the negative public images of mathematics. In the research, mathematics is considered as a discipline full of intuition, insight, and aesthetic qualities. In his critique of the new math movement driven by the modernists, Thom (1973) emphasized the person-ness of mathematics. He contended the formalist views of mathematics as arguing that mathematics cannot be reduced to de-personalized formalization with rigor. Thom (ibid.) argued that each step of mathematical reasoning is driven by a mathematician's choice out of a large number of possibilities and that the choice is guided by the mathematician's own intuitive interpretation of a mathematical context under inquiry. So the rigour of a proof depends on how comprehensible each step of its arguments is. In other words, the rigour in mathematics is rather a matter of sense-making than that of formality. From his point of view, formalization in mathematics results from the fundamental diversity of the world of mathematical meaning constructed by an individual mathematician in order to help meaningful communication among mathematicians of different backgrounds.

Hersh (1986) also argued that mathematics deals with ideas. Mathematics is invented by creative humans working on existing mathematical objects. Therefore, as he said, mathematicians are working on a plenty of uncertainty instead of absolute truth. In the bibliographic research of a famous mathematician Marston Morse, Snow and Hoover (2010) argued that logic is subservient to inspiration and intuition in mathematical inquiry. Following the insight by inspiration and intuition, a mathematician chooses one path for beauty's sake to find a proof and puts the idea in a logical form. Tymoczko (1986) offered a community-centered theory of mathematics to argue that mathematicians as social beings have internalized the social roles of mathematics to participate into the communal enterprise of mathematical inquiry. This participation is a dialogical process in which the different mathematicians play different roles. Proofs grow in response to dialogical conversation among the participants concerned with making conjectures and criticism, generalization, application, reinterpretation and aesthe- tic considerations. Burton (1995) adapted a feminist epistemology for the reconsideration of mathematics as a discipline with person-relatedness, full of aesthetics and intuition, celebrating diversity and globality.

These epistemological debates concerning the nature of mathematics provide perspectives that portrays mathematics as inclusive, accessible and encompassing of a wide range of styles of understanding and of doing. However, these are theory-based discourse on mathematics and the negative images of the discipline is still pervasive in our society. Then, how can we overcome the gap between the two contrasting perspectives on mathematics? Where are these negative images of mathematics and mathematicians originated from? Most people develop their images of mathematics and mathematicians through pictures, teachers or television. In particular, mass media is one of the most influential in formulating public images of mathematics (Renssa, 2006). As Brown and Porter (2001) pointed out, there are lacks of role models outside the community of mathematics. Thus, the public need role models that can spread enthusiasm and explain research in mathematics in an understandable way (Ernest, 1996; Lim, 2002; Renssa, 2006).

This research approaches this task by listening to insiders' perspectives on mathematics, in other words, professional mathematicians' views on the epistemological norms and values about mathematics such as what is the relationship between knowing and knower, and how knowing is achieved. This is a significant task since students' beliefs and attitudes toward mathematics are derived from their social experience through school, peers, parents, or mass media (Ernest, 1996; Greer, Verschaffel, & De Corte, 202; Lim, 2002; Renssa, 2006). While a belief system is a construct within an individual, it is also a sociocultural system that provides complex and consistent feedback toward an individual (de Corte, Verschaffel, & Op't Eynde, 2002). Due to this interface between an individual and the social, it is important to understand what mathematics is, what mathematicians are doing, how they are doing mathematics, in order to enlighten the perception of mathematics in our society by providing a relevant model of mathematics and of mathematicians. In this respect, this research purports to investigate the beliefs about mathematics held by professional mathema- ticians, that is, their own understanding of the nature of the knowledge that they were working on, and the relation between mathematicians and mathematics.

III. Research Methods

This research was conducted to investigate conceptions of mathematics from the viewpoint of professional mathematicians, specifically, focusing on their beliefs about the nature of mathematical knowledge, the relationship between mathematics and themselves as knowers. For the purpose, this research took an interpretive approach to the data collected from a university mathematics department. The department was one of the most highly renowned research institutions of mathematics in the US. The department consisted of people of various levels of expertise in mathematics such as faculty, graduate students, undergraduate students, and visiting scholars. The department was connected with a larger mathematics community through diverse networks such as conferencing, publishing, employing, etc. Thus, the department could be considered as a microcosm of the larger community of professional mathematicians, and as a site where professional mathematicians' conception of mathematics could be investigated.

In order to address the questions of this research, various types of data had been collected through direct class observation, written documents, in-depth and open-ended interview. In this study, the interview data constitute the major part of data. 23 members of the mathematics department were interviewed: 5 faculty members and 18 Ph.D students. The interviewees were selected to represent professional mathematicians of a various expertise levels and areas of specialty in order to collect data that could represent broadly the conception of mathematics shared in the department. Indeed, there was no clear-cut definition about who could be entitled as "mathematician"in the department. Some of the Ph.D students said that they were not mathematicians because they were not conferred doctoral degrees yet. However, in this research, Ph.D students were regarded as professional mathematics since mathematics is central part of their daily life and the object of their profession.

The interviews were conducted as semi-structured. A sequence of open-ended questions were posed to ask the interviewees' experience with mathematics, their definitions of mathematics and of a mathematician, teaching philosophy in order investigate the interviewees' conceptions of mathematics from various angles. All the interviews except one were done in person about 40 minutes to 1 hour. All the interviews were audio-taped and transcribed for detailed analysis.

In addition to interview data, mathematics courses of various levels taught by a professor, who was one of the interviewees, were observed. During the first semester, the professor taught the introductory calculus course and the undergraduate mathematics course. In the second semester, the professor taught graduate mathematics course. All of his class sessions were observed and video-recorded. The class video-recordings were transcribed for analysis. Field notes were taken after the observation. Informal conversation with the professor concerning his classes were recorded and used for cross-checking the researcher's interpretation of other data.

A variety of university documents concerning the history of the university, the department curriculum and regulation had been collected. The data from university documents and class observation were analyzed to provide a broader picture of social context in which the practice of mathematics in the mathematics department was contextualized. The analysis of the data from class observation and the documents complemented and supplemented the analysis of the interview data. Specifically, the analysis focused on the questions regarding what kind of knowledge and competence were valued in the department, how to cultivate them, what kind of language codes were used to express, what kind of social needs were incorporated into the educational practice of the department, and so on.

IV. Mathematicians' Conception of Mathematics: Mathematics as Practice

Most often, people consider mathematics as a package of immutable procedural skills and abstract principles. The data analysis of this research showed that the professional mathematicians shared this notion of mathematics. The notion of mathematics as a fixed set of knowledge, or mathematics as product could be mostly observed in the department regulation and curriculum organization. The mathematics department offered courses with a specific title such as algebra, analysis, geometry, topology, etc. The department had course requirements for majors given in those concrete titles. And each of the courses consisted of a set of mathematical objects such as definitions, axioms, theorems, and procedures. University regulations required a student to acquire the body of knowledge by a certain time. The usual evidence of this achievement was a record on a university official transcript stating that a student enrolled and completed a certain course with an acceptable grade. Based on this official acknowledgement, a student could advance to a next status of learning mathematics in the department. According to the university documents, everything seemed to follow standards clearly drawing a line between what is acceptable as mathematics and what is not. In this regard, mathematics was thought of as a logically structured closed system of fixed objective knowledge independent of how people think.

However, the data analysis revealed another facet of mathematics, which could be witnessed in the following conversation:

Interviewer	:	How do you define mathematics?
Interviewee	:	How do I define mathematics? Hum...I don't think that mathematics can be defined. Mathe- matics is what we do. It is one of something that you recognize when you see it. But it is holistic.

In interview, many interviewees of the mathematics department had difficulty for answering the question of how to define mathematics. Some interviewees even refused answering the question. They thought that the question did not make sense or irrelevant since they considered mathematics as holistic of what they do. Indeed, the notion of mathematics was prevalent in the department of mathematics and most of the mathematicians endorsed this notion in the interview. More importantly, this notion of mathematics as holistic human practice entails different features of mathematics viewed as a fixed final product. Followings are these features that the notion of mathematics as practice implies.

1. Mathematics as personal interpretive practice

While mathematics is often regarded as absolute and de-personalized objective knowledge, the professional mathematicians emphasized the close relationship between mathematics and a mathematician, in particular, what kind of a question, a perspective, methods a mathematician had. In the graduate mathematics classes, there were students who had already taken those courses and revisited the identical courses taught by different lecturers. If mathematics were fixed knowledge, for what did they come back to learn the same things over again?:

> "I already took the class last semester with another professor. But I knew that the professor is teaching it again this spring. And I thought that because every professor has a different point of view. It is just like humanity. Everybody has a different point of view and different experience that they bring to any class. And I knew the professor has a great expertise in the subject. So I knew that he would invaluably have many insights and he would have wealth of experience to share with us. So although I was taking the class before, I want to sit in just to hear his point of view."

This transcript reveals that the kind of knowledge mathematicians appreciate is not confined to mathematical facts such as definitions, theorems, and principles. In this conversation, the interviewee highlighted "point of view", "insights", and "exeprience" as valuable kinds of knowledge that she could encounter in the class. When the interviewees were asked about what makes "a good mathematician", they mentioned knowledgeability in one's research area as one of criteria. However, they valued the creativity as the most critical quality of being a good mathematician.

In the interviews, there came up discussion concerning who is entitled as a mathematician. For instance, some graduate students refused the interview. Their reason was that they were not mathematicians because they haven't acuiqre a Ph.D yet. They were asked why a doctoral degree mattered to be a mathematician. They answered that it was a little sign of their contribution to the community of mathematics by offering insightful perspectives on mathematics. This tells that what is essential in doing mathematics is making a creative perspective to provide an insightful understanding of mathematics under inquiry.

In general, the interviewee depicted a mathematics class as a place where participants share their stories of mathematics. What the story delivers is not only the objective mathematical facts. More importantly, in the class, the participants were engaged with sharing the story-teller's insights out of one's own experience with mathematics. This is why the students came back. The courses with an identical title did not necessarily deal with the identical set of mathematical knowledge. Even though lecturers were dealing with almost the same contents, the way how each lecturer presented them (e.g., what was highlighted, how it was plotted, what kind of mathematical codes were used, and so forth) was different from one another and deeply situated in one's own experience with mathematics. This means that teaching is not a simple regurgitation of a set of mathematical knowledge given in textbooks or in curricula:

"The professor spends lots of lectures going over Gromov things. He certainly didn't prove everything. Even there were things he said 'OK. I am not going to prove it. This is a fact.' But he is very very good at giving the insight on why things are being told in the way they are, why the story goes in the way it goes, why the flow of logic...you know? And I understand at the end of it the student is expected to retain the larger picture and he leaves details that she can solve by herself."

In addition to the presentation of the content, the majority of time in the classes was spent for interpreting mathematical facts. The most priority of class were placed upon sharing perspectives on the mathematical object under discussion by addressing questions such as what does a concept or a theorem tell? how is it connected to a broader structure of mathematics? what is its implication for the future development of the subject? This is a kind of knowledge that is rarely found in books partly because those interpretive activities were shaped by the specific interest and by the mathematical background of an individual mathematician:

"For example, they might be algebraists. And they might be doing the subject from the standpoint of algebraists. Namely, they may focus on algebraic properties and get more of algebraic proofs."

Even though mathematics exists in an objective form with some logical connections, the connections are not simply logical since they determined by the kinds of mathematical tools, backgrounds, and taste of what is more "natural" from the perspective of an individual mathematician. This suggests that a logical configuration of mathematics system is not independent of those such as purposes, methods, and codes used by a mathematician. The system is constructed through choice made by mathematicians. In other words, a logic is a choice of a mathematician according to what s/he thinks meaningful from his/her own mathematical point of view.

2. Mathematics as emergent practice

As described so far, mathematics taught in class is different from mathematics given in "a book" or in the official descriptions of a course given in the general catalog of the university. The professional mathematicians depicted mathematics as the crystalization of what a mathematician sees, tells, thinks, in general, what a mathematician does in the context of mathematical problem solving, interpretation, and communication. So mathematics as practice were deeply entrenched into a mathematician's personal interests, purposes, back- grounds, styles, and experience with mathematics under inquiry. Therefore, although a professor planned a class before s/he came to class to teach, actual teaching was not a simple reproduction of that plan. Mathematics practiced in class was determined by the audience as well.

For instance, students might raise questions. Then, a professor had to change his plan to address the issues that the students' questions brought up. Most of the time, a professor was talking while students were listening and taking notes quitely. However, students' little head nodding, smile of appreciation, hollow silence, subtle traces of eye focus signaling curiosity, misunderstanding, or boredome influence what and how to talk about mathematics in class. Therefore, mathematics is not fixed but emerged among people in communication at a certain time and space. In this regard, it is interesting to note that the professional mathematicians often used metaphors which described mathematics as a landscape:

> "We have to start from a place that we have a problem and then you have to start walking that path."

> "On the one hand, he has presented well developed theorems so to speak. Then, he holds your hand and takes you along the path that already other people had made for you. But on the other hand, he points at the direction that people have not gone."

For mathematicians, mathematics is a landscape that they are about to walk in. At an overview, there are some roads that have been already nicely laid by other mathematicians. However, as they are getting into the world and walking along the path, mathematicians approach the details of the terrain and encounter something unknown. As the journey endlessly unfolds the new facet of the landscape somewhat unanticipated, an understanding gets sophisticated:

> "A mathematician is somehow never quite good enough for a field, which is to say that the material you're working on always has surprises for you that you can never fully understand. There's always going to be something that you don't understand."

When conceptualizing mathematical inquiry as personal engagement, this leads to the perspectives of mathematics as a structure of mathematical knowledge that is continually evolving through the engagement. Inquiry into mathematics is never an exhaustive process because the world of mathematics always has a surprise that a mathematician can never fully understand.

Mathematics as practice suggests that mathematics is not fixed but keeps changing every moment through intrapersonal and interpersonal conversation among mathematicians. Mathematical meaning is always under negotiation in the context of practice. While mathematics exists as a stream-lined system, the system is never completed and it is always subject to transformation by a mathematician exploring the wonderland in one's own way.

3. Mathematics as dialogical practice

In its description of the notions of mathematics shared in the mathematics, this paper began with contrasting two differing notions of mathematics: mathematics as a fixed set of concepts, theorems, and procedures vs. mathematics as holistic on-going practice. In the interview, most of the professional mathematicians endorsed the notion of mathematics as practice. So far, this paper has provided an argument that the notion of mathematics as practice highlights the aspect of mathematics as personal interpretive activity. However, it is important to note that this does not suggest any dichotomy over the two notions of mathematics.

In the interview, the professional mathematicians most often represented mathematics as practice full of creativity and insight. They agreed that knowledgeability is not sufficient but only necessary condition to be "a good mathematician". However, they never underestimated the aspect of mathematics as a product. In mathematics class, the mathematicians taught mathematical objects such as definitions, theorems, algorithms, computing procedures. Mathematics-major students tried to memorize definitions, algorithms, theorems and their proofs. This tells that practice of a mathematician is also involved with mathematics as a product. For instance, whiile professional mathematicians valued creativity over technical perfection in their practice of mathematics, technical development cannot be separated from developing mathematical creativity:

"Somebody comes up with some ideas and the idea itself somehow brings some form already and another whole set of questions."

As this interviewee described, a mathematician begins his/her creative inquiry with a questions based on ar a rather vague idea. Through creative mathematical investigation, the question evolves into "a form", that is, a mathematical product such as a theorem. In turn, the form initiates another mathematical inquiry to lead to another mathematical discovery, and so forth. This recursive process suggests that mathematical creativity is the origin of the formal objective mathematical proposition.

As described earlier, personal understanding of mathematics developed through a journey into the landscape. And the personal understanding is shared among mathematicians to reconfigure the structure of objective mathematics. Mathematics is not merely a closed logical network among closed nodes of mathematical objects. Rather, it is a web of subtle meanings emerging as people work through the objects. However, mathematics certainly exists in a form of a bounded logical network of discrete objects. This objective mathematics frames the mathematics communicated in the context of practice and in turn, the skeleton of literal objective mathematics is fleshed out and brought to life by personal insightful imagination of a mathematician in practice. Through interpretive activities, the meaning of a mathematical object is always renegotiated, connections become refined, and the boundary of the network is redefined. In this regard, mathematics as a product is a snapshot of mathematics as practice.

Moreover, while an individual mathematician decides how to do mathematics according to his/her own personal interests, purposes and styles, the product of his/her mathematical inquiry eventually becomes shared with other mathematicians through communication such as publication, and presentation in conference. For sharing, an individual mathematician's idea has to be translated into shared language in the community of mathematics. Therefore, formal mathematical knowledge is a language to encode a creative mathematical idea of a mathematician. In other words, mathematical creativity is firmly grounded on a structure of mathematics as a product:

"There are a number of possible combinations of axioms, for example. It's infinite. And if we make some random deduction and publish a paper, that's silly. There is so to speak a sixth sense that tells you whether something is significant or not."

A mathematical discovery is usually the object of examination in the community of mathematics. The members of the mathematics community scrutinize its logical completeness and meaningfulness of a mathematical discovery with respect to the current system of mathematics, and more importantly, its creativity and productivity for future development of mathematics, in other words, judging whether an individual mathematician's practice has something new to offer to the mathematics community. In this regard, mathematical products such as computational skills and theorems are the culmination of the cultural norms of doing mathematics. They have acquired its social status as a consequence of on-going social review based on the social norms. Thus, not only a step by step guideline to solve a particular problem, a mathematician learns communal voice behind technicality such as what it is concerned with, what is the idea behind it, what and how it suggests doing to further a mathematical idea, and how to put a creative idea in a culturally meaningful way.

Therefore, a mathematical product such as techniques become a departing point for a future practice of mathematics. Furthermore, mathematical products are cultural language to materialize creative vision of a mathematician for future practice of mathematics. This suggests that the practice of mathematics is a dialogical process. The practice of mathematics as dialogical process, on one hand, refers to the interaction among mathematicians who are engaged with a shared enterprise in mathematics. On the other hand, the dialogical process highlights the connection between a product and a practice. Mathematics as a product is a snapshot of mathematics as practice at a specific moment. Mathematics as a product is the culmination of mathematics as practice and in turn, it provides a new departing point for future practice of mathematics as well as methods and visions for future inquiry. Mathematics as practice emerges every moment and mathematics as a product become transformed through the practice of mathematics.

This dialogical relation highlights the connection between mathematics as a product and human practice, in other words, the mutually constituting relation between a mathematician and the discipline. This suggests that mathematicians as practitioners are deeply engaged with mathematics. Mathematics is not an alienated, absolute, immutable knowledge but emergent, revisable knowledge produced in the context of dialogical engagement. In this regard, the notion of mathematics as practice reveals the deep connection between the discipline and a mathematician as a human.

V. Conclusion

Academic mathematics has traditionally formed the foundation of the school mathematics curricula. Academic mathematics, as a received science in the contemporary society, is based on absolutism and objectivism, which have represented mathematics as value-neutral and transcendenthal knowledge. In the perspectives, mathematics separates an observer from the observed and alienates the product of inquiry for possession. It conveys intrusive and mechanistic image of the discipline. As this reductive conception of mathematics has traditionally dominated the educational practice of school mathematics, mathematics has been taught with a heavy reliance upon textbook. As a consequence, learners have been positioned as passive beings in mathematics class and learning leads to competition among students (Burton, 1995).

As Burton (1995) noted, those beliefs have been integral to our learning experiences of mathematics and almost impossible to construct in our imaginations alternatives to the processes which we have been taught and with which we have gained success. This suggests that it is critical for the improvement of educational practice in school to make the learning of mathematics for all students more accessible by reconsidering the objectivist beliefs about mathematics. When considering that teaching and learning practice is deeply embedded within belief system concerning the subject matter, it is necessary to critically debate on the nature of knowledge in order to transform our consciousness for the reconstruction of our lived world including schools.

In this context, this research began by problematizing this serious misrepresentation of mathematics in our school and society. Through an interpretive inquiry of professional mathematicians who are intensely involved with the discipline, this research examined the relationship between the discipline and its knowers. This inquiry provides evidence for arguing against the myth of pure objectivity in mathematics which has exercised enormous power over educational practice of school mathematics. The analysis showed that the professional mathematicians represented mathematics as holistic practice such as what people talk about, listen to, ask questions of, seek for answers for, in general, "what people do". The professional mathematicians considered a mathematician as someone who "does" mathematics rather than who simply "knows" it as an object of possession or "acquires" a degree in the discipline. In their conception of mathematics, mathematics is a discipline of the dialogue that emerges in the context of playful engagement rather than of objective investigation.

Therefore, this research provides an alternative conception of mathematics. This conception of mathematics identified in this research would change students' beliefs about mathematics as immutable and fixed truth. Also, this alternative conception of mathematics would influence students to develop their self-identity as active inquirers rather than as passive consumers of mathematics. Not only students, this findings of this research would help teachers critically reflect upon their beliefs about mathematics. Since teaching practice is deeply embedded within a teacher's own belief system, this reflection is of essence for the change of teaching practice in mathematics class to promote more meaningful learning experience of mathematics and a student's development as a mathematically literate person.

The conception of mathematics identified in this research would provide an alternative perspective to the school mathematics curriculum development. In fact, the recent national curriculum of school mathematics echoes the conception of mathematics that is presented in this research. It advocate the conception of mathematics as human inquiry and as a science of patterns with problem solving activity. Also, since the 1990s, the world-wide reform movements in mathematics education produced discourse emphasizing active participation, sense-making, and sharing in teaching and learning of mathematics. In this context, this research would contribute to actualizing the ideal of the reform movement in school challenging reductive beliefs about mathematics to extend our understanding of what mathematics really is and ultimately to make the learning of mathematics more accessible for all students .

References

Brown, R., & Porter, T. (2001). The methodology of mathematics. *European Mathematical Society Newsletter, June. Retrived September 29, 2010, from http://www.bangor.ac.uk/~mas010/methmat.html.*

Burton, L. (1995). Moving towards a feminist epistemology of mathematics. Educational Studies in Mathematics, 28, 275-291.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44-48.

Ernest, P. (1996). Popularisation: Myths, mass media and modernism. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (eds.), Interantional Handbook of Research in Mathematics Education (pp. 785-817). Drodrecht, Boston, London: Kluwer Academic Publishers.

Ernest, P. (2009). New philosophy of mathematics: Implications for mathematics education. In B. Greer, S. Mulhopadhyay, A. B. Powell, & S. Nelson-Barber (eds.), *Culturally responsive* *mathematics education* (pp. 43-64). New York & London: Routledge.

Fennema, E., & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization, and sociocultural factors. American Educational Research Journal, 14, 51-71.

Furinghetti, F., & Pehkonen, E. (2002). Rethinking characterizations of beliefs. In G. C. Leder, E. Pehkonen, & G. Torner (eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 39-57). Dordrecht: Kluwer Academic Publishers.

Greer, B., Verschaffel, L., De Corte, E. (2002). "The answer is really 4.5": Beliefs about word problems. In G. C. Leder, E. Pehkonen, & G. Torner (eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 271-292). Dordrecht: Kluwer Academic Publishers.

Hersh, R. (1986). Some proposals for revising the philosophy of mathematics. In T. Tymoczko (ed.), New direction in the philosophy of mathematics (pp. 9-28). Boston: Birkhauser.

Kim, K.-H., Kim. S.-J., Kim, N.-H., Park, S.-Y., Kim., J.-Y., Park, H.-H., & Jung, S. (2008). Characteristics of achievement trend in Korea's middle and high school students from International Achievement Assessment (TIMSS/PISA). Research Roport RRE 2008- 3-1. KICE.

Kloosterman, P. (1996). Students' beliefs about knowing and learning mathematics: Implications for motivation. In M. Carr (ed.), Motivation in mathematics (pp. 131-156). Cresskill, NJ: Hampton Press.

Lave, J. (1988). *Cognition in practice: Mind, mathematics, and culture in everyday life*. New York: Cambridge University Press.

Leder, G. C., Pehkonen, E., & Torner, G. (2002). (eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 13-37). Dordrecht: Kluwer Academic Publishers.

Lim, C. S. (2002). Public image of mathematics. *Philosophy of Mathematics Education Journal, 15*. Retrieved September 29, 2010, from http://www.people.ex.ac.uk/PErnest.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 575-596). New York: Macmillan.

McLeod, D. B. & Adams, V. M. (1989). *Affect and mathematical problem solving: A new perspective*. New York: Springer-Verlag.

Mullis, I. V. S., Martin, O. M. & Foy, P. (2008). *TIMSS 2007 International mathematics report*. IEA.

Nunes, T. (1992). Ethnomathematics and everyday cognition. In D. A. Grouws (ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 557-574). New York: Simon & Schuster Macmillan.

OECD (2007). *PISA 2006 Science competencies for tomorrow's world*. Volume 1: Anaysis. Paris: OECD Pulbication.

Op't Eynde, P., de Corte, E., & Verschafel, L. (2002). Framing students' mathematics-related beliefs: A quest for conceptual clarity and a comprehensive categorization. In G. C. Leder,

E. Pehkonen, & G. Torner (eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 13-37). Dordrecht: Kluwer Academic Publishers.

Renssa, R. J. (2006). The image of a mathematician. *Philosophy of Mathematics Educational Journal, 19*. Retrieved September 15, 2010, from http://www.people.ex.ac.uk/ PErnest.

Restivo, S. (1994). The social life of mathematics. In P. Ernest (ed.), *Mathematics, education, and philosophy: An international perspective* (pp. 209-220). London and Washington, D. C.: The Falmer Press.

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula (ed.), Handbook of research on teacher education (2nd ed., pp. 102-119). New York: Simon & Schuster Macmillan.

Scribner, S. (1977). Modes of thinking and ways of speaking: Culture and logic reconsidered. In P. N. Johnson-Laird & P. C. Wason (eds.), *Thinking: Readings in cognitive science* (pp. 483-500). New York: Cambridge University Press.

Snow, J. E., & Hoover, C. M. (2010). Mathematician as an artist: Marston Morse. *The mathematical intelligencer, 32*(2), 11-18.

Stanic, G. (1986). The growing crisis in mathematics education in the early twenties century. *Journal for Research in Mathematics Education, 17*(3), 190-205.

Stehr, N. (1994). *Knowledge society*. Thousand Oaks, CA: SAGE.

Thom, R. (1973). Modern mathematics: Does it exist?. In K. Howson (Ed.), Developments in mathematical education: Proceedings of the second international congress on mathematics education (pp. 194-209). New York, NY: Cambridge University Press.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Thompson, A. G. (1992). Teacher's beliefs and conceptions: A sysnthesis of the research. In D. A. Grouws (ed.), Handbook of research on mathematics teaching and learning (pp. 127-146). Reston, VA: NCTM.

Tymoczko, T. (1986). Making room for mathematicians in the philosophy of mathematics. *The mathematical intelligencer, 8*(3), 44-50.

Underhill, R. (1988). Mathematics learners' beliefs: A review. *Focus on Learning Problems in Mathematics, 10*, 55-69.

Yachek, E. & Rasmussen, C. (2002). Beliefs and norms in the mathematics classroom. In G. C. Leder, E. Pehkonen, & G. Torner (eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 313-330). Dordrecht: Kluwer Academic Publishers.

전문수학자의 수학에 대한 신념

주미경 (한양대학교)

본 연구는 전문수학자들의 면담을 통해 그들이 수학에 대해 가지고 있는 신념체계를 질적으로 탐구하였다. 구체적으로, 자료분석은 지식체계로서 수학에 대한 규정, 수학적 지식의 특성, 및 수학적 지식과 수학자 사이의 관계에 대한 관점에 대한 수학자들의 관점 분석을 중심으로 이루어졌다. 자료분석 결과, 전문수학자들은 수학을 고정된 지식체계로 보기보다는 수학자 개인의 해석적 실천과 주관적 몰입 과정을 통해 발생하는 지식체계로 개념화하는 것으로 나타났다. 이러한 관점에서 수학자들은 수학을 소유적인 앎의 대상이 아닌 몰입과 공유를 통해 세계에 대한 새로운 안목을 이끌어 내는 매개체로 생각하는 것으로 나타났다. 본 연구가 제시하는 수학에 대한 수학자들의 관점은 수학이 기성의 지식으로 다루어지는 학교수학에 대하여 대안적 관점을 제공하여 학생들의 수학에 대한 정의적 태도를 개선하는데 기여할 수 있을 것이라고 기대한다.

* Key Words : conception of mathematics (수학에 대한 신념), professional mathematicians (전문수학자), engagement (몰입)

논문접수: 2010. 10. 7.
논문수정: 2010. 11. 1.
심사완료: 2010. 11. 9.