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Practical Design Issues in a Linear Feedback Control System with a Notch Filter

김 려 화* · 김 영 철†
(Lihua Jin · Young Chol Kim)

Abstract - This paper presents some practical design issues that should be carefully considered when a notch filter is included in a linear feedback controller. A notch filter is generally used to compensate the effects of resonant modes that may result in poor performance. It is very common that the practical engineers prefer to add such a notch filter after having previously designed a feedback controller without the filter. It is known that the resulting performance by this approach is not seriously different from when a feedback controller is designed for a plant previously compensated by a notch filter. However, we will point out that there are some cases where both approaches have different performances. In order to show this, a low-order controller design using the partial model matching method has been applied to a linear time invariant (LTI) model. The results suggest that there is a tendency to achieve much better time responses in terms of reducing the overshoot and shortening the settling time, and in the frequency domain characteristics such as the sensitivity function and the stability margins when the design of a feedback controller after including a notch filter is carried out.

Key Words : Notch filter, Partial model matching, Frequency response, Time response.

1. INTRODUCTION

Classical circuit components are often used in the design of feedback control systems. Lead/lag compensators and notch filters constitute two such examples, with notch filters being used to reduce the excitation of resonant modes. When the plant includes lightly damped poles, the use of notch filters has been found to improve loop shaping [1], [2].

There are essentially five different configurations of a feedback system depending on where a notch filter is placed in a two-degree of freedom (DOF) controller structure [3-5]. Even though all the configurations share a common open-loop transfer function, due mainly to the effects of the controller zeros and notch filter resonances, the time responses of the corresponding closed-loop systems may be quite different. Among all these configurations, we will show that the one illustrated in Fig. 1(a) has the best time domain performance.

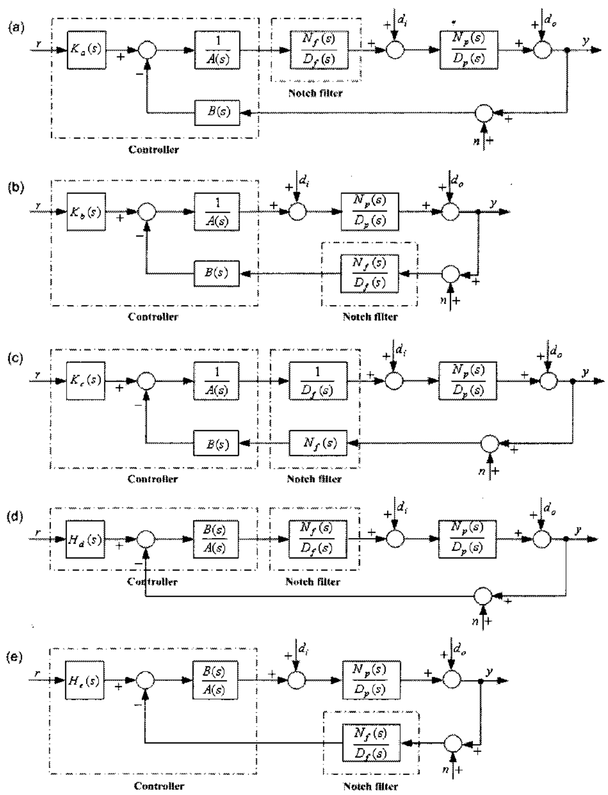

Fig. 1 The different configurations of the feedback control system.

* 정 회 원 : 충북대 전자공학과 박사과정
† 교신저자, 시니어회원 : 충북대 전자공학과 교수 · 공박
E-mail : yckim@chungbuk.ac.kr
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The control engineers in the industry often prefer to add a notch filter after having designed a feedback controller for the original plant. The expectation is that the loop transfer function obtained should be similar to the one obtained when a feedback controller is designed for a plant that has been compensated with a notch filter beforehand.

In this paper, we will investigate what differences, if any, there are between these two approaches. To this end, the partial model matching method [6] is employed to design a low-order controller. For a given fixed-order controller structure as shown in Fig. 1(a), we first select a target closed-loop transfer function that meets the desired specifications. Then the controller parameters are computed so that the resulting closed-loop transfer function approximates the target transfer function as closely as possible. This is what "partial model matching method" means in this paper. Two examples are given to compare the time and frequency domain characteristics of both approaches.

2. THE FEEDBACK CONTROL SYSTEM WITH A NOTCH FILTER

The configurations of feedback control system that use different controller structures and notch filter positions are illustrated as Fig. 1(a)-(e). The open- and closed-loop transfer functions of the feedback systems are derived and their time and frequency responses are compared later in this section.

The plant model is given by

$$G(s)=\frac{N_p(s)}{D_p(s)}=\frac{N_p(s)}{\overline{D_p}(s)R(s)}, \tag{1}$$

where $\overline{D_p}(s)$ denotes the weakly damped poles and $R(s)$ is the remainder of $D_p(s)$.

The notch filter can be described by

$$F(s)=\frac{N_f(s)}{D_f(s)}=\frac{s^2+2\zeta\omega_n s+\omega_n^2}{s^2+2\zeta^*\omega_n s+\omega_n^2}, \tag{2}$$

where ω_n, ζ, ζ^* are the undamped natural frequency, actual and desired damping ratio of resonance, respectively.

The low-order feedback controller to be designed is

$$C(s)=\frac{B(s)}{A(s)}. \tag{3}$$

If we let

$$\overline{D_p}(s)=N_f(s), \tag{4}$$

the loop transfer function and sensitivity function corresponding to Fig. 1 are given by

$$L(s)=\frac{N_p(s)N_f(s)B(s)}{D_p(s)D_f(s)A(s)}=\frac{N_p(s)B(s)}{R(s)D_f(s)A(s)}, \tag{5}$$

$$\begin{aligned}S(s)&=\frac{D_p(s)D_f(s)A(s)}{D_p(s)D_f(s)A(s)+N_p(s)N_f(s)B(s)}\\&=\frac{R(s)D_f(s)A(s)}{R(s)D_f(s)A(s)+N_p(s)B(s)}.\end{aligned} \tag{6}$$

Consequently, the resonant term $\overline{D_p}(s)$ is exactly cancelled by $N_f(s)$ in the open-loop system.

The closed-loop transfer functions of Fig. 1 from r to y are as follows:

(i) Fig. 1(a):

$$T_a(s)=\frac{K_a(s)N_p(s)}{R(s)D_f(s)A(s)+N_p(s)B(s)}, \tag{7}$$

(ii) Fig. 1(b):

$$T_b(s)=\frac{K_b(s)N_p(s)D_f(s)}{[R(s)D_f(s)A(s)+N_p(s)B(s)]N_f(s)}, \tag{8}$$

(iii) Fig. 1(c):

$$T_c(s)=\frac{K_c(s)N_p(s)}{[R(s)D_f(s)A(s)+N_p(s)B(s)]N_f(s)}, \tag{9}$$

(iv) Fig. 1(d):

$$T_d(s)=\frac{H_d(s)N_p(s)B(s)}{R(s)D_f(s)A(s)+N_p(s)B(s)}, \tag{10}$$

(v) Fig. 1(e):

$$T_e(s)=\frac{H_e(s)N_p(s)B(s)D_f(s)}{[R(s)D_f(s)A(s)+N_p(s)B(s)]N_f(s)}. \tag{11}$$

It should be noted that all the closed-loop transfer functions are different from each other, whereas the open-loop transfer functions and the sensitivity functions are not dependant on the system configuration. In particular, the resonant poles $N_f(s)$ are not included in (7) and (10), while they do exist in (8), (9) and (11). Comparing Fig. 1(a)-(c) with (d)-(e), the zeros of the controller directly appear in the closed-loop transfer function for configurations (d) and (e), as well as for (10) and (11). Consequently, we say that the configuration of Fig. 1(a) is preferable. However, when a cascade controller structure is to be used, one should select Fig. 1(d).

Example 1:

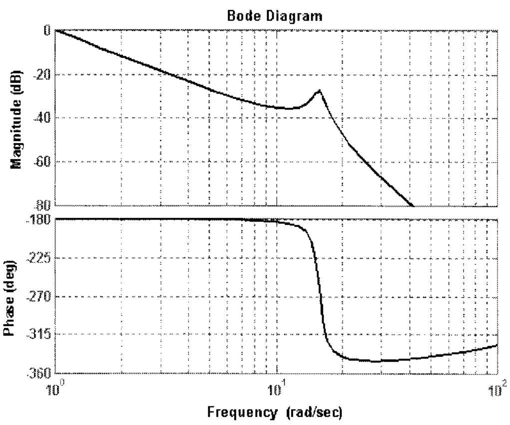

Fig. 2 The Bode diagram of the plant in Example 1.

Let us consider a simple feedback control system including a notch filter F(s), where the plant model in [7] is used

$$G(s) = \frac{N_p(s)}{D_p(s)} = \frac{2\zeta_1\omega_1 s + \omega_1^2}{s^2(s^2 + 2\zeta_1\omega_1 s + \omega_1^2)}, \tag{12}$$

where $\zeta_1 = 0.05$, $\omega_1 = 5\pi$. Let us examine how performance is influenced by the particular feedback structure.

Suppose that we intend to design a first-order controller leading to a closed-loop step response exhibiting almost no overshoot and a 2% settling time of less than 6 sec. Applying the synthesis method found in [8] and [9], we have a target model

$$T^*(s) = \frac{K \cdot N_p(s)}{\delta^*(s)}, \tag{13}$$

where

$\delta^*(s) = s^5 + 28.736s^4 + 294.904s^3 + 1336.061s^2 + 267.123s + 1908.659$,

and K is the feed-forward gain.

To reduce the frequency response magnitude at $\omega = \omega_1$ the following notch filter has been selected

$$F(s) = \frac{s^2 + 1.5708s + 246.74}{s^2 + 25.133s + 246.74}. \tag{14}$$

Let us find a first-order controller based on a posteriori notch compensation (the details will be shown in section 3.1 later). Then we have obtained

$$C_1(s) = \frac{10.78s + 7.736}{s + 5.346}, \qquad K_1 = 7.7356. \tag{15}$$

Here we have

$$y = T_j(s)r + S(s)d_o + S(s)G(s)d_i + [1 - S(s)]n, \tag{16}$$

for $j = a, b, c, d, e$ where r, y, d_i, d_o, n represents the reference input, closed-loop output, input and output disturbance, and measurement noise, respectively. Also, $S(s)$ and $T_j(s)$ given in (6), (7)-(11) show that only the response to the reference input is dependant on the structure whereas the sensitivity functions are never changed even when different configurations as Fig. 1(a)-(e) are in place.

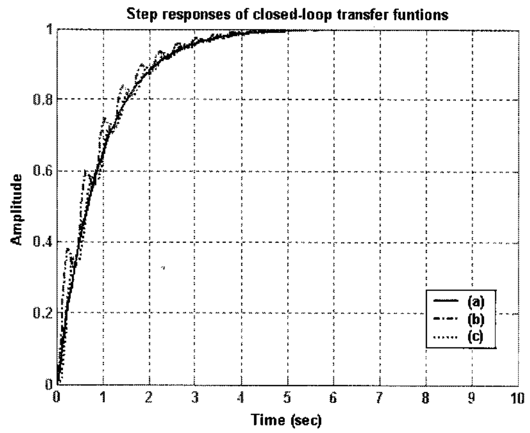

Fig. 3 The step responses of the closed-loop transfer functions of (7), (8) and (9).

Figs. 3, 4 and 5, 6 show the time and frequency responses of the closed-loop transfer functions (7)-(11). We can see that the closed-loop transfer functions in Fig. 1(b), Fig. 1(c) and Fig. 1(e) are influenced by the resonances of the notch filter and Fig. 1(d), Fig. 1(e) are affected by the controller zeros while that is not the case for the system in Fig. 1(a).

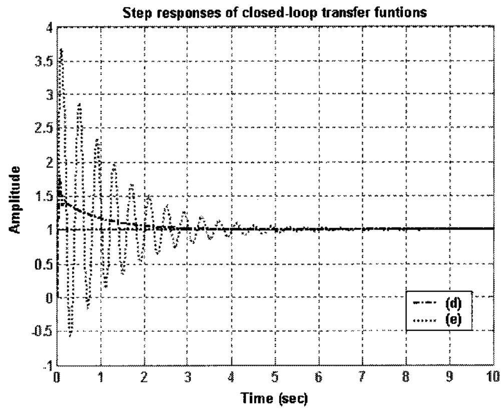

Fig. 4 The step responses of the closed-loop transfer functions of (10) and (11).

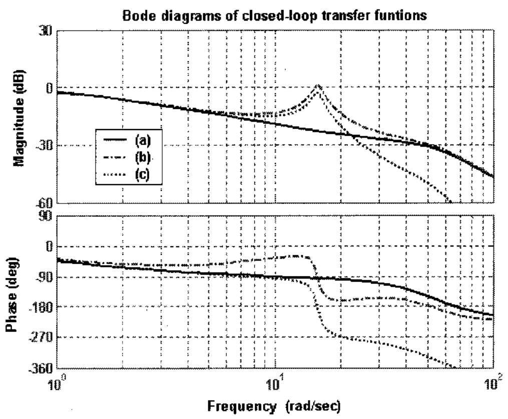

Fig. 5 The Bode diagrams of the closed-loop transfer functions of (7), (8) and (9).

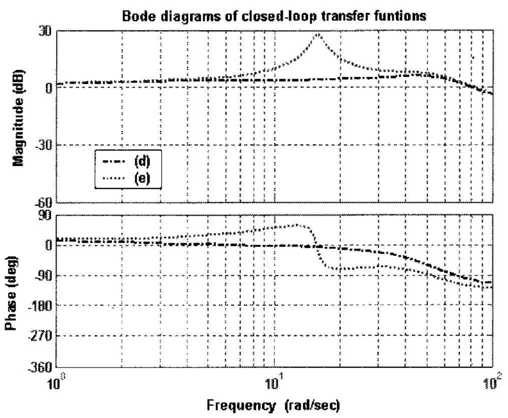

Fig. 6 The Bode diagrams of the closed-loop transfer functions of (10) and (11).

Therefore we conclude that the Fig. 1(a) configuration is more desirable to use than the others.

In the above design, the a posteriori notch compensation approach was used for the controller design. Now, if we design the controller by means of the a priori notch compensation (see section 3.2 for the details), we have

$$C_2(s)=\frac{10.78s+7.735}{s+5.344},\qquad K_2=7.7355. \tag{17}$$

It can be easily seen that both controllers are almost identical. Apparently, the frequency and time responses of both designs are very similar. The question is whether or this similarity is true for all cases.

This is the problem that we are going to investigate in this paper. We take only the effects on the design order of the notch filter into consideration. The other issues related to cancellation or shifting in the design can be found in [10].

3. A COMPARATIVE STUDY ON NOTCH COMPENSATOR DESIGN

Let us consider a practical design problem of a feedback control system with a notch filter under a two-parameter configuration, as shown in Fig. 1(a). The closed-loop transfer function between the reference input r and the closed-loop output y are given by (7). In this section, we show two approaches to the controller design by using the partial model matching method in parameter space. Depending on whether or not the notch filter is designed during or after the feedback controller design, they are called either the "a posteriori compensation approach" or the "a priori compensation approach".

3.1. The a posteriori compensation approach

In this approach, we first design a feedback controller in the absence of a notch filter. The controller design is performed using the feedback system structure shown in Fig. 7 (which does not coincide with Fig. 1(a)). Later the overall system is implemented by using the feedback controller and a notch filter that is selected independently of the feedback controller.

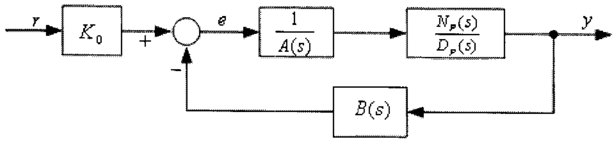

Fig. 7 The feedback control system without a notch filter.

Thus, in this design procedure the closed-loop transfer function is given by

$$T_0(s)=\frac{K_0\cdot N_p(s)}{\delta_1(s)}=\frac{K_0\cdot N_p(s)}{R(s)\overline{D}_p(s)A_1(s)+N_p(s)B_1(s)}, \tag{18}$$

and the target model shall be selected in a similar form as (18), that is,

$$T_1^*(s)=\frac{K_0\cdot N_p(s)}{\delta_1^*(s)}. \tag{19}$$

The design objective is then to find a controller $\{A_1(s),B_1(s),K_0\}$ so that $T_0(s)\cong T_1^*(s)$. Replacing this controller in the feedback system shown in Fig. 1(a), the loop transfer function and the sensitivity function of the overall system are given by

$$L_1(s)=\frac{N_p(s)B_1(s)}{R(s)D_f(s)A_1(s)}, \tag{20}$$

$$S_1(s)=\frac{R(s)D_f(s)A_1(s)}{R(s)D_f(s)A_1(s)+N_p(s)B_1(s)}. \tag{21}$$

Also, the closed-loop transfer function of the overall system is given by

$$T_1(s)=\frac{K_1N_p(s)}{R(s)D_f(s)A_1(s)+N_p(s)B_1(s)}. \tag{22}$$

Note that the controller $\{A_1(s),B_1(s),K_0\}$ obtained by a posteriori compensation does not guarantee closed-loop stability because the denominator of (18) is not identical to the closed-loop characteristic polynomial.

3.2. The a priori compensation approach

In this approach, after a notch filter compensates the weakly damped modes of the plant, the controller design is carried out with the plant including the notch filter. Then, from (7), the resulting closed-loop transfer function becomes

$$T_2(s)=\frac{K_2N_p(s)}{R(s)D_f(s)A_2(s)+N_p(s)B_2(s)}. \tag{23}$$

We select the target model in a similar form as

$$T_2^*(s)=\frac{K_2\cdot N_p(s)}{\delta_2^*(s)}. \tag{24}$$

As for section 3.1, we will find the controller parameters $\{A_2(s),B_2(s),K_2\}$ so that $T(s)\cong T_2^*(s)$.

Then we have

$$L_2(s)=\frac{N_p(s)B_2(s)}{R(s)D_f(s)A_2(s)}, \tag{25}$$

$$S_2(s)=\frac{R(s)D_f(s)A_2(s)}{R(s)D_f(s)A_2(s)+N_p(s)B_2(s)}. \tag{26}$$

The stability problem mentioned in section 3.1 does not exist in the current approach. Nevertheless, comparing (22) with (23), we can see that the overall system properties may be quite different.

4. AN ILLUSTRATIVE EXAMPLE

In this section, we are going to show the differences between both approaches through a numerical example.

Example 2:

We consider an optical model whose numerator and denominator polynomials are given by

$$N_p(s) = 1.7295\times10^{10}s^3 + 2.7080\times10^{13}s^2 + 1.3239\times10^{17}s + 1.8980\times10^{20},$$

$$D_p(s) = s^6 + 1.8449\times10^3 s^5 + 2.2896\times10^7 s^4 + 3.2197\times10^{10}s^3 + 1.0759\times10^{14}s^2 + 7.9767\times10^{16}s + 1.3171\times10^{20}.$$

This plant has two resonance modes as shown in Table 1.

Table 1 The natural frequencies and damping ratios of the resonances.

	1st resonance	2nd resonance
Natural frequency (rad/sec)	1.7503×10^3	4.0623×10^3
Damping ratio	0.0606	0.0344

Suppose that the objective is to design a feedback control system satisfying:

i) Overshoot $\le 20\%$;
ii) Settling time ≤ 0.02sec;
iii) Sensitivity peak$\le 10dB$.

Let us assume that the first resonance is to be compensated by a notch filter. According to (2) and Table 1, the following notch filter with $\zeta = 0.38$ was selected

$$F(s) = \frac{s^2 + 2.12\times10^2 s + 3.06\times10^6}{s^2 + 1.33\times10^3 s + 3.06\times10^6}. \tag{27}$$

The Bode magnitude diagrams of the original and compensated plants are shown in Fig. 8.

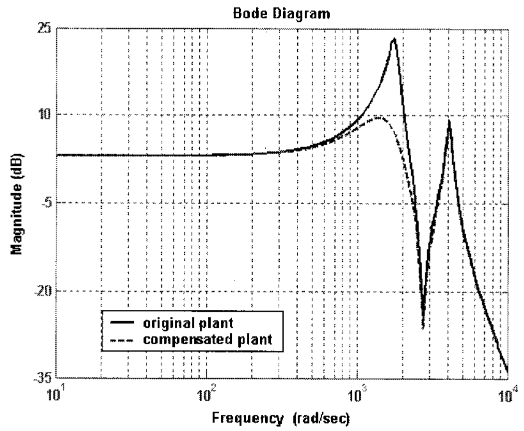

Fig. 8 The Bode magnitude diagrams of the original and compensated plants from Example 2.

We consider a modified I-PD controller in the form of Fig. 1(a) where

$$C(s) = \frac{K_d s^2 + K_p s + K_i}{s(T_d s + 1)}. \tag{28}$$

To fairly compare the time and frequency responses of the two approaches, the denominator polynomials of the target models in (19) and (24) are assigned to be the same, that is,

$$\delta^*(s) = \delta_1^*(s) = \delta_2^*(s). \tag{29}$$

A target polynomial $\delta^*(s)$ can be easily generated by using the *K-polynomial* [8], [9] with $\alpha_1 = 2.1$ and $\tau = 0.0038$.

$$\delta^*(s) = s^8 + 9.729\times10^3 s^7 + 4.507\times10^7 s^6 + 1.290\times10^{11}s^5 + 2.448\times10^{14}s^4 + 3.304\times10^{17}s^3 + 2.955\times10^{20}s^2 + 1.633\times10^{23}s + 4.297\times10^{25}.$$

4.1. A controller design based on the a posteriori compensation approach

According to (18) and (19), the controller is computed as

$$C_1(s) = \frac{0.4762s^2 - 789.7s + 2.255\times10^5}{s(s+2152)} \tag{30}$$

with $K_1 = 0.2255$.

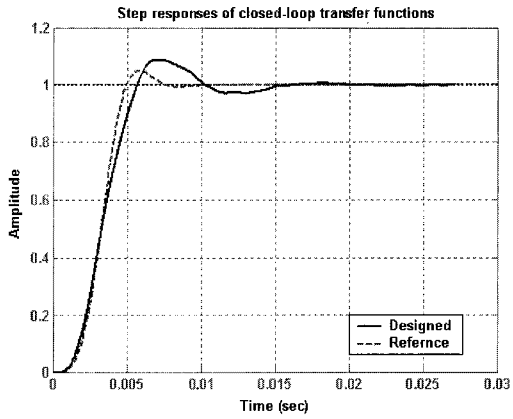

Fig. 9 The step responses of the closed-loop transfer functions of (18) and (19).

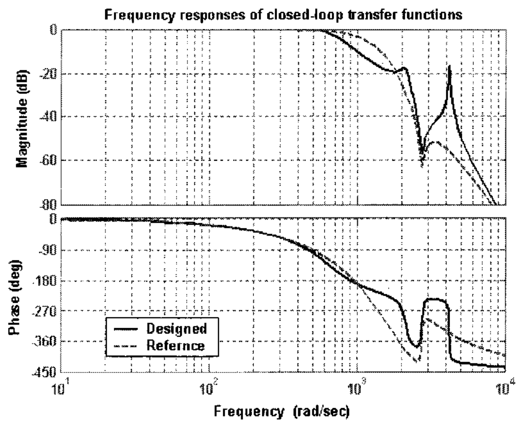

Fig. 10 The frequency responses of the closed-loop transfer functions of (18) and (19).

Figs. 9 and 10 show the step and frequency responses of the designed and target transfer functions found in (18) and (19). The closed-loop transfer function (22) of the overall system has the time response shown in Fig. 13.

4.2. A controller design based on the a priori compensation approach

Substituting $\delta^*(s)$ above into (24) and using the partial model matching method [6] with (23), the controller is obtained by

$$C_2(s) = \frac{0.0098s^2 - 501.3s + 2.258\times 10^5}{s(s+1736)} \tag{31}$$

with the feed-forward gain $K_2 = 0.2258$.

The time and frequency responses of (23) and (24) are shown in Figs. 11 and 12, respectively.

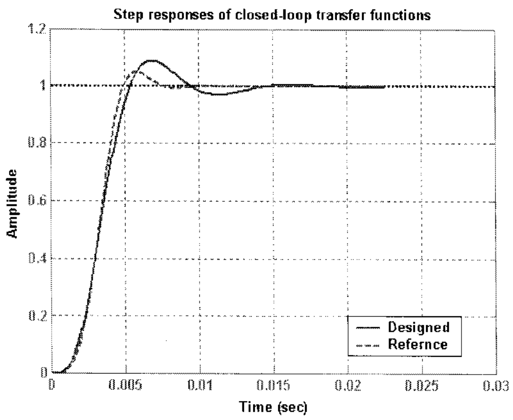

Fig. 11 The step responses of the closed-loop transfer functions of (23) and (24).

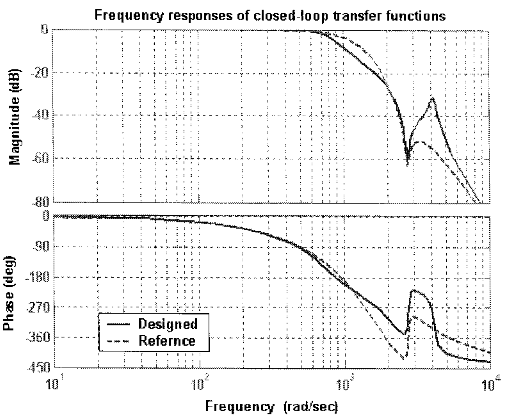

Fig. 12 The frequency responses of the closed-loop transfer functions of (23) and (24).

4.3. The performance comparison

The time responses of (18) and (23), as shown in Fig. 9 and Fig. 11, and the similar frequency responses in Fig. 10 and Fig. 12, reveal overall similar performances. However, the closed-loop transfer function of the overall system obtained via the a posteriori compensation is given by (22) not (18). Comparing the time responses of (22) and (23), as shown in Fig. 13, it is clear that the a priori compensation gives a much better time domain performance in terms of reduced overshoot and settling time.

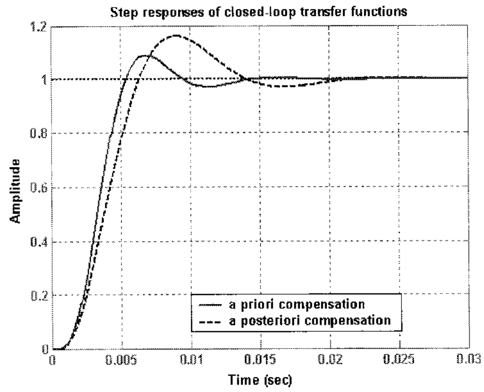

Fig. 13 The step responses of the closed-loop transfer functions of (22) and (23).

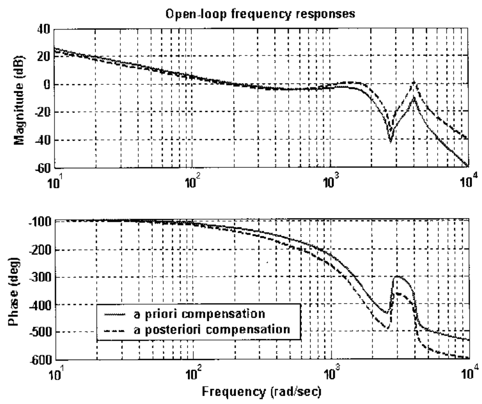

Fig. 14 The frequency responses of the closed-loop transfer functions of (20) and (25).

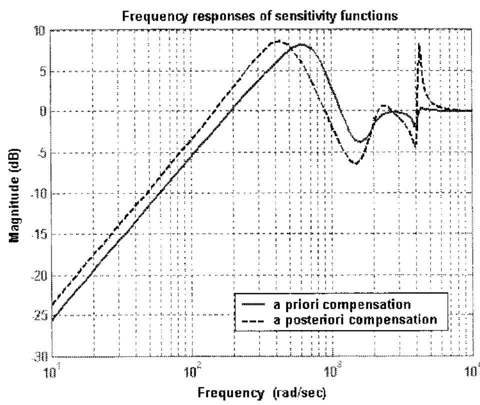

Fig. 15 The frequency responses of the sensitivity transfer functions of (21) and (26).

Figs. 14 and 15 show the frequency responses of the open-loop transfer functions of (20) and (21), and of the sensitivity functions of (25) and (26), respectively. It is clear from these figures that the gain margins are similar in both designs but the a priori compensation approach gives a better phase margin and sensitivity.

Subject to the same target model that meets the desired specifications, the characteristics of the designed systems are shown in Table 2.

Table 2 The time and frequency performances of both approaches.

		A posteriori approach	A priori approach
Time response	overshoot	16.1%	8.84%
	settling time	0.0188sec	0.0125sec
Frequency response	gain margin	4.07*dB*	4.30*dB*
	phase margin	37.1*dB*	54.4*dB*
	sensitivity peak	8.56*dB*	8.13*dB*

5. CONCLUDING REMARKS

This paper has described some practical design issues that should be carefully considered when a notch filter is included in a linear feedback controller. Two issues have been mainly investigated. One issue is in what position of a notch filter provides better performance. The other is related to the design procedure of notch filter. Five kinds of feedback configurations associated with the different positions of the notch filter have been considered. The closed-loop system performances are considerably affected by the position of the notch filter whereas the open-loop transfer function and sensitivity function are not dependent on the configuration of system. As a result, we concluded that the structure shown in Fig. 1(a) is the best among them.

It is very common that the practical engineers in the industry prefer to add a notch filter after having previously designed a feedback controller without a filter. Intuition may suggest that the resulting performance should not be different from when a feedback controller is designed for a plant that has been previously compensated by a notch filter. To compare the performance of both approaches, we have considered a low-order controller design using the partial model matching method. The results show that both the a posteriori compensation and the a priori compensation approaches for notch filter design give a similar performances in many cases. However in the case where the resonance of the plant is close to the desired closed-loop frequency, the a priori compensation approach has a tendency to achieve much better time and frequency domain characteristics than the a posteriori approach. It is noticeable that canceling a weakly damped pole by using notch filter may give a highly sensitivity design.

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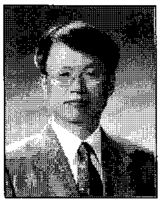
BIOGRAPHIES

Lihua Jin (金 麗 花)
was born in Jilin, China in 1978. She received the B.S. degree in the department of Electronic Engineering from Yanbian University, China in June 1999, and the M.S. degree in the department of Electronics Engineering from Chungbuk National University, Korea in August 2007. She is currently working towards Ph.D. degree on Electronics Engineering. Her research interests include low-order controller design, model-free/non-parametric model design, and system identification.

Tel : 043-273-3978

Fax : 043-272-2475

E-mail : jinlihua@chungbuk.ac.kr



Young Chol Kim (金 永 喆)
received the B.S. degree in Electrical Engineering from Korea University, Korea in 1981, and the M.S. and the Ph.D. degrees in Electrical Engineering from Seoul University, Korea in 1983 and 1987 respectively. Since March 1988, he has been with the school of Electrical & Computer Engineering, Chungbuk National University, Korea, where he is currently a professor. He was a post-doctoral fellow at Texas A&M University, 1992-1993, and a visiting research fellow at COE-ISM, Tennessee State Univ./Vanderbilt Univ., 2001-2002. He was a director of the Technical Society of Control and Instrument, 2004-2007. His research interests include robust control In parameter space and low-order controller design based on the Characteristic Ratio Assignment.

Tel : 043-271-2475

Fax : 043-272-2475

E-mail : yckim@chungbuk.ac.kr