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A SYSTEM OF NONLINEAR SET-VALUED IMPLICIT VARIATIONAL INCLUSIONS IN REAL BANACH SPACES

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Abstract. In this paper, we introduce and study a system of nonlinear set-valued implicit variational inclusions (SNSIVI) with relaxed cocoercive mappings in real Banach spaces. By using resolvent operator technique for M-accretive mapping, we construct a new class of iterative algorithms for solving this class of system of set-valued implicit variational inclusions. The convergence of iterative algorithms is proved in q-uniformly smooth Banach spaces. Our results generalize and improve the corresponding results of recent works.

1. Introduction

In [4], Fang and Huang first introduced the concept of H-accretive operators in Banach spaces. Obviously, the class of H-accretive operators provides a unifying frameworks for classes of maximal monotone operators, m-accretive operators and H-monotone operator [5]. Very recently, Chang, Joseph Lee, and Chan [2] introduced and studied a class of generalized system for relaxed cocoercive variational inequalities in Hilbert spaces which extended and improved the main results in [6, 8, 9].

Inspired and motivated by the results in [1, 2, 4, 6], the purpose of this paper is to introduce and study a system of nonlinear implicit variational inclusions in Banach spaces. By using the resolvent operator technique for the M-accretive mapping, we develop a new class of iterative algorithms to solve a class of relaxed cocoercive set-valued implicit variational inclusions associated with M-accretive mappings in q-uniformly smooth Banach spaces. Our results improve and extend the corresponding results of Chang, Joseph Lee, and Chan [2].

Let E be an arbitrary real Banach space and let J_q $(q > 1)$ denote the generalized duality mapping from E into 2^{E^*} given by

$$J_q(x) = \{f \in E^* : \langle x, f\rangle = \|x\|^q,\ \|f\| = \|x\|^{q-1}\},$$

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where E^* denotes the dual space of E and $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. In particular, J_2 is called the normalized duality mapping and it is usually denoted by J.

The modulus of smoothness of E is the function $\rho_E : [0, \infty) \to [0, \infty)$ defined by

$$\rho_E(t) = \sup \left\{ \frac{1}{2}(\|x + y\| + \|x - y\|) - 1 : \|x\| \leq 1, \|y\| \leq t \right\}.$$

A Banach space E is called uniformly smooth if $\lim_{t\to 0} \frac{\rho_E(t)}{t} = 0$. E is called q-uniformly smooth if there exists a constant $c > 0$ such that

$$\rho_E(t) \leq ct^q, \quad q > 1.$$

It is known that, J_q is single-valued if E is uniformly smooth.

Definition 1.1. Let E be a real uniformly smooth Banach space, and $M : E \to E$ be a single-valued operator. Then M is said to be:

(i) accretive, if

$$\langle Mx - My, J_q(x - y) \rangle \geq 0, \quad \forall x, y \in E;$$

(ii) strictly accretive, if M is accretive and

$$\langle Mx - My, J_q(x - y) \rangle = 0 \quad \text{if and only if } x = y;$$

(iii) strongly accretive, if there exists a constant $r > 0$ such that

$$\langle Mx - My, J_q(x - y) \rangle \geq r\|x - y\|^q, \quad \forall x, y \in E.$$

Let $C(E)$ denote the family of all nonempty compact subsets of E. We introduce a new definition as follows.

Definition 1.2. Let E be a real uniformly smooth Banach space, $T : E \times E \to E$ and $g : E \to E$ be two single-valued mappings, and $V : E \to C(E)$ be a set-valued mapping.

(i) T is said to be relaxed (ω, r)-cocoercive with respect to (V, g) in the first variable, if there exist constants $w, r > 0$ such that for all $x_1, x_2 \in E$, $u_1 \in V(x_1)$ and $u_2 \in V(x_2)$

$$\begin{aligned}&\langle T(u_1, v_1) - T(u_2, v_2), J_q(g(x_1) - g(x_2)) \rangle \\ &\geq (-\omega)\|T(u_1, v_1) - T(u_2, v_2)\|^q + r\|x_1 - x_2\|^q, \quad \forall v_1, v_2 \in E.\end{aligned}$$

(ii) T is said to be μ-Lipschitz continuous, if there exists a constant $\mu > 0$ such that for all $u_1, u_2 \in E$

$$\|T(u_1, v_1) - T(u_2, v_2)\| \leq \mu\|g(u_1) - g(u_2)\|, \quad \forall v_1, v_2 \in E.$$

Remark 1.1. If $E = H$ is a Hilbert space, $g = I$ (the identity map on E) and $V : E \to E$ is a single-valued with $V = I$, then (i) of Definition 2.2 reduces to the definition of relaxed (γ, r)-cocoercive mappings in [2].

Definition 1.3. The set-valued operator $W : E \to C(E)$ is said to be $\widetilde{H}$-Lipschitz continuous, if there exists a constant $\xi > 0$ such that

$$\widetilde{H}(W(u), W(v)) \le \xi \|u - v\|, \quad \forall u, v \in E,$$

where $\widetilde{H}(\cdot, \cdot)$ is the Hausdorff metric on $C(E)$.

Definition 1.4. Let $M : E \to E$ be a single-valued operator and $A : E \to 2^E$ be a multivalued operators. A is said to be M-accretive if A is accretive and $(M + \lambda A)(E) = E$ holds for all $\lambda > 0$.

Remark 1.2. From [4], it is easily established that if $M = I$ (the identity map on E), then the definition of I-accretive operators is that of m-accretive operators. Conversely, the Example 2.1 in [4] shows that an m-accretive operator need not be M-accretive for some M.

Remark 1.3. It is well known that, if $E = H$ is a Hilbert space, then $A : D(A) \subset H \to 2^H$ is an m-accretive mapping if and only if $A : D(A) \subset H \to 2^H$ is a maximal monotone mapping (see, for example, [3]).

Let $M, g : E \to E$, $T : E \times E \to E$ be three single-valued operators, $V, W : E \to C(E)$ be two set-valued mappings and $A : E \to 2^E$ be a M-accretive operator. We consider a system of nonlinear implicit set-valued variational inclusion (abbreviated as SNISVI) problem as follows: to find $x^*, y^* \in E$, $u^* \in V(x^*)$, $v^* \in W(y^*)$ such that

$$\theta \in \rho T(v^*, u^*) + M(x^*) - g(y^*) + A(x^*) \quad \text{for } \rho > 0, \tag{1}$$

$$\theta \in \eta T(u^*, v^*) + M(y^*) - g(x^*) + A(y^*) \quad \text{for } \eta > 0, \tag{2}$$

where θ is a zero element in E. Below are some special cases of the SNISVI problem (1) and (2).

(i) If $V, W : E \to E$ are two single-valued mappings, then the SNISVI problem (1) and (2) can be replaced by finding $x^*, y^* \in E$ such that

$$\theta \in \rho T(W(y^*), V(x^*)) + M(x^*) - g(y^*) + A(x^*) \quad \text{for } \rho > 0, \tag{3}$$

$$\theta \in \eta T(V(x^*), W(y^*)) + M(y^*) - g(x^*) + A(y^*) \quad \text{for } \eta > 0. \tag{4}$$

(ii) If $V = W = I$, then the system of nonlinear implicit variational inclusion (abbreviated as SNIVI) problem (3) and (4) is equivalent to finding $x^*, y^* \in E$ such that

$$\theta \in \rho T(y^*, x^*) + M(x^*) - g(y^*) + A(x^*) \quad \text{for } \rho > 0, \tag{5}$$

$$\theta \in \eta T(x^*, y^*) + M(y^*) - g(x^*) + A(y^*) \quad \text{for } \eta > 0. \tag{6}$$

(iii) If $E = H$ is a Hilbert space, $M = I$ and $A = \partial\phi$, where $\phi : H \to R \cup \{+\infty\}$ is a proper convex lower semicontinuous function on H and $\partial\phi$ denotes the subdifferential of function ϕ, then the SNIVI problem (5) and (6) is equivalent to finding $x^*, y^* \in H$ such that

$$\langle \rho T(y^*, x^*) + x^* - g(y^*),\ x - x^* \rangle \ge \phi(x^*) - \phi(x), \quad \forall\, x \in H \text{ and } \rho > 0, \tag{7}$$

(8) $\langle \eta T(x^*, y^*) + y^* - g(x^*),\ x - y^* \rangle \geq \phi(y^*) - \phi(x), \quad \forall x \in H$ and $\eta > 0$.

(iv) If $E = H$ is a Hilbert space, $M = g = I$ and ϕ is the indicator function of a closed convex subset K in H, that is,

$$\phi(u) = I_K(u) = \begin{cases} 0, & u \in K, \\ +\infty, & \text{other}, \end{cases}$$

then the SNIVI problem (7) and (8) is equivalent to finding $x^*, y^* \in K$ such that

$$\langle \rho T(y^*, x^*) + x^* - y^*,\ x - x^* \rangle \geq 0, \quad \forall\, x \in H \text{ and } \rho > 0, \tag{9}$$

$$\langle \eta T(x^*, y^*) + y^* - x^*,\ x - y^* \rangle \geq 0, \quad \forall\, x \in H \text{ and } \eta > 0, \tag{10}$$

which is called a system of nonlinear variational inequality (SNVI) introduced and studied by Chang, Joseph Lee, and Chan [2].

Let $M : E \to E$ be a strictly accretive operator and $A : E \to 2^E$ is a M-accretive operator. Fang and Huang [4] defined the resolvent operator $J_M^{A,\lambda} : E \to E$ associated with A, M and λ as follows:

$$J_{A,\lambda}^M(u) = (M + \lambda A)^{-1}(u), \quad \forall u \in E.$$

From the proof of Theorem 2.3 in [4], it is easy to obtain the following result.

Lemma 1.5. *Let $M : E \to E$ be a strongly accretive operator with constant $k > 0$ and $A : E \to 2^E$ is a M-accretive operator. Then the resolvent operator $J_{A,\lambda}^M : E \to E$ is Lipschitz continuous with constant $\frac{1}{k}$, i.e.,*

$$\| J_{A,\lambda}^M(x) - J_{A,\lambda}^M(y) \| \leq \frac{1}{k} \|x - y\|, \quad \forall x, y \in E.$$

2. Main results

Throughout this section, we always let E a real q-uniformly smooth Banach space. In [10], Xu proved the following result.

Lemma 2.1. *If E is a real q-uniformly smooth Banach space, then there exists a constant $c_q \geq 1$ such that*

$$\|x + y\|^q \leq \|x\|^q + q\langle y,\ J_q(x) \rangle + c_q \|y\|^q, \quad \forall x, y \in E.$$

For convenience, let $J_{A,1}^M := J_A^M$. To obtain the approximate solution of the SNISVI problem (1) and (2), we first give a characterization the solution of problem (1) and (2) by using the resolvent operator J_A^M.

Lemma 2.2. *Let $A : E \to 2^E$ is a M-accretive. Then (x^*, y^*, u^*, v^*) is a solution to the SNISVI problem (1) and (2) if and only if (x^*, y^*, u^*, v^*) satisfies*

$$x^* = J_A^M[g(y^*) - \rho T(v^*, u^*)] \quad \textit{for } \rho > 0,$$

$$y^* = J_A^M[g(x^*) - \eta T(u^*, v^*)] \quad \textit{for } \eta > 0.$$

Proof. By using the definition of J_A^M, we can prove this lemma immediately. □

Based on Lemma 2.2, we construct the following iterative algorithm for solving problem (1) and (2).

Algorithm 2.1. For given $x_0, y_0 \in E$, $u_0 \in V(x_0), v_0 \in W(y_0)$, compute the sequences $\{x_n\}$, $\{y_n\}$, $\{u_n\}$ and $\{v_n\}$ from the iterative schemes

$$\begin{aligned} u_n \in V(x_n) &: \|u_n - u_{n+1}\| \leq \widetilde{H}(V(x_n), V(x_{n+1})), \\ v_n \in W(y_n) &: \|v_n - v_{n+1}\| \leq \widetilde{H}(W(y_n), W(y_{n+1})), \\ x_{n+1} &= (1-\alpha_n)x_n + \alpha_n J_A^M[g(y_n) - \rho T(v_n, u_n)] \quad \text{for } \rho > 0, \\ y_n &= (1-\beta_n)x_n + \beta_n J_A^M[g(x_n) - \eta T(u_n, v_n)] \quad \text{for } \eta > 0, \end{aligned}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[0,1]$.

If $T : E \to E$ is a univariate mapping and $\beta_n = 1$, then the Algorithm 2.1 reduces to the following.

Algorithm 2.2. For given $x_0, y_0 \in E$, $u_0 \in V(x_0), v_0 \in W(y_0)$, compute the sequences $\{x_n\}$, $\{y_n\}$, $\{u_n\}$ and $\{v_n\}$ from the iterative schemes

$$\begin{aligned} u_n \in V(x_n) &: \|u_n - u_{n+1}\| \leq \widetilde{H}(V(x_n), V(x_{n+1})), \\ v_n \in W(y_n) &: \|v_n - v_{n+1}\| \leq \widetilde{H}(W(y_n), W(y_{n+1})), \\ x_{n+1} &= (1-\alpha_n)x_n + \alpha_n J_A^M[g(y_n) - \rho T(v_n)] \quad \text{for } \rho > 0, \\ y_n &= J_A^M[g(x_n) - \eta T(u_n)] \quad \text{for } \eta > 0, \end{aligned}$$

where $\alpha_n \in [0,1]$, $\forall n \geq 0$.

Remark 2.1. If $E = H$ is a Hilbert space, $M = g = I$ and $A = \partial\phi$, where ϕ is the indicator function of a closed convex subset K in H, then Algorithm 2.1 reduces to the Algorithm 2.1 in [2].

In order to prove our main results we need the following lemma:

Lemma 2.3 ([2]). *Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be three nonnegative real sequences satisfying the following conditions*:

$$a_{n+1} \leq (1-\lambda_n)a_n + b_n + c_n, \quad \forall n \geq n_0,$$

where n_0 is some nonnegative integer, $\lambda_n \in (0,1)$ with $\sum_{n=0}^{\infty} \lambda_n = \infty$, $b_n = o(\lambda_n)$ and $\sum_{n=0}^{\infty} c_n < \infty$, then $a_n \to 0$ (as $n \to \infty$).

We now present, based on Algorithm 2.1, the approximation-solvability of the SNISVI problem (1) and (2) involving a mapping $T : E \times E \to E$ which is relaxed (γ, r)-cocoercive with respect to (V, g) and (W, g) in the first variable, respectively.

Theorem 2.4. *Let E be a real q-uniformly smooth Banach space, $M : E \to E$ be a strongly accretive operator with constant k and $A : E \to 2^E$ be a M-accretive operator. Let $g : E \to E$ be σ-Lipschitz continuous. Assume that $V, W : E \to C(E)$ be $\widetilde{H}$-Lipschitz continuous with constant $\beta > 0$ and $\xi > 0$, respectively. Let $T : E \times E \to E$ be relaxed (γ, r)-cocoercive with respect to (V, g) and (W, g) in the first variable, respectively. And let $T : E \times E \to E$*

be μ-Lipschitz continuous in the first variable. Suppose that $(x^*, y^*, u^*, v^*) \in E \times E \times E \times E$ is a solution to the SNISVI problem (1) *and* (2) *and that $\{x_n\}$, $\{y_n\}$, $\{u_n\}$, and $\{v_n\}$ are the sequences generated by Algorithm 2.1. If the following conditions are satisfied*:

(i) $\sum_{n=0}^{\infty} \alpha_n = \infty$;

(ii) $\sum_{n=0}^{\infty} (1-\beta_n) < \infty$;

(iii) $0 < \rho < \sqrt[q-1]{\dfrac{q(r-\gamma\mu^q\xi^q)}{c_q\mu^q\xi^q}}, \quad 0 < \eta < \sqrt[q-1]{\dfrac{q(r-\gamma\mu^q\beta^q)}{c_q\mu^q\beta^q}}$;

(iv) $r > \max\{\gamma\mu^q\xi^q, \gamma\mu^q\beta^q\}, \quad \sigma \le k$,

then the sequence $\{x_n\}$, $\{y_n\}$, $\{u_n\}$ and $\{v_n\}$ converges strongly to x^*, y^*, u^* and v^* in E, respectively.

Proof. Since (x^*, y^*, u^*, v^*) is a solution to the SNISVI problem (1) and (2), it follows from Lemma 2.2 that

$$x^* = J_A^M[g(y^*) - \rho T(v^*, u^*)], \qquad y^* = J_A^M[g(x^*) - \eta T(u^*, v^*)].$$

It follows from Algorithm 2.1 and Lemma 1.5 that

$$\begin{aligned} \|x_{n+1} - x^*\| &= \|(1-\alpha_n)x_n + \alpha_n J_A^M[g(y_n) - \rho T(v_n, u_n)] \\ &\quad - (1-\alpha_n)x^* - \alpha_n J_A^M[g(y^*) - \rho T(v^*, u^*)]\| \\ &\le (1-\alpha_n)\|x_n - x^*\| + \alpha_n \|J_A^M[g(y_n) - \rho T(v_n, u_n)] \\ &\quad - J_A^M[g(y^*) - \rho T(v^*, u^*)]\| \\ &\le (1-\alpha_n)\|x_n - x^*\| \\ &\quad + \frac{\alpha_n}{k}\|g(y_n) - g(y^*) - \rho(T(v_n, u_n) - T(v^*, u^*))\|. \end{aligned} \tag{11}$$

Since T is relaxed (γ, r)-cocoercive with respect to (W, g) and μ-Lipschitz continuous in the first variable, respectively, we have by Lemma 2.1 and the $\xi - \widetilde{H}$-Lipschitz continuity of W that

$$\begin{aligned} &\|g(y_n) - g(y^*) - \rho(T(v_n, u_n) - T(v^*, u^*))\|^q \\ &\le \|g(y_n) - g(y^*)\|^q - \rho q\langle T(v_n, u_n) - T(v^*, u^*),\ J_q(g(y_n) - g(y^*))\rangle \\ &\quad + c_q\rho^q\|T(v_n, u_n) - T(v^*, u^*)\|^q \\ &\le \sigma^q\|y_n - y^*\|^q + c_q\rho^q\mu^q\|v_n - v^*\|^q + \rho q\gamma\|T(v_n, u_n) - T(v^*, u^*)\|^q \\ &\quad - q\rho r\|y_n - y^*\|^q \\ &\le \sigma^q\|y_n - y^*\|^q + c_q\rho^q\mu^q\xi^q\|y_n - y^*\|^q \\ &\quad + \rho q\gamma\mu^q\|v_n - v^*\|^q - q\rho r\|y_n - y^*\|^q \\ &\le \sigma^q\left[1 + \frac{c_q\rho^q\mu^q\xi^q + \rho q\gamma\mu^q\xi^q - q\rho r}{\sigma^q}\right]\|y_n - y^*\|^q. \end{aligned} \tag{12}$$

Substituting (12) into (11), we get by conditions (iii) and (iv) that

$$\begin{aligned} \|x_{n+1} - x^*\| &\leq (1-\alpha_n)\|x_n - x^*\| + \frac{\sigma}{k}\alpha_n \Omega_1 \|y_n - y^*\| \\ &\leq (1-\alpha_n)\|x_n - x^*\| + \alpha_n \Omega_1 \|y_n - y^*\|, \end{aligned} \tag{13}$$

where $\Omega_1 = \sqrt[q]{1 + \frac{c_q \rho^q \mu^q \xi^q + \rho q \gamma \mu^q \xi^q - q\rho r}{\sigma^q}} < 1$ (By condition (iii)).

Next, we make an estimation for $\|y_n - y^*\|$. Again applying Algorithm 2.1 and Lemma 1.5, we have

$$\begin{aligned} \|y_n - y^*\| &= \|(1-\beta_n)x_n + \beta_n J_A^M[g(x_n) - \eta T(u_n, v_n)] \\ &\quad - (1-\beta_n)y^* - \beta_n J_A^M[g(x^*) - \eta T(u^*, v^*)]\| \\ &\leq (1-\beta_n)\|x_n - y^*\| + \frac{\beta_n}{k}\|g(x_n) - g(x^*) - \eta(T(u_n,v_n) - T(u^*,v^*))\| \\ &\leq (1-\beta_n)\|x_n - x^*\| + (1-\beta_n)\|x^* - y^*\| \\ &\quad + \frac{\beta_n}{k}\|g(x_n) - g(x^*) - \eta(T(u_n,v_n) - T(u^*,v^*))\|. \end{aligned} \tag{14}$$

Since T is relaxed (γ, r)-cocoercive with respect to (V, g) and μ-Lipschitz continuous in the first variable respectively, and V is $\beta - \widetilde{H}$-Lipschitz continuous, similar to the proof of (12), we obtain that

$$\begin{aligned} &\|g(x_n) - g(x^*) - \eta(T(u_n,v_n) - T(u^*,v^*))\|^q \\ &\leq \sigma^q \left[1 + \frac{c_q \eta^q \mu^q \beta^q + \eta q \gamma \mu^q \beta^q - q\eta r}{\sigma^q}\right] \|x_n - x^*\|^q. \end{aligned} \tag{15}$$

Combining (14), (15) and condition (iv), we have

$$\begin{aligned} \|y_n - y^*\| &\leq (1-\beta_n)\|x_n - x^*\| + (1-\beta_n)\|x^* - y^*\| + \frac{\sigma}{k}\beta_n \Omega_2 \|x_n - x^*\| \\ &\leq (1-\beta_n)\|x_n - x^*\| + (1-\beta_n)\|x^* - y^*\| + \beta_n \Omega_2 \|x_n - x^*\| \\ &\leq \|x_n - x^*\| + (1-\beta_n)\|x^* - y^*\|, \end{aligned} \tag{16}$$

where $\Omega_2 = \sqrt[q]{1 + \frac{c_q \eta^q \mu^q \beta^q + \eta q \gamma \mu^q \beta^q - q\eta r}{\sigma^q}} < 1$ (By condition (iii)).
Substituting (16) into (13), we get

$$\begin{aligned} \|x_{n+1} - x^*\| &\leq (1-\alpha_n)\|x_n - x^*\| + \alpha_n \Omega_1 [\|x_n - x^*\| + (1-\beta_n)\|x^* - y^*\|] \\ &\leq (1 - \alpha_n(1-\Omega_1))\|x_n - x^*\| + \Omega_1(1-\beta_n)\|x^* - y^*\|. \end{aligned} \tag{17}$$

Taking $a_n = \|x_n - x^*\|$, $\lambda_n = \alpha_n(1-\Omega_1)$, $b_n = 0$ and $c_n = \Omega_1(1-\beta_n)\|x_* - y^*\|$ in Lemma 2.3, we know that all conditions in Lemma 2.3 are satisfied, and so $x_n \to x^* \in E$ (as $n \to \infty$). Since V is β-Lipschitz continuous, we obtain

$$\|u_n - u^*\| \leq \widetilde{H}(V(x_n), V(x^*)) \leq \beta\|x_n - x^*\| \to 0, \quad n \to \infty,$$

i.e., $u_n \to u^* \in E$ (as $n \to \infty$). Similarly, we have that $v_n \to v^*,\ n \to \infty$.

We now show that $u^* \in V(x^*)$ and $v^* \in W(y^*)$. In fact,

$$\begin{aligned} d(u^*, V(x^*)) &\le \|u^* - u_n\| + d(u_n, V(x^*)) \\ &\le \|u^* - u_n\| + \widetilde{H}(V(x_n), V(x^*)) \\ &\le \|u^* - u_n\| + \beta\|x_n - x^*\| \to 0 \quad \text{as } n \to \infty, \end{aligned}$$

where $d(u^*, V(x^*)) = \inf\{\|u^* - z\| : z \in V(x^*)\}$. This implies that $u^* \in V(x^*)$. In a similar way, one show that $v^* \in W(y^*)$. This completes the proof. □

Remark 2.2. If $E = H$ is a Hilbert space, $M = g = I$ and $A = \partial\phi$, where ϕ is the indicator function of a closed convex subset K in H, then $q = 2$, $c_2 = 1$ (by [10]), $\sigma = k = \xi = \beta = 1$, and Theorem 2.4 reduces to Theorem 3.1 in [2]. Obviously, Theorem 2.4 extends and improves the main results in Verma [8, 9].

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References

[1] C. Z. Bai and J. X. Fang, *A system of nonlinear variational inclusions in real Banach spaces*, Bull. Korean Math. Soc. **40** (2003), no. 3, 385–397.

[2] S. S. Chang, H. W. Joseph Lee, and C. K. Chan, *Generalized system for relaxed cocoercive variational inequalities in Hilbert spaces*, Appl. Math. Lett. **20** (2007), no. 3, 329–334.

[3] K. Deimling, *Nonlinear Functional Analysis*, Springer-Verlag, Berlin, 1985.

[4] Y. P. Fang and N. J. Huang, *H-accretive operators and resolvent operator technique for solving variational inclusions in Banach spaces*, Appl. Math. Lett. **17** (2004), no. 6, 647–653.

[5] ———, *H-Monotone operator and resolvent operator technique for variational inclusions*, Appl. Math. Comput. **145** (2003), no. 2-3, 795–803.

[6] H. Nie, Z. Liu, K. H. Kim, and S. M. Kang, *A system of nonlinear variational inequalities involving strongly monotone and pseudocontractive mappings*, Adv. Nonlinear Var. Inequal. **6** (2003), no. 2, 91–99.

[7] M. Aslam Noor, *Three-step iterative algorithms for multivalued quasi variational inclusions*, J. Math. Anal. Appl. **255** (2001), no. 2, 589–604.

[8] R. U. Verma, *Generalized system for relaxed cocoercive variational inequalities and its projection methods*, J. Optim. Theory Appl. **121** (2004), no. 1, 203–210.

[9] ———, *General convergence analysis for two-step projection methods and applications to variational problem*, Appl. Math. Lett. **18** (2005), no. 11, 1286–1292.

[10] H. K. Xu, *Inequalities in Banach spaces with applications*, Nonlinear Anal. **16** (1991), no. 12, 1127–1985.

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