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COMMON FIXED POINT THEOREM OF SEMI-COMPATIBLE MAPS ON INTUITIONISTIC FUZZY METRIC SPACE

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Abstract. In this paper, we prove common fixed point theorems for semi-compatible mappings on intuitionistic fuzzy metric space with different some conditions of Park and Kim [10]. This research extended and generalized the results of Singh and Chauhan [14].

1. Introduction

The concept of fuzzy set was developed extensively by many authors and used in various fields. Several authors have defined fuzzy metric space ([5] etc.) with various methods to use this concept in analysis. Jungck ([3], [4]) researched the more generalized concept compatibility than commutativity and weak commutativity in metric space and proved common fixed point theorems, and Singh and Chauhan [14] introduced the concept of compatibility in fuzzy metric space and studied common fixed point theorems for four compatible mappings.

Recently, Park et. al. [7] defined the upgraded intuitionistic fuzzy metric space and Park et. al. ([8], [9], [11], [12]) studied several theories in this space. Also, Park and Kim [10] proved common fixed point theorem for self maps in intuitionistic fuzzy metric space.

In this paper, we prove common fixed point theorems for semi-compatible mappings on intuitionistic fuzzy metric space with different some conditions of Park and Kim [10]. This research extended and generalized the results of Singh and Chauhan [14].

2. Preliminaries

We give some definitions and properties of intuitionistic fuzzy metric space. Throughout this paper, $\mathbb{N}$ will denote the set of all positive integers.

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Let us recall (see [13]) that a continuous t-norm is a binary operation $* : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions:

(a) $*$ is commutative and associative;

(b) $*$ is continuous;

(c) $a * 1 = a$ for all $a \in [0,1]$;

(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a,b,c,d \in [0,1])$.

Similarly, a continuous t-conorm is a binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions:

(a) $\diamond$ is commutative and associative;

(b) $\diamond$ is continuous;

(c) $a \diamond 0 = a$ for all $a \in [0,1]$;

(d) $a \diamond b \geq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a,b,c,d \in [0,1])$.

Also, let us recall (see [6]) that the following conditions are satisfied:

(a) For any $r_1, r_2 \in (0,1)$ with $r_1 > r_2$, there exist $r_3, r_4 \in (0,1)$ such that $r_1 * r_3 \geq r_2$ and $r_4 \diamond r_2 \leq r_1$;

(b) For any $r_5 \in (0,1)$, there exist $r_6, r_7 \in (0,1)$ such that $r_6 * r_6 \geq r_5$ and $r_7 \diamond r_7 \leq r_5$.

Definition 2.1 ([7])**.** The 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that

(a) $M(x,y,t) > 0$,

(b) $M(x,y,t) = 1 \Longleftrightarrow x = y$,

(c) $M(x,y,t) = M(y,x,t)$,

(d) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,

(e) $M(x,y,\cdot) : (0,\infty) \to (0,1]$ is continuous,

(f) $N(x,y,t) > 0$,

(g) $N(x,y,t) = 0 \Longleftrightarrow x = y$,

(h) $N(x,y,t) = N(y,x,t)$,

(i) $N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s)$,

(j) $N(x,y,\cdot) : (0,\infty) \to (0,1]$ is continuous.

Note that (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Definition 2.2 ([12])**.** Let X be an intuitionistic fuzzy metric space. Then

(a) A sequence $\{x_n\} \subset X$ is convergent to x in X if and only if for each $\epsilon > 0,\ t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x, t) > 1 - \epsilon$, $N(x_n, x, t) < \epsilon$ for all $n \geq n_0$.

(b) A sequence $\{x_n\} \subset X$ is called Cauchy sequence if and only if for each $\epsilon > 0,\ t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$, $N(x_n, x_m, t) < \epsilon$ for all $n, m \geq n_0$.

(c) X is complete if every Cauchy sequence in X is convergent.

Definition 2.3 ([10])**.** Let A, B be mappings from intuitionistic fuzzy metric space X into itself.

(a) (A, B) are said to be compatible if and only if

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1, \ \lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$$

for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

(b) (A, B) are said to be semi-compatible if and only if

$$\lim_{n\to\infty} M(ABx_n, Bx, t) = 1, \ \lim_{n\to\infty} N(ABx_n, Bx, t) = 0$$

for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Lemma 2.4 ([10])**.** *Let A, B be self mappings on intuitionistic fuzzy metric space X. If B is continuous, then (A, B) is semi-compatible if and only if (A, B) is compatible.*

3. Main results

Theorem 3.1. *Let A, B, S and T be self maps of complete intuitionistic fuzzy metric space X with t-norm $*$ and t-conorm $\diamond$ defined by $\alpha * \beta = \min\{\alpha, \beta\}$ and $\alpha \diamond \beta = \max\{\alpha, \beta\}$, $\alpha, \beta \in [0, 1]$, satisfying*

(a) *(A, S) and (B, T) are semi-compatible pairs of maps,*

(b) *S and T are continuous,*

(c) *$A^a(X) \subset T^t(X)$, $B^b(X) \subset S^s(X)$,*

(d)

$$\begin{aligned} & M(A^a x, B^b y, kt) \\ \geq\ & \min\{M(S^s x, T^t y, t), M(A^a x, S^s x, t), M(B^b y, T^t y, t), M(B^b y, S^s x, 2t), \\ & \qquad M(A^a x, T^t y, t)\}, \\ & N(A^a x, B^b y, kt) \\ \leq\ & \max\{N(S^s x, T^t y, t), N(A^a x, S^s x, t), N(B^b y, T^t y, t), N(B^b y, S^s x, 2t), \\ & \qquad N(A^a x, T^t y, t)\}. \end{aligned}$$

(e) $\lim_{t\to\infty} M(x, y, t) = 1$, $\lim_{t\to\infty} N(x, y, t) = 0$ *for all $x, y \in X$, $t > 0$ and $a, b, s, t \in \mathbb{N}$.*

Then A, B, S and T have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point in X, we can inductively construct a sequence $\{y_n\} \subset X$ such that $y_{2n-1} = T^t x_{2n-1} = A^a x_{2n-2}$, $y_{2n} = S^s x_{2n} = B^b x_{2n-1}$ for $n = 1, 2, 3, \ldots$.

First, we prove that $\{y_n\}$ is Cauchy sequence. From (d), we have

$$\begin{aligned} & M(y_{2n+1}, y_{2n+2}, kt) \\ =\ & M(A^a x_{2n}, B^b x_{2n+1}, kt) \end{aligned}$$

$$
\begin{aligned}
&\geq \min\{M(S^s x_{2n}, T^t x_{2n+1}, t), M(A^a x_{2n}, S^s x_{2n}, t), M(B^b x_{2n+1}, T^t x_{2n+1}, t)\\
&\qquad M(B^b x_{2n+1}, S^s x_{2n}, 2t), M(A^a x_{2n}, T^t x_{2n+1}, t)\}\\
&\geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+2}, y_{2n+1}, t),\\
&\qquad\quad M(y_{2n+2}, y_{2n}, 2t), M(y_{2n+1}, y_{2n+1} t)\}\\
&\geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+2}, y_{2n+1}, t), 1\},\\
&\ N(y_{2n+1}, y_{2n+2}, kt)\\
&= N(A^a x_{2n}, B^b x_{2n+1}, kt)\\
&\leq \max\{N(S^s x_{2n}, T^t x_{2n+1}, t), N(A^a x_{2n}, S^s x_{2n}, t), N(B^b x_{2n+1}, T^t x_{2n+1}, t)\\
&\qquad\quad N(B^b x_{2n+1}, S^s x_{2n}, 2t), N(A^a x_{2n}, T^t x_{2n+1}, t)\}\\
&\leq \max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n}, t), N(y_{2n+2}, y_{2n+1}, t),\\
&\qquad\quad N(y_{2n+2}, y_{2n}, 2t), N(y_{2n+1}, y_{2n+1} t)\}\\
&\leq \max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+2}, y_{2n+1}, t), 0\}
\end{aligned}
$$

which implies

$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t),\ N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t).$

Generally,

$$M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t), \quad N(y_n, y_{n+1}, kt) \leq M(y_{n-1}, y_n, t).$$

Therefore

$$
\begin{aligned}
M(y_n, y_{n+1}, t) &\geq M\left(y_{n-1}, y_n, \frac{t}{k}\right)\\
&\geq \cdots\cdots\cdots\\
&\geq M\left(y_0, y_1, \frac{t}{k^n}\right)\\
&\to 1 \ \text{ as } \ n \to \infty,\\
N(y_n, y_{n+1}, t) &\leq N\left(y_{n-1}, y_n, \frac{t}{k}\right)\\
&\leq \cdots\cdots\cdots\\
&\leq N\left(y_0, y_1, \frac{t}{k^n}\right)\\
&\to 0 \ \text{ as } \ n \to \infty.
\end{aligned}
$$

Hence for $t > 0$ and $\epsilon \in (0,1)$, we can choose $n_0 \in \mathbb{N}$ such that

$$M(y_n, y_{n+1}, t) > 1 - \epsilon, \quad N(y_n, y_{n+1}, t) < \epsilon$$

for all $n \geq n_0$.

Suppose that for m,

$$M(y_n, y_{n+m}, t) > 1 - \epsilon, \quad N(y_n, y_{n+m}, t) < \epsilon$$

for all $n \geq n_0$ and for every $m \in \mathbb{N}$. Then

$$\begin{aligned}
&M(y_n, y_{n+m+1}, t)\\
\geq\ &\min\left\{M\left(y_n, y_{n+m}, \frac{t}{2}\right), M\left(y_{n+m}, y_{n+m+1}, \frac{t}{2}\right)\right\}\\
>\ &1-\epsilon,\\
&N(y_n, y_{n+m+1}, t)\\
\leq\ &\max\left\{N\left(y_n, y_{n+m}, \frac{t}{2}\right), N\left(y_{n+m}, y_{n+m+1}, \frac{t}{2}\right)\right\}\\
<\ &\epsilon.
\end{aligned}$$

Therefore $\{y_n\} \subset X$ is a Cauchy sequence.

Second, we prove that A^a, B^b, S^s and T^t have a unique common fixed point. Since $\{y_n\}$ converges to some point x from completeness of X, $A^a x_{2n} \to x$, $S^s x_{2n} \to x$, $B^b x_{2n-1} \to x$ and $T^t x_{2n-1} \to x$. Since S is continuous, hence $S^s(A^a x_{2n}) \to S^s(x)$. Thus for $t > 0$ and $\epsilon \in (0,1)$, there exists an $n_0 \in \mathbb{N}$ such that

$$M\left(S^s(A^a x_{2n}), S^s(x), \frac{t}{2}\right) > 1-\epsilon, \quad N\left(S^s(A^a x_{2n}), S^s(x), \frac{t}{2}\right) < \epsilon$$

for all $n \geq n_0$. Also, since (A,S) and (B,T) are semi-compatible pairs, by Lemma 2.4, (A,S) and (B,T) are compatible pairs. Therefore (A^a, S^s) and (B^b, T^t) are compatible pairs for all $a, b, s, t \in \mathbb{N}$. From (a), we have

$$\begin{aligned}
&\lim_{n\to\infty} M\left(A^a(S^s x_{2n}), S^s(A^a x_{2n}), \frac{t}{2}\right) = 1,\\
&\lim_{n\to\infty} N\left(A^a(S^s x_{2n}), S^s(A^a x_{2n}), \frac{t}{2}\right) = 0.
\end{aligned}$$

Hence

$$\begin{aligned}
&M(S^s(A^a x_{2n}), S^s(x), t)\\
\geq\ &\min\left\{M\left(A^a(S^s x_{2n}), S^s(A^a x_{2n}), \frac{t}{2}\right), M\left(S^s A^a(x_{2n}), S^s x, \frac{t}{2}\right)\right\}\\
>\ &1-\epsilon,\\
&N(S^s(A^a x_{2n}), S^s(x), t)\\
\leq\ &\max\left\{M\left(A^a(S^s x_{2n}), S^s(A^a x_{2n}), \frac{t}{2}\right), M\left(S^s A^a(x_{2n}), S^s x, \frac{t}{2}\right)\right\}\\
<\ &\epsilon
\end{aligned}$$

for all $n \geq n_0$. Therefore $\lim_{n\to\infty} A^a S^s x_{2n} = S^s x$. Also, since $\lim_{n\to\infty} B^b x_{2n-1} = x$ and T is continuous, $\lim_{n\to\infty} T^t(B^b x_{2n-1}) = T^t x$. Thus for $t > 0$ and

$\epsilon \in (0,1)$, there exists an $n_0 \in \mathbb{N}$ such that

$$M\left(T^t(B^b x_{2n-1}), T^t(x), \frac{t}{2}\right) > 1-\epsilon, \quad N\left(T^t(B^b x_{2n-1}), T^t(x), \frac{t}{2}\right) < \epsilon$$

for all $n \geq n_0$.

From (a), we have

$$\lim_{n\to\infty} M\left(B^b(T^t x_{2n-1}), T^t(B^b x_{2n-1}), \frac{t}{2}\right) = 1,$$
$$\lim_{n\to\infty} N\left(B^b(T^t x_{2n-1}), T^t(B^b x_{2n-1}), \frac{t}{2}\right) = 0.$$

Hence

$$\begin{aligned} & M(B^b(T^t x_{2n-1}), T^t x, t) \\ \geq\ & \min\left\{M\left(B^b(T^t x_{2n-1}), T^t(B^b x_{2n-1}), \frac{t}{2}\right), M\left(T^t(B^b x_{2n-1}), T^t x, t\right)\right\} \\ \geq\ & 1-\epsilon, \\ & N(B^b(T^t x_{2n-1}), T^t x, t) \\ \leq\ & \max\left\{N\left(B^b(T^t x_{2n-1}), T^t(B^b x_{2n-1}), \frac{t}{2}\right), N\left(T^t(B^b x_{2n-1}), T^t x, t\right)\right\} \\ \leq\ & \epsilon \end{aligned}$$

for all $n \geq n_0$. Therefore $\lim_{n\to\infty} B^b(T^t x_{2n-1}) = T^t x$.

Using (d), we have

$$\begin{aligned} & M(A^a(S^s x_{2n}), B^b(T^t x_{2n-1}), kt) \\ \geq\ & \min\{M(S^s(S^s x_{2n}), T^t(T^t x_{2n-1}), t), M(A^a(S^s x_{2n}), S^s(S^s x_{2n}), t), \\ & \quad M(B^b(T^t x_{2n-1}), T^t(T^t x_{2n-1}), t), M(B^b(T^t x_{2n-1}), S^s(S^s x_{2n}), 2t), \\ & \quad M(A^a(S^s x_{2n}), T^t(T^t x_{2n-1}), t)\}, \\ & N(A^a(S^s x_{2n}), B^b(T^t x_{2n-1}), kt) \\ \leq\ & \max\{N(S^s(S^s x_{2n}), T^t(T^t x_{2n-1}), t), N(A^a(S^s x_{2n}), S^s(S^s x_{2n}), t), \\ & \quad N(B^b(T^t x_{2n-1}), T^t(T^t x_{2n-1}), t), N(B^b(T^t x_{2n-1}), S^s(S^s x_{2n}), 2t), \\ & \quad N(A^a(S^s x_{2n}), T^t(T^t x_{2n-1}), t)\}. \end{aligned}$$

Taking the limit as $n \to \infty$, and using above results,

$$M(S^s x, T^t x, kt) \geq M(S^s x, T^t x, t), \quad N(S^s x, T^t x, kt) \leq N(S^s x, T^t x, t)$$

which implies $S^s x = T^t x$.

Now,

$$\begin{aligned} & M(A^a x, B^b(T^t x_{2n-1}), kt) \\ \geq\ & \min\{M(S^s x, T^t(T^t x_{2n-1}), t), M(A^a x, S^s x, t), M(B^b(T^t x_{2n-1}), \\ & \quad T^t(T^t x_{2n-1}), t), M(B^b(T^t x_{2n-1}), S^s(S^s x), 2t), \end{aligned}$$

$$M(A^a x, T^t(T^t x_{2n-1}), t)\},$$

$$\begin{aligned} & N(A^a x, B^b(T^t x_{2n-1}), kt) \\ \leq\ & \max\{N(S^s x, T^t(T^t x_{2n-1}), t), N(A^a x, S^s x, t), N(B^b(T^t x_{2n-1}), \\ & T^t(T^t x_{2n-1}), t), N(B^b(T^t x_{2n-1}), S^s(S^s x), 2t), \\ & N(A^a x, T^t(T^t x_{2n-1}), t)\}. \end{aligned}$$

Taking the limit as $n \to \infty$, and using above results,

$$M(A^a x, T^t x, kt) \geq M(A^a x, T^t x, t), \quad N(A^a x, T^t x, kt) \leq N(A^a x, T^t x, t)$$

which implies $A^a x = T^t x$. Also, since

$$M(A^a x, B^b x, kt) \geq M(A^a x, B^b x, t), \quad N(A^a x, B^b x, kt) \leq N(A^a x, B^b x, t).$$

Hence $A^a x = B^b x$. Therefore $A^a x = B^b x = S^s x = T^t x$. Furthermore, since

$$\begin{aligned} & M(A^a x_{2n}, B^b x, kt) \\ \geq\ & \min\{M(S^s x_{2n}, T^t x, t), M(A^a x_{2n}, S^s x_{2n}, t), M(B^b x, T^t x, t), \\ & \quad M(B^b x, S^s x_{2n}, 2t), M(A^a x_{2n}, T^t x, t)\}, \\ & N(A^a x_{2n}, B^b x, kt) \\ \leq\ & \max\{N(S^s x_{2n}, T^t x, t), N(A^a x_{2n}, S^s x_{2n}, t), N(B^b x, T^t x, t), \\ & \quad N(B^b x, S^s x_{2n}, 2t), N(A^a x_{2n}, T^t x, t)\}. \end{aligned}$$

Taking the limit as $n \to \infty$,

$$M(x, B^b x, kt) \geq M(x, B^b x, t), \quad N(x, B^b x, kt) \leq N(x, B^b x, t)$$

which implies $x = B^b x$. Therefore $x = B^b x = A^a x = S^s x = T^t x$. That is, x is a common fixed point of A^a, B^b, S^s and T^t. Let z be another common fixed point of maps. Then

$$\begin{aligned} & M(A^a x, B^b z, kt) \\ \geq\ & \min\{M(S^s x, T^t z, t), M(A^a x, S^s x, t), M(B^b z, T^t z, t), \\ & \quad M(B^b z, S^s z, 2t), M(A^a x, T^t z, t)\} \\ \geq\ & \min\{M(x, z, t), M(x, x, t), M(z, z, t), \\ & \quad M(z, z, 2t), M(x, z, t)\} \\ \geq\ & M(x, z, t), \\ & N(A^a x, B^b z, kt) \\ \leq\ & \max\{N(S^s x, T^t z, t), N(A^a x, S^s x, t), N(B^b z, T^t z, t), \\ & \quad N(B^b z, S^s z, 2t), N(A^a x, T^t z, t)\} \\ \leq\ & \max\{N(x, z, t), N(x, x, t), N(z, z, t), \\ & \quad N(z, z, 2t), N(x, z, t)\} \\ \leq\ & N(x, z, t). \end{aligned}$$

Hence x is a unique common fixed point of maps.

Third, we prove that this point x is a common fixed point of A, B, S and T. Since $Ax = A(A^a x) = A^a(Ax)$ and $Ax = A(S^s x) = S^s(Ax)$ from (a), hence Ax is a common fixed point of A^a and S^s. Also, since $Bx = B(B^b x) = B^b(Bx)$ and $Bx = B(T^t x) = T^t(Bx)$ from (a), hence Bx is a common fixed point of B^b and T^t. Now, letting $x = Ax$ and $y = Bx$ in (d), we have

$$\begin{aligned}
& M(Ax, Bx, kt) \\
&= M(A^a(Ax), B^b(Bx), kt) \\
&\geq \min\{M(S^s(Ax), T^t(Bx), t), M(A^a(Ax), S^s(Ax), t), M(B^b(Bx), T^t(Bx), t), \\
&\qquad M(B^b(Bx), S^s(Ax), 2t), M(A^a(Ax), T^t(Bx), t)\} \\
&= \min\{M(Ax, Bx, t), M(Ax, Ax, t), M(Bx, Bx, t), \\
&\qquad M(Bx, Ax, 2t), M(Ax, Bx, t)\} \\
&\geq M(Ax, Bx, t), \\
& N(Ax, Bx, kt) \\
&= N(A^a(Ax), B^b(Bx), kt) \\
&\leq \max\{N(S^s(Ax), T^t(Bx), t), N(A^a(Ax), S^s(Ax), t), N(B^b(Bx), T^t(Bx), t), \\
&\qquad N(B^b(Bx), S^s(Ax), 2t), N(A^a(Ax), T^t(Bx), t)\} \\
&= \max\{N(Ax, Bx, t), N(Ax, Ax, t), N(Bx, Bx, t), \\
&\qquad N(Bx, Ax, 2t), N(Ax, Bx, t)\} \\
&\leq N(Ax, Bx, t).
\end{aligned}$$

Therefore $Ax = Bx$. Also, we have

$$\begin{aligned}
& M(Sx, Tx, kt) \\
&= M(S^s(Sx), T^t(Tx), kt) \\
&\geq \min\{M(S^s(Sx), T^t(Tx), t), M(A^a(Sx), S^s(Sx), t), M(B^b(Tx), T^t(Tx), t), \\
&\qquad M(B^b(Tx), S^s(Sx), 2t), M(A^a(Sx), T^t(Tx), t)\} \\
&= \min\{M(Sx, Tx, t), M(Sx, Sx, t), M(Tx, Tx, t), \\
&\qquad M(Tx, Sx, 2t), M(Sx, Tx, t)\} \\
&\geq M(Sx, Tx, t), \\
& N(Sx, Tx, kt) \\
&= N(S^s(Sx), T^t(Tx), kt) \\
&\leq \max\{N(S^s(Sx), T^t(Tx), t), N(A^a(Sx), S^s(Sx), t), N(B^b(Tx), T^t(Tx), t), \\
&\qquad N(B^b(Tx), S^s(Sx), 2t), N(A^a(Sx), T^t(Tx), t)\} \\
&= \max\{N(Sx, Tx, t), N(Sx, Sx, t), N(Tx, Tx, t), \\
&\qquad N(Tx, Sx, 2t), N(Sx, Tx, t)\}
\end{aligned}$$

$\leq N(Sx, Tx, t)$.

Therefore $Sx = Tx$. Since x is a unique common fixed point of A^a, B^b, S^s, T^t, hence $Ax = Bx$ is common fixed points of A^a, S^s and $Sx = Tx$ is common fixed points of B^b, T^t. Hence $x = Ax = Bx = Sx = Tx$. That is, x is a common fixed point of A, B, S and T. □

Corollary 3.2 ([14]). *Let A, B, S and T be self maps of complete fuzzy metric space $(X, M, *)$ with continuous t-norm $*$ defined by $a * b = \min\{a, b\}$, $a, b \in [0, 1]$, satisfying the following conditions*:

(a) $A(X) \subset T(X)$, $B(X) \subset S(X)$.
(b) *S and T are continuous.*
(c) *(A, S), (B, T) are compatible pairs of maps.*
(d) *For all $x, y \in X$, $k \in (0, 1)$, $t > 0$,*

$$\begin{aligned} &M(Ax, By, kt) \\ \geq\ &\min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), \\ &\quad M(By, Sx, 2t), M(Ax, Ty, t)\}. \end{aligned}$$

(e) *For all $x, y \in X$, $\lim_{t \to \infty} M(x, y, t) = 1$.*

Then A, B, S and T have a unique common fixed point in X.

Proof. For fuzzy metric space X, if $a = b = s = t = 1$ in Condition (d) of Theorem 3.1, then the proof follows from Theorem 3.1. □

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