An approach based on the generalized ILOWHM operators to group decision making

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Abstract

In this paper, we define generalized induced linguistic aggregation operator called generalized induced linguistic ordered weighted harmonic mean (GILOWHM) operator. Each object processed by this operator consists of three components, where the first component represents the importance degree or character of the second component, and the second component is used to induce an ordering, through the first component, over the third components which are linguistic variables and then aggregated. It is shown that the induced linguistic ordered weighted harmonic mean (ILOWHM) operator and linguistic ordered weighted harmonic mean (LOWHM) operator are the special cases of the GILOWHM operator. Based on the GILOWHM and LWHM operators, we develop an approach to group decision making with linguistic preference relations. Finally, a numerical example is used to illustrate the applicability of the proposed approach.

Key words : Group decision making, linguistic variable, generalized induced linguistic ordered weighted harmonic mean (GILOWHM) operator, operational laws.

1. Introduction

Yager and Filev [25] introduced an induced aggregation operator called the induced ordered weighted averaging (IOWA) operator, which takes as its argument pairs, called OWA pairs, in which one component is used to induce an ordering over the second components which are exact numerical values and then aggregated. Later, some new induced aggregation operators have been developed, including the induced ordered weighted geometric (IOWG) operator [20], induced fuzzy integral aggregation (IFIA) operator [23] and induced choquet ordered averaging (ICOA) operator [24]. Xu and Da [20] introduced two more general aggregation techniques called generalized IOWA (GIOWA) and generalized IOWG (GIOWG) operators, and proved that the OWA and IOWA operators are the special cases of the GIOWA operator, and that the OWG and IOWG operators are the special cases of the GIOWG operator. All this operators have been used in situations in which the input arguments are numerical values. In some situations, however, the input arguments take the form of linguistic variables or uncertain linguistic variables rather than numerical ones because of time pressure, lack of knowledge, and the decision maker's limit attention and information processing capabilities [2, 4, 5, 7, 8, 10, 11, 27, 28, 29]. Recently, Xu [18] developed various generalized induced linguistic aggregation operators, such as generalized induced linguistic ordered weighted averaging (GILOWA) and generalized induced linguistic ordered weighted geometric (GILOWG) operator, both of which can be used to deal with the linguistic information. In this paper, we shall develop some generalized induced linguistic aggregation operator called generalized induced linguistic ordered weighted harmonic mean (GILOWHM) opera-

This work was supported by Pukyong National University Research Fund in 2007(PK-2007-018).
Corresponding Author: Jin Han Park
접수일자 : 2010년 4월 3일
완료일자 : 2010년 5월 17일
감사의 글 :
본 논문은 본 학회 2010년도 춘계학술대회에서 선정된 우수논문입니다.

tor, which can be used to deal with linguistic information. Each object processed by this operator consists of three components, where the first component represents the importance degree or character of the second component, and the second component is used to induce an ordering, through the first component, over the third components which are linguistic variables and then aggregated. It is shown that the induced linguistic ordered weighted harmonic mean (ILOWHM) [11] operator and linguistic ordered weighted harmonic mean (LOWHM) [10] operator are the special cases of the GILOWHM operator. Based on the GILOWHM and LWHM [10] operators, we propose a practical method for multiple attribute group decision making with linguistic preference relations. Finally, an illustrative example demonstrates the practicality and effectiveness of the proposed method.

2. Preliminaries

The linguistic approach is approximate technique, which represents qualitative aspects as linguistic values by means of linguistic variables [27, 28, 29].

Let $S = \{s_i : i = 1, 2, \ldots, t\}$ be a finite and totally ordered discrete label set, whose cardinality value is odd. Any label, s_i, represents a possible value for a linguistic variable, and it must have the following characteristics [15]: (1) the set is ordered: $s_i \geq s_j$ if $i \geq j$, and (2) there is the negation operator: $\text{neg}(s_i) = s_{t-i+1}$. We call this linguistic label set S the additive linguistic scale. For example, S can be defined as:

$$\begin{aligned} S = \{&s_1 = \text{extremely poor}, s_2 = \text{very poor}, \\ &s_3 = \text{poor}, s_4 = \text{slightly poor}, s_5 = \text{fair}, \\ &s_6 = \text{slightly good}, s_7 = \text{good}, \\ &s_8 = \text{very good}, s_9 = \text{extremely good}\}. \end{aligned}$$

To preserve all the given information, we extend the discrete linguistic label set S to a continuous linguistic label set $\bar{S} = \{s_\alpha : \alpha \in [-q, q]\}$, where q ($q > t$) is a sufficiently large positive integer. If $s_\alpha \in S$, then we call s_α an original linguistic label, otherwise, we call s_α the virtual linguistic label [15]. The decision maker, in general, uses the original linguistic labels to evaluate alternatives, and the virtual linguistic labels can only appear in operations.

Consider any three linguistic variables s_α, s_{α_1}, $s_{\alpha_2} \in \bar{S}$, then we define the operations $s_{\alpha_1} \oplus s_{\alpha_2}$, λs_α and $\frac{1}{s_\alpha}$ as follows:

(1) $s_{\alpha_1} \oplus s_{\alpha_2} = s_{\alpha_1 + \alpha_2}$;

(2) $\lambda s_\alpha = s_{\lambda\alpha}$, where $\lambda \in [0, 1]$;

(3) $\frac{1}{s_\alpha} = s_{\frac{1}{\alpha}}$.

3. Generalized induced LOWHM operators

Definition 3.1. [10] Let $\text{LWHM} : \bar{S}^n \to \bar{S}$, if

$$\text{LWHM}_w(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \frac{1}{\frac{w_1}{s_{\alpha_1}} \oplus \frac{w_2}{s_{\alpha_2}} \oplus \cdots \oplus \frac{w_n}{s_{\alpha_n}}} = \frac{1}{s_{\sum_{i=1}^n \frac{w_i}{\alpha_i}}}, \quad (1)$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of the s_{α_i} with $w_i \in [0, 1]$, $\sum_{i=1}^n w_i = 1$, $s_{\alpha_i} \in \bar{S}$, then LWHM is called the linguistic weighted harmonic mean (LWHM) operator. Especially, if $w_i = 1$ and $w_j = 0, j \neq i$, then $\text{LWHM}(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = s_{\alpha_i}$; if $w = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, then LWHM operator is called the linguistic harmonic mean (LHM) operator, i.e.,

$$\text{LHM}(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \frac{n}{s_{\sum_{i=1}^n \frac{1}{\alpha_i}}}. \quad (2)$$

The fundamental aspect of the LWHM operator is that it compute an aggregated value taking into the importance of the sources of information.

Definition 3.2. [10] A LOWHM operator of dimension n is a mapping $\text{LOWHM} : \bar{S}^n \to \bar{S}$, which has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$ with $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$, such that

$$\text{LOWHM}_w(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \frac{1}{\frac{w_1}{s_{\beta_1}} \oplus \frac{w_2}{s_{\beta_2}} \oplus \cdots \oplus \frac{w_n}{s_{\beta_n}}} = \frac{1}{s_{\sum_{i=1}^n \frac{w_i}{\beta_i}}}, \quad (3)$$

where s_{β_i} is the ith largest of the s_{α_i}.

The weighted vector $w = (w_1, w_2, \ldots, w_n)^T$ can be determined by using some weight determining methods like the normal distribution based method.

Definition 3.3. [11] An ILOWHM operator is defined as follows:

$$\text{ILOWHM}_w(\langle u_1, s_{\alpha_1} \rangle, \langle u_2, s_{\alpha_2} \rangle, \ldots, \langle u_n, s_{\alpha_n} \rangle) = \frac{1}{\frac{w_1}{s_{\beta_1}} \oplus \frac{w_2}{s_{\beta_2}} \oplus \cdots \oplus \frac{w_n}{s_{\beta_n}}} = \frac{1}{s_{\sum_{i=1}^n \frac{w_i}{\beta_i}}}, \quad (4)$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is a weighting vector, such that $w_i \in [0, 1], i = 1, 2, \ldots, n$, $\sum_{i=1}^n w_i = 1$, s_{β_i} is the s_{α_i} value of the LOWHM pair $\langle u_i, s_{\alpha_i} \rangle$ having the ith largest u_i, and u_i in $\langle u_i, s_{\alpha_i} \rangle$ is referred to as the order inducing variable and s_{α_i} as the linguistic argument variable. Especially, if $w = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, then ILOWHM is reduced to the LHM operator; if $u_i = s_{\alpha_i}$, for all i, then ILOWHM is reduced to the LOWHM operator; if u_i =No. i, for all i, where No. i is the ordered position of the a_i, then ILOWHM is the LWHM operator.

However, if there is a tie between $\langle u_i, s_{\alpha_i}\rangle$, $\langle u_j, s_{\alpha_j}\rangle$ with respect to order-inducing variables, in this case, we can follow the policy presented by Yager and Filov [25] - to replace the arguments of the tied objects by the mean of the arguments of the tied objects (i.e., we replace the argument component of each of $\langle u_i, s_{\alpha_i}\rangle$ and $\langle u_j, s_{\alpha_j}\rangle$ by their average $(s_{\alpha_i} \oplus s_{\alpha_j})/2$). If k items are tied, we replace these by k replicas of their average.

In the following, we shall give example to specify the special cases with respect to the inducing variables.

Example 3.4. Consider the following collection of LOWHM pairs:

$$\langle s_4, s_3\rangle, \langle s_6, s_7\rangle, \langle s_3, s_1\rangle, \langle s_5, s_4\rangle.$$

Performing the ordering the LOWHM pairs with respect to the first component, we have:

$$\langle s_6, s_7\rangle, \langle s_5, s_4\rangle, \langle s_4, s_3\rangle, \langle s_3, s_1\rangle.$$

This ordering induces the ordered linguistic arguments:

$$s_{\beta_1} = s_7, s_{\beta_2} = s_4, s_{\beta_3} = s_3, s_{\beta_4} = s_1.$$

If the weighting vector $w = (0.3, 0.1, 0.4, 0.2)^T$, then we get an aggregated value:

$$\begin{aligned} &\mathrm{ILOWHM}_w(\langle s_4, s_3\rangle, \langle s_5, s_7\rangle, \langle s_3, s_1\rangle, \langle s_6, s_4\rangle) \\ &= \frac{1}{\frac{0.3}{s_7} \oplus \frac{0.1}{s_4} \oplus \frac{0.4}{s_3} \oplus \frac{0.2}{s_1}} \\ &= \frac{1}{s_{\frac{0.3}{7}} \oplus s_{\frac{0.1}{4}} \oplus s_{\frac{0.4}{3}} \oplus s_{\frac{0.2}{1}}} \\ &= s_{2.49}. \end{aligned}$$

An important feature of the ILOWHM operator is that the argument ordering process is guided by a variable called the order inducing value. This operator essentially aggregate objects, which are pairs, and provide a very general family of aggregations operators. In some situations, however, when we need to provide more information about the objects, i.e. each object may consist of three components, a direct locator, an indirect locator and a prescribed value, it is unsuitable to use this induced aggregation operator as an aggregation tool. In following we shall present some more general linguistic aggregation technique.

Definition 3.5. A generalized induced LOWHM (GILOWHM) operator is given by

$$\begin{aligned} &\mathrm{GILOWHM}_w(\langle v_1, u_1, s_{\alpha_1}\rangle, \ldots, \langle v_n, u_n, s_{\alpha_n}\rangle) \\ &\qquad = \frac{1}{\frac{w_1}{s_{\beta_1}} \oplus \frac{w_2}{s_{\beta_2}} \oplus \cdots \oplus \frac{w_n}{s_{\beta_n}}} \end{aligned} \quad (5)$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the associated weighting vector with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, the object $\langle v_i, u_i, s_{\alpha_i}\rangle$ consists of three components, where the first component v_i represents the importance degree or character of second component u_i, and the second component u_i is used to induce an ordering, through the first component v_i, over the third component s_{α_i} which are then aggregated. Here, s_{β_j} is the s_{α_i} value of the object having the jth largest v_i. In discussing the object $\langle v_i, u_i, s_{\alpha_i}\rangle$, because of its role we shall refer to the v_i as the direct order inducing variable, the u_i as the indirect inducing variable, and s_{α_i} as the linguistic argument variable.

Especially, if $v_i = u_i$, for all i, then the GILOWHM operator is reduced to the ILOWHM operator; if $v_i = s_{\alpha_i}$, for all i, then the GILOWHM operator is reduced to the LOWHM operator; if $v_i = \text{No. } i$, for all i, where No. i is the ordered position of the s_{α_i}, then the GILOWHM operator is reduced to the LWHM operator; if $w = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, then the GILOWHM operator is reduced to the LHM operator.

Example 3.6. Consider the collection of the objects

$$\begin{aligned} &\langle \text{No. 3}, \text{Kim}, s_1\rangle, \langle \text{No. 1}, \text{Park}, s_7\rangle, \\ &\langle \text{No. 2}, \text{Lee}, s_2\rangle, \langle \text{No. 4}, \text{Jung}, s_5\rangle. \end{aligned}$$

By the first component, we get the ordered objects

$$\begin{aligned} &\langle \text{No. 1}, \text{Park}, s_7\rangle, \langle \text{No. 2}, \text{Lee}, s_2\rangle, \\ &\langle \text{No. 3}, \text{Kim}, s_1\rangle, \langle \text{No. 4}, \text{Jung}, s_5\rangle. \end{aligned}$$

The ordering induces the ordered arguments $s_{\beta_1} = s_7$, $s_{\beta_2} = s_2$, $s_{\beta_3} = s_1$, $s_{\beta_4} = s_5$. If the weighting vector for this aggregation is $w = (0.3, 0.1, 0.2, 0.4)^T$, then we get

$$\begin{aligned} &\mathrm{GILOWHM}_w(\langle \text{No. 3}, \text{Kim}, s_1\rangle, \langle \text{No. 1}, \text{Park}, s_7\rangle, \\ &\qquad \langle \text{No. 2}, \text{Lee}, s_2\rangle, \langle \text{No. 4}, \text{Jung}, s_5\rangle) \\ &\qquad = \frac{1}{\frac{0.3}{s_7} \oplus \frac{0.1}{s_2} \oplus \frac{0.2}{s_1} \oplus \frac{0.4}{s_5}} \\ &\qquad = \frac{1}{s_{\frac{0.3}{7}} \oplus s_{\frac{0.1}{2}} \oplus s_{\frac{0.2}{1}} \oplus s_{\frac{0.4}{5}}} \\ &\qquad = s_{2.70}. \end{aligned}$$

However, if we replace the objects in Example 3.6 with

$$\begin{aligned} &\langle \text{No. 3}, \text{Kim}, s_1\rangle, \langle \text{No. 1}, \text{Park}, s_7\rangle, \\ &\langle \text{No. 2}, \text{Lee}, s_2\rangle, \langle \text{No. 3}, \text{Jung}, s_5\rangle \end{aligned}$$

then there is a tie between $\langle \text{No. } 3, \text{Kim}, s_1\rangle$ and $\langle \text{No. } 3, \text{Jung}, s_5\rangle$ with respect to order direct inducing variable, in this case, we can follow the policy: we replace the linguistic argument component of each of $\langle \text{No. } 3, \text{Kim}, s_1\rangle$ and $\langle \text{No. } 3, \text{Jung}, s_5\rangle$ by their average $(s_1 \oplus s_5)/2 = s_3$. This substitution gives us ordered arguments $s_{\beta_1} = s_7$, $s_{\beta_2} = s_2$, $s_{\beta_3} = s_3$, $s_{\beta_4} = s_3$. Thus

$$\begin{aligned}\text{GILOWHM}_w(&\langle \text{No. } 3, \text{Kim}, s_1\rangle, \langle \text{No. } 1, \text{Park}, s_7\rangle,\\ &\langle \text{No. } 2, \text{Lee}, s_2\rangle, \langle \text{No. } 3, \text{Jung}, s_5\rangle)\\ &= \frac{1}{s_{\frac{0.3}{7}} \oplus s_{\frac{0.1}{2}} \oplus s_{\frac{0.2}{3}} \oplus s_{\frac{0.4}{3}}}\\ &= s_{3.44}.\end{aligned}$$

In the following we shall make an investigation on some desirable properties of the GILOWHM operator.

(1) (Commutativity) If $(\langle v'_1, u'_1, s'_{\alpha_1}\rangle, \ldots, \langle v'_n, u'_n, s'_{\alpha_n}\rangle)$ is any permutation of $(\langle v_1, u_1, s_{\alpha_1}\rangle, \ldots, \langle v_n, u_n, s_{\alpha_n}\rangle)$, then:

$$\begin{aligned}&\text{GILOWHM}_w(\langle v_1, u_1, s_{\alpha_1}\rangle, \ldots, \langle v_n, u_n, s_{\alpha_n}\rangle)\\ &\quad = \text{GILOWHM}_w(\langle v'_1, u'_1, s'_{\alpha_1}\rangle, \ldots, \langle v'_n, u'_n, s'_{\alpha_n}\rangle).\end{aligned}$$

(2) (Idempotency) If $s_{\alpha_i} = s_\alpha$, for all i, then:

$$\text{GILOWHM}_w(\langle v'_1, u'_1, s'_{\alpha_1}\rangle, \ldots, \langle v'_n, u'_n, s'_{\alpha_n}\rangle = s_\alpha.$$

(3) (Monotonicity) If $s_{\alpha_i} \leq s'_{\alpha_i}$, for all i, then:

$$\begin{aligned}&\text{GILOWHM}_w(\langle v_1, u_1, s_{\alpha_1}\rangle, \ldots, \langle v_n, u_n, s_{\alpha_n}\rangle)\\ &\quad \leq \text{GILOWHM}_w(\langle v_1, u_1, s'_{\alpha_1}\rangle, \ldots, \langle v_n, u_n, s'_{\alpha_n}\rangle).\end{aligned}$$

(4) (Boundedness)

$$\begin{aligned}\min_i(s_{\alpha_i}) \leq \text{GILOWHM}_w(&\langle v_1, u_1, s_{\alpha_1}\rangle,\\ &\ldots, \langle v_n, u_n, s_{\alpha_n}\rangle) \leq \max_i(s_{\alpha_i}).\end{aligned}$$

4. An approach to group decision making with linguistic preference relations

For a group decision making with linguistic information, let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of alternatives, and $G = \{G_1, G_2, \ldots, G_m\}$ be the set of attributes, and $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ be the weight vector of attributes, where $\omega_i \geq 0$, $i = 1, 2, \ldots, m$, $\sum_{i=1}^m \omega_i = 1$. Let $D = \{d_1, d_2, \ldots, d_m\}$ be a set of decision makers, and $V = \{v_1, v_2, \ldots, v_l\}$ be the set of direct order inducing variables, and $U = \{u_1, u_2, \ldots, u_l\}$ be the set of indirect order inducing variables. Suppose that $A^{(k)} = (a_{ij}^{(k)})_{m\times n}$ is the additive linguistic decision matrix, where $a_{ij}^{(k)} \in \bar{S}$ is preference value, which takes the form of additive linguistic variables, given by the decision maker $d_k \in D$, for the alternative $x_j \in X$ with respect to the attribute $G_i \in G$, for all $k = 1, 2, , \ldots, l$; $i = 1, 2 \ldots, m$; $j = 1, 2 \ldots, n$.

In the following, we apply the GILOWHM operator (whose exponential weighting vector $w = (w_1, w_2, \ldots, w_l)^T$, $w_k \geq 0$, $k = 1, 2, \ldots l$, $\sum_{k=1}^l w_k = 1$) and the LWHM operator to group decision making with linguistic information:

Step 1: Utilize the GILOWHM operator

$$\begin{aligned}a_{ij} = \text{GILOWHM}_w(&\langle v_1, u_1, a_{ij}^{(1)}\rangle, \ldots, \langle v_l, u_l, a_{ij}^{(l)}\rangle),\\ i = 1, 2, &\ldots, m;\ j = 1, 2, \ldots, n\end{aligned}$$

to aggregate all the decision matrices $A^{(k)}$ $(k = 1, 2, \ldots, l)$ into a collective decision matrix $A = (a_{ij})_{m\times n}$, where v_k $(k = 1, 2, \ldots l)$ are direct order inducing variables and u_k $(k = 1, 2, \ldots l)$ are indirect order inducing variables.

Step 2: Utilize the decision information given in matrix A, and the LWHM operator

$$\begin{aligned}a_j &= \text{LWHM}_\omega(a_{1j}, a_{2j}, \ldots, a_{mj})\\ &= \frac{1}{\frac{\omega_1}{a_{1j}} \oplus \frac{\omega_2}{a_{2j}} \oplus \cdots \oplus \frac{\omega_m}{a_{mj}}},\ j = 1, 2, \ldots, n\end{aligned}$$

to derive the collective overall preference values a_j of the alternative x_j, where $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ be the weight vector of attributes.

Step 3: Rank all the alternatives x_j $(j = 1, 2, \ldots, n)$ and select the best one(s) in accordance with the collective overall preference values a_j $(j = 1, 2, , \ldots, n)$.

Step 4: End.

5. Illustrative example

Let us suppose an investment company, which wants to invest a sum of money in the best option (adapted by Herrera et al. [6]). There is a panel with five possible alternatives in which to invest the money:

(1) x_1 is a car industry;
(2) x_2 is a food company;
(3) x_3 is a computer company;
(4) x_4 is an arms company;
(5) x_5 is a TV company.

The investment company must make a decision according to the following four attributes (suppose that the weight vector of four attributes is $\omega = (0.3, 0.4, 0.2, 0.1)^T$):

(1) G_1 is the risk analysis;
(2) G_2 is the growth analysis;

(3) G_3 is the social-political impact analysis;
(4) G_4 is the environmental impact analysis.

There is three decision makers to evaluate five alternatives as follows: u_1 : Anderson, u_2 : Smith and u_3 : Brown are names of three decision makers, and v_1 = No. 3, v_2 = No. 2 and v_3 = No. 1 are important degrees of decision makers, respectively. The five possible alternatives x_j $(j = 1, 2, 3, 4, 5)$ are evaluated using the multiplicative linguistic scale:

$$S = \{s_1 = \text{extremely poor}, s_2 = \text{very poor}, s_3 = \text{poor}, s_4 = \text{slightly poor}, s_5 = \text{fair}, s_6 = \text{slightly good}, s_7 = \text{good}, s_8 = \text{very good}, s_9 = \text{extremely good}\}.$$

by three decision makers under the above four attributes, and construct, respectively, the decision matrices $A^{(k)} = (a_{ij}^{(k)})_{4\times5}$ $(k = 1, 2, 3)$ as listed in Tables 1-3.

Table 1. Decision matrix $A^{(1)}$

	x_1	x_2	x_3	x_4	x_5
G_1	s_6	s_9	s_4	s_3	s_6
G_2	s_3	s_7	s_8	s_8	s_4
G_3	s_7	s_4	s_6	s_8	s_7
G_4	s_2	s_4	s_6	s_7	s_8

Table 2. Decision matrix $A^{(2)}$

	x_1	x_2	x_3	x_4	x_5
G_1	s_6	s_8	s_4	s_7	s_3
G_2	s_3	s_6	s_8	s_8	s_4
G_3	s_7	s_4	s_6	s_7	s_9
G_4	s_2	s_3	s_4	s_6	s_8

Table 3. Decision matrix $A^{(3)}$

	x_1	x_2	x_3	x_4	x_5
G_1	s_6	s_8	s_4	s_7	s_2
G_2	s_4	s_6	s_8	s_7	s_4
G_3	s_7	s_3	s_7	s_9	s_8
G_4	s_3	s_4	s_4	s_7	s_7

To get the best alternative(s), the following steps are involved:

Step 1: Utilize the GILOWHM operator (whose weight vector $w = (0.3, 0.4, 0.3)^T$)

$$a_{ij} = \text{GILOWHM}_w(\langle v_1, u_1, a_{ij}^{(1)}\rangle, \langle v_2, u_2, a_{ij}^{(2)}\rangle, \langle v_3, u_3, a_{ij}^{(3)}\rangle), \quad i = 1, 2, 3, 4; j = 1, 2, 3, 4, 5$$

to aggregate all the decision matrices $A^{(k)}$ $(k = 1, 2, 3)$ into a collective decision matrix $A = (a_{ij})_{4\times5}$ (Table 4).

Table 4. Decision matrix A

	x_1	x_2	x_3	x_4	x_5
G_1	$s_{6.0}$	$s_{8.3}$	$s_{4.0}$	$s_{5.0}$	$s_{3.0}$
G_2	$s_{3.2}$	$s_{6.3}$	$s_{8.0}$	$s_{7.7}$	$s_{4.0}$
G_3	$s_{7.0}$	$s_{3.6}$	$s_{6.3}$	$s_{7.8}$	$s_{8.0}$
G_4	$s_{2.2}$	$s_{3.5}$	$s_{4.4}$	$s_{6.6}$	$s_{7.7}$

Step 2: Utilize the decision information given in matrix A, and the LWHM operator

$$a_j = \text{LWHM}_\omega(a_{1j}, a_{2j}, a_{3j}, a_{4j}) = \frac{1}{\frac{\omega_1}{a_{1j}} \oplus \frac{\omega_2}{a_{2j}} \oplus \frac{\omega_3}{a_{3j}} \oplus \frac{\omega_4}{a_{4j}}}, \ j = 1, 2, 3, 4, 5$$

to derive the collective overall preference values a_j of the alternative x_j:

$$a_1 = \frac{1}{\frac{0.3}{s_{6.0}} \oplus \frac{0.4}{s_{3.2}} \oplus \frac{0.2}{s_{7.0}} \oplus \frac{0.1}{s_{2.2}}} = s_{4.02},$$
$$a_2 = \frac{1}{\frac{0.3}{s_{8.3}} \oplus \frac{0.4}{s_{6.3}} \oplus \frac{0.2}{s_{3.6}} \oplus \frac{0.1}{s_{3.5}}} = s_{5.44},$$
$$a_3 = \frac{1}{\frac{0.3}{s_{4.0}} \oplus \frac{0.4}{s_{8.0}} \oplus \frac{0.2}{s_{6.3}} \oplus \frac{0.1}{s_{4.4}}} = s_{5.57},$$
$$a_4 = \frac{1}{\frac{0.3}{s_{5.0}} \oplus \frac{0.4}{s_{7.7}} \oplus \frac{0.2}{s_{7.8}} \oplus \frac{0.1}{s_{6.6}}} = s_{6.55},$$
$$a_5 = \frac{1}{\frac{0.3}{s_{3.0}} \oplus \frac{0.4}{s_{4.0}} \oplus \frac{0.2}{s_{8.0}} \oplus \frac{0.1}{s_{7.7}}} = s_{4.20}.$$

Step 3: Rank all the alternatives x_j $(j = 1, 2, 3, 4, 5)$ and select the best one(s) in accordance with the collective overall preference values a_j $(j = 1, 2, 3, 4, 5)$:

$$x_4 \succ x_3 \succ x_2 \succ x_1 \succ x_5$$

thus the best alternative is x_4.

6. Concluding remarks

In this paper, we have developed generalized ILOWHM (GILOWHM) operators, which takes their argument objects, in which first component presents the degree of second component and second component is used to induce an ordering through the first component over third components which are linguistic values and then aggregated. We have studied some desirable properties of the GILOWHM operator and applied the GILOWHM operator to group decision making with linguistic information. In the future, we shall continue working in the application and extension of the GILOWHM operators in to other domain.

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