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구간치 퍼지측도와 관련된 수게노적분에 의해 모델화된 언어 정량자에 관한 연구

A note on Linguistic quantifiers modeled by Sugeno integral with respect to an interval-valued fuzzy measures

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Abstract

Ying[M.S. Ying, Linguistic quantifiers modeled by Sugeno integrals, Artificial Intelligence 170(2006) 581-606] studied a framework for modeling quantifiers in natural languages in which each linguistic quantifier is represented by a family of fuzzy measures and the truth value of a quantified proposition is evaluated by using Sugeno integral.

In this paper, we consider interval-valued fuzzy measures and interval quantifiers which are the generalized concepts of fuzzy measures and quantifiers, respectively. We also investigate logical properties of a first order language with interval quantifiers modeled by the Sugeno integral with respect to an interval-valued fuzzy measures.

Key Words : Linguistic quantifiers, Sugeno integrals, interval-valued fuzzy measures

1. Introduction

Linguistic terms can be characterized as linguistic variables via fuzzy sets proposed by Zadeh in 1965([16]). The existing definitions of linguistic quantifiers are all based on fuzzy sets, the non-decreasing fuzzy quantifiers most, almost all, and at least half with membership function

$$m(r)=\begin{cases} 0 & \text{if} \quad r<a \\ \dfrac{r-a}{b-a} & \text{if } a \le r \le b \\ 1 & \text{if} \quad r>b \end{cases}$$

characterized by parameters $(a,b)=(0.3,0.8)$, $(0,0.5)$ and $(0.5,1)$, respectively. M.S. Ying [15] studied that ligquistic quantifiers modeled by Sugeno integral which was defined by Sugeno[11,12].

In order to characterize the higher level uncertainty associated to linguistic weights, it is more reasonable to define a linguistic quantifier by using interval-valued fuzzy measures. Note that we will define Sugeno integral with respect to an interval-valued fuzzy measure (see [1-10,14,18]).

In this paper, we study interval-valued quantifiers based on Sugeno integral with respect to an interval-valued fuzzy measure. In section 2, we list the definitions and some properties of fuzzy measures, linguistic quantifiers, and Sugeno integrals. In section 3, we consider interval-valued fuzzy measures and interval-valued quantifiers and investigates some properties of them.

2. Sugeno integrals and linguistic quantifiers

In this section, we list some notations and fundamental results needed in the sequel from the theory of fuzzy measures and Sugeno integrals. Let $I=[0,1]$ and $(X,\wp)$ be a measurable space.

Definition 2.1 ([1-18]) (1) A set function $m:\wp\to I$ is called a fuzzy measure if it satisfies the following properties

(i) $m(\varnothing)=0$ and $m(X)=1$;

(ii) If $E,F\in\wp$ and $E\subset F$, then $m(E)\le m(F)$

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(2) A fuzzy measure m is continuous if $E_n \in \wp$ for $1 \leq n < \infty$ and $\{E_n\}$ is monotone, then

$$m(\lim_{n\to\infty} E_n) = \lim_{n\to\infty} m(E_n)$$

The notion of dual fuzzy measure is required when dealing with dual linguistic quantifier.

Definition 2.2 Let $(X,\wp,m)$ be a fuzzy measure space. Then the dual set function $m^*: \wp \to I$ of m is defined by

$$m^*(E) = 1 - m(X - E)$$
$$\text{for each } E \in \wp$$

It is well known that m^* is a fuzzy measure. We introduce the Sugeno integral with respect to a fuzzy measure m.

Definition 2.3 Let $(X,\wp,m)$ be a fuzzy measure space. If $A \in \wp$ and $h: X \to I$ is a $\wp$-measurable function, then the Sugeno integral of h over A with respect to a fuzzy measure m is defined by

$$\int_A h\, dm =$$

where $H_\lambda = \{x \in X \mid h(x) \geq \lambda\}$ for each $\lambda \in I$

The next lemma gives an alternative definition of Sugeno's integral for the case that the Borel field in a measurable space is taken to be the power set of a set X.

Lemma 2.4 ([12]) If the Borel field $\wp$ in the fuzzy measure space $(X,\wp,m)$ is the power set 2^X of X, then for any function $h: X \to I$, we have

$$\int_A h\, dm = \sup_{F \in 2^X} \min[\inf_{x \in 2^X} h(x), m(A \cap F)]$$

We remark that $m(E)$ expresses someones's subjective evaluation of the statement "x is in E" in a situation in which he guesses whether x is in E. We consider fuzzy quntified statements, namly, proposition of the form "QXs are As". Let $M(X,\wp)$ be the set of all fuzzy measures. Then we will define a fuzzy quantifier and introduce a partial order between fuzzy quantifiers and three operations of fuzzy quantifiers as followings.

Definition 2.5 ([15,17]) A fuzzy quantifier(or quantifier for short) consists of the following two items;

(i) for each nonempty set X, a Borel field $\wp_X$ over X is equipped; and

(ii) a Choice function

$$Q: (X, \wp_X) \mapsto Q_{(X,\wp_X)} \in M(X, \wp_X)$$

of the (proper) class

$$\{M(X, \wp_X) \mid (X, \wp_X) \text{ is a measurable space}\}$$

Definition 2.6 ([15,17]) Let Q, Q_1 and Q_2 be quantifiers.

(1) We say that Q_1 is stronger than Q_2, written $Q_1 \sqsubseteq Q_2$, if for any nonempty set X and for any $E \in \wp_X$, we have

$$Q_{1X}(E) \leq Q_{2X}(E).$$

(2) The dual Q^* of Q, and the meet $Q_1 \sqcap Q_2$ and the union of Q_1 and Q_2 are defined respectively as follows; for any nonempty set X and for any $E \in \wp_X$,

$$Q^*_X(E) = 1 - Q_X(X - E),$$
$$(Q_1 \sqcap Q_2)_X(E) = \min(Q_{1X}(E), Q_{2X}(E)),$$
$$(Q_1 \sqcup Q_2)_X(E) =$$

We also introduce a first order logical language $\mathbb{L}_q$ with linguistic quantifiers. The alphabet of our language $\mathbb{L}_q$ is given as follows:

(i) A denumerable set of individual variables: $x_0, x_1, \cdots$;

(ii) A set $\mathbb{F} = \cup_{n=0}^{\infty} \mathbb{F}_n$ of predicate symbols, where $\mathbb{F}_n$ is the set of all n-place predicate symbols for each $n \geq 0$. It is assumed that $\cup_{n=0}^{\infty} \mathbb{F}_n \neq \varnothing$;

(iii) Propositional connectives: $\sim, \wedge$; and

(iv) Parentheses:(,).

The syntax of the language $\mathbb{L}_q$ is then presented by the following definition.

Definition 2.7 ([15, 17]) The set Wff of well-formed formulas is the smallest set of symbol strings satisfying the following conditions:

(i) If $n \geq 0$, $F \in \mathbb{F}_n$, and $y_1, \cdots, y_n$ are individual variables, then $F(y_1, \cdots, y_n) \in Wff$;

(ii) If Q is a quantifier, x is an individual variable, and $\varphi, \varphi_1, \varphi_2 \in Wff$, then

(iii) If $\varphi, \varphi_1, \varphi_2 \in Wff$, then $\sim\varphi, \varphi_1 \wedge \varphi_2 \in Wff$.

Definition 2.8 ([15,17]) An interpretation I of logical language consists of the following items:

(i) A measurable space $(X,\wp)$, called the domain of I;

(ii) For each $n \geq 0$, we associate the individual variable x_i, with an element x_i^I in X;

(iii) For each $n \geq 0$ and for any $F \in F_n$.

Definition 2.9 ([15,17]) Let I be an interpretation. Then the truth value $T_I(\varphi)$ of a formula φ under I is defined recursively as follows:

(i) If $\varphi = F(y_1, \cdots, y_n)$, then

$$T_I(\varphi) = F^I(y_1^I, \cdots, y_n^I).$$

(ii) If $\varphi = (Q_x)\psi$, then

$$T_I(\varphi) = \int T_{I\{\cdot/x\}}(\psi)\, d\, Q_X$$

where X is the domain of I, $T_{I\{\cdot/x\}}(\psi) : X \to [0,1]$ is a mapping such that

$$T_{I\{\cdot/x\}}(\varphi)(u) = T_{I\{u/x\}}(\varphi)$$

for all $u \in X$, and $I\{u/x\}$ is the ineterpretation which differs from I only in the assignment of the individual variable x, that is, $y^{I\{u/x\}} = y^I$ for all $y \neq x$ and $x^{I\{u/x\}} = u$;

(iii) If $\varphi = \sim \psi$, then

$$T_I(\varphi) = 1 - T_I(\psi)$$

and if $\varphi = \varphi_1 \wedge \varphi_2$, then

$$T_I(\varphi) = \min(T_I(\varphi_1), T_I(\varphi_2))$$

The following proposition establishes a close link between the truth evaluation of fuzzy quantified statement and the extension of fuzzy measure on fuzzy sets.

Proposition 2.10 ([15]) Let Q be a fuzzy quantifier and x an individual variable, and let $\varphi \in Wff$. Then for any interpretation I,

$$T_I((Q_X)\varphi) = \widetilde{Q}_X(T_I(\varphi)),$$

where $\widetilde{Q_X}$ is the extension of Q_X on fuzzy set.

Proposition 2.11 ([15]) Let X be a finite set, let I be an interpretation with X as its domain, and let $\lambda \in I$. Then for any fuzzy quantifier Q and $\varphi \in Wff$, we have

(i) $T_I((Q_X)\varphi) \geq \lambda$ if and only if

$$Q_X(\{u \in X:\ T_{I\{u/x\}}(\varphi) \geq \lambda\}) \geq \lambda.$$

(ii) $T_I((Q_X)\varphi) \leq \lambda$ if and only if

1 $$Q_X(\{u \in X:\ T_{I\{u/x\}}(\varphi) \geq \lambda\}) \leq \lambda.$$

3. interval-valued Linguistic quantifiers and interval-valued fuzzy measures

Throughout the paper, $[0,1]$ is the unit interval and

$$[I] = \{[a^-, a^+] \mid a^-, a^+ \in I \text{ and } a^- \leq a^+\}$$

For any $a \in I$, we define $a = [a,a]$. Obviously, $a \in [I]$.

Definition 3.1 ([3-10]) If $\bar{a}, \bar{b} \in [I], k \in I$, then we define

(1) $\bar{a} + \bar{b} = [a^- + b^-, a^+ + b^+]$,

(2) $k\bar{a} = [ka^-, ka^+]$,

(3) $\bar{a} \wedge \bar{b} = [a^- \wedge b^-, a^+ \wedge b^+]$,

(4) $\bar{a} \vee \bar{b} = [a^- \vee b^-, a^+ \vee b^+]$,

(5) $\bar{a} \leq \bar{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$,

(6) $\bar{a} < \bar{b}$ if and only if $\bar{a} \leq \bar{b}$ and $\bar{a} \neq \bar{b}$,

(7) $\bar{a} \subset \bar{b}$ if and only if $b^- \leq a^-$ and $a^+ \leq b^+$.

Theorem 3.2 ([3-10]) Let $\bar{a}, \bar{b} \in I(R^+)$. Then the followings hold.

(1) idempotent law: $\bar{a} \wedge \bar{a} = \bar{a}, \bar{a} \vee \bar{a} = \bar{a}$,

(2) commutative law: $\bar{a} \wedge \bar{b} = \bar{b} \wedge \bar{a}, \bar{a} \vee \bar{b} = \bar{b} \vee \bar{a}$,

(3) associative law: $(\bar{a} \wedge \bar{b}) \wedge \bar{c} = \bar{a} \wedge (\bar{b} \wedge \bar{c})$,
$(\bar{a} \vee \bar{b}) \vee \bar{c} = \bar{a} \vee (\bar{b} \vee \bar{c})$,

(4) absorption law: $\bar{a} \wedge (\bar{a} \vee \bar{b}) = \bar{a} \vee (\bar{a} \wedge \bar{b}) = \bar{a}$,

(5) distributive law: $\bar{a} \wedge (\bar{b} \vee \bar{c}) = (\bar{a} \wedge \bar{b}) \vee (\bar{a} \wedge \bar{c})$,
$\bar{a} \vee (\bar{b} \vee \bar{c}) = (\bar{a} \vee \bar{b}) \wedge (\bar{a} \vee \bar{c})$,

Definition 3.3 ([3-10]) A set function $d_H : [I] \times [I] \to [0, \infty]$ is called the Hausdorff metric if

$$d_H(A,B) = \max\{\sup_{x \in A} \inf_{y \in B} |x-y|,$$
$$\sup_{y \in B} \inf_{x \in A} |x-y|\},$$

for all $A, B \in [I]$.

Theorem 3.4 ([3-10]) If $d_H : [I] \times [I] \to [0, \infty]$ is the Hausdorff metric, then for $\bar{a} = [a^-, a^+], \bar{b} = [b^-, b^+] \in [I]$

$$d_H(\bar{a}, \bar{b}) = \max\{|a^- - b^-|, |a^+ - b^+|\}.$$

Definition 3.5 Let $(X, \wp)$ be a measurable space. If an interval-valued set function $\overline{m} = [m_1, m_2] : \wp \to [I]$ is called an interval-valued fuzzy measure if m_1 and m_2 are fuzzy measures. $Q_1 \sqcup Q_2$

Intuitively, $\overline{m}(E)$ express some one's subjective evaluation of the statement "x is in E" in a situation in which he guesses whether x is in E. Let $M(X, \wp)$ be the set of all fuzzy measures and $\overline{M}(X, \wp)$the set of all interval-valued fuzzy measures.

Definition 3.6 ([3-10]) A closed set-valued function $\bar{h}$ is said to be measurable if for each open set $O \subset R^+$,

$$\bar{h}^{-1}(O) = \{x \in X \mid \bar{h}(x) \cap O \neq \varnothing \in \wp\}.$$

Definition 3.7 The Sugeno integral of h over A with respect to an interval-valued fuzzy measure $\overline{m} = [m_1, m_2]$ is defined by

$$\int_A h\, d\overline{m} = [\int_A h\, dm_1, \int_A h\, dm_2].$$

Definition 3.8 An interval quantifier $\overline{Q}$ is defined by

$$\overline{Q}=[Q^{-},Q^{+}],$$

where Q^{-} and Q^{+} are quantifiers in the meaning of Definition 3.4.

Remark 3.9 From Definitions 2.4, 2.5 and 2.6, for $\overline{Q}=[Q^{-},Q^{+}]$, $\overline{Q_1}=[Q_1^{-},Q_1^{+}]$, and $\overline{Q_2}=[Q_2^{-},Q_2^{+}]$, we easily have the following some properties:

$\overline{Q_1}\sqsubseteq\overline{Q_2}$ if and only if $Q_1^{-}\sqsubseteq Q_2^{-}$ and $Q_1^{+}\sqsubseteq Q_2^{+}$.

$\overline{Q_X^{*}}(E)=[Q_X^{-*}(E),Q_X^{+*}(E)]$,

$(\overline{Q_1}\sqcap\overline{Q_2})_X(E)=[Q_1^{-}\sqcap Q_2^{-},Q_1^{+}\sqcap Q_2^{+}]$,

$(\overline{Q_1}\sqcup\overline{Q_2})_X(E)=[Q_1^{-}\sqcup Q_2^{-},Q_1^{+}\sqcup Q_2^{+}]$

Theorem 3.10 Let $\overline{Q}$ be an interval quantifier and x an individual variable, and let $\varphi\in Wff$. Then for any interpretation I,

$$T_I((\overline{Q_x})\varphi)=[T_I((Q_x^{-})\varphi),T_I((Q_x^{+})\varphi)],$$

where $T_I(Q_x^{-})\varphi=\int T_{I\{\cdot/x\}}(\varphi)\,dQ_X^{-}$ and

$T_I(Q_x^{+})\varphi=\int T_{I\{\cdot/x\}}(\varphi)\,dQ_X^{+}$.

Proof. By Definition 2.9, we have

$$T_I(Q_x^{-})\varphi=\int T_{I\{\cdot/x\}}(\varphi)\,dQ_X^{-},$$
$$T_I(Q_x^{+})\varphi=\int T_{I\{\cdot/x\}}(\varphi)\,dQ_X^{+}$$

Then, Definition 3.5 implies the following equation:

$$\begin{aligned}T_I((\overline{Q_x})\varphi)&=\int T_{I\{\cdot/x\}}(\varphi)\,d\overline{Q_X}\\&=[\int T_{I\{\cdot/x\}}(\varphi)dQ_X^{-},\int T_{I\{\cdot/x\}}(\varphi)\,dQ_X^{+}]\\&=[T_I((Q_x^{-})\varphi),T_I((Q_x^{+})\varphi)].\end{aligned}$$

Clearly, the above Theorem 3.10 implies the following corollary.

Corollary 3.11 Let $\overline{Q}$ be an interval-valued fuzzy quantifier and x an individual variable, and let $\varphi\in Wff$. Then for any interpretation I,

$$T_I((\overline{Q_X})\varphi)=\widetilde{\overline{Q_X}}(T_I(\varphi)),$$

where $\widetilde{\overline{Q_X}}=[\widetilde{Q_X^{-}},\widetilde{Q_X^{+}}]$ and $\widetilde{Q_X^{-}}$, $\widetilde{Q_X^{+}}$ are the extensions of Q_X^{-}, Q_X^{+} on interval-valued fuzzy set, respectively.

Example 3.12 We first consider the simplest case of interval-valued fuzzy quantification. For any interval-valued fuzzy quantifier $\overline{Q}=[Q^{-},Q^{+}]$ and for any $\varphi\in Wff$, if I is an interpretation with the domain being a singleton $X=\{u\}$, Then for any inetrval-valued fuzzy quantifire $\overline{Q}$,

$$T_I((\overline{Q_X})\varphi)=T_I(\varphi).$$

This means that fuzzy quantification degenerates on a singleton discourse universe.

Example 3.13 This example shows that the Sugeno integral evaluation of universally and existentially interval-valued fuzzy quantified statements coincide with the standard way, and so gives a witness for reasonablenss of Sugeno integral semantics of interval-valued quantification. Let $\overline{Q}$ be an interval-valued fuzzy quantifier and x an individual variable, and $\varphi\in Wff$. Then for any interpretation I with the domain X and for any $E\subset X$, the universal interval-valued fuzzy quantifier $\overline{\forall}=[\forall^{-},\forall^{+}]$="all" and the existential interval-valued fuzzy quantifier $\overline{\exists}=[\exists^{-},\exists^{+}]$="some" are defined as follows:

$$\overline{\forall_X}=\begin{cases}[0.8,1], & \text{if } E=X\\ 0, & \text{otherwise,}\end{cases}$$

$$\overline{\exists_X}=\begin{cases}[0.8,1], & \text{if } E\neq\varnothing\\ 0, & \text{otherwise}\end{cases}$$

and then we have

$$T_I((\overline{\forall_X})\varphi)=[\int T_{I\{u/x\}}(\varphi)\,d\forall_X^{-},\int T_{I\{u/x\}}(\varphi)\,d\forall_X^{+}]$$

and

$$T_I((\overline{\exists_X})\varphi)=[\int T_{I\{u/x\}}(\varphi)\,d\exists_X^{-},\int T_{I\{u/x\}}(\varphi)\,d\exists_X^{+}].$$

Example 3.14 This example shows that the existing definitions of linguistic quantifiers are based on interval-avlued fuzzy sets, the non-decreasing interval-valued fuzzy quantifiers: most, almost all, and at least half with membership function

$$\overline{Q_X}=\begin{cases}[0,\frac{1}{4}] & \text{if } x<l_1\\ [\frac{3}{4}\frac{x-l_1}{l_2-l_1},\frac{3}{4}\frac{x-l_1}{l_2-l_1}+\frac{1}{4} & \text{if } l_1\le x\le l_2\\ [\frac{3}{4},1] & \text{if } x>l_2\end{cases}$$

characterized by parameters $(l_1,l_2)=(0.2,0.7),(0,0.4)$ and $(0.5,0.9)$, respectively.

To conclude this section, in the case of finite discourse universe we give a necessary and sufficient condition under which the truth value of an interval-valued quantified proposition is bounded by a given threshold value from up and below. By the same method of the proof of Theorem 3.10, we easily obtain the following proposition.

Proposition 3.15 Let X be a finite set, let I be an interpretation with X as its domain, and let $\overline{\lambda}=[\lambda^-,\lambda^+]\in[I]$. Then for any interval-valued fuzzy quantifier $\overline{Q}$ and $\varphi\in Wff$, we have

(i) $T_I((\overline{Q_X})\varphi)\geq\overline{\lambda}$ if and only if

$$Q_X^-(\{u\in X\colon T_{I\{u/x\}}(\varphi)\geq\lambda^-\})\geq\lambda^- \text{ and}$$
$$Q_X^+(\{u\in X\colon T_{I\{u/x\}}(\varphi)\geq\lambda^+\})\geq\lambda^+.$$

(ii) $T_I((\overline{Q_X})\varphi)\leq\overline{\lambda}$ if and only if

$$Q_X^-(\{u\in X\colon T_{I\{u/x\}}(\varphi)\geq\lambda^-\})\leq\lambda^- \text{ and}$$
$$Q_X^+(\{u\in X\colon T_{I\{u/x\}}(\varphi)\geq\lambda^+\})\leq\lambda^+.$$

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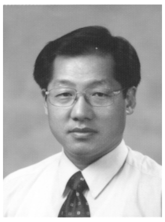

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